# Njang/jàng Xayma

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Ci wéeru ñaari junni ak ñaar fukk ak ñaar (2022)

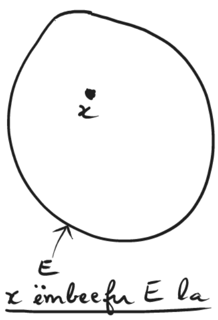
## 1 Ëmb

Téeki

Nañu wowee ëmb, beep saamu cër[a] yu ñuy wowee ëmbeef. Amal been ëmb $E$, da ñuy né $x$ ëmbeefu $E$ la, ta ñu ko bindé $x \in E$[b], bu féké ni $x$ ci biir $E$ la nek/la bok.

[a] élément
[b] Mën na ñu ko liré "$x$ mu ngi ci biir $E$"

- $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ ay ëmb la ñu.
- $\{a, b, c, d, ...z\}$ moy ëmb bu am ab ëmbeef $a$, $b$, $c$, ..., ba $z$
- $\emptyset$ moy mbindu ëmb bu amul tus, manam ëmb bu amul been ëmbeef.
- $\{a, b\} = \{b, a\}$




### 1.1 Wàllu ëmb

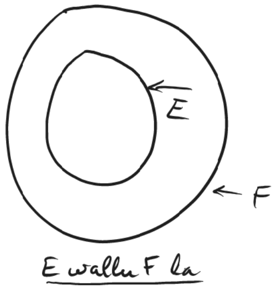
Téeki

Amal ëmb $E$ ak ëmb $F$, $E$ mu ngi ci biir $F$ bu féké ni rek[a] ëmbeef $x$ yëp yu nek ci $E$ ñu ngi ci biir $F$. Da ñuy wax itam $E$ wàllu $F$ la, di ko bindé $E \subset F$.

[a] si et seulement si = bu féké ni rek ?

Nañu mandargal wàllu ëmb:

- Amal ëmb $E = \{1, 2, 3\}$, $F = \{1, 2, 3, 4, 5, 6\}$ ak $G = \{1, 2, 4, 5, 6\}$, kon $E \subset F$, wayé $E$ nekkoul wàllu $G$, ñu koy bindé $E \not\subset G$, ndaxté $3$ mu ngi ci biir $E$, wanté $3$ nekkul ci biir $G$.




## 1.2 Ëmbu ay wàllu been ëmb

Téeki

Amal been ëmb $E$, ëmbu wàllu $E$ yi, mo di saamu wàllu $E$ yëp, ñu di ko bindé $\mathcal{W}(E)$.

Nañu mandargal ëmbu ay wàllu been ëmb:

- Amal ëmb $E = \{a, b, c\}$, $\mathcal{W}(E) = \big\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\big\}$

Seetlu

- Amal been ëmb $E$: $E \subset E$[a], manaam $E$ wàllu $E$ la
- Ëmbeefu $\mathcal{W}(E)$ ay ëmb la ñu.

[a]ñu di ko bindé itam $E \in \mathcal{W}(E)$

## 1.3 Sëfu Xayma ci ëmb yi

### 1.3.1 Selebe(yoon) ay ëmb (intersection d'ensembles)

Téeki

Amal ëmb $E$ ak ëmb $F$, saamu ëmbeef $x$ yëp yu bok ci $E$ té bok itam ci $F$, ñu di ko bindé $E \cap F$ la ñuy wowee selebe wu $E$ ak $F$

Nañu mandaargal[1] selebe ay ëmb.

- Amal ñaari ëmb $E = \{1, 2, 3\}$ ak $F = \{4, 5\}$, $E \cap F = \emptyset$ (manaam $E$ ak $F$ bokku ñu been ëmbeef.)
- Amal $E = \emptyset = F$, $E \cap F = \emptyset$
- Amal $E = \emptyset$, $F = \{0, 1\}$, $E \cap F = \emptyset$

### 1.3.2 Lëkkalé/Mboole ay ëmb (union d'ensembles)

Téeki

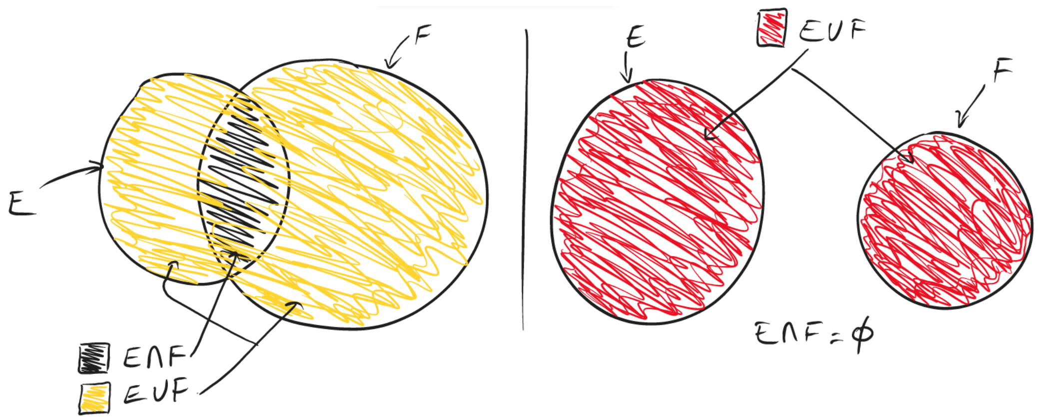
Amal ëmb $E$ ak ëmb $F$, saamu ëmbeef $x$ yëp yu bok ci $E$ wala bok ci $F$, ñu di ko bindé $E \cup F$ la ñuy wowee mboolo $E$ ak $F$.

Nañu mandaargal mbollo ëmb:

- Amal ñaari ëmb $E = \{1, 2, 3, 4, 5\}$ ak $F = \{4, 5, 6, 7, 8, 9\}$, $E \cup F = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

[1]Exemple ?

