From Last Time

- Permittivity (ε) is the diagnostic physical property but electrical conductivity (σ) plays an important role.

- Radiowaves propagate at different speeds in different materials:

$$v = \frac{c}{\sqrt{\epsilon_r}}$$

- Radiowaves attenuate (lose amplitude) while they propagate:

<u>Skin depth:</u>

$$\delta \approx \begin{cases} 503\sqrt{\dfrac{1}{\sigma f}} & \text{for } \omega\varepsilon \ll \sigma \\ 0.0053\dfrac{\sqrt{\varepsilon_r}}{\sigma} & \text{for } \sigma \ll \omega\varepsilon \end{cases}$$

From Last Time

- Radiowaves reflect at boundaries where the velocity/dielectric permittivity changes:

$$R = \frac{\textbf{Reflected Amplitude}}{\textbf{Incident Amplitude}} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

- Conductors are large reflectors of radiowaves

- Snell's law applies to GPR:

$$\frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2}$$

$$\sqrt{\varepsilon_1}\,\sin\theta_1 = \sqrt{\varepsilon_2}\,\sin\theta_2$$

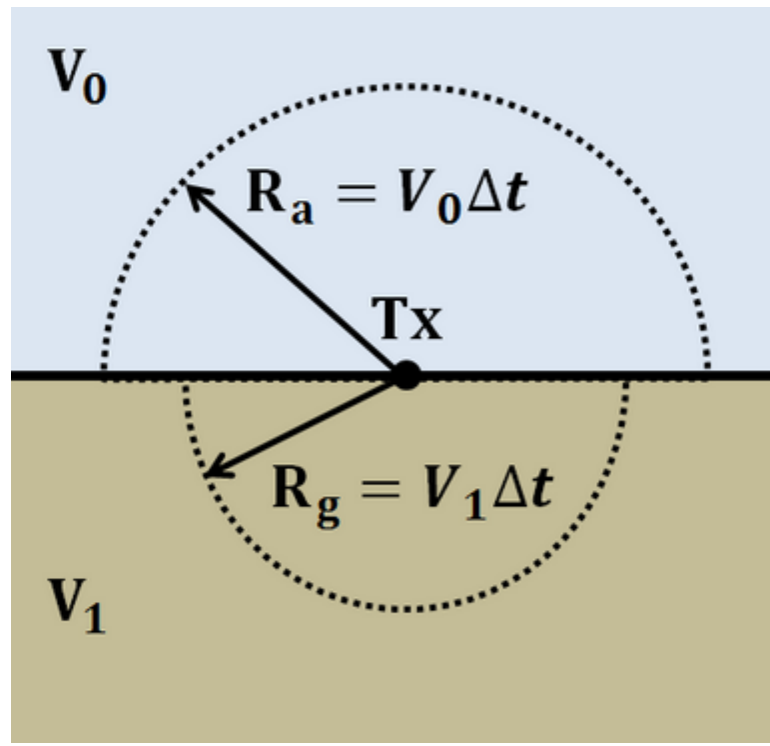
Outline

- Common survey configurations and some applications
- The source wavelet signal
- Resolution
- GPR App
- Probing distance
- Sources of Noise

Geometrical Spreading

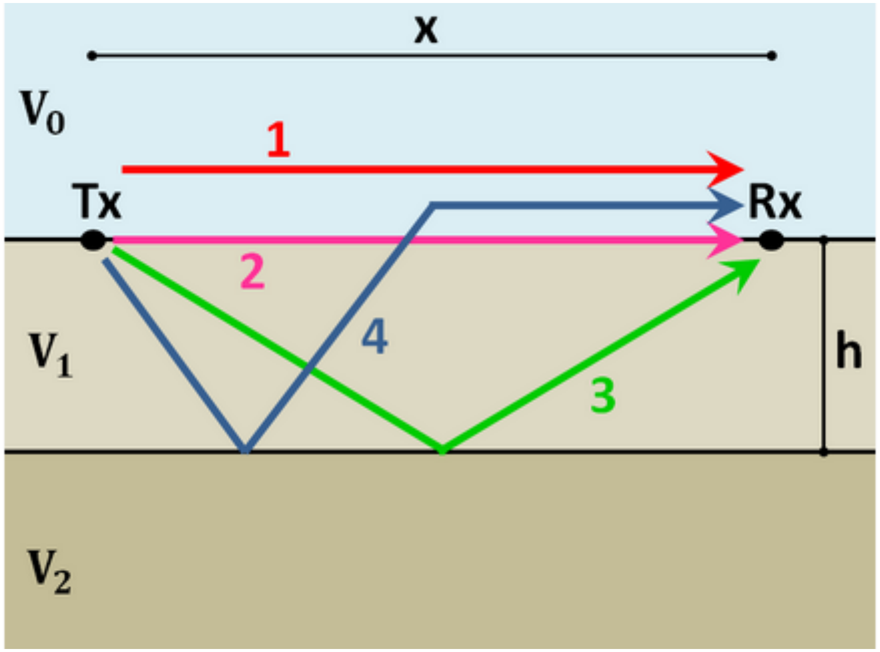
- As the wave front travels, it spreads geometrically
- The rate of geometrical spreading depends on the velocity
- Spreading causes the radiowave to lose amplitude

