Njang/jàng Xayma

SARR Georges Mbissane

Ci wéeru ñaari junni ak ñaar fukk ak ñaar (2022)

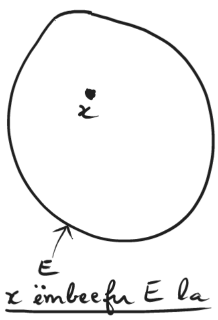
1 Ëmb

Téeki

Nañu wowee ëmb, beep saamu cër[a] yu ñuy wowee ëmbeef. Amal been ëmb E, da ñuy né x ëmbeefu E la, ta ñu ko bindé $x \in E$[b], bu féké ni x ci biir E la nek/la bok.

[a] élément
[b] Mën na ñu ko liré "x mu ngi ci biir E"

- $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ ay ëmb la ñu.
- $\{a, b, c, d, ...z\}$ moy ëmb bu am ab ëmbeef a, b, c, ..., ba z
- $\emptyset$ moy mbindu ëmb bu amul tus, manam ëmb bu amul been ëmbeef.
- $\{a, b\} = \{b, a\}$



1.1 Wàllu ëmb

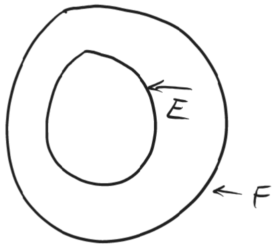
Téeki

Amal ëmb E ak ëmb F, E mu ngi ci biir F bu féké ni rek[a] ëmbeef x yëp yu nek ci E ñu ngi ci biir F. Da ñuy wax itam E wàllu F la, di ko bindé $E \subset F$.

[a] si et seulement si = bu féké ni rek ?

Nañu mandargal wàllu ëmb:

- Amal ëmb $E = \{1, 2, 3\}$, $F = \{1, 2, 3, 4, 5, 6\}$ ak $G = \{1, 2, 4, 5, 6\}$, kon $E \subset F$, wayé E nekkoul wàllu G, ñu koy bindé $E \not\subset G$, ndaxté 3 mu ngi ci biir E, wanté 3 nekkul ci biir G.



E wallu F la

1.2 Ëmbu ay wàllu been ëmb

Téeki

Amal been ëmb E, ëmbu wàllu E yi, mo di saamu wàllu E yëp, ñu di ko bindé $\mathcal{W}(E)$.

Nañu mandargal ëmbu ay wàllu been ëmb:

- Amal ëmb $E = \{a, b, c\}$, $\mathcal{W}(E) = \big\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\big\}$

Seetlu

- Amal been ëmb E: $E \subset E$[a], manaam E wàllu E la
- Ëmbeefu $\mathcal{W}(E)$ ay ëmb la ñu.

[a]ñu di ko bindé itam $E \in \mathcal{W}(E)$

1.3 Sëfu Xayma ci ëmb yi

1.3.1 Selebe(yoon) ay ëmb (intersection d'ensembles)

Téeki

Amal ëmb E ak ëmb F, saamu ëmbeef x yëp yu bok ci E té bok itam ci F, ñu di ko bindé $E \cap F$ la ñuy wowee selebe wu E ak F

Nañu mandaargal[1] selebe ay ëmb.

- Amal ñaari ëmb $E = \{1, 2, 3\}$ ak $F = \{4, 5\}$, $E \cap F = \emptyset$ (manaam E ak F bokku ñu been ëmbeef.)
- Amal $E = \emptyset = F$, $E \cap F = \emptyset$
- Amal $E = \emptyset$, $F = \{0, 1\}$, $E \cap F = \emptyset$

1.3.2 Lëkkalé/Mboole ay ëmb (union d'ensembles)

Téeki

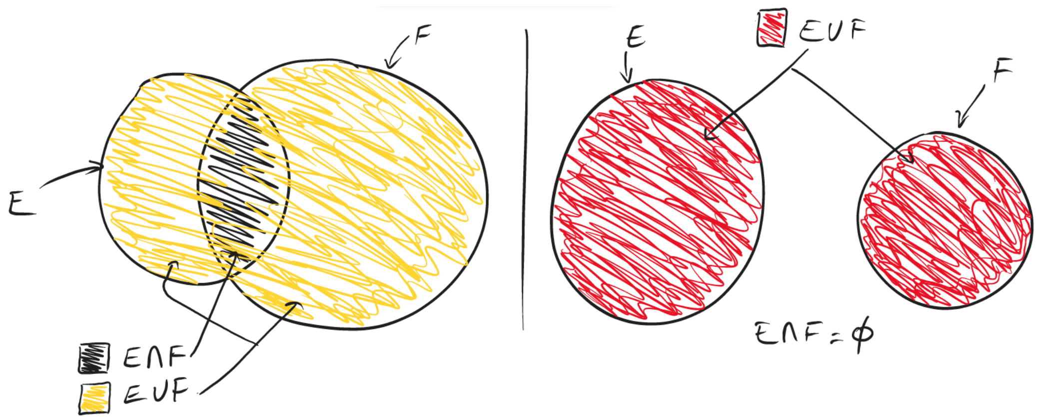
Amal ëmb E ak ëmb F, saamu ëmbeef x yëp yu bok ci E wala bok ci F, ñu di ko bindé $E \cup F$ la ñuy wowee mboolo E ak F.

Nañu mandaargal mbollo ëmb:

- Amal ñaari ëmb $E = \{1, 2, 3, 4, 5\}$ ak $F = \{4, 5, 6, 7, 8, 9\}$, $E \cup F = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

[1]Exemple ?