- Amal $E = \emptyset = F$, $E \cup F = \emptyset$
- Amal $E = \emptyset$, $F = \{0, 1\}$, $E \cup F = \{0, 1\}$




### 1.3.3 Full carteseng ay ëmb

**Téeki**

Amal ñaari ëmb $E$ ak $F$, fullu $E$ ak $F$, ñu koy bindé $E \times F$, mo di beep tank-tank[a] $(x, y)$ bu deme ni[b] $x \in E$ ak $y \in F$

[a] couple
[b] tel que

- Amal $E = \{1, 2, 3\}$ ak $F = \{4, 5\}$, kon $E \times F = \big\{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\big\}$

**Téeki**

Amal been limukaay $n \in \mathbb{N}$ bu eup $2$[a]. Amal itam $n$ ëmb $E_1, E_2, ..., E_n$, fullu $E_1, E_2, ..., E_n$, ñu koy bindé $E_1 \times ... \times E_n$, mo di beep ëmbeef $x = (x_1, .., x_n)$ bu deme ni[b] $x_1 \in E_1$, $x_2 \in E_2$, ..., $x_n \in E_n$

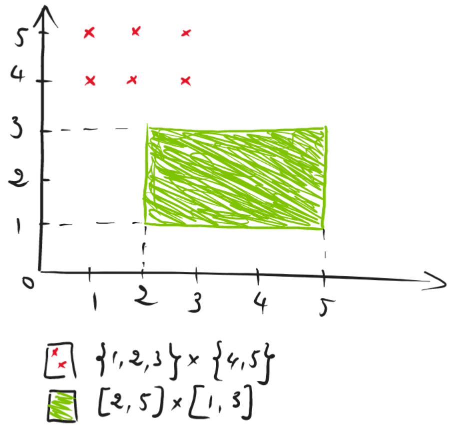
[a] $n \geq 2$
[b] tel que

**Seetlu**

Amal ëmb $E$ ak ëmb $F$;

- Bu féké $E$ ak $F$ ay ëmb yu wuté la ñu, ñuy binde $E \neq F$, té itam $E \neq \emptyset$ ak $F \neq \emptyset$, kon fullu $E$ ak $F$ wuté na ak fullu $F$ ak $E$, ñuy bind $E \times F \neq F \times E$
- Bu féké $E = F$, manaam $E$ ak $F$ been la ñu, mën na ñu binde $E \times F = E \times E = E^2$
- Amal ñaari tank-tank $(u, v)$ ak $(x, y)$ ci biir $E \times F$, da ñuy né $(u, v) = (x, y)$ bu féké rek $u = x$ ak $v = y$. Ngir leeral, $(1, 2) \in \mathbb{N}^2$ ak $(1, 3) \in \mathbb{N}^2$ wuté na ñu, ndaxté $2$ wuté na ak $3$.
- Bu féké ni limu ëmbeefu $E$ ak limu ëmbeefu $F$ da ñuy jex[a], kon ëmbeefu $E \times F$ maat nañu lu tolu ci limu ëmbeefu $E$ nga ful ko ak limu ëmbeefu $F$. Ngir leeral wax ji, amal $E = \{1, 2, 3\}$ ak $F = \{4, 5\}$, kon $E$ amna ñaat ëmbeef (kon limu ëmbeefu $E$, manam ñaat, day jex), $F$ amna ñaari ëmbeef (kon limu ëmbeefu $F$, manam ñaar, day jex), limu ëmbeefu $E \times F$ mo di ñaat nga full ko ak ñaar, manaan $3 \times 2 = 6$. Leneen lu koy woné mo di: $E \times F = \big\{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\big\}$ amna bu bax juròom-been (6) ëmbeef.
- Amal $n$ ëmb $E_1, E_2, ..., E_n$, bu féké ni $E = E_1 = E_2 = ... = E_n$, mên na ñu binde, $E_1 \times ... \times E_n = E^n$

[a] $E$ et $F$ sont des ensembles finis.




## 2 Doxalin

**Téeki**

Been [a]doxalin $f$ mo di beep ñaati ëmb $E$, $F$ ak $\mathcal{G}$. $E$ moy ëmbef bu doxalin $f$ di tambali, $F$ moy ëmbef bu muy agsi. Da ñuy bindë $\mathcal{G} = \{(x, y) \in E \times F, y = f(x)\}$. Ëmb $\mathcal{G}$, ñu di ko wowé graaf, moy lëkkalé beep ëmbeef $x \in E$ ak been ëmbeef $y \in F$ rek. Doxalin $f$ ñio ngi koy bindé:

$$f : \left\{ \begin{array}{l} E \to F \\ x \mapsto f(x) \end{array} \right.$$

wala ñuy binde

$$f : E \to F, \text{ ak } f(x) = \ldots$$

wala itam

$$f : x \mapsto \ldots \quad \text{ak } f : E \to F$$

Beep ëmbeef $x \in E$ warna am been natal[b] rek ci $f$, nataal gogu ñu di ko bindé $f(x)$

[a] Fonction ou application
[b] Image par une fonction

Nañu binde $\mathbb{R}_+ = \{x \in \mathbb{R}, x \geq 0\}$ manaam ëmbeefu limu $\mathbb{R}$ yu "positif" yi walla tolo ak tus, ak $\mathbb{R}_+^* = \{x \in \mathbb{R}, x > 0\}$, manaam ëmbeefu limu $\mathbb{R}$ yu "positif" yi té weesu/ëp tus.

- Doxalin $f : \left\{ \begin{array}{l} \mathbb{R} \to \mathbb{R} \\ x \mapsto x^2 + 1 \end{array} \right.$ ak doxalin $g : \left\{ \begin{array}{l} \mathbb{R} \to \mathbb{R}_+ \\ x \mapsto x^2 + 1 \end{array} \right.$ wuté nañu, ndaxté ëmbeef yu ñuy agsi wuté nañu: $\mathbb{R} \neq \mathbb{R}_+$
- $f : \left\{ \begin{array}{l} \mathbb{R} \to \mathbb{R}_+^* \\ x \mapsto x^2 \end{array} \right.$ nekkul doxalin, ndaxté $0^2 = 0 \notin \mathbb{R}_+^*$