$$\frac{|\mathbf{A}|}{|\mathbf{A_0}|} \propto \frac{1}{R}$$

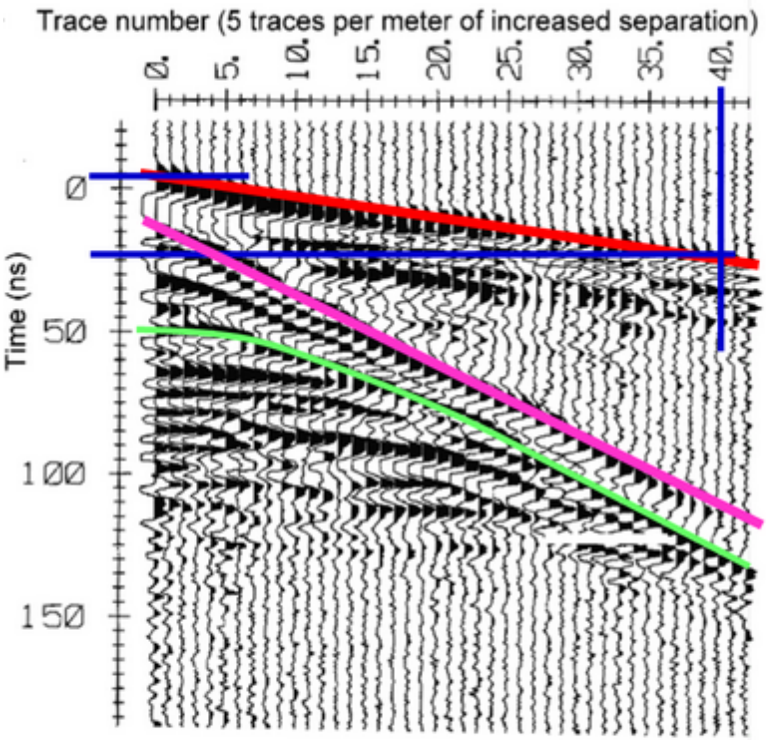


2-Layer Example

Model

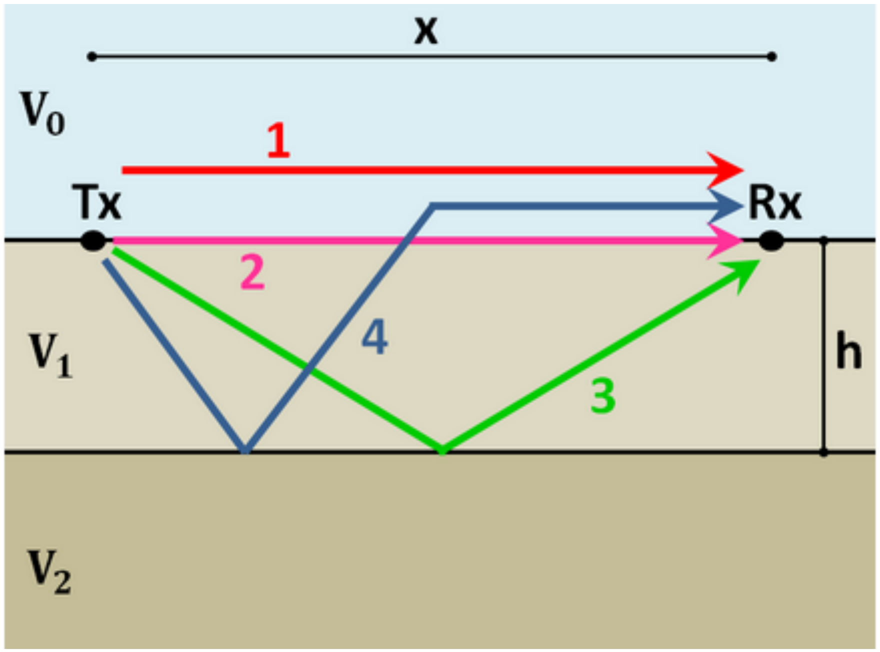


Radargram

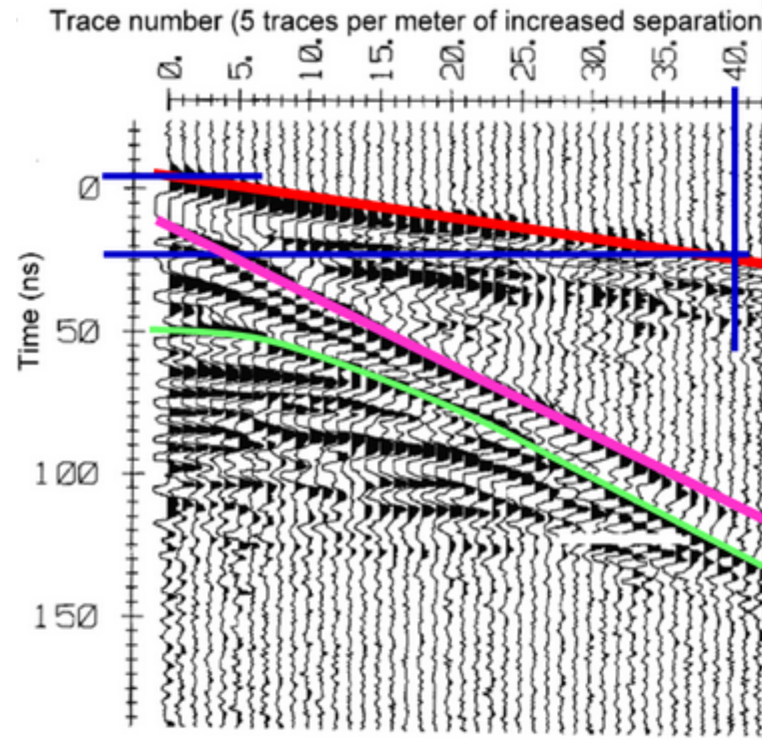


2-Layer Example

Model



Radargram



2) Direct Ground Wave

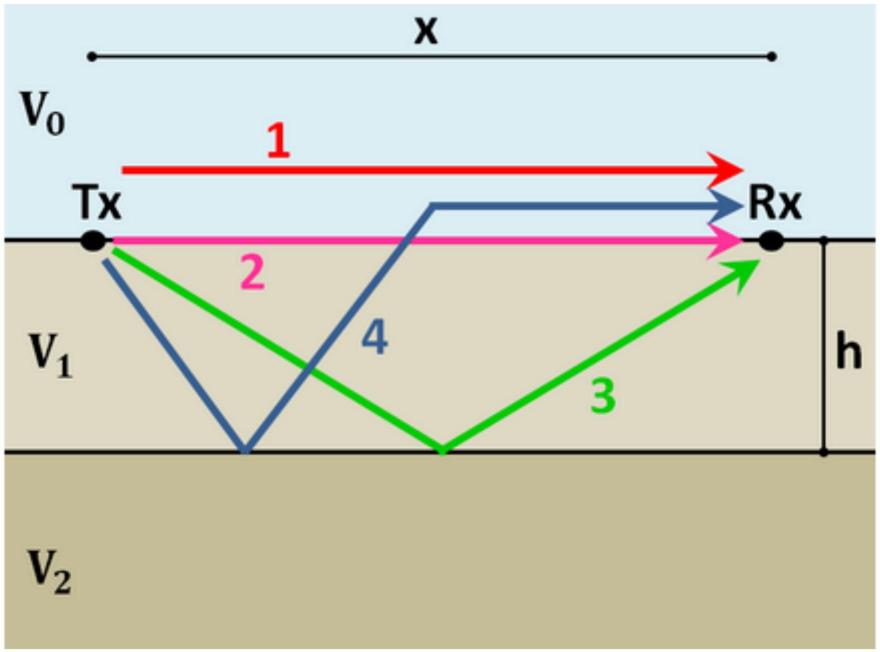
Travel Time: $$t_{air} = \frac{x}{c}$$

$$V = c/\sqrt{\varepsilon_r}$$

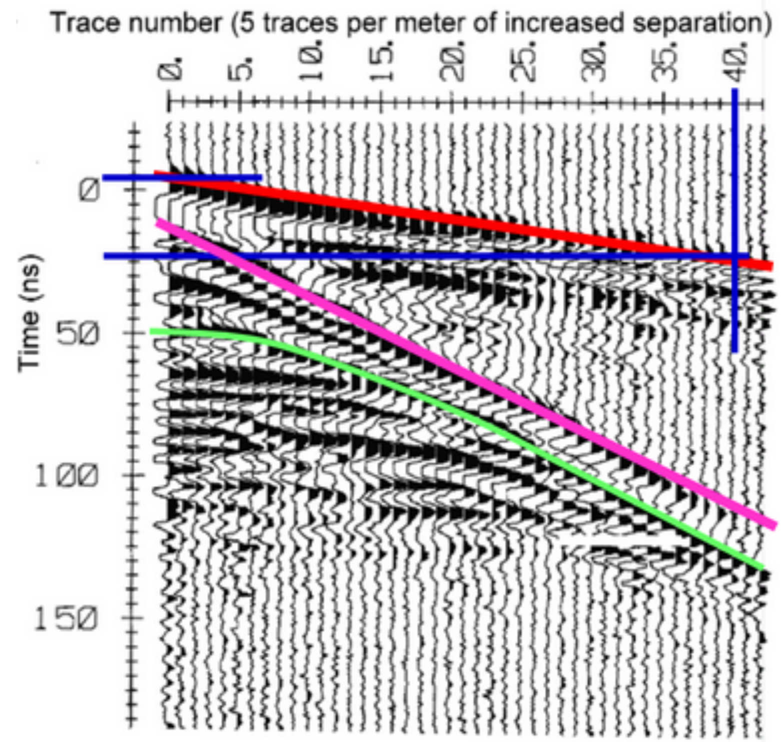
$$c = 3.00 \times 10^8 \text{ m/s}$$

2-Layer Example

Model



Radargram



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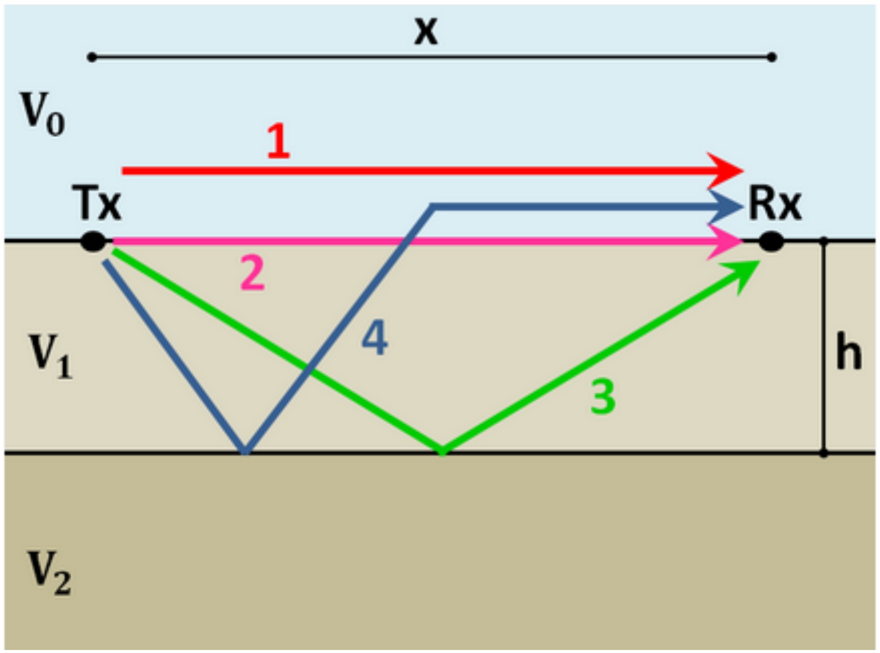
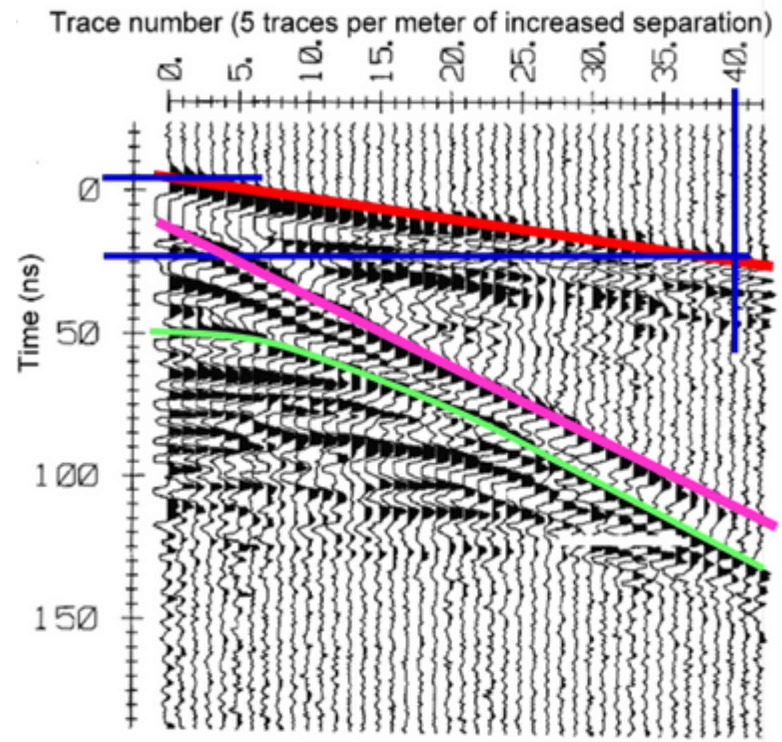
Model



Radargram



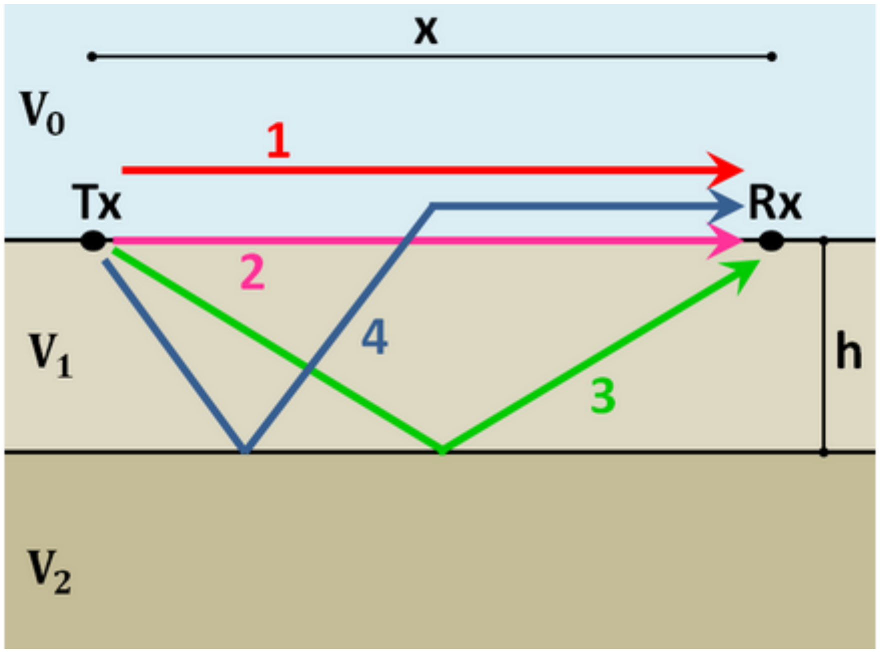
3) Reflected Wave

Travel Time:

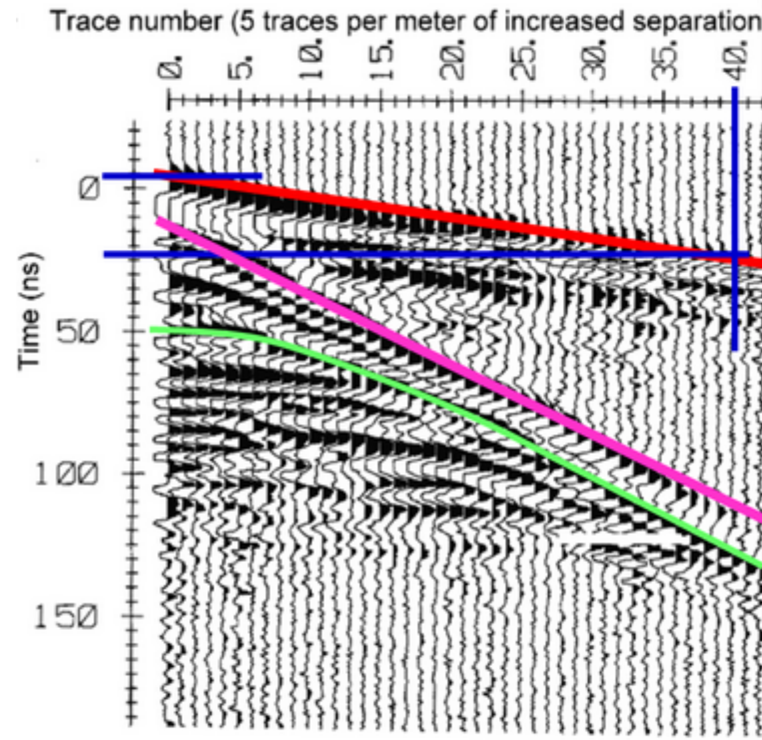
$$t_{ref} = \frac{\sqrt{x^2 + 4h^2}}{V_1}$$

2-Layer Example

Model



Radargram

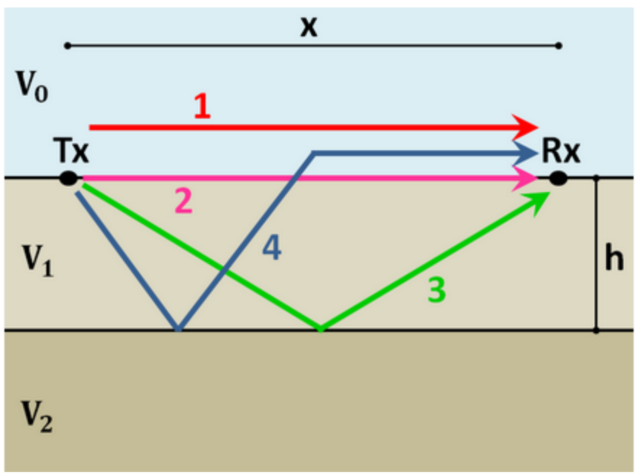


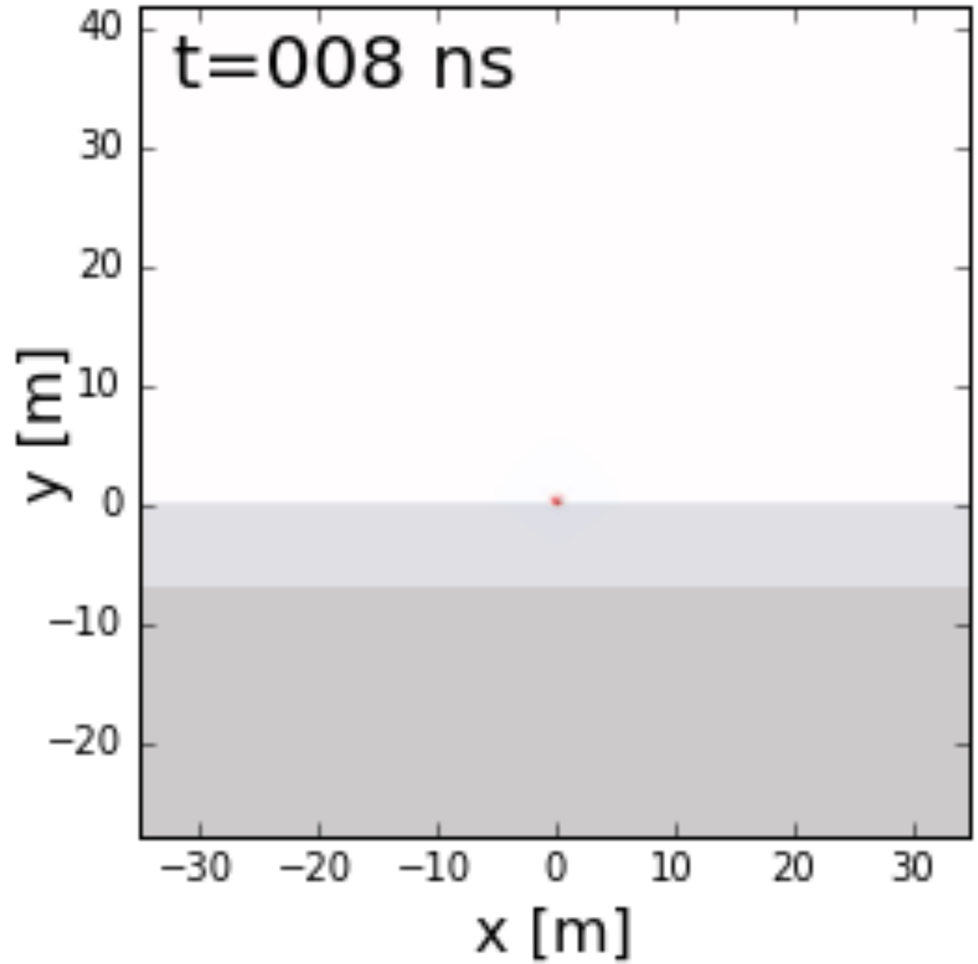
4) Refracted Wave

Travel Time: $$t_c = \frac{x}{c} + \text{Constant}$$

$$V_1 < V_0$$

Can you recognize ray paths?





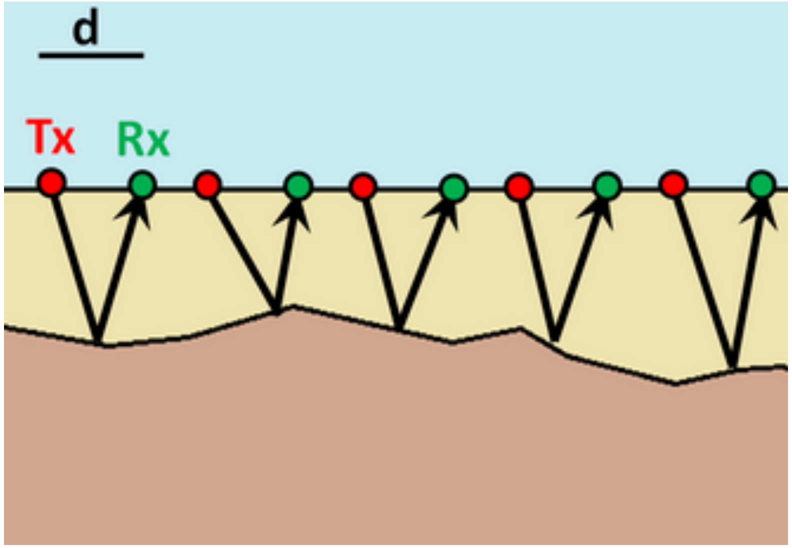
Survey and Data

Reading on the GPG:

https://gpg.geosci.xyz/content/GPR/GPR_survey_data.html#

Common Offset Survey

- Tx-Rx distance is fixed
- Tx-Rx is moved for every shot
- Most common GPR survey
- Good for:
  - Finding horizontal interfaces
  - Locating discrete objects
- Zero offset survey has Tx-Rx coincident (same location)



