Bayesian Statistics III/IV (MATH3361/4071) Michaelmas term 2021

Description of the course

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The following details concern the module Bayesian Statistics III/IV (MATH3361/4071) in Michaelmas term 2021. The description below is informal and aims at helping students organize his/her study. The official description of the course can be found here for the Level III, and here for the Level IV.

1 Description of the course

Aim

This course provides an introduction to the Bayesian statistical theory and methods. We will gain the necessary background to (i.) justify why it is desirable to use Bayesian statistical analysis and what are the associated challenges, (ii.) specify a suitable Bayesian statistical model, and (iii.) perform inference in the Bayesian framework.

Intended learning outcomes

The students will be able to

[ILO1] Explain the foundations and theoretical basis of the Bayesian paradigm, as well as possible challenges and related paradoxes

[ILO2] Explain, and specify a Bayesian model (parametric, prior, posterior, and predictive model)

[ILO3] Specify a prior model for a given statistical model

[ILO4] Explain, and apply Bayesian parametric and predictive inference, as well as design the necessary appropriate inferential tools.

[ILO5] Explain, extend, and apply theoretical properties of Bayesian concepts

[ILO6] Use appropriate software to facilitate Bayesian statistical analysis

[ILO7] Explain, theorize, derive, apply "Variational inference methods in Bayesian statistics" ; this is given as an additional reading material for Level IV and postgraduate students. `https://github.com/georgios-stats/Bayesian_Statistics_Michaelmas_2021/blob/main/Additional_reading_material/Variational_Bayes_Methods.pdf`

Teaching and learning activities

[TLA1] Lectures

Students will be introduced to the theory, and be exposed to a small number of examples.

- Major focus [ILOs 1-4]

[TLA2] Problem Class

Students will learn how to implement the theory and methods introduced in Lectures. Examples will be given, and exercises will be solved in the blackboard.

- Major focus [ILOs 1-4]

[TLA3] ~~Computer practicals~~ Merged with Problem Class

Students will learn how to use R and JAGS statistical software to facilitate Bayesian statistical analysis.

- Major focus [ILOs 4 & 6]

[TLA4] Office hours

Students will ask further questions. Students should have itemize their questions to be discussed in a bullet list on a piece of paper.

- Major focus [ILO 1-7]

Assessment tasks / activities

Formative assessment

[FA1] Four homework assignments will be assigned regularly. The homework sheet will contain a number of problems which have to be assessed and returned. Homework problems and solutions will be available from Blackboard Ultra. The submission of the solution will be done Gradescope. Feedback will be given via Gradescope and emails. Major focus [ILOs 1-6].

Sumative assessment

[SA1] ILOs 1-5 will be assessed in the end-of-year examination. Level IV and postgraduate students will be assessed in ILO 7 as well.

2 Syllabus

Foundations of Bayesian statistics:

- Subjective probability, Bayesian paradigm, sufficiency, ~~stopping rules~~, likelihood principle, decision theory elements [ILO1]

Bayesian inference:

- prior model specification (conjugate, conditional-conjugate, hierarchical, mixture priors, Jeffrey, maximum entropy), [ILOs 1-3]
- Bayes point estimation, credible intervals, hypothesis tests [ILO 4]
- prior/posterior/predictive distribution, estimates, and intervals [ILO 4]
- inference under model uncertainty, [ILO 3-5]
- asymptotic behavior of the posterior and predictive distributions; [ILO 5]
- Hierarchical Bayes, and Empirical Bayes [ILOs 2-5]
- Variational inference methods in Bayesian statistics [ILO 7] –Level IV and postgraduate only

Statistical Modeling:

- Exchangeable model [ILO 1]
- Non-identifiability [ILO 3]
- Bayesian regression models [ILOs 2-4]

3 Reading list

Lecturer's teaching material:

- `https://github.com/georgios-stats/Bayesian_Statistics_Michaelmas_2021#details`

Go to `https://rl.talis.com/3/durham/lists/3F704A29-C94F-7B1D-49C0-DC78119CFE91.html`

- Berger, J. O. (2013). Statistical decision theory and Bayesian analysis. Springer Science & Business Media.
  - The main textbook of the course which contains almost all the Bayesian theory concepts. For the Bayesian model selection which is covered superficially you can check O'Hagan, A., & Forster, J. J. (2004) or Robert, C. (2007)
- Robert, C. (2007). The Bayesian choice: from decision-theoretic foundations to computational implementation. Springer Science & Business Media.
  - The second main textbook of the course. It covers more or less the Bayesian material covered by Berger, J. O. (2013), with less details but still rigorously enough. Also it is may uses examples based on more advanced models that those you have been thought.
- Raiffa, H., & Schlaifer, R. (1961). Applied statistical decision theory.
  - Companion textbook from which you can focus on concepts: Conjugate priors (great details), decision theory (applied)
- DeGroot, M. H. (2005). Optimal statistical decisions (Vol. 82). John Wiley & Sons.
  - Companion textbook from which you can focus on concepts: Bayesian paradigm, subjective probability, decision theory (theory)
- O'Hagan, A., & Forster, J. J. (2004). Kendall's advanced theory of statistics, volume 2B: Bayesian inference (Vol. 2). Arnold.
  - Companion textbook from which you can focus on concepts: Bayesian model selection/uncertainty, Linear regression model.
- Reich, B. J., & Ghosh, S. K. (2019). Bayesian statistical methods. CRC Press.
  - Supplementary textbook from which you can focus on the implementation of the above concepts: Bayesian statistical tools (brief description), examples in RJAGS, and WINBUGS
- Lecture notes and textbooks used in the courses: Statistical Concepts II, Probability I, Analysis of Many Variables II, Calculus I, Analysis I, Linear Algebra I

Additional reading material for for Level IV and postgraduate

- Bishop, Christopher M. (2006). Pattern recognition and machine learning. New York :Springer
  - subsection 10.1 'Variational Inference' as core/compulsory material