# 3 Xalaat ci Xayma

## 3.1 Baat

Téeki

Been **baat**[a] ci Xayma mo di beep kaadu gi mëna nekka dëgg[b], walla nekka lu dul dëgg[c].
Dëgg (D), ak lu dëggul (L) la ñuy wowee xayma dëgg[d]. Bu féké ñaari baat $\mathcal{B}$ ak $\mathcal{C}$ ño bok xayma dëgg, kon da ñuy né ño niro, di ko bindé: $\mathcal{B} \sim \mathcal{C}$, seeni xayma dëg bu ñu wuté wé, da ñuy bindë $\mathcal{B} \nsim \mathcal{C}$

[a] assertion, prédicat
[b] vrai
[c] faux
[d] valeur de vérité

Amal $(0, 1, 2) \in \mathbb{N}^3$, kon baat $\mathcal{B} = "1 \geq 0"$ dëgg la, wanté, baat $\mathcal{C} = "2 + 1 \leq 2"$ du dëgg, kon bok $\mathcal{B} \nsim \mathcal{C}$. Baat $\mathcal{B} = "2 = 0"$ ak baat $\mathcal{C} = "1 > 2"$ dëgguñu, kon $\mathcal{B} \sim \mathcal{C}$.

## 3.2 Muk/Deet

Téeki

**Muk been baat**[a]$\mathcal{B}$, ñu di ko bindé *muk* $\mathcal{B}$ (wala $\neg\mathcal{B}$) baatu dëgg la, bu féké ni $\mathcal{B}$ du dëgg. Té itam, baat bu dëggul la, bu féké ni $\mathcal{B}$ dëgg la.

[a] La négation d'une assertion

Amal $x \in \mathbb{R}$, $\mathcal{B}(x) = "x \leq 0"$, kon $\neg\mathcal{B}(x) \sim "x > 0"$.

Tèg[a]

[a] Proposition

Amal been baat $\mathcal{B}$, boba/kon $\neg(\neg\mathcal{B}) \sim \mathcal{B}$, manaam muk (muk $\mathcal{B}$) ak $\mathcal{B}$ ño book xayma dëgg.

**Woné:** Amal been baat $\mathcal{B}$

| $\mathcal{B}$ | $\neg\mathcal{B}$ | $\neg(\neg\mathcal{B})$ |
|---|---|---|
| D | L | D |
| L | D | L |

## 3.3 Takhalé ak tékhalé ay baat

Téeki[a]

[a] Définition

Takhalo[a] ñaari baat $\mathcal{B}$ ak $\mathcal{C}$, ñu di ko bindé $\mathcal{B} \wedge \mathcal{C}$, wala $\mathcal{B}$ *ak* $\mathcal{C}$, baatu dëgg la bu féké ni rek $\mathcal{B}$ dëgg la, té $\mathcal{C}$ dëgg la itam.

Tékhalo[b] ñaari baat $\mathcal{B}$ ak $\mathcal{C}$, ñu di ko bindé $\mathcal{B} \vee \mathcal{C}$, wala $\mathcal{B}$ *wala* $\mathcal{C}$, dëgg la bu féké ni rek $\mathcal{B}$ dëgg la, wala $\mathcal{C}$ dëgg la.

| $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{B} \wedge \mathcal{C}$ | $\mathcal{B} \vee \mathcal{C}$ |
|---|---|---|---|
| D | D | D | D |
| D | L | L | D |
| L | D | L | D |
| L | L | L | L |

[a]Conjonction
[b]Disjonction

Tèg[a]

[a]Proposition

Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$, boba/kon:

$$\neg(\mathcal{B} \wedge \mathcal{C}) \sim (\neg\mathcal{B}) \vee (\neg\mathcal{C})$$

$$\neg(\mathcal{B} \vee \mathcal{C}) \sim (\neg\mathcal{B}) \wedge (\neg\mathcal{C})$$

**Woné:**
Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$, nañu bindë këralegu/natalu[2] xayma dëgg baat yi.

| $\mathcal{B}$ | $\mathcal{C}$ | $\neg\mathcal{B} \vee \neg\mathcal{C}$ | $\neg(\mathcal{B} \wedge \mathcal{C})$ | $\neg(\mathcal{B} \vee \mathcal{C})$ | $(\neg\mathcal{B}) \wedge (\neg\mathcal{C})$ |
|---|---|---|---|---|---|
| D | D | L | L | L | L |
| D | L | D | D | L | L |
| L | D | D | D | L | L |
| L | L | D | D | D | D |

Mën nañu gis ci ñaatel jeñ[3] gi ak ñeentel gi, né $\neg(\mathcal{B} \wedge \mathcal{C}) \sim (\neg\mathcal{B}) \vee (\neg\mathcal{C})$. Juròomeel jeñ gi ak juròomeel-beeneel gi, woné nañu né $\neg(\mathcal{B} \vee \mathcal{C}) \sim (\neg\mathcal{B}) \wedge (\neg\mathcal{C})$.

Tègtal[a]

[a]Indication ?

Ngir woné né ñaari baat ño bok xayma dëgg, mën naño jëfandikoo këralegu xayma dëgg yi.

**Jéemantu**
Amal ñaat baat $\mathcal{A}, \mathcal{B}, \mathcal{C}$, wonéel ni:

$$\mathcal{B} \wedge \mathcal{B} \sim \mathcal{B}$$
$$\mathcal{B} \vee \mathcal{B} \sim \mathcal{B}$$
$$(\mathcal{B} \wedge \mathcal{C}) \wedge \mathcal{A} \sim \mathcal{B} \wedge (\mathcal{C} \wedge \mathcal{A})$$
$$(\mathcal{B} \vee \mathcal{C}) \vee \mathcal{A} \sim \mathcal{B} \vee (\mathcal{C} \vee \mathcal{A})$$
$$(\mathcal{B} \wedge \mathcal{C}) \vee \mathcal{A} \sim (\mathcal{B} \vee \mathcal{A}) \wedge (\mathcal{C} \vee \mathcal{A})$$
$$(\mathcal{B} \vee \mathcal{C}) \wedge \mathcal{A} \sim (\mathcal{B} \wedge \mathcal{A}) \vee (\mathcal{C} \wedge \mathcal{A})$$

## 3.4 Baat bi yobualé/andi been baat

Téeki

Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$. Da ñuy né $\mathcal{B}$ *da yobualé* $\mathcal{C}$[a] dëgg la, bu féké ni rek $\mathcal{C}$ mënul bagna nek dëgg bu $\mathcal{B}$ néké dëgg.
$\mathcal{B}$ *da yobualé* $\mathcal{C}$ ñu di ko bindé $\mathcal{B} \implies \mathcal{C}$.