Common Offset Survey

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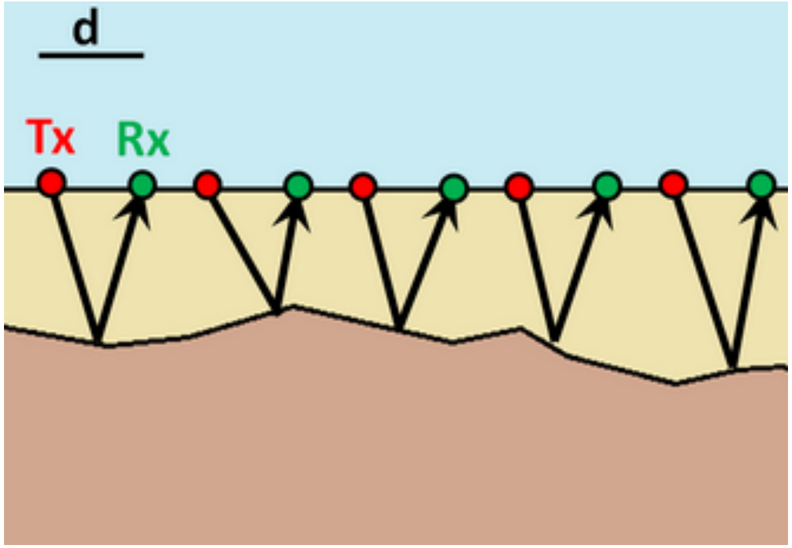
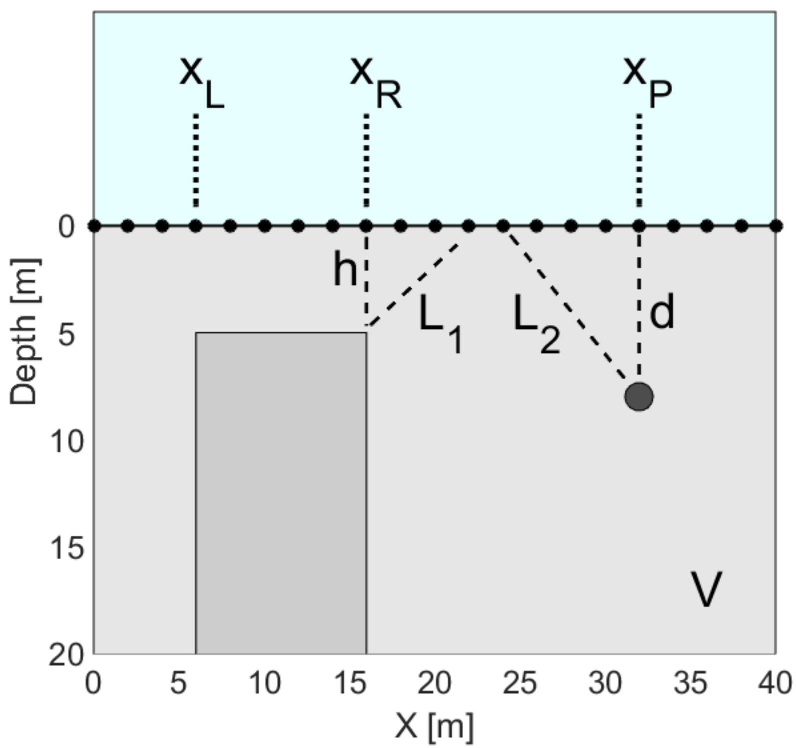
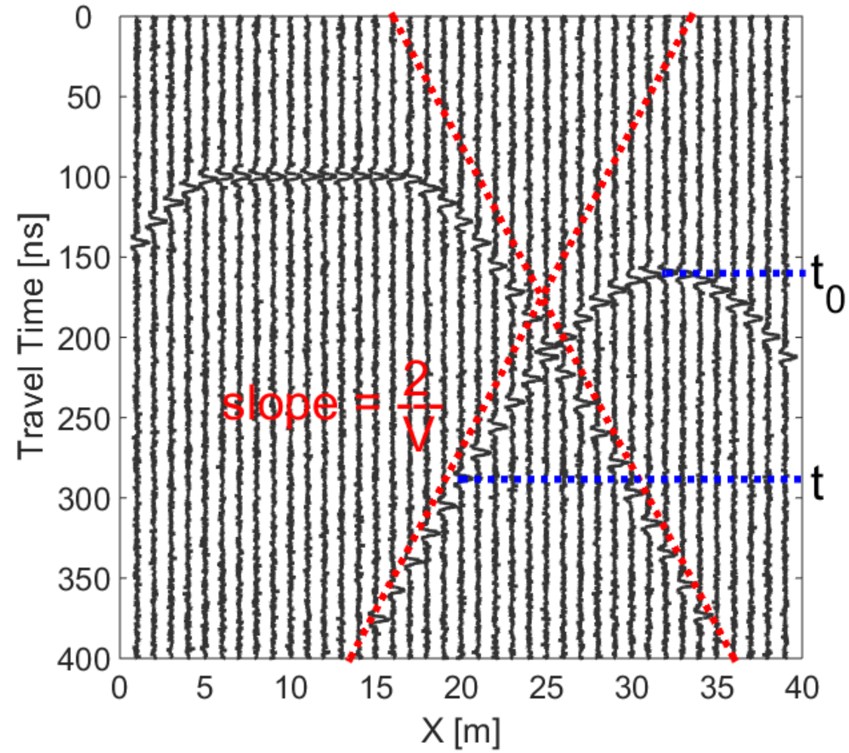


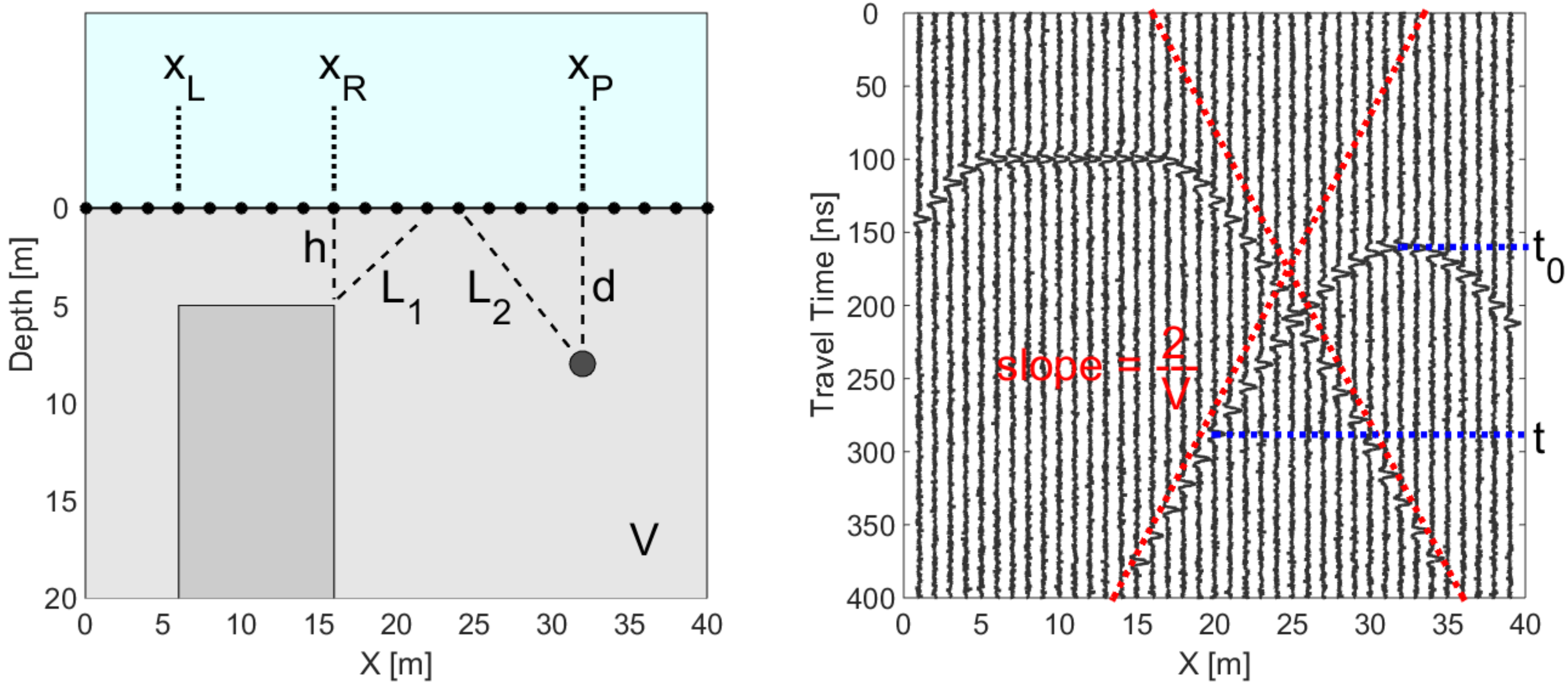
Zero Offset Survey: Finding Objects





- A thin pipe at x = 32 m and depth 8 m
- A block between xL and xR at 5 m depth

Zero Offset Survey: Finding Objects



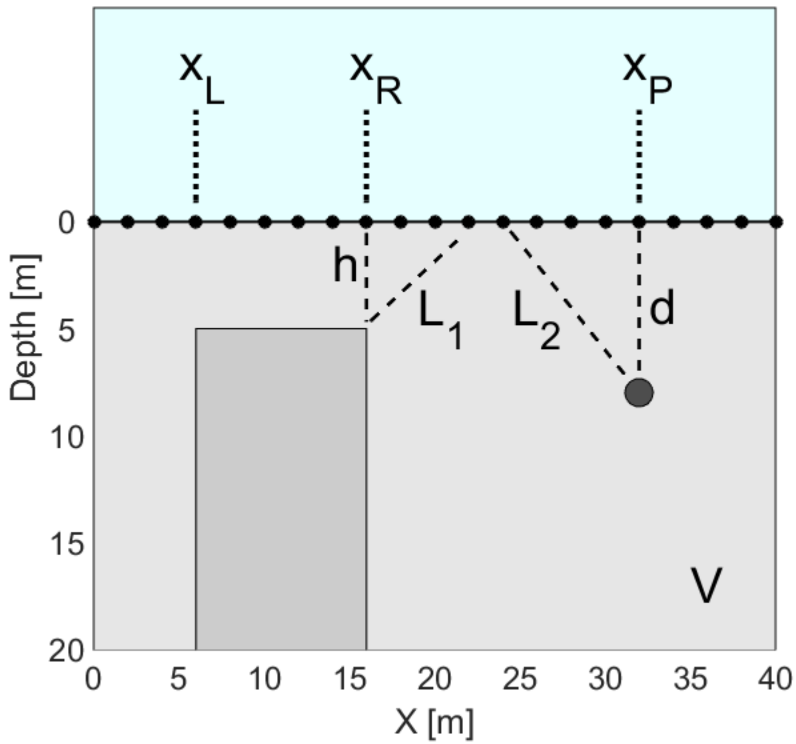
Travel time for the pipe:

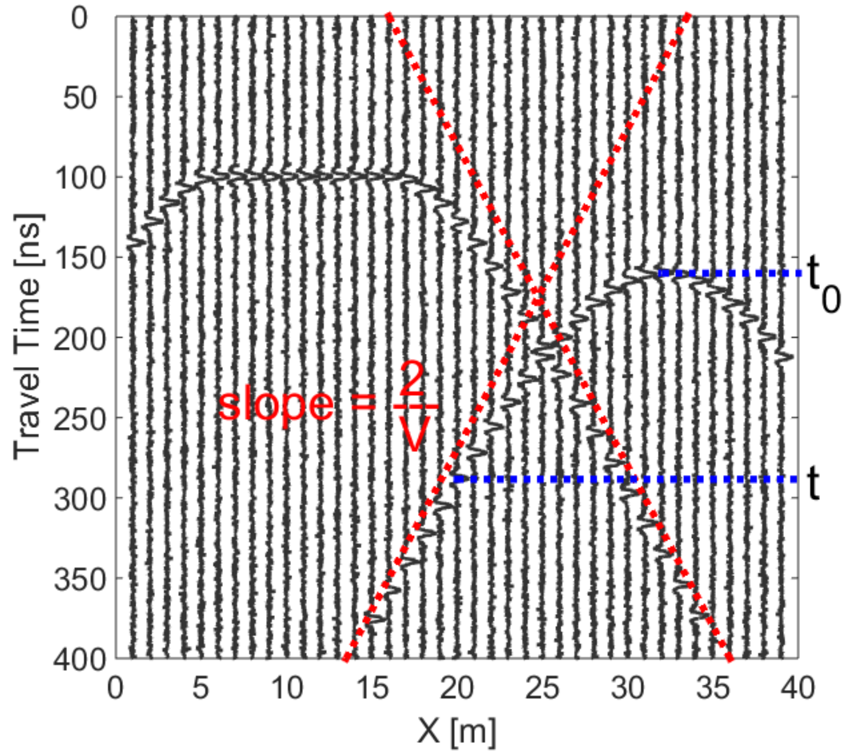
$$t_p = \frac{2L_2}{V} = \frac{2\sqrt{(x-x_p)^2+d^2}}{V}$$