- Amal $E = \emptyset = F$, $E \cup F = \emptyset$
- Amal $E = \emptyset$, $F = \{0, 1\}$, $E \cup F = \{0, 1\}$



1.3.3 Full carteseng ay ëmb

Téeki

Amal ñaari ëmb E ak F, fullu E ak F, ñu koy bindé $E \times F$, mo di beep tank-tank[a] (x, y) bu deme ni[b] $x \in E$ ak $y \in F$

[a] couple
[b] tel que

- Amal $E = \{1, 2, 3\}$ ak $F = \{4, 5\}$, kon $E \times F = \big\{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\big\}$

Téeki

Amal been limukaay $n \in \mathbb{N}$ bu eup 2[a]. Amal itam n ëmb $E_1, E_2, ..., E_n$, fullu $E_1, E_2, ..., E_n$, ñu koy bindé $E_1 \times ... \times E_n$, mo di beep ëmbeef $x = (x_1, .., x_n)$ bu deme ni[b] $x_1 \in E_1$, $x_2 \in E_2$, ..., $x_n \in E_n$

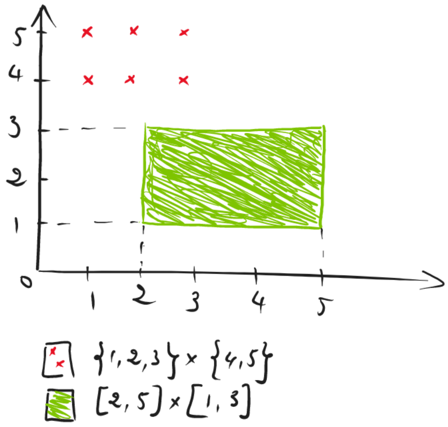
[a] $n \geq 2$
[b] tel que

Seetlu

Amal ëmb E ak ëmb F;

- Bu féké E ak F ay ëmb yu wuté la ñu, ñuy binde $E \neq F$, té itam $E \neq \emptyset$ ak $F \neq \emptyset$, kon fullu E ak F wuté na ak fullu F ak E, ñuy bind $E \times F \neq F \times E$
- Bu féké $E = F$, manaam E ak F been la ñu, mën na ñu binde $E \times F = E \times E = E^2$
- Amal ñaari tank-tank (u, v) ak (x, y) ci biir $E \times F$, da ñuy né $(u, v) = (x, y)$ bu féké rek $u = x$ ak $v = y$. Ngir leeral, $(1, 2) \in \mathbb{N}^2$ ak $(1, 3) \in \mathbb{N}^2$ wuté na ñu, ndaxté 2 wuté na ak 3.
- Bu féké ni limu ëmbeefu E ak limu ëmbeefu F da ñuy jex[a], kon ëmbeefu $E \times F$ maat nañu lu tolu ci limu ëmbeefu E nga ful ko ak limu ëmbeefu F. Ngir leeral wax ji, amal $E = \{1, 2, 3\}$ ak $F = \{4, 5\}$, kon E amna ñaat ëmbeef (kon limu ëmbeefu E, manam ñaat, day jex), F amna ñaari ëmbeef (kon limu ëmbeefu F, manam ñaar, day jex), limu ëmbeefu $E \times F$ mo di ñaat nga full ko ak ñaar, manaan $3 \times 2 = 6$. Leneen lu koy woné mo di: $E \times F = \big\{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\big\}$ amna bu bax juròom-been (6) ëmbeef.
- Amal n ëmb $E_1, E_2, ..., E_n$, bu féké ni $E = E_1 = E_2 = ... = E_n$, mên na ñu binde, $E_1 \times ... \times E_n = E^n$

[a] E et F sont des ensembles finis.



2 Doxalin

Téeki

Been [a]doxalin f mo di beep ñaati ëmb E, F ak $\mathcal{G}$. E moy ëmbef bu doxalin f di tambali, F moy ëmbef bu muy agsi. Da ñuy bindë $\mathcal{G} = \big\{(x, y) \in E \times F, y = f(x)\big\}$. Ëmb $\mathcal{G}$, ñu di ko wowé graaf, moy lëkkalé beep ëmbeef $x \in E$ ak been ëmbeef $y \in F$ rek. Doxalin f ñio ngi koy bindé:

$$f : \left\{ \begin{array}{l} E \to F \\ x \mapsto f(x) \end{array} \right.$$

wala ñuy binde

$$f : E \to F, \text{ ak } f(x) = \ldots$$

wala itam

$$f : x \mapsto \ldots \quad \text{ak } f : E \to F$$

Beep ëmbeef $x \in E$ warna am been natal[b] rek ci f, nataal gogu ñu di ko bindé $f(x)$

[a] Fonction ou application
[b] Image par une fonction

Nañu binde $\mathbb{R}_+ = \{x \in \mathbb{R}, x \geq 0\}$ manaam ëmbeefu limu $\mathbb{R}$ yu "positif" yi walla tolo ak tus, ak $\mathbb{R}_+^* = \{x \in \mathbb{R}, x > 0\}$, manaam ëmbeefu limu $\mathbb{R}$ yu "positif" yi té weesu/ëp tus.

- Doxalin $f : \left\{ \begin{array}{l} \mathbb{R} \to \mathbb{R} \\ x \mapsto x^2 + 1 \end{array} \right.$ ak doxalin $g : \left\{ \begin{array}{l} \mathbb{R} \to \mathbb{R}_+ \\ x \mapsto x^2 + 1 \end{array} \right.$ wuté nañu, ndaxté ëmbeef yu ñuy agsi wuté nañu: $\mathbb{R} \neq \mathbb{R}_+$
- $f : \left\{ \begin{array}{l} \mathbb{R} \to \mathbb{R}_+^* \\ x \mapsto x^2 \end{array} \right.$ nekkul doxalin, ndaxté $0^2 = 0 \notin \mathbb{R}_+^*$

3 Xalaat ci Xayma

3.1 Baat

Téeki

Been **baat**[a] ci Xayma mo di beep kaadu gi mëna nekka dëgg[b], walla nekka lu dul dëgg[c].
Dëgg (D), ak lu dëggul (L) la ñuy wowee xayma dëgg[d]. Bu féké ñaari baat $\mathcal{B}$ ak $\mathcal{C}$ ño bok xayma dëgg, kon da ñuy né ño niro, di ko bindé: $\mathcal{B} \sim \mathcal{C}$, seeni xayma dëg bu ñu wuté wé, da ñuy bindë $\mathcal{B} \nsim \mathcal{C}$

[a] assertion, prédicat
[b] vrai
[c] faux
[d] valeur de vérité

Amal $(0, 1, 2) \in \mathbb{N}^3$, kon baat $\mathcal{B} = "1 \geq 0"$ dëgg la, wanté, baat $\mathcal{C} = "2 + 1 \leq 2"$ du dëgg, kon bok $\mathcal{B} \nsim \mathcal{C}$. Baat $\mathcal{B} = "2 = 0"$ ak baat $\mathcal{C} = "1 > 2"$ dëgguñu, kon $\mathcal{B} \sim \mathcal{C}$.