Xayma dëgg $\mathcal{B} \implies \mathcal{C}$ mu ngi ni:

[2]tableau ?
[3]Colonne = jiñ = jeñ ?

| $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{B} \implies \mathcal{C}$ |
|---|---|---|
| D | D | D |
| D | L | L |
| L | D | D |
| L | L | D |

[a] $\mathcal{B}$ implique $\mathcal{C}$

Ak beep $x \in \mathbb{R}$, bo bindé $\mathcal{B}(x) = "x >= 2"$, $\mathcal{C}(x) = "x^2 >= 4"$, kon $\mathcal{B}(x) \implies \mathcal{C}(x)$ dëgg la.

Téeki

Bu $\mathcal{B} \implies \mathcal{C}$ néké dëgg, kon da ñuy né

- *$\mathcal{B}$ baat bu doy $\mathcal{C}$* la, wala *$\mathcal{B}$ doy na $\mathcal{C}$*.[a]
- *$\mathcal{B}$ da soxla $\mathcal{C}$*.[b]

$\mathcal{C} \implies \mathcal{B}$ mo di *wëlbati wu* [c] baat $\mathcal{B} \implies \mathcal{C}$.

[a] $\mathcal{B}$ est une condition suffisante pour $\mathcal{C}$
[b] $\mathcal{C}$ est une condition nécessaire pour $\mathcal{B}$
[c] implication réciproque

Tèg[a]

[a] Proposition

Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$, boba:

$$(\mathcal{B} \implies \mathcal{C}) \sim (\neg\mathcal{B}) \vee \mathcal{C}$$
$$(\mathcal{B} \implies \mathcal{C}) \sim \big((\neg\mathcal{C}) \implies (\neg\mathcal{B})\big)$$

$(\neg\mathcal{C}) \implies (\neg\mathcal{B})$ la ñuy wowee *contaraposee*[a] wu $\mathcal{B} \implies \mathcal{C}$

[a] Contraposée

**Woné:**

| $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{B} \implies \mathcal{C}$ | $(\neg\mathcal{B}) \vee \mathcal{C}$ |
|---|---|---|---|
| D | D | D | D |
| D | L | L | L |
| L | D | D | D |
| L | L | D | D |

Mën na ñu gis né $(\mathcal{B} \implies \mathcal{C}) \sim (\neg\mathcal{B}) \vee \mathcal{C}$ ci ñaateel jeñ ak ñeenteel gi, kon bok bu ñu wécanté $\mathcal{B}$ ak $\neg\mathcal{C}$, té wécanté itam $\mathcal{C}$ ak $\neg\mathcal{B}$, ñu am $\big((\neg\mathcal{C}) \implies (\neg\mathcal{B})\big) \sim \big((\neg(\neg\mathcal{C})) \vee (\neg\mathcal{B})\big) \sim \big(\mathcal{C} \vee (\neg\mathcal{B})\big) \sim (\neg\mathcal{B}) \vee \mathcal{C} \sim (\mathcal{B} \implies \mathcal{C})$, fi la ñuy jexalé woné gi.

Mënon nañu woné itam $(\mathcal{B} \implies \mathcal{C}) \sim \big((\neg\mathcal{C}) \implies (\neg\mathcal{B})\big)$ ak natal xayma dëgg yi.

## 3.5 Baat yu yèm

Téeki

Amal ñaari baat $\mathcal{B}$, $\mathcal{C}$.
*$\mathcal{B}$ mo yèm ak $\mathcal{C}$*[a], ñuy bindë $\mathcal{B} \iff \mathcal{C}$, mo di baat $(\mathcal{B} \implies \mathcal{C}) \wedge (\mathcal{C} \implies \mathcal{B})$, manaam

$$\mathcal{B} \iff \mathcal{C} \sim \big((\mathcal{B} \implies \mathcal{C}) \wedge (\mathcal{C} \implies \mathcal{B})\big)$$

Natal xayma dëgg bu yèmalé ñaari baat mo ngi ni:

| $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{B} \implies \mathcal{C}$ | $\mathcal{C} \implies \mathcal{B}$ | $\mathcal{B} \iff \mathcal{C}$ |
|---|---|---|---|---|
| D | D | D | D | D |
| D | L | L | D | L |
| L | D | D | L | L |
| L | L | D | D | D |

[a] L'équivalence de deux assertions

Seetlu

Amal ñaari baat $\mathcal{B}$, $\mathcal{C}$.

- Bu $\mathcal{B} \iff \mathcal{C}$ néké dëgg, kon $\mathcal{B} \sim \mathcal{C}$.
- Bu féké $\mathcal{B} \iff \mathcal{C}$ dëggul, kon $\mathcal{B} \nsim \mathcal{C}$

Seetlu yi muj, woné nañu né **yémalé ay baat ak nirolé lèn been lañu**. Manam wax $\mathcal{B} \iff \mathcal{C}$ been la ak wax $\mathcal{B} \sim \mathcal{C}$, manaam:

$$(\mathcal{B} \iff \mathcal{C}) \sim (\mathcal{B} \sim \mathcal{C})$$
$$(\mathcal{B} \iff \mathcal{C}) \iff (\mathcal{B} \sim \mathcal{C})$$

Tèg[a]

[a] Proposition

Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$, boba:

$$(\mathcal{B} \iff \mathcal{C}) \sim ((\neg\mathcal{C}) \iff (\neg\mathcal{B}))$$

**Jéemantu:** Woneel tèg bi muj

Téeki

Bu $\mathcal{B} \iff \mathcal{C}$ néké dëgg, kon da ñuy né

- *$\mathcal{B}$ baat bu soxla té doy $\mathcal{C}$* la, [a]

[a] $\mathcal{B}$ est une condition nécessaire et suffisante pour $\mathcal{C}$

## 3.6 Natakat baat

Téeki

Amal been ëmb $E$. Ak beep $x \in E$, amal been baat $\mathcal{B}(x)$ bu nék surgau $x^a$.
Da ñuy wax *ak beep $x \in E$, $\mathcal{B}(x)$ dëgg la*, té di bindë

$$\forall x \in E, \mathcal{B}(x)$$

bu féké ni rek $\mathcal{B}(x)$ dëgg la ak beep ëmbeef $x$ bu nék ci biir $E$.