where

$$t_p(x_p) = \frac{2d}{V}$$

Zero Offset Survey: Finding Objects

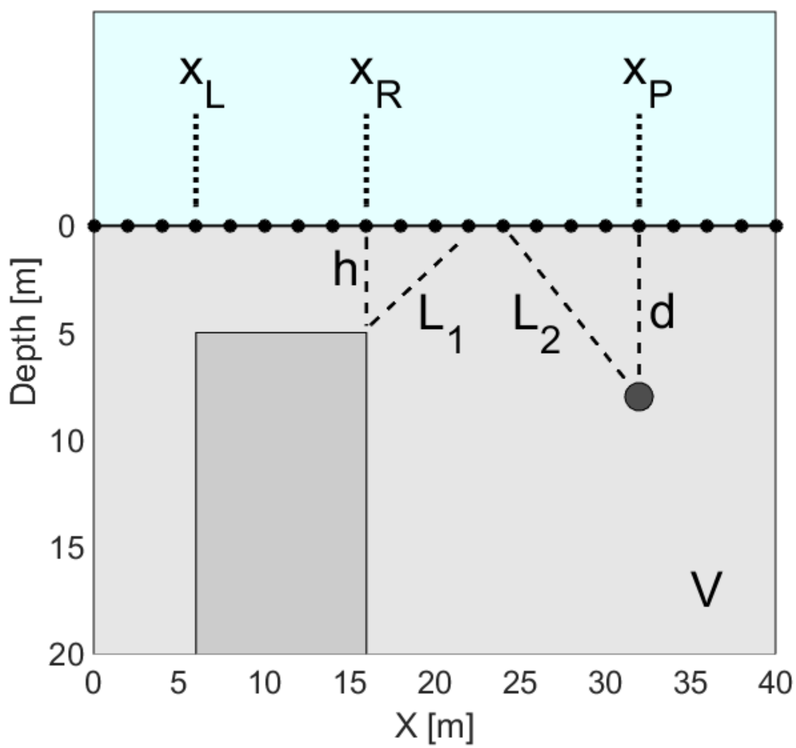


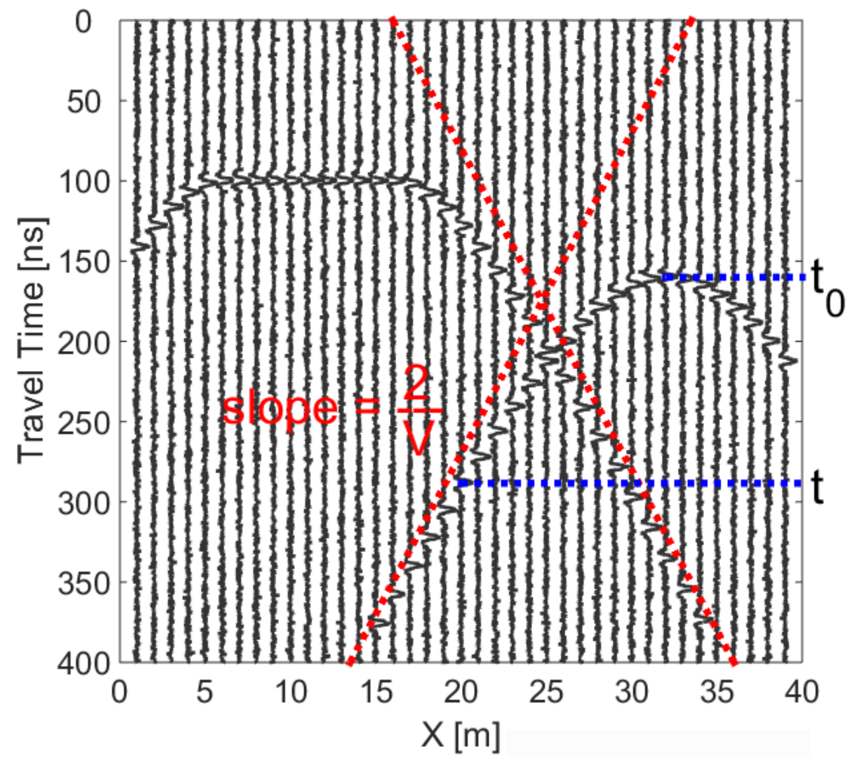


For the block:

$$t_b = \begin{cases} \dfrac{2\sqrt{(x-x_L)^2+h^2}}{V} & \text{for} \quad x < x_L \\ \dfrac{2h}{V} & \text{for} \quad x_L \le x \le x_R \\ \dfrac{2\sqrt{(x-x_R)^2+h^2}}{V} & \text{for} \quad x > x_R \end{cases}$$

Finding Objects (Method 1)

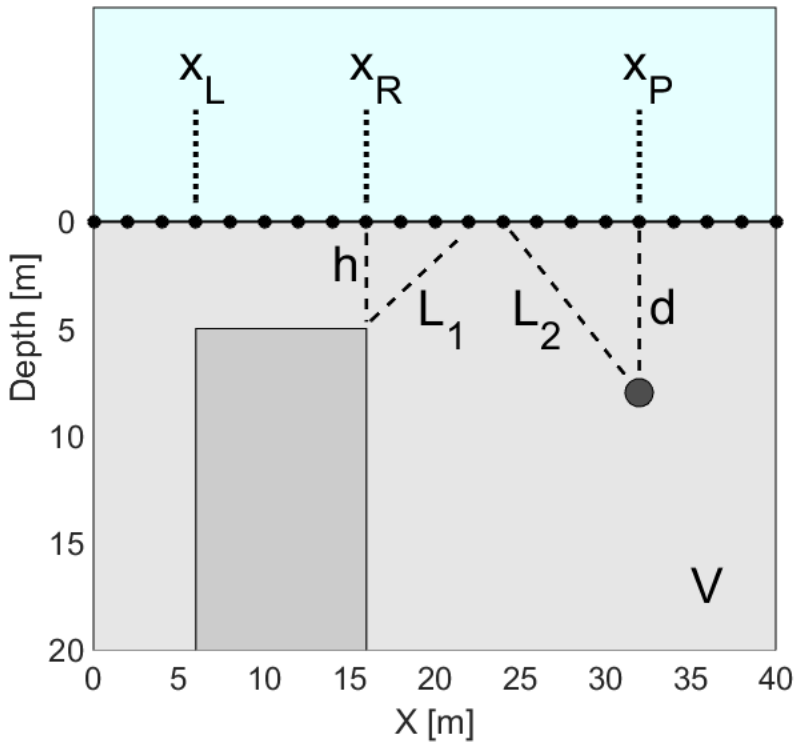


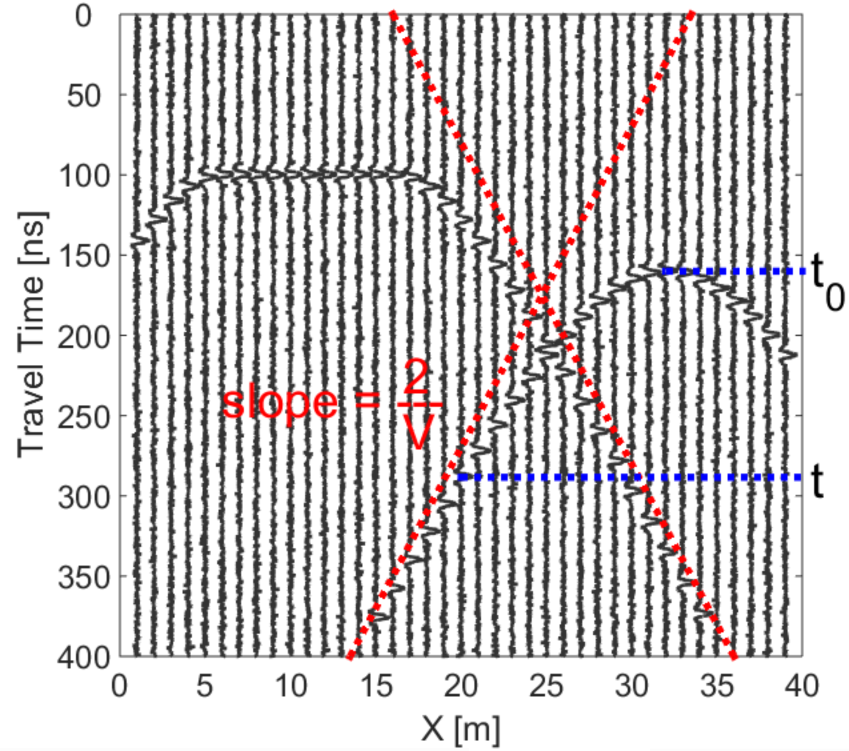


Slope (red dashed): $\pm 2/V$ where $t_0 = \dfrac{2d}{V}$

1) Get velocity from slope
2) Get depth to object

Finding Objects (Method 2)

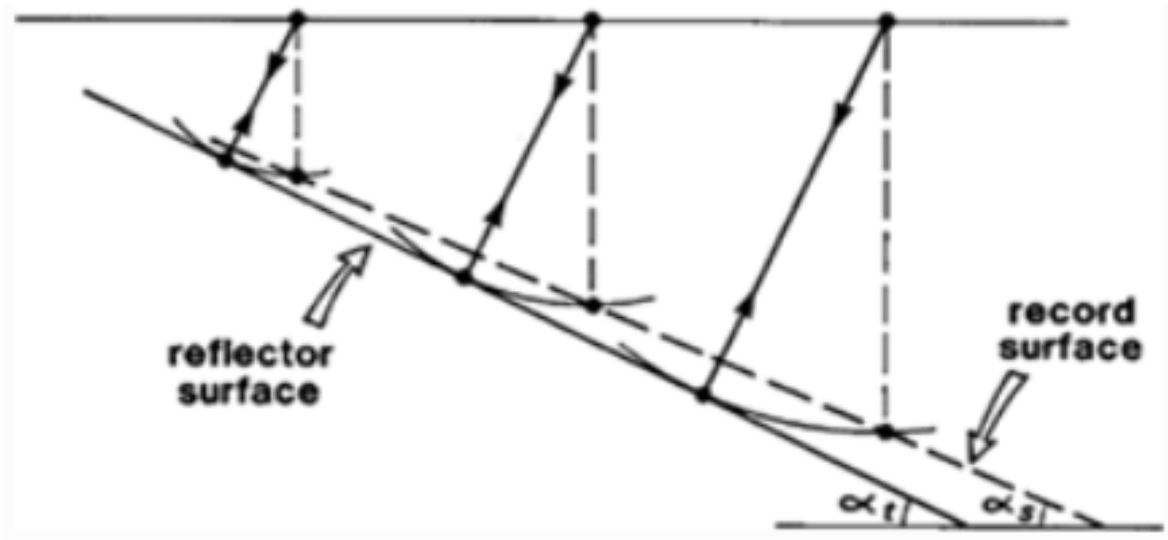




Using a point (blue dashed): $$V = 2\sqrt{\frac{(x - x_p)^2}{t^2 - t_0^2}}$$ where $$t_0 = \frac{2d}{V}$$

1) Get velocity from a point on the curve
2) Get depth to object

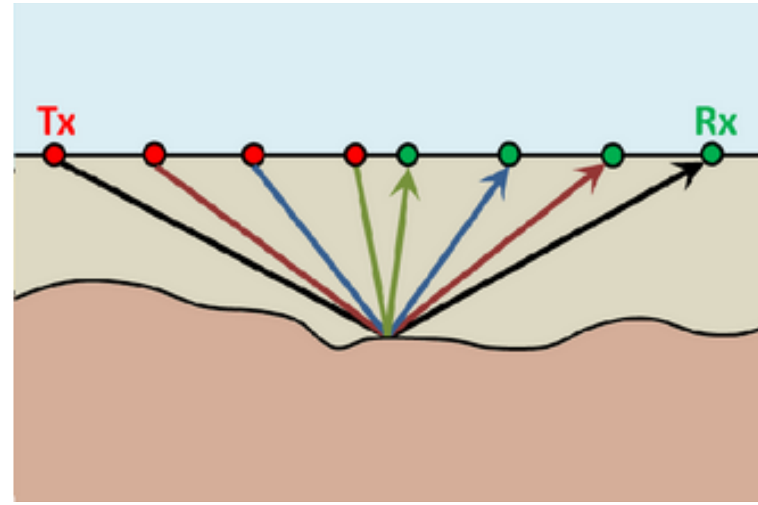
Common Offset Survey: Dipping Layers



- Zero offset reflection is perpendicular to surface
- Can lead to underestimate of depth and slope of layer
- Can be corrected using migration correction (GPG)

Common Midpoint Survey

- Tx-Rx distance varies
- Midpoint between Tx-Rx is left constant
- Good for:
  - Finding horizontal interfaces



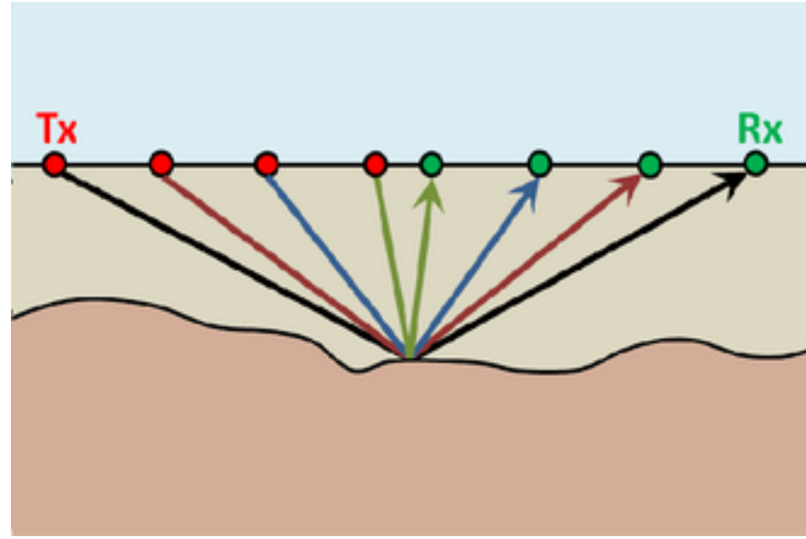
Common Midpoint Survey

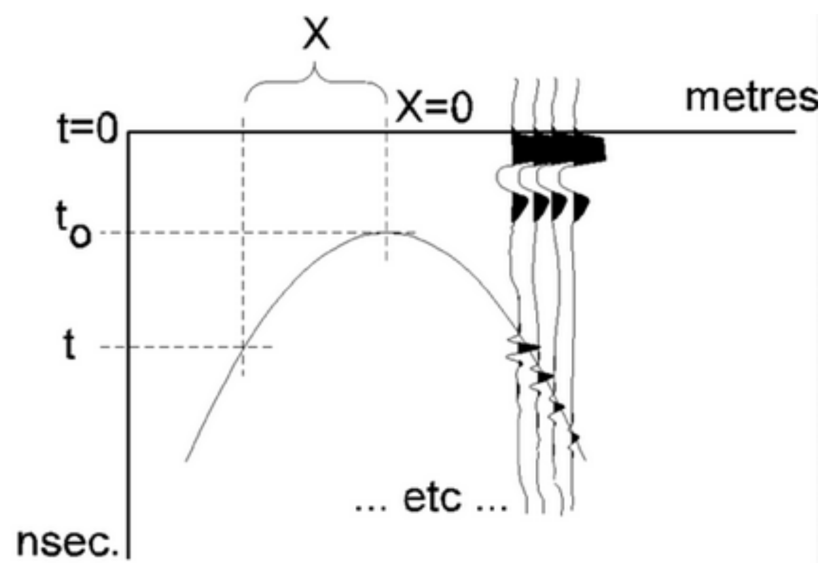
- Travel time off same reflection point make a hyperbola:

$$t = \frac{2\left(x^2 + d^2\right)^{1/2}}{V}$$

- Can use hyperbola to get velocity and layer depth

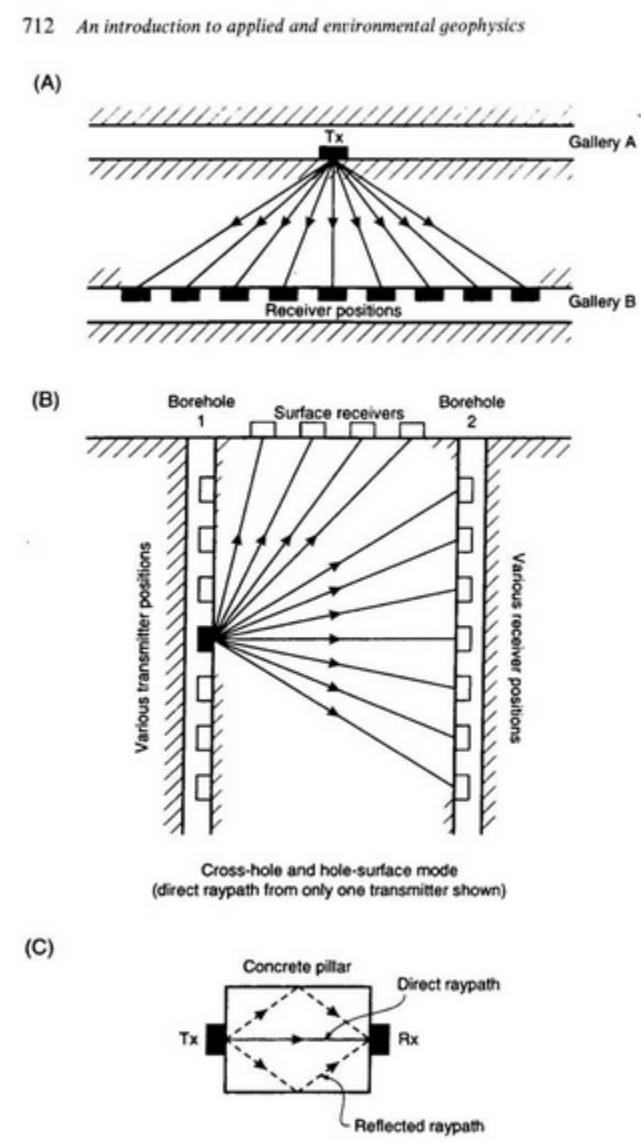
- Reading not hyperbola:
  - Indicates uneven/dipping interface