3.2 Muk/Deet

Téeki

Muk been baat[a]$\mathcal{B}$, ñu di ko bindé *muk* $\mathcal{B}$ (wala $\neg\mathcal{B}$) baatu dëgg la, bu féké ni $\mathcal{B}$ du dëgg. Té itam, baat bu dëggul la, bu féké ni $\mathcal{B}$ dëgg la.

[a] La négation d'une assertion

Amal $x \in \mathbb{R}$, $\mathcal{B}(x) = "x \leq 0"$, kon $\neg\mathcal{B}(x) \sim "x > 0"$.

Tèg[a]

[a] Proposition

Amal been baat $\mathcal{B}$, boba/kon $\neg(\neg\mathcal{B}) \sim \mathcal{B}$, manaam muk (muk $\mathcal{B}$) ak $\mathcal{B}$ ño book xayma dëgg.

Woné: Amal been baat $\mathcal{B}$

$\mathcal{B}$	$\neg\mathcal{B}$	$\neg(\neg\mathcal{B})$
D	L	D
L	D	L

3.3 Takhalé ak tékhalé ay baat

Téeki[a]

[a] Définition

Takhalo[a] ñaari baat $\mathcal{B}$ ak $\mathcal{C}$, ñu di ko bindé $\mathcal{B} \wedge \mathcal{C}$, wala $\mathcal{B}$ *ak* $\mathcal{C}$, baatu dëgg la bu féké ni rek $\mathcal{B}$ dëgg la, té $\mathcal{C}$ dëgg la itam.

Tékhalo[b] ñaari baat $\mathcal{B}$ ak $\mathcal{C}$, ñu di ko bindé $\mathcal{B} \vee \mathcal{C}$, wala $\mathcal{B}$ *wala* $\mathcal{C}$, dëgg la bu féké ni rek $\mathcal{B}$ dëgg la, wala $\mathcal{C}$ dëgg la.

$\mathcal{B}$	$\mathcal{C}$	$\mathcal{B} \wedge \mathcal{C}$	$\mathcal{B} \vee \mathcal{C}$
D	D	D	D
D	L	L	D
L	D	L	D
L	L	L	L

[a]Conjonction
[b]Disjonction

Tèg[a]

[a]Proposition

Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$, boba/kon:

$$\neg(\mathcal{B} \wedge \mathcal{C}) \sim (\neg\mathcal{B}) \vee (\neg\mathcal{C})$$

$$\neg(\mathcal{B} \vee \mathcal{C}) \sim (\neg\mathcal{B}) \wedge (\neg\mathcal{C})$$

Woné:
Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$, nañu bindë këralegu/natalu[2] xayma dëgg baat yi.

$\mathcal{B}$	$\mathcal{C}$	$\neg\mathcal{B} \vee \neg\mathcal{C}$	$\neg(\mathcal{B} \wedge \mathcal{C})$	$\neg(\mathcal{B} \vee \mathcal{C})$	$(\neg\mathcal{B}) \wedge (\neg\mathcal{C})$
D	D	L	L	L	L
D	L	D	D	L	L
L	D	D	D	L	L
L	L	D	D	D	D

Mën nañu gis ci ñaatel jeñ[3] gi ak ñeentel gi, né $\neg(\mathcal{B} \wedge \mathcal{C}) \sim (\neg\mathcal{B}) \vee (\neg\mathcal{C})$. Juròomeel jeñ gi ak juròomeel-beeneel gi, woné nañu né $\neg(\mathcal{B} \vee \mathcal{C}) \sim (\neg\mathcal{B}) \wedge (\neg\mathcal{C})$.

Tègtal[a]

[a]Indication ?

Ngir woné né ñaari baat ño bok xayma dëgg, mën naño jëfandikoo këralegu xayma dëgg yi.

Jéemantu
Amal ñaat baat $\mathcal{A}, \mathcal{B}, \mathcal{C}$, wonéel ni:

$$\mathcal{B} \wedge \mathcal{B} \sim \mathcal{B}$$
$$\mathcal{B} \vee \mathcal{B} \sim \mathcal{B}$$
$$(\mathcal{B} \wedge \mathcal{C}) \wedge \mathcal{A} \sim \mathcal{B} \wedge (\mathcal{C} \wedge \mathcal{A})$$
$$(\mathcal{B} \vee \mathcal{C}) \vee \mathcal{A} \sim \mathcal{B} \vee (\mathcal{C} \vee \mathcal{A})$$
$$(\mathcal{B} \wedge \mathcal{C}) \vee \mathcal{A} \sim (\mathcal{B} \vee \mathcal{A}) \wedge (\mathcal{C} \vee \mathcal{A})$$
$$(\mathcal{B} \vee \mathcal{C}) \wedge \mathcal{A} \sim (\mathcal{B} \wedge \mathcal{A}) \vee (\mathcal{C} \wedge \mathcal{A})$$

3.4 Baat bi yobualé/andi been baat

Téeki

Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$. Da ñuy né $\mathcal{B}$ *da yobualé* $\mathcal{C}$[a] dëgg la, bu féké ni rek $\mathcal{C}$ mënul bagna nek dëgg bu $\mathcal{B}$ néké dëgg.
$\mathcal{B}$ *da yobualé* $\mathcal{C}$ ñu di ko bindé $\mathcal{B} \implies \mathcal{C}$.