Da ñuy wax *ak been $x \in E$, $\mathcal{B}(x)$ dëgg la*, té di bindë

$$\exists x \in E, \mathcal{B}(x)$$

bu féké ni rek $\mathcal{B}(x)$ dëgg la ak lu mu tuti tuti been ëmbeef $x$ bu nék ci biir $E$.

Da ñuy wax *ak been $x \in E$ rek, $\mathcal{B}(x)$ dëgg la*, té di bindë

$$\exists! x \in E, \mathcal{B}(x)$$

bu féké ni rek $\mathcal{B}(x)$ dëgg la ak been ëmbeef $x$ dong bu nék ci biir $E$.

[a]Une assertion qui dépend de $x$

Mën nañu né $\forall x \in \mathbb{R}, \exp(x) > 0$, ak $\forall x \in \mathbb{R}, \exists! n \in \mathbb{Z}, n \leq x < n+1$

Amal been doxalin $f$ bi jogé ci $\mathbb{R}$ té agsi ci $\mathbb{R}$, kon bok:
Da ñuy wax $f$ moy doxalinu dara/tus [4] bu féké ni rek

$$\forall x \in \mathbb{R}, f(x) = 0$$

Da ñuy wax $f$ di na agsi ci tus[5] bu féké ni rek

$$\exists x \in \mathbb{R}, f(x) = 0$$

Da ñuy wax $f$ di na agsi been yoon ci tus bu féké ni rek [6]

$$\exists! x \in \mathbb{R}, f(x) = 0$$

Da ñuy wax $f$ ci $\mathbb{R}_+$ rek la mëna tolo ak tus[7] bu féké ni rek

$$\forall x \in \mathbb{R}, f(x) = 0 \implies x \in \mathbb{R}_+$$

wala itam

$$\forall x \in \mathbb{R}, x \notin \mathbb{R}_+ \implies f(x) \neq 0$$

Tèg[a]

[a]Proposition

$$\neg(\forall x \in E, \mathcal{B}(x)) \sim \exists x \in E, \neg\big(\mathcal{B}(x)\big)$$
$$\neg(\exists x \in E, \mathcal{B}(x)) \sim \forall x \in E, \neg\big(\mathcal{B}(x)\big)$$

**Jéemantu:** Woneel tèg yi muj

Waxanté[a]

[a]Convention

Beep baat buy tambalé ak $\exists x \in \emptyset$ dëggul, kon beep baat buy tambalé ak $\forall x \in \emptyset$ dëgg la.

**Jéemantu bu am solo ci xam-xam ci koompuutar**[8] Amal been doxalin $f$ bi jogé ci been full carteseng ëmb xayma dëgg yi $\{0,1\}^n$, té di agsi ci ëmbu xayma dëgg yi, $0$ di téeki lu dëggul $(L)$, $1$ di téeki dëgg $(D)$, ñuy bindë

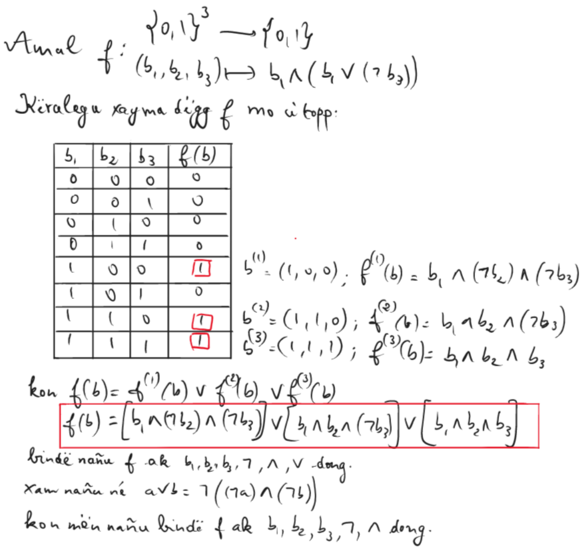
$$f : \{0,1\}^n \to \{0,1\}$$

mën nañu woné né $f$ mën nañu ko bindé ak sëfu xayma dëgg *takhalo* (manaam $\wedge$) ak sëfu xayma dëgg *deet* (manaam $\neg$). Manam bo jëlé $b = (b_1, b_2, ...b_n) \in \{0,1\}^n$, kon $f(b)$ mën nañu kon bindé ak $b_1, b_2, ...b_n$ ak $\wedge$ ak $\neg$ dong.
*Tégtal: ngir woné tèg bi muj*

[4]La fonction $f$ est la fonction nulle.
[5]La fonction $f$ s'annule
[6]La fonction $f$ s'annule une seule fois
[7]La fonction $f$ ne s'annule que sur $\mathbb{R}_+$
[8]Exercice important en informatique

- Mën nañ woné ni amna been $0 \leq k \leq 2^n$ ak $f_1$, ..., $f_k$ ay doxalin yuy jogé ci $\{0,1\}^n$ té agsi ci $\{0,1\}$, yu mel ni
$$f(b) = f^{(1)}(b) \vee f^{(2)}(b) \vee ... \vee f^{(k)}(b)$$
té $f^{(l)}(b) = \begin{cases} 1 \text{ bu féké ni } b = b^{(l)} \\ 0 \text{ bu féké ni } b \neq b^{(l)} \end{cases}$, ak $b^{(l)}$ been néekin $b$ ci këralegu xayma dëggu $f$, té $f(b^{(l)}) = 1$. Ngir léeral, bu féké $b = (1,0,1,0,0,0,1)$ ci liñ $l$ këralegu xayma dëgg doxalin $f$, té $f(b^{(l)}) = 1$, mën nañu bindë $b^{(l)} = (1,0,1,0,0,0,1)$ té bindë $f_l(b) = b_1 \wedge (\neg b_2) \wedge b_3 \wedge (\neg b_4) \wedge (\neg b_5) \wedge (\neg b_6) \wedge b_7$. Bu féké $f = 0$ (manaam $k = 0$), kon mën nañu bindë $f(b) = b_1 \wedge (\neg b_1)$