Transillumination Survey

- Tx and Rx are placed on opposing sides of a target.
- Sometimes many Tx and Rx
- Used for:
  - Structural integrity of mine shafts
  - Borehole surveys
  - Finding internal structures within objects

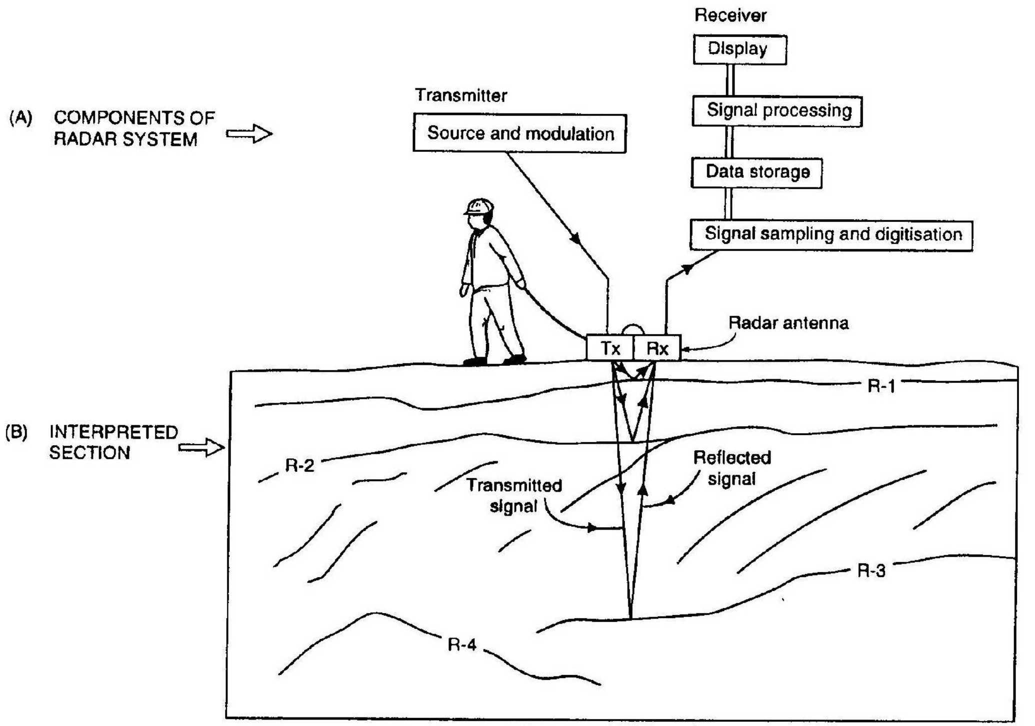


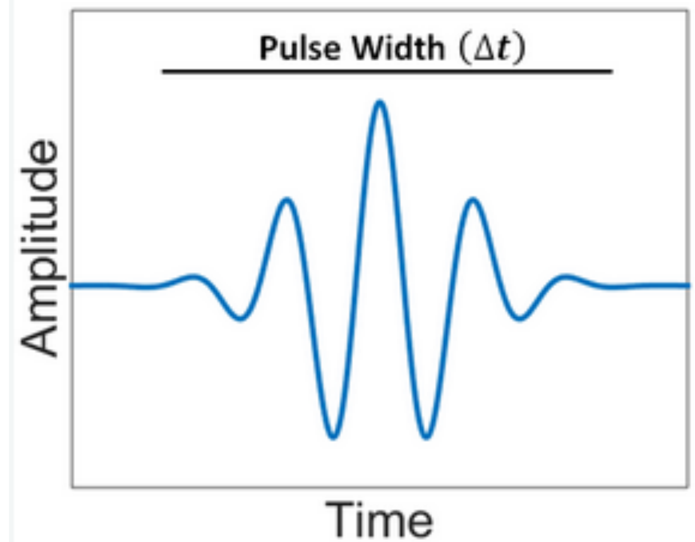
Recap Questions

Q: What is the most commonly used survey configuration?

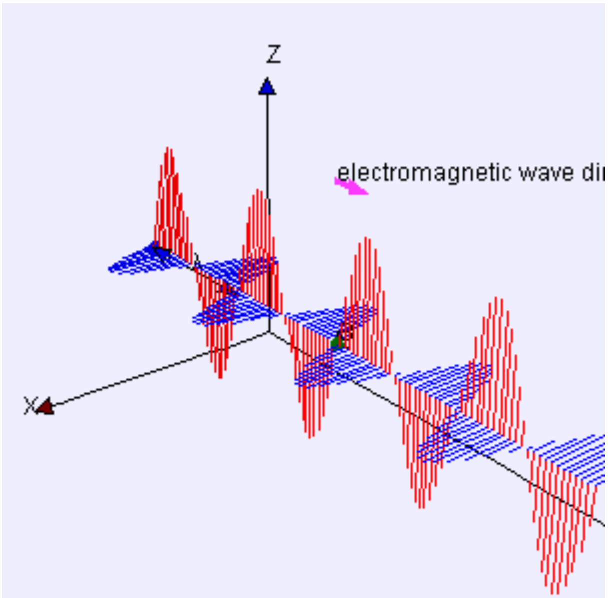
Q: What kind of signatures do objects make in radargrams?

GPR Source Signal





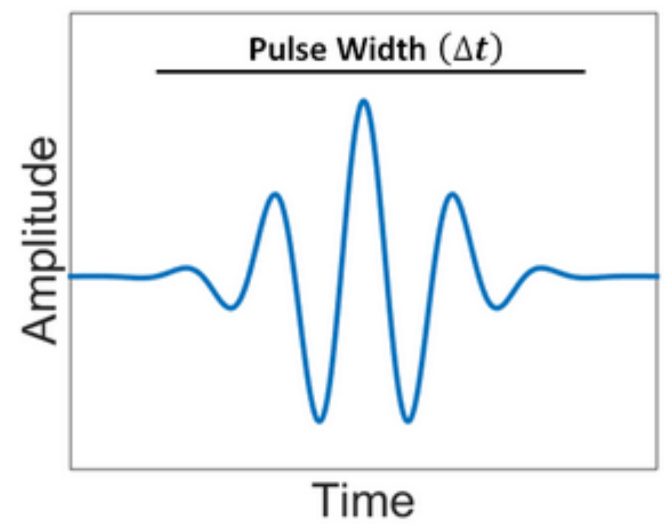
Examine properties of the source pulse



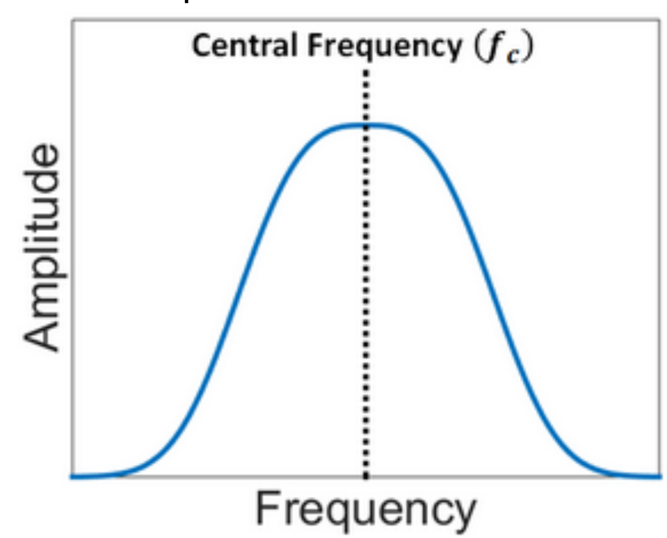
GPR Source Signal: Wavelet

- Wavelet: A wave-like oscillation of short duration
- Bandwidth: Range of frequencies in the wavelet
- Pulse Width: Time-duration of wavelet
- Spatial Length: Wavelength of the wavelet
- Central Frequency: Operating frequency of GPR survey

Wavelet



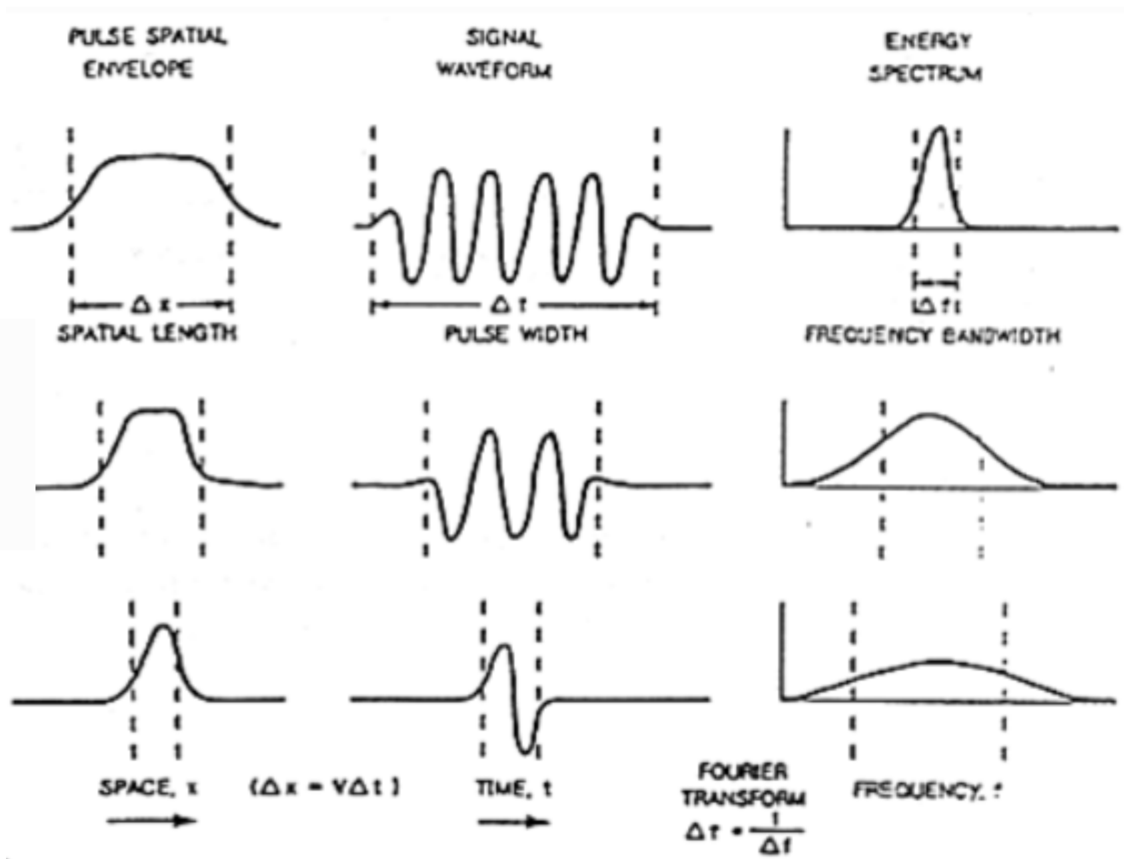
Frequencies in Wavelet



GPR Source Signal: Wavelet

- Shorter pulse overall contain higher frequencies
- Spatial length increases as pulse length increases
- Shorter pulses contain a wider range of frequencies

$$f_c = \frac{1}{\Delta t}$$



GPR Source Signal: Spatial Length

- The spatial length (wavelength) of the GPR pulse is dependent on the central frequency and velocity

$$\lambda = \frac{V}{f_c} = \frac{c}{f_c\sqrt{\varepsilon_r}}$$

- When the GPR signal at some frequency is transmitted across an interface, it can be stretched or contracted

- Lower velocity $\Rightarrow$ Shorter spatial length
- Lower frequency $\Rightarrow$ Larger spatial length

GPR Source Signal: Spatial Length

- Since $f_c = 1/\Delta t$, the spatial width is given by:

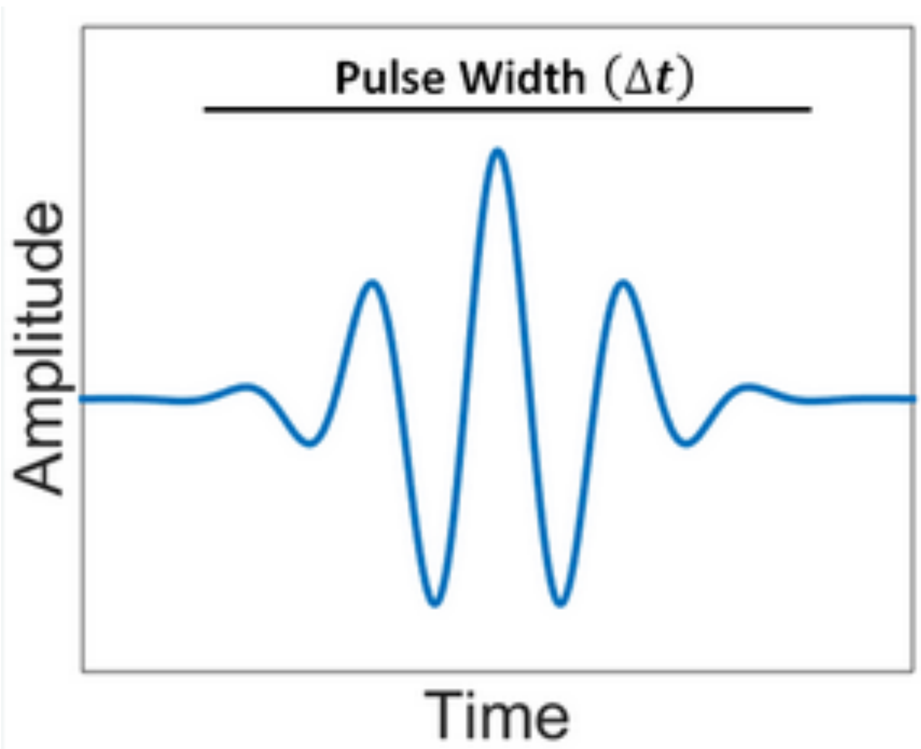
$$\lambda = V\,\Delta t = \frac{c\,\Delta t}{\sqrt{\varepsilon_r}}$$