Xayma dëgg $\mathcal{B} \implies \mathcal{C}$ mu ngi ni:

[2]tableau ?
[3]Colonne = jiñ = jeñ ?

$\mathcal{B}$	$\mathcal{C}$	$\mathcal{B} \implies \mathcal{C}$
D	D	D
D	L	L
L	D	D
L	L	D

[a] $\mathcal{B}$ implique $\mathcal{C}$

Ak beep $x \in \mathbb{R}$, bo bindé $\mathcal{B}(x) = "x >= 2"$, $\mathcal{C}(x) = "x^2 >= 4"$, kon $\mathcal{B}(x) \implies \mathcal{C}(x)$ dëgg la.

Téeki

Bu $\mathcal{B} \implies \mathcal{C}$ néké dëgg, kon da ñuy né

- $\mathcal{B}$ *baat bu doy* $\mathcal{C}$ la, wala $\mathcal{B}$ *doy na* $\mathcal{C}$.[a]
- $\mathcal{B}$ *da soxla* $\mathcal{C}$.[b]

$\mathcal{C} \implies \mathcal{B}$ mo di *wëlbati wu* [c] baat $\mathcal{B} \implies \mathcal{C}$.

[a] $\mathcal{B}$ est une condition suffisante pour $\mathcal{C}$
[b] $\mathcal{C}$ est une condition nécessaire pour $\mathcal{B}$
[c] implication réciproque

Tèg[a]

[a] Proposition

Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$, boba:

$$(\mathcal{B} \implies \mathcal{C}) \sim (\neg\mathcal{B}) \vee \mathcal{C}$$
$$(\mathcal{B} \implies \mathcal{C}) \sim \big((\neg\mathcal{C}) \implies (\neg\mathcal{B})\big)$$

$(\neg\mathcal{C}) \implies (\neg\mathcal{B})$ la ñuy wowee *contaraposee*[a] wu $\mathcal{B} \implies \mathcal{C}$

[a] Contraposée

Woné:

$\mathcal{B}$	$\mathcal{C}$	$\mathcal{B} \implies \mathcal{C}$	$(\neg\mathcal{B}) \vee \mathcal{C}$
D	D	D	D
D	L	L	L
L	D	D	D
L	L	D	D

Mën na ñu gis né $(\mathcal{B} \implies \mathcal{C}) \sim (\neg\mathcal{B}) \vee \mathcal{C}$ ci ñaateel jeñ ak ñeenteel gi, kon bok bu ñu wécanté $\mathcal{B}$ ak $\neg\mathcal{C}$, té wécanté itam $\mathcal{C}$ ak $\neg\mathcal{B}$, ñu am $\big((\neg\mathcal{C}) \implies (\neg\mathcal{B})\big) \sim \big((\neg(\neg\mathcal{C})) \vee (\neg\mathcal{B})\big) \sim \big(\mathcal{C} \vee (\neg\mathcal{B})\big) \sim (\neg\mathcal{B}) \vee \mathcal{C} \sim (\mathcal{B} \implies \mathcal{C})$, fi la ñuy jexalé woné gi.

Mënon nañu woné itam $(\mathcal{B} \implies \mathcal{C}) \sim \big((\neg\mathcal{C}) \implies (\neg\mathcal{B})\big)$ ak natal xayma dëgg yi.

3.5 Baat yu yèm

Téeki

Amal ñaari baat $\mathcal{B}$, $\mathcal{C}$.
$\mathcal{B}$ *mo yèm ak* $\mathcal{C}$[a], ñuy bindë $\mathcal{B} \iff \mathcal{C}$, mo di baat $(\mathcal{B} \implies \mathcal{C}) \wedge (\mathcal{C} \implies \mathcal{B})$, manaam

$$\mathcal{B} \iff \mathcal{C} \sim \big((\mathcal{B} \implies \mathcal{C}) \wedge (\mathcal{C} \implies \mathcal{B})\big)$$

Natal xayma dëgg bu yèmalé ñaari baat mo ngi ni:

$\mathcal{B}$	$\mathcal{C}$	$\mathcal{B} \implies \mathcal{C}$	$\mathcal{C} \implies \mathcal{B}$	$\mathcal{B} \iff \mathcal{C}$
D	D	D	D	D
D	L	L	D	L
L	D	D	L	L
L	L	D	D	D

[a] L'équivalence de deux assertions

Seetlu

Amal ñaari baat $\mathcal{B}$, $\mathcal{C}$.

- Bu $\mathcal{B} \iff \mathcal{C}$ néké dëgg, kon $\mathcal{B} \sim \mathcal{C}$.
- Bu féké $\mathcal{B} \iff \mathcal{C}$ dëggul, kon $\mathcal{B} \not\sim \mathcal{C}$

Seetlu yi muj, woné nañu né **yémalé ay baat ak nirolé lèn been lañu**. Manam wax $\mathcal{B} \iff \mathcal{C}$ been la ak wax $\mathcal{B} \sim \mathcal{C}$, manaam:

$$(\mathcal{B} \iff \mathcal{C}) \sim (\mathcal{B} \sim \mathcal{C})$$
$$(\mathcal{B} \iff \mathcal{C}) \iff (\mathcal{B} \sim \mathcal{C})$$

Tèg[a]

[a] Proposition

Amal baat $\mathcal{B}$ ak baat $\mathcal{C}$, boba:

$$(\mathcal{B} \iff \mathcal{C}) \sim ((\neg\mathcal{C}) \iff (\neg\mathcal{B}))$$

Jéemantu: Woneel tèg bi muj

Téeki

Bu $\mathcal{B} \iff \mathcal{C}$ néké dëgg, kon da ñuy né

- *$\mathcal{B}$ baat bu soxla té doy $\mathcal{C}$* la, [a]

[a] $\mathcal{B}$ est une condition nécessaire et suffisante pour $\mathcal{C}$

3.6 Natakat baat

Téeki

Amal been ëmb E. Ak beep $x \in E$, amal been baat $\mathcal{B}(x)$ bu nék surgau x[a].
Da ñuy wax *ak beep $x \in E$, $\mathcal{B}(x)$ dëgg la*, té di bindë

$$\forall x \in E, \mathcal{B}(x)$$

bu féké ni rek $\mathcal{B}(x)$ dëgg la ak beep ëmbeef x bu nék ci biir E.