- Fatéliku itam né $\vee$ mën nañu ko bindë ak $\wedge$ ak $\neg$




## 3.7 Kadum sago

Ngir wax né been baat, baatu dëgg la ci Xayma, war na ñu ko firndeel/woral ak been woné, manaam ci Xayma, woné rek moy dëggal been baat. Baatu dëgg bu nék, **tèg**[9] la tud. Ci tèg yi, amna yu ci gëna am solo, ñu len di wowée **téorèm**[10]. Wanté amna ay baat yu ñu dul woné té nangu né ay baati dëgg la ñu, ñu léen di wowée **ñalém**, wala **baatu dëga yu wor**[11]. Baat yoyu lé, mën na ñu lèna jappé ay sart[12] yuy lal xalaat ci Xayma.

Nañu lim woné yu ñuy tama jëfandiko ngir woral ay baat.

### 3.7.1 Woné ab contaraposee

**Téeki**

Mën na ñu woné been baat $\mathcal{B} \implies \mathcal{C}$ dëgg la, bu ñu woné $(\neg\mathcal{C}) \implies (\neg\mathcal{B})$ dëgg la. Woné gi la ñuy wowée **woné contaraposee**[a]

[a]Démonstration par contraposition

[9]Proposition
[10]Théorème
[11]Axiomes
[12]Règles

### 3.7.2 Tofal

Téeki

Mën na ñu woné been baat $\mathcal{C}$ dëgg la, bu ñu tambali wé ci béneen baat $\mathcal{B}$ bu nek dëgg té woné ni $\mathcal{B} \implies \mathcal{C}$ dëgg la. Woné gi la ñuy wowée **tofal**[a].

[a]Tirer une conséquence.

Nañu woné

$$\forall x \in \mathbb{R}, x^2 + 1 > 0$$

ak tofal.

Amal $x \in \mathbb{R}$. Xam na ñu $x^2 \geq 0$ ak $1 > 0$ wanté xamna ñu itam sa su jëlé $a$ ak $b$ ay ëmbeefu $\mathbb{R}$,

$$(a \geq 0) \text{ ak } b > 0 \implies a + b > 0$$

kon itam $x^2 + 1 > 0$ (bu ñu jëlé $a = x^2$ ak $b = 1$).

### 3.7.3 Tofal ak tékhalé ay baat / ak nékin yëp yu wuté

Téeki

Mën nañu woné $\mathcal{C}$ dëgg la, bu ñu tambali wé ak béneen baat $\mathcal{B}$, té woné $\mathcal{B} \implies \mathcal{C}$ ak $(\neg\mathcal{B}) \implies \mathcal{C}$ dëgg la. Woné gi mo tud **tofal ak tékhalé ay baat**[a]

[a]Disjonction des cas

Nañu woné

$$\forall n \in \mathbb{N}, \frac{n(n+1)}{2} \in \mathbb{N}$$

di jëfandiko tofal ak tékhalé ay baat.

Amal $n \in \mathbb{N}$, xam na ñu amna been $k \in \mathbb{N}$ bu mel ni $n = 2k$, wala $n = 2k + 1$.

Bu féké $n = 2k$, kon

$$\frac{n(n+1)}{2} = \frac{2k(2k+1)}{2} = k(2k+1) \in \mathbb{N}$$

Bu féké itam $n = 2k + 1$, kon

$$\frac{n(n+1)}{2} = \frac{(2k+1)(2k+2)}{2} = (2k+1)(k+1) \in \mathbb{N}$$

fi la woné gi jexé.

### 3.7.4 Wédi

Téeki

Mën nañu woné $\mathcal{B}$ dëgg la, bu ñu woné ni amna béneen baat $\mathcal{C}$ bu dëggul, té woné itam $(\neg\mathcal{B}) \implies \mathcal{C}$ dëgg la.

Nañu woné ni amul been $N \in \mathbb{N}$, bu gën ëp[13] beep $n \in \mathbb{N}$ di ko bindé itam

$$\exists N \in \mathbb{N}, \forall n \in \mathbb{N}, N > n$$

dëggul. Nañu ko wédi, manaan né amna been $N \in \mathbb{N}$, bu gën ëp beep $n \in \mathbb{N}$, kon $N$ gogu mo gën ëp $N+1$, ndaxté $N + 1 \in \mathbb{N}$, kon dé

$$1 = (N+1) - N < 0$$

Li mënul nék, ndaxté xam nañu $1 > 0$ ci biir $\mathbb{N}$

### 3.7.5 Topalanté ay baat

[13]Strictement supérieur

Téeki

Amal $n_0 \in \mathbb{N}$ ak ay baat $\mathcal{B}(n)$, $n \in \mathbb{N}, n \geq n_0$.
Bu féké $\mathcal{B}(n_0)$ dëgg la, té itam

$$\forall n \geq n_0,\ \mathcal{B}(n) \implies \mathcal{B}(n+1)$$

kon

$$\forall n \geq n_0,\ \mathcal{B}(n)$$

Nañu woné

$$\forall n \in \mathbb{N}^*, 1+2+3+...+n = \frac{n(n+1)}{2}$$

ak topalanté ay baat.
Amal $n = 1$, kon $1+2+3+..+n = 1$, té itam $\frac{n(n+1)}{2} = 1$, kon dëgg la ak $n = 1$.
Amal been $n \in \mathbb{N}^*$. Bu féké ni $1+2+3+...+n = \frac{n(n+1)}{2}$, kon

$$1+2+3+...+(n+1) = (1+2+3+...+n)+(n+1) = \frac{n(n+1)}{2}+n+1 = \frac{(n+1)\big((n+1)+1\big)}{2}$$

fi la woné gi jexé.