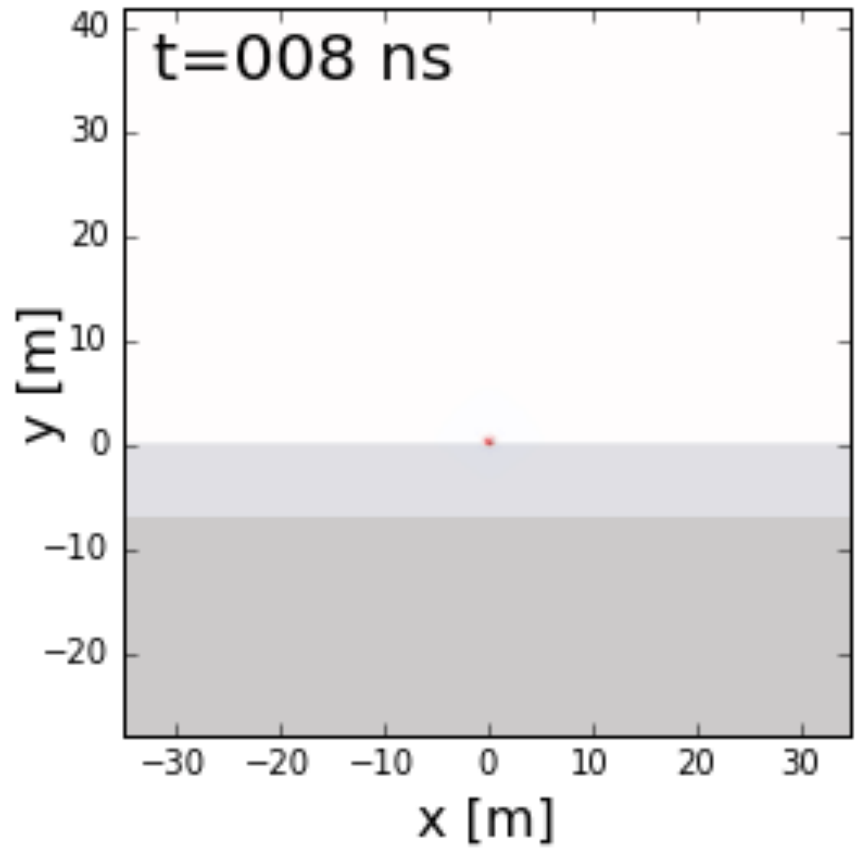
- Shorter pulse width → Higher frequencies

→ Shorter pulse length

Spatial Length: 2D Example

- Does the reflected signal coming up to the surface becomes stretched or contracted?
- Why is this?





Resolution of GPR Surveys

- Resolution: Smallest features which can be distinguished using the survey.

- Resolution depends on:
  - The frequency of the GPR signal
  - The physical properties of the ground
  - The dimensions and separations of features

Resolution of GPR Surveys: Layers

¼ wavelength rule:

The thickness of a layer must be at least ¼ the wavelength of the GPR signal.

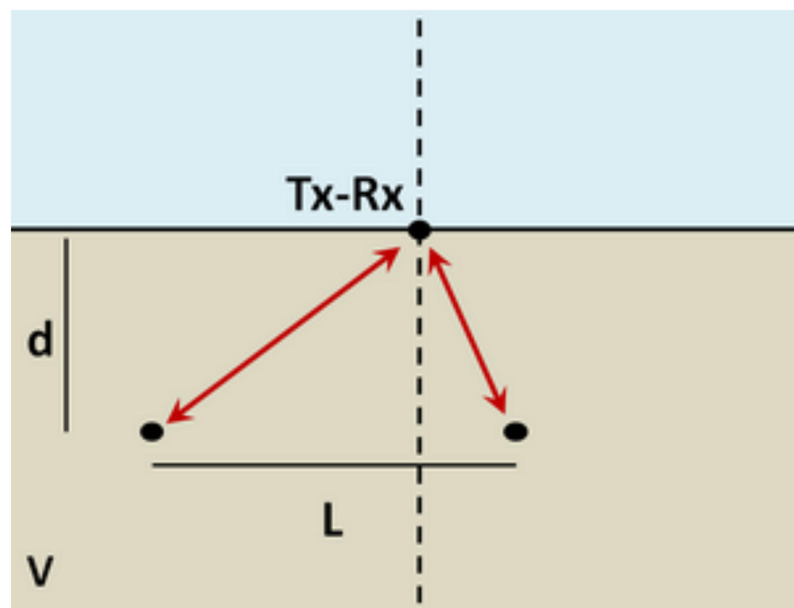
$$L > \frac{c}{4 f_c \sqrt{\varepsilon_r}} = \frac{c \Delta t}{4 \sqrt{\varepsilon_r}}$$

Resolution of GPR Surveys: Separation

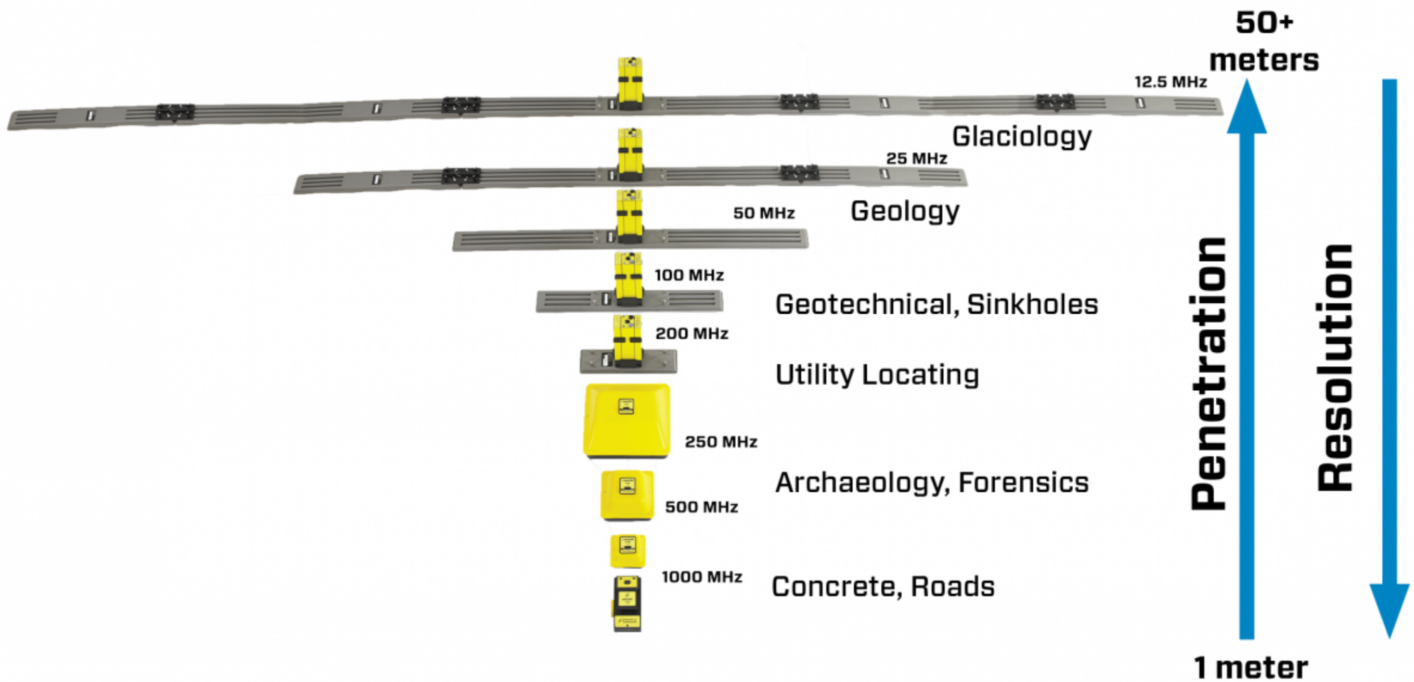
- If objects are too close to one another:
  - The two way travel time is almost the same
  - The two returning wavelet signals will overlap
  - They will appear to be one object

- For zero offset survey

$$L > \sqrt{\frac{V\,d}{2f_c}}$$



Antenna length



$$L \simeq \frac{\lambda}{2} = \frac{c}{2f_c}$$

Wavelenth: $\lambda = \dfrac{c}{f_c}$

Probing Distance (Depth of Investigation)

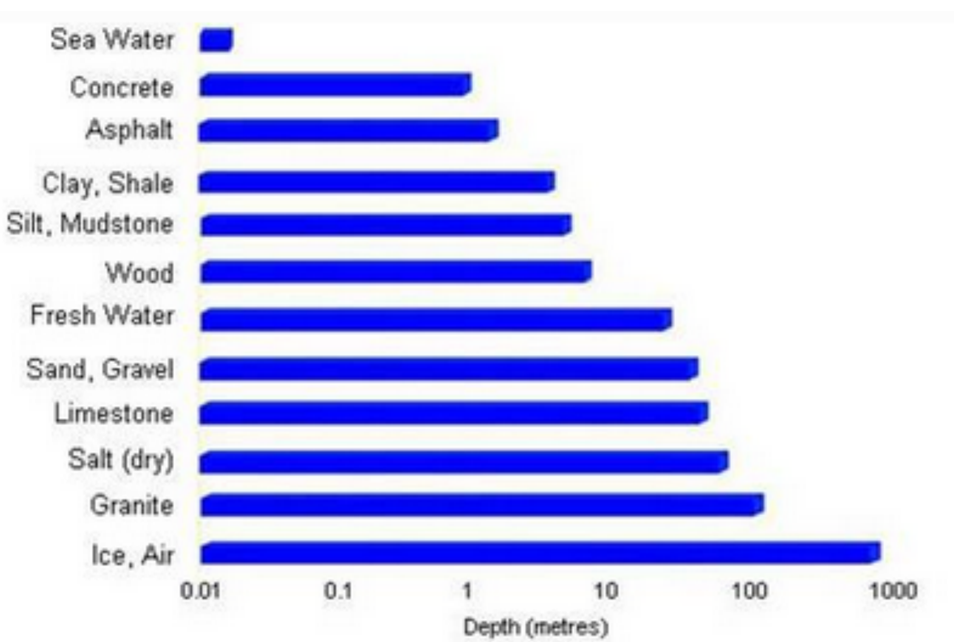
- Maximum depth at which GPR can be used to get information about subsurface

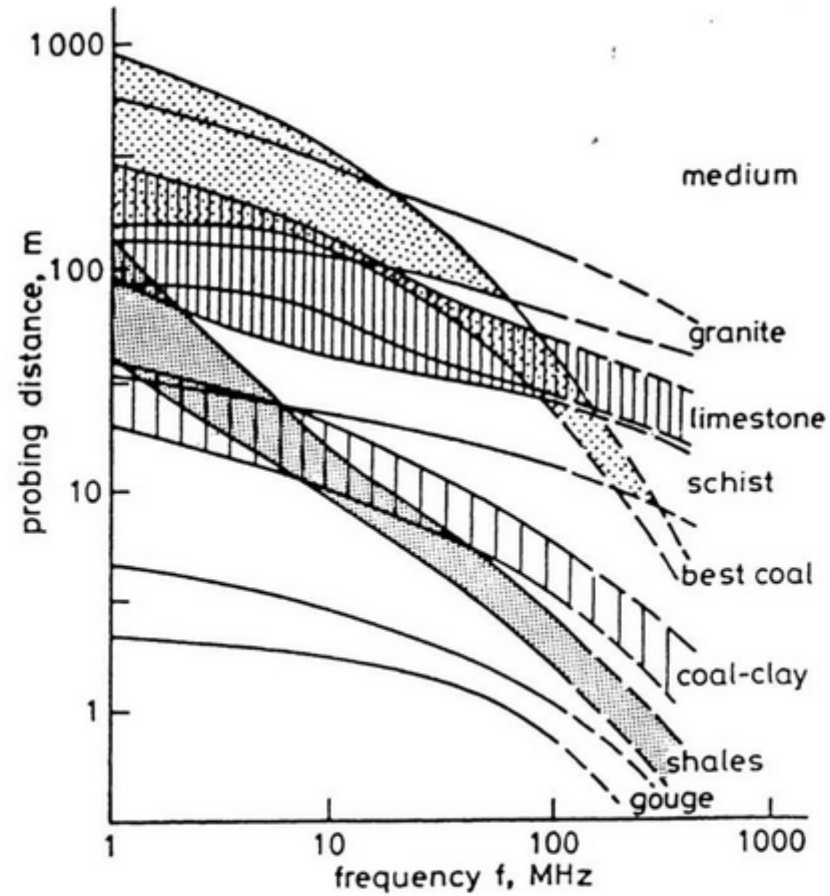
- Probing distance is approximation 3 skin depths:

$$D = 3\delta \approx \begin{cases} 1510\sqrt{\dfrac{1}{\sigma f}} & \text{for } \omega\varepsilon \ll \sigma \\ 0.0159\dfrac{\sqrt{\varepsilon_r}}{\sigma} & \text{for } \omega\varepsilon \gg \sigma \end{cases}$$

Probing Distance (Depth of Investigation)