Da ñuy wax *ak been $x \in E$, $\mathcal{B}(x)$ dëgg la*, té di bindë

$$\exists x \in E, \mathcal{B}(x)$$

bu féké ni rek $\mathcal{B}(x)$ dëgg la ak lu mu tuti tuti been ëmbeef x bu nék ci biir E.

Da ñuy wax *ak been $x \in E$ rek, $\mathcal{B}(x)$ dëgg la*, té di bindë

$$\exists! x \in E, \mathcal{B}(x)$$

bu féké ni rek $\mathcal{B}(x)$ dëgg la ak been ëmbeef x dong bu nék ci biir E.

[a]Une assertion qui dépend de x

Mën nañu né $\forall x \in \mathbb{R}, \exp(x) > 0$, ak $\forall x \in \mathbb{R}, \exists! n \in \mathbb{Z}, n \leq x < n+1$

Amal been doxalin f bi jogé ci $\mathbb{R}$ té agsi ci $\mathbb{R}$, kon bok:
Da ñuy wax f moy doxalinu dara/tus [4] bu féké ni rek

$$\forall x \in \mathbb{R}, f(x) = 0$$

Da ñuy wax f di na agsi ci tus[5] bu féké ni rek

$$\exists x \in \mathbb{R}, f(x) = 0$$

Da ñuy wax f di na agsi been yoon ci tus bu féké ni rek [6]

$$\exists! x \in \mathbb{R}, f(x) = 0$$

Da ñuy wax f ci $\mathbb{R}_+$ rek la mëna tolo ak tus[7] bu féké ni rek

$$\forall x \in \mathbb{R}, f(x) = 0 \implies x \in \mathbb{R}_+$$

wala itam

$$\forall x \in \mathbb{R}, x \notin \mathbb{R}_+ \implies f(x) \neq 0$$

Tèg[a]

[a]Proposition

$$\neg(\forall x \in E, \mathcal{B}(x)) \sim \exists x \in E, \neg\big(\mathcal{B}(x)\big)$$
$$\neg(\exists x \in E, \mathcal{B}(x)) \sim \forall x \in E, \neg\big(\mathcal{B}(x)\big)$$

Jéemantu: Woneel tèg yi muj

Waxanté[a]

[a]Convention

Beep baat buy tambalé ak $\exists x \in \emptyset$ dëggul, kon beep baat buy tambalé ak $\forall x \in \emptyset$ dëgg la.

Jéemantu bu am solo ci xam-xam ci koompuutar[8] Amal been doxalin f bi jogé ci been full carteseng ëmb xayma dëgg yi $\{0,1\}^n$, té di agsi ci ëmbu xayma dëgg yi, 0 di téeki lu dëggul (L), 1 di téeki dëgg (D), ñuy bindë

$$f : \{0,1\}^n \to \{0,1\}$$

mën nañu woné né f mën nañu ko bindé ak sëfu xayma dëgg *takhalo* (manaam $\wedge$) ak sëfu xayma dëgg *deet* (manaam $\neg$). Manam bo jëlé $b = (b_1, b_2, ...b_n) \in \{0,1\}^n$, kon $f(b)$ mën nañu kon bindé ak $b_1, b_2, ...b_n$ ak $\wedge$ ak $\neg$ dong.
Tégtal: ngir woné tèg bi muj

[4]La fonction f est la fonction nulle.
[5]La fonction f s'annule
[6]La fonction f s'annule une seule fois
[7]La fonction f ne s'annule que sur $\mathbb{R}_+$
[8]Exercice important en informatique

- Mën nañ woné ni amna been $0 \leq k \leq 2^n$ ak f_1, ..., f_k ay doxalin yuy jogé ci $\{0,1\}^n$ té agsi ci $\{0,1\}$, yu mel ni
$$f(b) = f^{(1)}(b) \vee f^{(2)}(b) \vee ... \vee f^{(k)}(b)$$
té $f^{(l)}(b) = \begin{cases} 1 \text{ bu féké ni } b = b^{(l)} \\ 0 \text{ bu féké ni } b \neq b^{(l)} \end{cases}$, ak $b^{(l)}$ been néekin b ci këralegu xayma dëggu f, té $f(b^{(l)}) = 1$. Ngir léeral, bu féké $b = (1,0,1,0,0,0,1)$ ci liñ l këralegu xayma dëgg doxalin f, té $f(b^{(l)}) = 1$, mën nañu bindë $b^{(l)} = (1,0,1,0,0,0,1)$ té bindë $f_l(b) = b_1 \wedge (\neg b_2) \wedge b_3 \wedge (\neg b_4) \wedge (\neg b_5) \wedge (\neg b_6) \wedge b_7$. Bu féké $f = 0$ (manaam $k = 0$), kon mën nañu bindë $f(b) = b_1 \wedge (\neg b_1)$

- Fatéliku itam né $\vee$ mën nañu ko bindë ak $\wedge$ ak $\neg$

Amal $f : \begin{array}{l} \{0,1\}^3 \longrightarrow \{0,1\} \\ (b_1, b_2, b_3) \longmapsto b_1 \wedge (b_1 \vee (\neg b_3)) \end{array}$

Këralegu xayma dëgg f mo ci topp:

b_1	b_2	b_3	$f(b)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	$\boxed{1}$
1	0	1	0
1	1	0	$\boxed{1}$
1	1	1	$\boxed{1}$

$b^{(1)} = (1,0,0)$; $f^{(1)}(b) = b_1 \wedge (\neg b_2) \wedge (\neg b_3)$

$b^{(2)} = (1,1,0)$; $f^{(2)}(b) = b_1 \wedge b_2 \wedge (\neg b_3)$

$b^{(3)} = (1,1,1)$; $f^{(3)}(b) = b_1 \wedge b_2 \wedge b_3$

kon $f(b) = f^{(1)}(b) \vee f^{(2)}(b) \vee f^{(3)}(b)$

$$\boxed{f(b) = [b_1 \wedge (\neg b_2) \wedge (\neg b_3)] \vee [b_1 \wedge b_2 \wedge (\neg b_3)] \vee [b_1 \wedge b_2 \wedge b_3]}$$

bindë nañu f ak $b_1, b_2, b_3, \neg, \wedge, \vee$ dong.