Seetlu

Amal $\mathcal{B}$ ak $\mathcal{C}$, natal xayma dëgg yi ñoy dëggal woné yi jal.

natal gi njëk:

| $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{B} \implies \mathcal{C}$ |
|---|---|---|
| D | D | D |
| D | L | L |
| L | D | D |
| L | L | D |

ñaareel natal gi:

| $\mathcal{B}$ | $\mathcal{C}$ | $\neg\mathcal{B} \implies \mathcal{C}$ |
|---|---|---|
| D | D | D |
| D | L | D |
| L | D | D |
| L | L | L |

- **Tofal**: liñ[a] bu njëk ci natal bu njëk bi moy woral woné ak *tofal*.
- **Tofal ak tékhalé ay baat**: liñ bu njëk ak ñaateel liñu ñaari natal yi, ñoy woral woné ak *tofal ak tékhalé ay baat*.
- **Wédi**: ñaareel liñ ñu ñaareel lu natal gi moy woral woné ak *wédi*
- **Topalanté ay baat**: amal $n_0 \in \mathbb{N}$ ak ay baat $\mathcal{B}(n)$, $n \in \mathbb{N}, n \geq n_0$.
  Bu féké $\mathcal{B}(n_0)$ dëgg la, té itam

  $$\forall n \geq n_0,\ \mathcal{B}(n) \implies \mathcal{B}(n+1)$$

  kon, ndaxté $\mathcal{B}(n_0)$ dëgg la, té $\mathcal{B}(n_0) \implies \mathcal{B}(n_0+1)$ dëgg la, woné ak *tofal* dëggal na $\mathcal{B}(n_0+1)$, *tofal* moy woral woné ak *topalanté ay baat*

[a]Ligne

# 4 Xayma ci ëmb yi

## 4.1 Wàllu

Téeki

Amal ëmb $E$ ak $F$, $E$ **wàllu** $F$ **la**, ñuy bindë $E \subset F$ bu féké ni, ak beep $x \in E$, $x \in F$ itam:

$$E \subset F \iff \forall x \in E, x \in F$$

kon $E$ **nekul wàllu** $F$, ñuy bindë $E \not\subset F$ bu féké ni amna been $x \in E$ té $x \notin F$

$$E \not\subset F \iff \exists x \in E, x \notin F$$

Seetlu

Amal been ëmb $E$ ak $F$, kon

- $\emptyset \subset F$ ndaxté $\forall x \in \emptyset, x \in F$ (ndaxté beep baat bu di tambali ak $\forall x \in \emptyset$ dëgg la)
- $E \subset E$, ndaxté $\forall x \in E, x \in E$ lu leer la

Tèg[a]

[a]Proposition

Amal ëmb $E$ ak $F$, kon

$$E = F \iff (E \subset F \text{ ak } F \subset E)$$

Woné:

Amal ëmb $E$ ak $F$. Ngir woné $E = F \iff E \subset F$ ak $F \subset E$, mën nañu woné $\mathcal{B} = "(E = F \implies E \subset F \text{ ak } F \subset E)"$ dëgg la, té woné itam $\mathcal{C} = "(E \subset F \text{ ak } F \subset E) \implies E = F"$ dëgg la.

Nañu woné $\mathcal{B}$. Xam nañu $E \subset E$ (seetlu bi muj mo ko wax). Bu féké $E = F$, kon $E \subset F$ (bu ñu wecé ñaareel $E$ bi ak $F$ ci diganté gi $E \subset E$). Nonu la ñuy woné itam $F \subset E$ bu féké $E = F$, kon $\mathcal{B}$ dëgg la.

Nañu woné $\mathcal{C}$ dëgg la ak contaraposee wam manaam $E \neq F \implies \neg(E \subset F \text{ ak } F \subset E)$. Bu féké $E \neq F$ (manaam $E$ wuté na ak $F$, kon $\exists x \in E, x \notin F$ mba/wala $\exists y \in F, y \notin E$, manaam *deet*$(\forall x \in E, x \in F \text{ ak } \forall y \in F, x \in E)$, manaam $\neg(E \subset F \text{ ak } F \subset E)$. Fi la woné gi jexé.

Tèg[a]

[a]Proposition

Amal ëmb $E$, $F$, $G$ kon

$$E \subset F \text{ ak } F \subset G \implies E \subset G$$

**Jéemantu:** Woneel tèg bi muj.

Téeki

Amal ëmb $E$, $\mathcal{W}(E)$ **mo di mbidu ëmb bi bolé rek wàllu** $E$ **yëp**, manaam amal been ëmb $A$

$$A \in \mathcal{W}(E) \iff A \subset E$$

Amal ëmb $E$, kon $\emptyset \in \mathcal{W}(E)$, $E \in \mathcal{W}(E)$, $\emptyset \subset \mathcal{W}(E)$, wanté nañu woytu[14] $E \subset \mathcal{W}(E)$ mën na baña nek dëgg.

## 4.2 Selebe(yoon) ak Mbolo ay ëmb

Téeki

Amal ëmb $A$, $B$ ay wàllu been ëmb $E$, $A \cap B$ **mo di selebe(yoon)** $A$ **ak** $B$ di saamu ëmbeef $x \in E$ yëp yu bok ci $A$ te bok itam ci $B$, manaan

$$A \cap B = \{x \in E / x \in A \text{ ak } x \in B\}$$

Téeki

Amal ëmb $A$, $B$ ay wàllu been ëmb $E$, $A \cup B$ **mo di mbolo** $A$ **ak** $B$ di saamu ëmbeef $x \in E$ yëp yu bok ci $A$ walla bok ci $B$, manaan

$$A \cup B = \{x \in E / x \in A \text{ walla } x \in B\}$$

Seetlu

Amal ëmb $A$ ak $B$ ay wàllu ëmb $E$, kon

- $A \cap B \subset A$ ak $A \subset A \cup B$
- $A \cap B \subset B$, ak $B \subset A \cup B$
- $A \cap A = A$, $A \cup A = A$
- $A \cup E = E$, $A \cap E = A$
- $A \cup \emptyset = A$, $A \cap \emptyset = \emptyset$

.

Tègtal[a]

[a]Indication ?