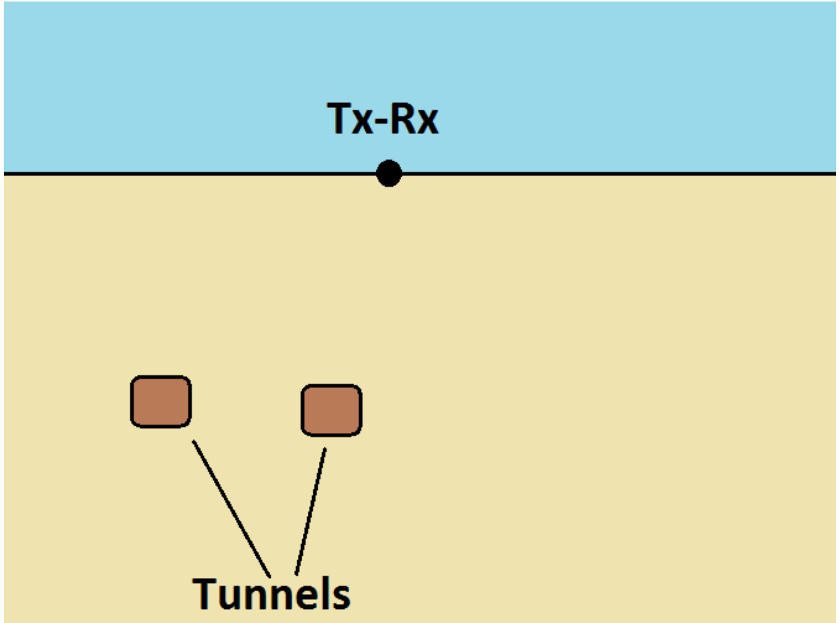
- Generally decreases as frequency increases
- Is lower for more conductive materials and non-dielectric materials





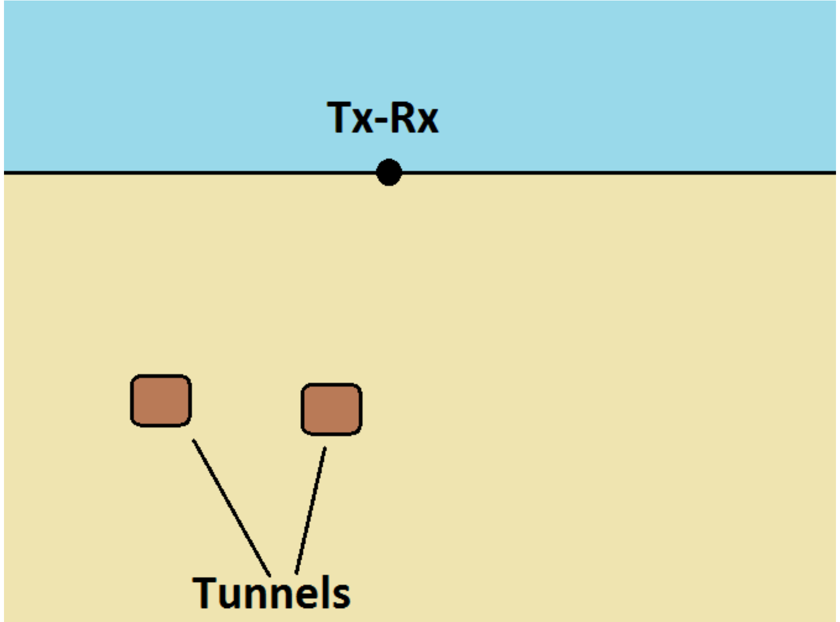
Probing Distance vs. Resolution

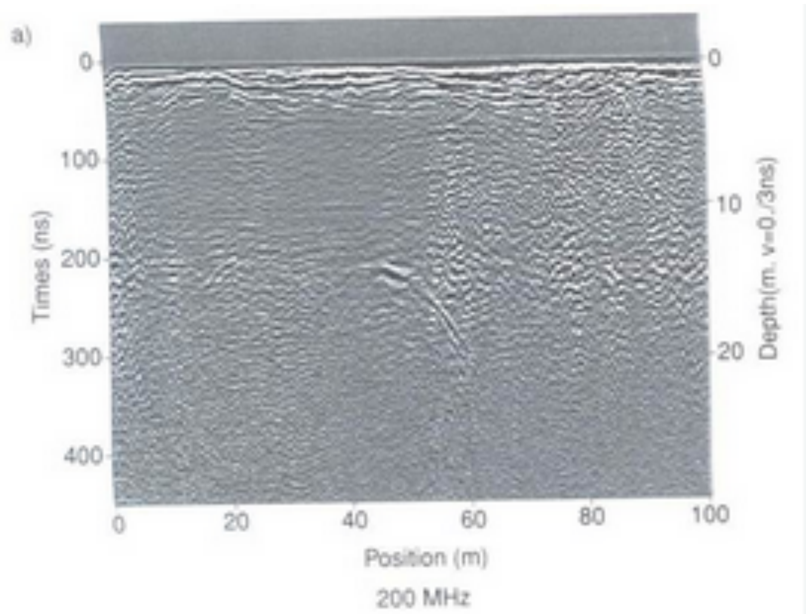
- Want to find two buried tunnels.
- Using a zero offset survey configuration.
- Higher frequencies give better resolution
- Lower frequencies give larger probing distance



Radargram 200 MHz

- Little to no useful signal after 200 ns
- Can't see features from the tunnels
- Too much attenuation of signal
- Probing distance insufficient

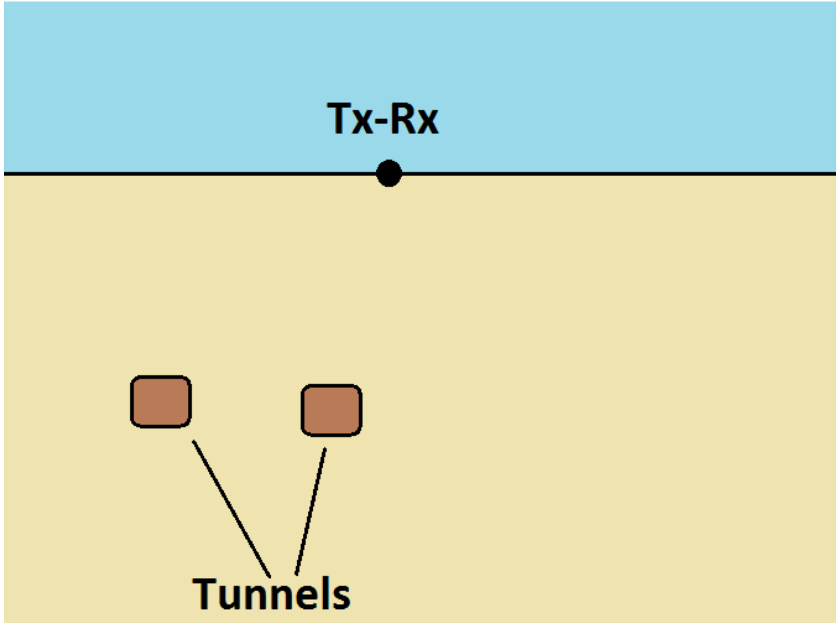


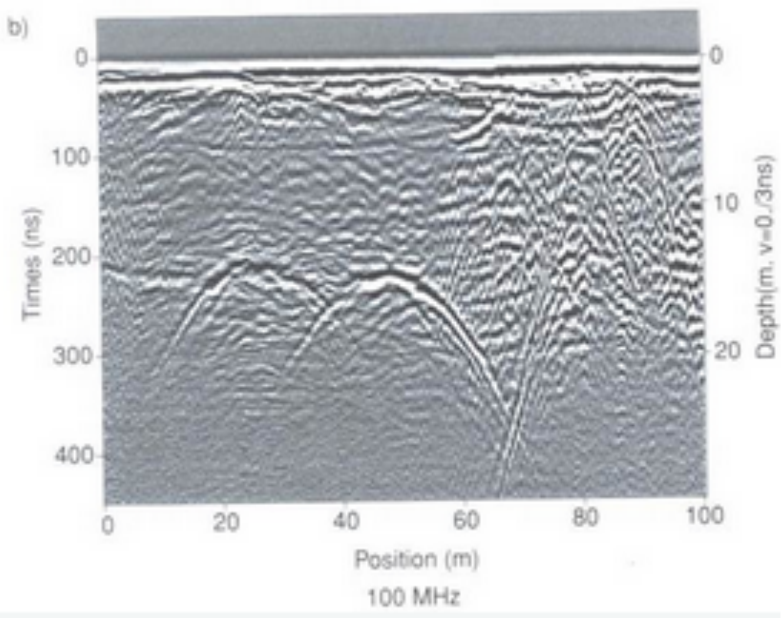


Radargram 100 MHz

- Useful signals up to 300 ns
- See top of hyperbolas from tunnels

- Lower resolution
- Can see tunnels





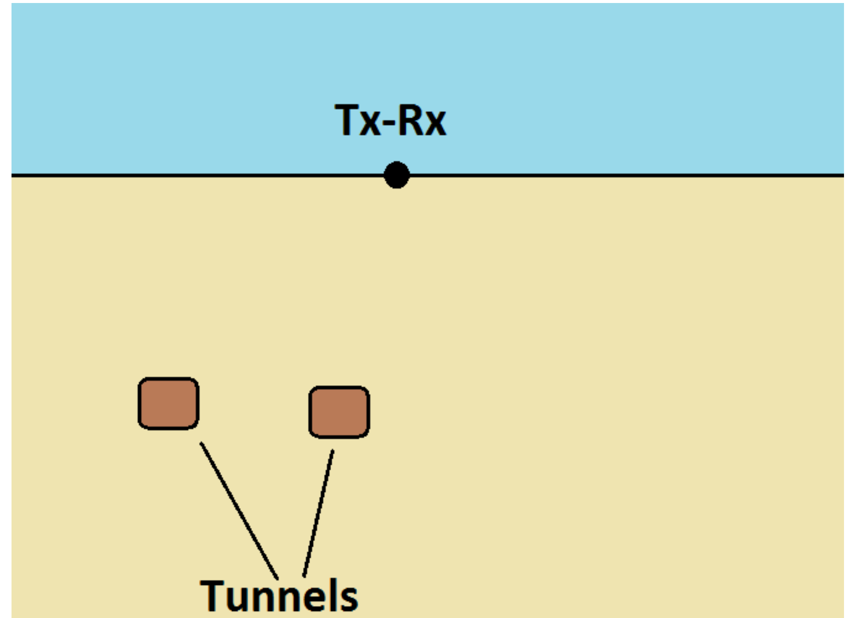
Radargram 50 MHz

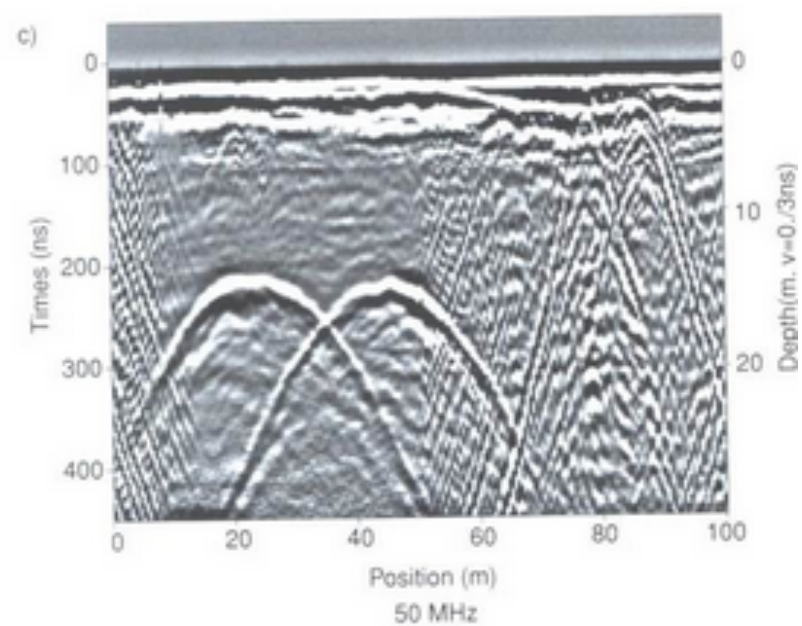
- Useful signals through 400 ns

- Well-defined hyperbolas from tunnels

- Lower resolution image

- Best frequency for what we want to observe





Recap of Material

- There is a compromise between resolution and probing distance:

Higher frequencies $\Rightarrow$ Better resolution

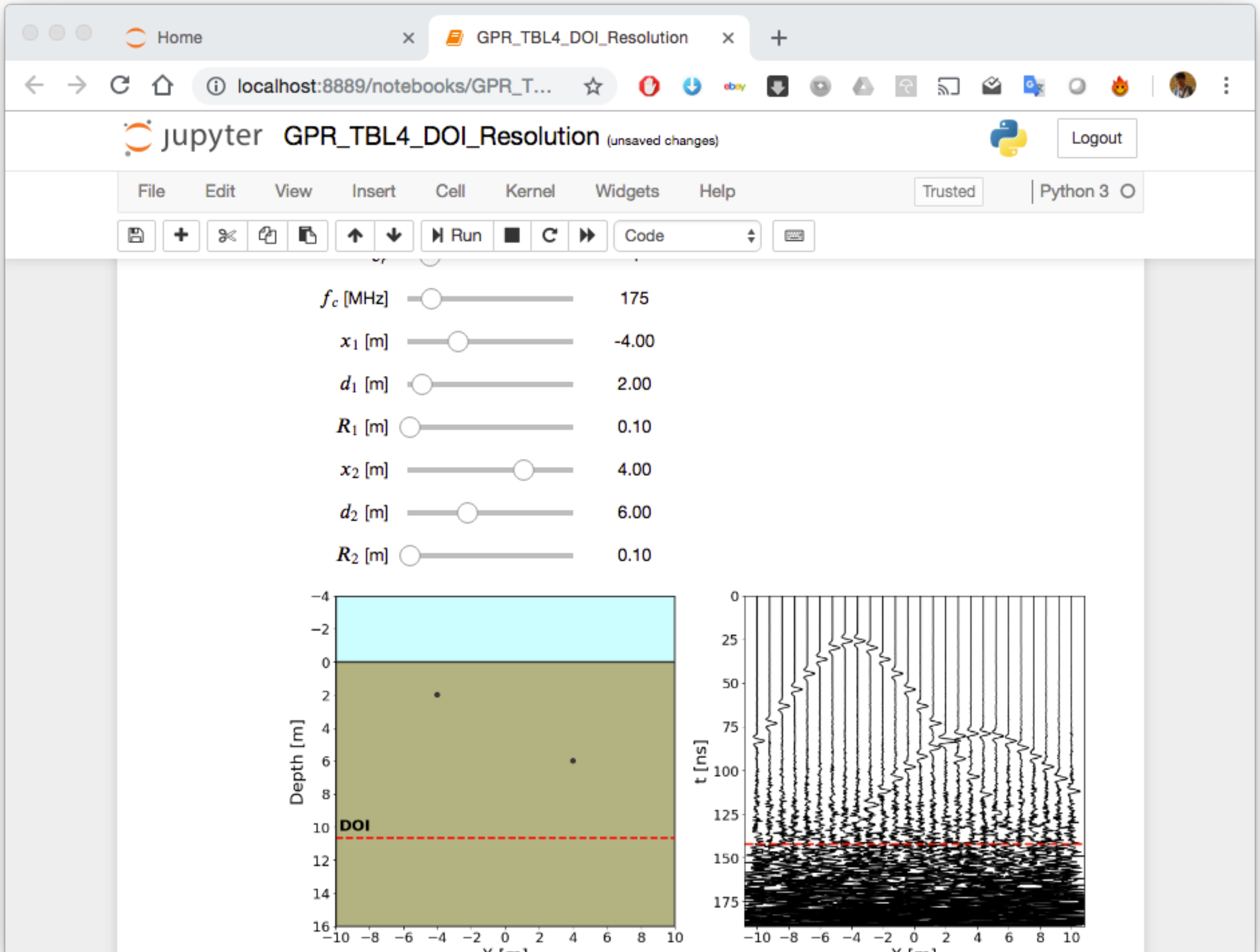
Layers: $$L > \frac{c}{4 f_c \sqrt{\varepsilon_r}} = \frac{c \Delta t}{4\sqrt{\varepsilon_r}}$$