Xam nañu né $a \vee b = \neg((\neg a) \wedge (\neg b))$

kon mën nañu bindë f ak $b_1, b_2, b_3, \neg, \wedge$ dong.

3.7 Kadum sago

Ngir wax né been baat, baatu dëgg la ci Xayma, war na ñu ko firndeel/woral ak been woné, manaam ci Xayma, woné rek moy dëggal been baat. Baatu dëgg bu ték, **tèg**[9] la tud. Ci tèg yi, amna yu ci gëna am solo, ñu len di wowée **téorèm**[10]. Wanté amna ay baat yu ñu dul woné té nangu né ay baati dëgg la ñu, ñu léen di wowée **ñalém**, wala **baatu dëga yu wor**[11]. Baat yoyu lé, mën na ñu lèna jappé ay sart[12] yuy lal xalaat ci Xayma.

Nañu lim woné yu ñuy tama jëfandiko ngir woral ay baat.

3.7.1 Woné ab contaraposee

Téeki

Mën na ñu woné been baat $\mathcal{B} \implies \mathcal{C}$ dëgg la, bu ñu woné $(\neg\mathcal{C}) \implies (\neg\mathcal{B})$ dëgg la. Woné gi la ñuy wowée **woné contaraposee**[a]

[a] Démonstration par contraposition

[9] Proposition
[10] Théorème
[11] Axiomes
[12] Règles

3.7.2 Tofal

Téeki

Mën na ñu woné been baat $\mathcal{C}$ dëgg la, bu ñu tambali wé ci béneen baat $\mathcal{B}$ bu nek dëgg té woné ni $\mathcal{B} \implies \mathcal{C}$ dëgg la. Woné gi la ñuy wowée **tofal**[a].

[a] Tirer une conséquence.

Nañu woné

$$\forall x \in \mathbb{R}, x^2 + 1 > 0$$

ak tofal.

Amal $x \in \mathbb{R}$. Xam na ñu $x^2 \geq 0$ ak $1 > 0$ wanté xamna ñu itam sa su jëlé a ak b ay ëmbeefu $\mathbb{R}$,

$$(a \geq 0) \text{ ak } b > 0 \implies a + b > 0$$

kon itam $x^2 + 1 > 0$ (bu ñu jëlé $a = x^2$ ak $b = 1$).

3.7.3 Tofal ak tékhalé ay baat / ak nékin yëp yu wuté

Téeki

Mën nañu woné $\mathcal{C}$ dëgg la, bu ñu tambali wé ak béneen baat $\mathcal{B}$, té woné $\mathcal{B} \implies \mathcal{C}$ ak $(\neg\mathcal{B}) \implies \mathcal{C}$ dëgg la. Woné gi mo tud **tofal ak tékhalé ay baat**[a]

[a] Disjonction des cas

Nañu woné

$$\forall n \in \mathbb{N}, \frac{n(n+1)}{2} \in \mathbb{N}$$

di jëfandiko tofal ak tékhalé ay baat.

Amal $n \in \mathbb{N}$, xam na ñu amna been $k \in \mathbb{N}$ bu mel ni $n = 2k$, wala $n = 2k + 1$.

Bu féké $n = 2k$, kon

$$\frac{n(n+1)}{2} = \frac{2k(2k+1)}{2} = k(2k+1) \in \mathbb{N}$$

Bu féké itam $n = 2k + 1$, kon

$$\frac{n(n+1)}{2} = \frac{(2k+1)(2k+2)}{2} = (2k+1)(k+1) \in \mathbb{N}$$

fi la woné gi jexé.

3.7.4 Wédi

Téeki

Mën nañu woné $\mathcal{B}$ dëgg la, bu ñu woné ni amna béneen baat $\mathcal{C}$ bu dëggul, té woné itam $(\neg\mathcal{B}) \implies \mathcal{C}$ dëgg la.

Nañu woné ni amul been $N \in \mathbb{N}$, bu gën ëp[13] beep $n \in \mathbb{N}$ di ko bindé itam

$$\exists N \in \mathbb{N}, \forall n \in \mathbb{N}, N > n$$

dëggul. Nañu ko wédi, manaan né amna been $N \in \mathbb{N}$, bu gën ëp beep $n \in \mathbb{N}$, kon N gogu mo gën ëp $N + 1$, ndaxté $N + 1 \in \mathbb{N}$, kon dé

$$1 = (N+1) - N < 0$$

Li mënul nék, ndaxté xam nañu $1 > 0$ ci biir $\mathbb{N}$

3.7.5 Topalanté ay baat

[13] Strictement supérieur

Téeki

Amal $n_0 \in \mathbb{N}$ ak ay baat $\mathcal{B}(n)$, $n \in \mathbb{N}, n \geq n_0$.
Bu féké $\mathcal{B}(n_0)$ dëgg la, té itam

$$\forall n \geq n_0,\ \mathcal{B}(n) \implies \mathcal{B}(n+1)$$

kon

$$\forall n \geq n_0,\ \mathcal{B}(n)$$

Nañu woné

$$\forall n \in \mathbb{N}^*, 1+2+3+...+n = \frac{n(n+1)}{2}$$

ak topalanté ay baat.
Amal $n=1$, kon $1+2+3+..+n=1$, té itam $\frac{n(n+1)}{2}=1$, kon dëgg la ak $n=1$.
Amal been $n \in \mathbb{N}^*$. Bu féké ni $1+2+3+...+n = \frac{n(n+1)}{2}$, kon

$$1+2+3+...+(n+1) = (1+2+3+...+n)+(n+1) = \frac{n(n+1)}{2}+n+1 = \frac{(n+1)\big((n+1)+1\big)}{2}$$

fi la woné gi jexé.