Amal ëmb $A$ ak $B$, ay wàllu been ëmb $E$, ngir woné $A = B$ been lañu mën nañu woné ni: ak beep ëmbeef $x \in E$, $x \in A \iff x \in B$. Manaam woné $A \subset B$ ak $B \subset A$

Tèg[a]

[a]Proposition

Amal ëmb $E$, $F$,

$$\begin{aligned}
A \cup B &= B \cup A \\
A \cap B &= B \cap A \\
(A \cup B) \cup C &= A \cup (B \cup C) \\
(A \cap B) \cap C &= A \cap (B \cap C) \\
(A \cap B) \cup C &= (A \cup C) \cap (B \cup C) \\
(A \cup B) \cap C &= (A \cap C) \cup (B \cap C)
\end{aligned}$$

**Jéemantu:** Woneel tèg bi muj.

[14]faire attention

**Seetlu**

Amal ëmb $A$, $B$ ak $C$, kon

- $(A \cup B) \cup C = A \cup (B \cup C)$ la ñuy bindë $A \cup B \cup C$
- $(A \cap B) \cap C = A \cap (B \cap C)$ la ñuy bindë $A \cap B \cap C$
- Wanté ken du bindë $A \cap B \cup C$ walla $A \cup B \cap C$ ndaxté léru ñu

**Tèg**[a]

[a]Proposition

Amal ëmb $A$, $B$ ak $C$, kon

$$A \subset C \text{ ak } B \subset C \implies A \cup B \subset C$$
$$C \subset A \text{ ak } C \subset B \implies C \subset A \cap B$$

Woné:
Amal ëmb $A$, $B$ ak $C$. Nañu woné $A \subset C$ ak $B \subset C \implies A \cup B \subset C$
Bu féké $A \subset C$ ak $B \subset C$. Amal $x \in A \cup B$, manaam $x \in A$ walla $x \in B$:

- Bu féké $x \in A$, kon, ndaxté $A \subset C$, $x \in C$ tamit
- Bu féké $x \in B$, kon, ndaxté $B \subset C$, $x \in C$ tamit

Woné nañu $A \subset C$ ak $B \subset C \implies A \cup B \subset C$ ci beep nékin.

Ak xeetu woné ji muj, mën nañu woné $C \subset A$ ak $C \subset B \implies C \subset A \cap B$

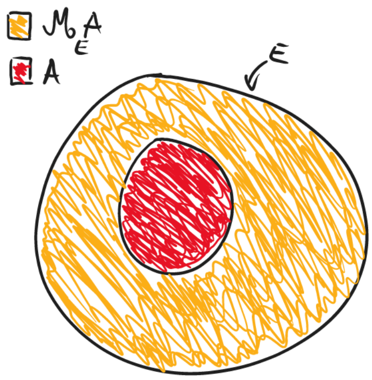
## 4.3 Motali been ëmb ci biir been ëmb

**Téeki**

Amal ëmb $A$ wàllu been ëmb $E$, **ëmb bi di motali $A$ ci biir $E$**, ñu di ko bindë

$$\mathcal{M}_E A = \{x \in E, \text{ té } x \notin A\}$$

mo di saamu ëmb yëp yu nék ci $E$ té nekku ñu ci $A$




A ak motali A ci biir E

**Seetlu**

Amal ëmb $A$ ab wàllu ëmb $E$. Bu féké ëmb bi di motali $A$ ci biir $E$ lu lér la, manaam munu ñu ko jaxasé ak leenen, kon mën nañu bindë $\bar{A}$ ngir wax ëmb bi motali ci biir $E$.

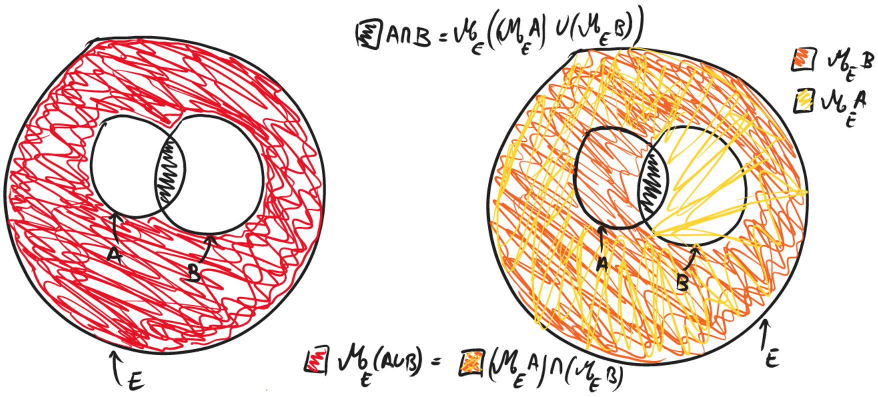
- $\mathcal{M}_E A \cap A = \emptyset$, $\mathcal{M}_E(\mathcal{M}_E A) = A$
- $\mathcal{M}_E E = \emptyset$, $\mathcal{M}_E \emptyset = E$

**Tèg**[a]

[a]Proposition

Amal ëmb $A$, $B$ ay wàllu ëmb $E$, kon

$$\begin{aligned} \mathcal{M}_E(A \cup B) &= (\mathcal{M}_E A) \cap (\mathcal{M}_E B) \\ \mathcal{M}_E(A \cap B) &= (\mathcal{M}_E A) \cup (\mathcal{M}_E B) \\ A \subset B &\iff (\mathcal{M}_E B) \subset (\mathcal{M}_E A) \end{aligned}$$




## 4.4 Wañi ëmb ci ëmb

**Téeki**

Amal ëmb $A$ ak $B$ ay wàllu been ëmb $E$, **ëmb bi di motali** $A$ **ci biir** $E$, ñu di ko bindë

$$A - B = \{x \in E, \text{ té } x \in A \text{ ak } x \notin B\}$$

mo di saamu ëmbeef yëp yu nék ci $A$ té nekku ñu ci $B$. Yèna say ñu bindé ko $A \backslash B$

**Seetlu**

Amal ëmb $A$ ak $B$ ay wàllu been ëmb $E$, boba

- $A \backslash B = \mathcal{M}_A(A \cap B) = A \cap \mathcal{M}_E(B)$

**Rëd fi been natal bu di mandargal kaadu yi muj.**