Seetlu

Amal $\mathcal{B}$ ak $\mathcal{C}$, natal xayma dëgg yi ñoy dëggal woné yi jal.

natal gi njëk:

$\mathcal{B}$	$\mathcal{C}$	$\mathcal{B} \implies \mathcal{C}$
D	D	D
D	L	L
L	D	D
L	L	D

ñaareel natal gi:

$\mathcal{B}$	$\mathcal{C}$	$\neg\mathcal{B} \implies \mathcal{C}$
D	D	D
D	L	D
L	D	D
L	L	L

- **Tofal**: liñ[a] bu njëk ci natal bu njëk bi moy woral woné ak *tofal*.
- **Tofal ak tékhalé ay baat**: liñ bu njëk ak ñaateel liñu ñaari natal yi, ñoy woral woné ak *tofal ak tékhalé ay baat*.
- **Wédi**: ñaareel liñ ñu ñaareel lu natal gi moy woral woné ak *wédi*
- **Topalanté ay baat**: amal $n_0 \in \mathbb{N}$ ak ay baat $\mathcal{B}(n)$, $n \in \mathbb{N}, n \geq n_0$.
  Bu féké $\mathcal{B}(n_0)$ dëgg la, té itam

  $$\forall n \geq n_0,\ \mathcal{B}(n) \implies \mathcal{B}(n+1)$$

  kon, ndaxté $\mathcal{B}(n_0)$ dëgg la, té $\mathcal{B}(n_0) \implies \mathcal{B}(n_0+1)$ dëgg la, woné ak *tofal* dëggal na $\mathcal{B}(n_0+1)$, *tofal* moy woral woné ak *topalanté ay baat*

[a] Ligne

4 Xayma ci ëmb yi

4.1 Wàllu

Téeki

Amal ëmb E ak F, E **wàllu** F **la**, ñuy bindë $E \subset F$ bu féké ni, ak beep $x \in E$, $x \in F$ itam:

$$E \subset F \iff \forall x \in E, x \in F$$

kon E **nekul wàllu** F, ñuy bindë $E \not\subset F$ bu féké ni amna been $x \in E$ té $x \notin F$

$$E \not\subset F \iff \exists x \in E, x \notin F$$

Seetlu

Amal been ëmb E ak F, kon

- $\emptyset \subset F$ ndaxté $\forall x \in \emptyset, x \in F$ (ndaxté beep baat bu di tambali ak $\forall x \in \emptyset$ dëgg la)
- $E \subset E$, ndaxté $\forall x \in E, x \in E$ lu leer la

Tèg[a]

[a]Proposition

Amal ëmb E ak F, kon

$$E = F \iff (E \subset F \text{ ak } F \subset E)$$

Woné:

Amal ëmb E ak F. Ngir woné $E = F \iff E \subset F$ ak $F \subset E$, mën nañu woné $\mathcal{B} = "(E = F \implies E \subset F \text{ ak } F \subset E)"$ dëgg la, té woné itam $\mathcal{C} = "(E \subset F \text{ ak } F \subset E) \implies E = F"$ dëgg la.

Nañu woné $\mathcal{B}$. Xam nañu $E \subset E$ (seetlu bi muj mo ko wax). Bu féké $E = F$, kon $E \subset F$ (bu ñu wecé ñaareel E bi ak F ci diganté gi $E \subset E$). Nonu la ñuy woné itam $F \subset E$ bu féké $E = F$, kon $\mathcal{B}$ dëgg la.

Nañu woné $\mathcal{C}$ dëgg la ak contaraposee wam manaam $E \neq F \implies \neg(E \subset F \text{ ak } F \subset E)$. Bu féké $E \neq F$ (manaam E wuté na ak F, kon $\exists x \in E, x \notin F$ mba/wala $\exists y \in F, y \notin E$, manaam *deet*$(\forall x \in E, x \in F \text{ ak } \forall y \in F, x \in E)$, manaam $\neg(E \subset F \text{ ak } F \subset E)$. Fi la woné gi jexé.

Tèg[a]

[a]Proposition

Amal ëmb E, F, G kon

$$E \subset F \text{ ak } F \subset G \implies E \subset G$$

Jéemantu: Woneel tèg bi muj.

Téeki

Amal ëmb E, $\mathcal{W}(E)$ **mo di mbidu ëmb bi bolé rek wàllu** E **yëp**, manaam amal been ëmb A

$$A \in \mathcal{W}(E) \iff A \subset E$$

Amal ëmb E, kon $\emptyset \in \mathcal{W}(E)$, $E \in \mathcal{W}(E)$, $\emptyset \subset \mathcal{W}(E)$, wanté nañu woytu[14] $E \subset \mathcal{W}(E)$ mën na baña nek dëgg.

4.2 Selebe(yoon) ak Mbolo ay ëmb

Téeki

Amal ëmb A, B ay wàllu been ëmb E, $A \cap B$ **mo di selebe(yoon)** A **ak** B di saamu ëmbeef $x \in E$ yëp yu bok ci A te bok itam ci B, manaan

$$A \cap B = \{x \in E / x \in A \text{ ak } x \in B\}$$

Téeki

Amal ëmb A, B ay wàllu been ëmb E, $A \cup B$ **mo di mbolo** A **ak** B di saamu ëmbeef $x \in E$ yëp yu bok ci A walla bok ci B, manaan

$$A \cup B = \{x \in E / x \in A \text{ walla } x \in B\}$$

Seetlu

Amal ëmb A ak B ay wàllu ëmb E, kon

- $A \cap B \subset A$ ak $A \subset A \cup B$
- $A \cap B \subset B$, ak $B \subset A \cup B$
- $A \cap A = A$, $A \cup A = A$
- $A \cup E = E$, $A \cap E = A$
- $A \cup \emptyset = A$, $A \cap \emptyset = \emptyset$

.

Tègtal[a]

[a]Indication ?

Amal ëmb A ak B, ay wàllu been ëmb E, ngir woné $A = B$ been lañu mën nañu woné ni: ak beep ëmbeef $x \in E$, $x \in A \iff x \in B$. Manaam woné $A \subset B$ ak $B \subset A$

Tèg[a]

[a]Proposition

Amal ëmb E, F,

$$\begin{aligned} A \cup B &= B \cup A \\ A \cap B &= B \cap A \\ (A \cup B) \cup C &= A \cup (B \cup C) \\ (A \cap B) \cap C &= A \cap (B \cap C) \\ (A \cap B) \cup C &= (A \cup C) \cap (B \cup C) \\ (A \cup B) \cap C &= (A \cap C) \cup (B \cap C) \end{aligned}$$

Jéemantu: Woneel tèg bi muj.

[14]faire attention

Seetlu

Amal ëmb A, B ak C, kon

- $(A \cup B) \cup C = A \cup (B \cup C)$ la ñuy bindë $A \cup B \cup C$
- $(A \cap B) \cap C = A \cap (B \cap C)$ la ñuy bindë $A \cap B \cap C$
- Wanté ken du bindë $A \cap B \cup C$ walla $A \cup B \cap C$ ndaxté léru ñu

Tèg[a]

[a] Proposition

Amal ëmb A, B ak C, kon

$$A \subset C \text{ ak } B \subset C \implies A \cup B \subset C$$
$$C \subset A \text{ ak } C \subset B \implies C \subset A \cap B$$

Woné:
Amal ëmb A, B ak C. Nañu woné $A \subset C$ ak $B \subset C \implies A \cup B \subset C$
Bu féké $A \subset C$ ak $B \subset C$. Amal $x \in A \cup B$, manaam $x \in A$ walla $x \in B$:

- Bu féké $x \in A$, kon, ndaxté $A \subset C$, $x \in C$ tamit
- Bu féké $x \in B$, kon, ndaxté $B \subset C$, $x \in C$ tamit

Woné nañu $A \subset C$ ak $B \subset C \implies A \cup B \subset C$ ci beep nékin.

Ak xeetu woné ji muj, mën nañu woné $C \subset A$ ak $C \subset B \implies C \subset A \cap B$

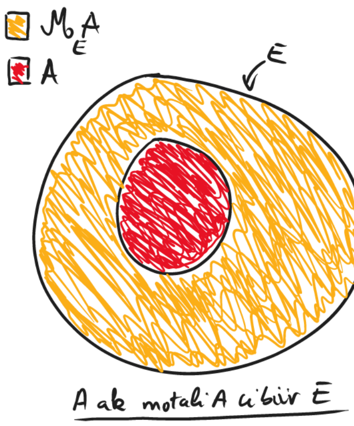
4.3 Motali been ëmb ci biir been ëmb

Téeki

Amal ëmb A wàllu been ëmb E, **ëmb bi di motali** A **ci biir** E, ñu di ko bindë

$$\mathcal{M}_E A = \{x \in E, \text{ té } x \notin A\}$$

mo di saamu ëmb yëp yu nék ci E té nekku ñu ci A



Seetlu

Amal ëmb A ab wàllu ëmb E. Bu féké ëmb bi di motali A ci biir E lu lér la, manaam munu ñu ko jaxasé ak leenen, kon mën nañu bindë $\bar{A}$ ngir wax ëmb bi motali ci biir E.

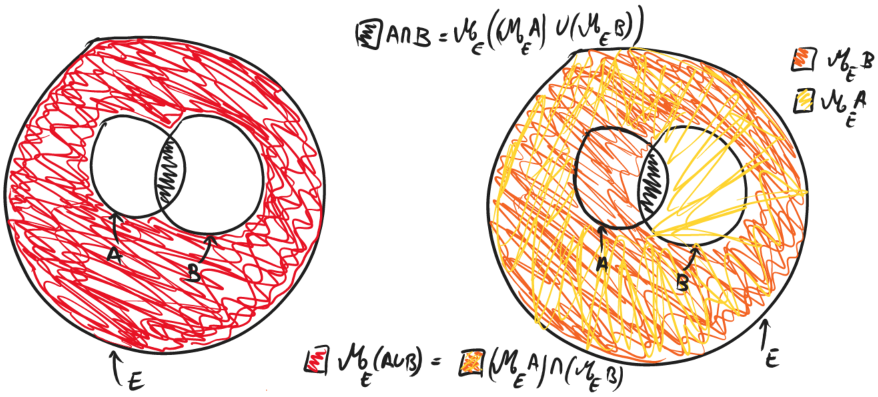
- $\mathcal{M}_E A \cap A = \emptyset$, $\mathcal{M}_E(\mathcal{M}_E A) = A$
- $\mathcal{M}_E E = \emptyset$, $\mathcal{M}_E \emptyset = E$

Tèg[a]

[a] Proposition

Amal ëmb A, B ay wàllu ëmb E, kon

$$\begin{aligned} \mathcal{M}_E(A \cup B) &= (\mathcal{M}_E A) \cap (\mathcal{M}_E B) \\ \mathcal{M}_E(A \cap B) &= (\mathcal{M}_E A) \cup (\mathcal{M}_E B) \\ A \subset B &\iff (\mathcal{M}_E B) \subset (\mathcal{M}_E A) \end{aligned}$$



4.4 Wañi ëmb ci ëmb

Téeki

Amal ëmb A ak B ay wàllu been ëmb E, **ëmb bi di motali** A **ci biir** E, ñu di ko bindë

$$A - B = \{x \in E, \text{ té } x \in A \text{ ak } x \notin B\}$$

mo di saamu ëmbeef yëp yu nék ci A té nekku ñu ci B. Yèna say ñu bindé ko $A \backslash B$

Seetlu

Amal ëmb A ak B ay wàllu been ëmb E, boba

- $A \backslash B = \mathcal{M}_A(A \cap B) = A \cap \mathcal{M}_E(B)$

Rëd fi been natal bu di mandargal kaadu yi muj.