Objects: $$L > \sqrt{\frac{V\, d}{2 f_c}}$$

Higher frequencies $\Rightarrow$ Lower probing distance

$$D = 3\delta \approx \begin{cases} 1510\sqrt{\dfrac{1}{\sigma f}} & \text{for } \ \omega\varepsilon \ll \sigma \\ 0.0159\dfrac{\sqrt{\varepsilon_r}}{\sigma} & \text{for } \ \omega\varepsilon \gg \sigma \end{cases}$$

GPR Applet



Home
GPR_TBL4_DOI_Resolution
localhost:8889/notebooks/GPR_T...
jupyter GPR_TBL4_DOI_Resolution (unsaved changes)
Logout
File Edit View Insert Cell Kernel Widgets Help
Trusted
Python 3
Run
Code
f_c [MHz] 175
x_1 [m] -4.00
d_1 [m] 2.00
R_1 [m] 0.10
x_2 [m] 4.00
d_2 [m] 6.00
R_2 [m] 0.10
Depth [m]
DOI
X [m]
t [ns]
X [m]

Recap Questions

Q: If a GPR signal contains more high frequency waves, is its pulse length longer or shorter?

$$f_c = \frac{1}{\Delta t}$$

Q: How thick does a layer need to be for us to see it?

Q: What happens when objects are too close together?

Recap Questions:

Q: Does probing distance increase/decrease as frequency increases?

$$f_c = \frac{1}{\Delta t}$$

Q: What are some things you want to know before choosing an operating frequency?

Source of noise

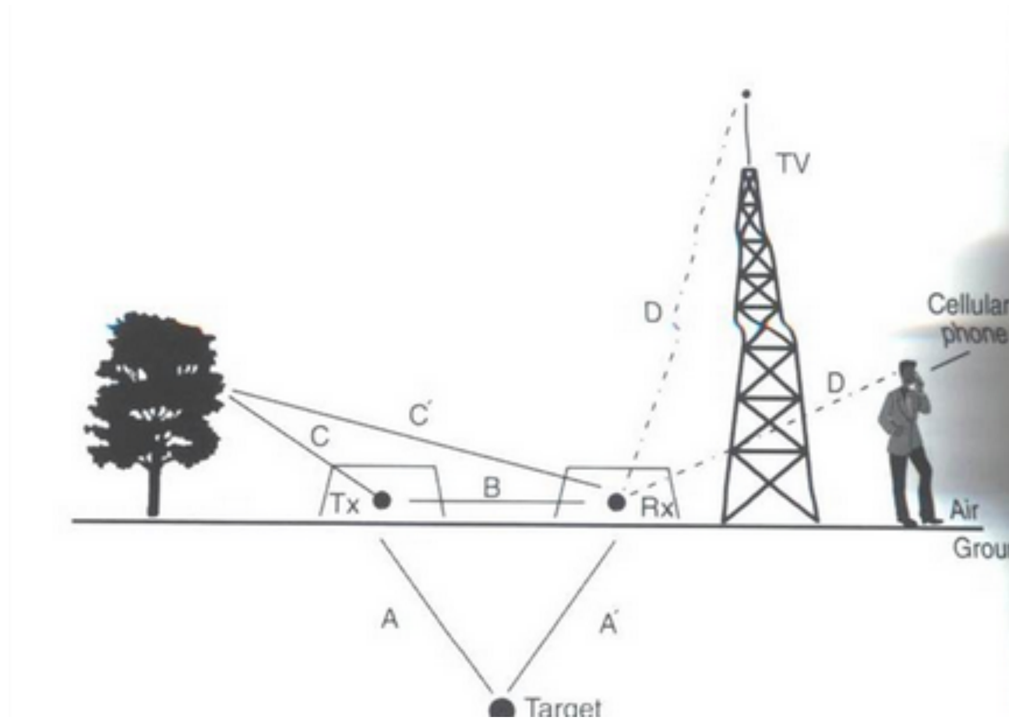
- Any signal which interferes from useful signals from GPR targets.

Examples:

- External radiowave sources
- Above ground objects
- Ringing
- Scattering

Interference from other Radiowave Sources

- Radio towers
- Cell phones
- Power Lines



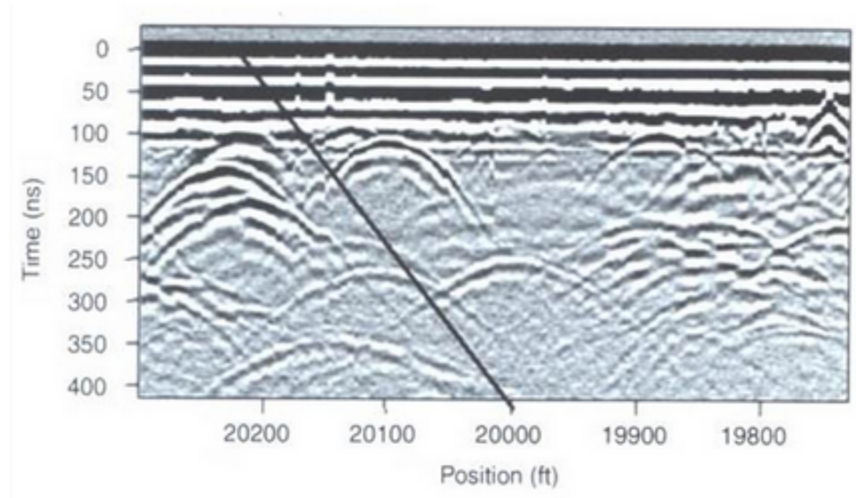
- Tx and Rx usually shielded to avoid these signals

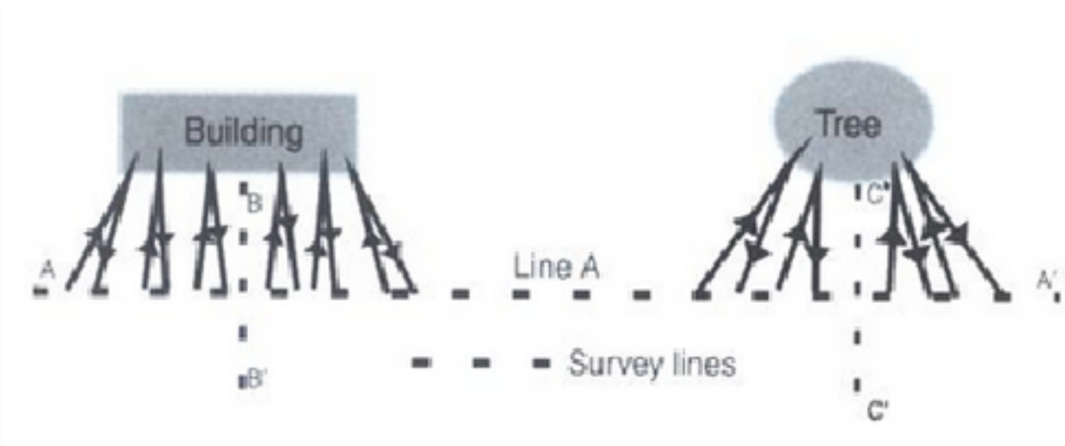
Noise from Above Ground Objects

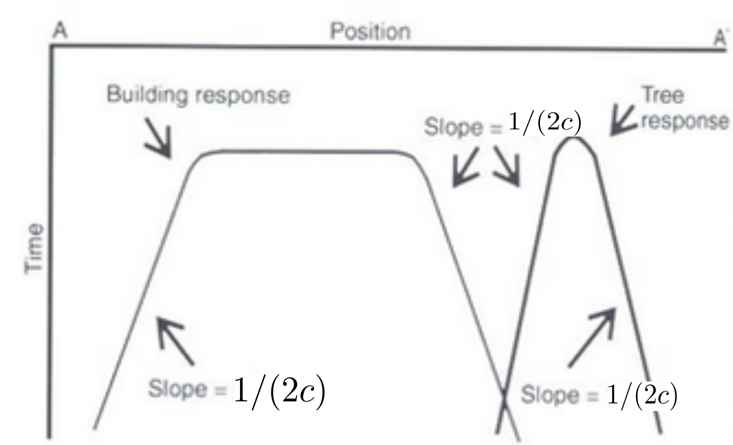
- Signals can reflect off neaby building and trees.
- Two-way travel time:

$$t = \frac{2\left(x^2 + d^2\right)^{1/2}}{V}$$

- Makes hyperbolas in zero offset surveys

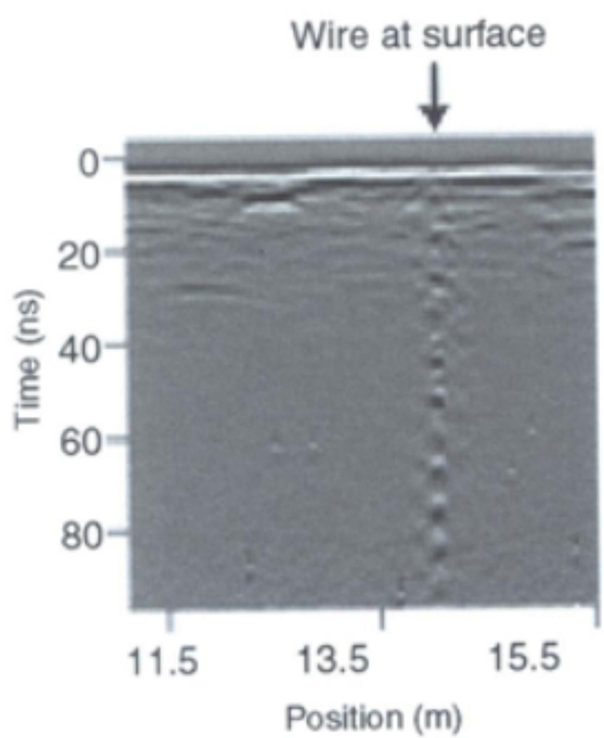




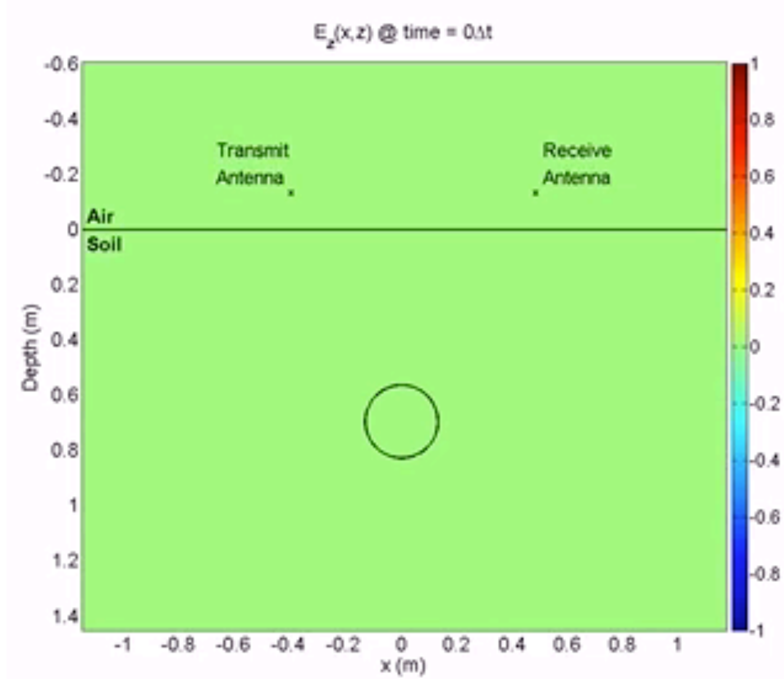


Noise from “Ringing”

- Caused when signals reverberate in regular fashion
- Signal repeatedly bounces within a layer or between objects.



Wire below surface



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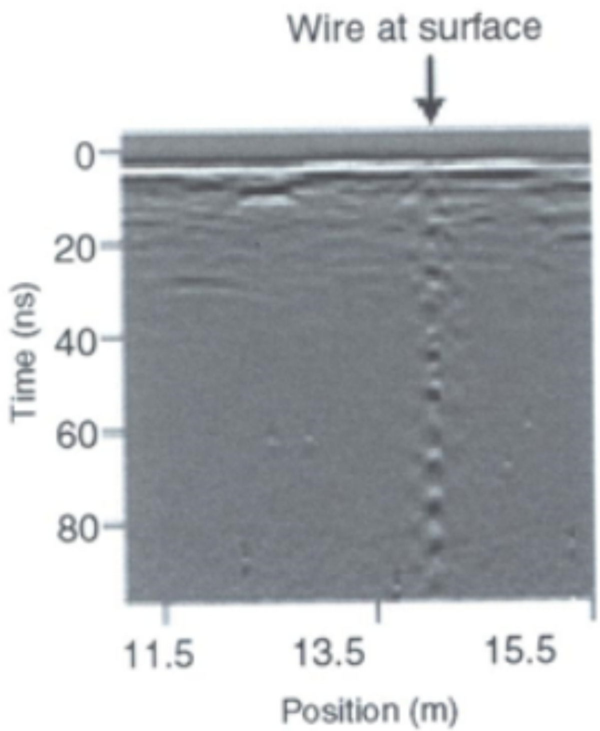
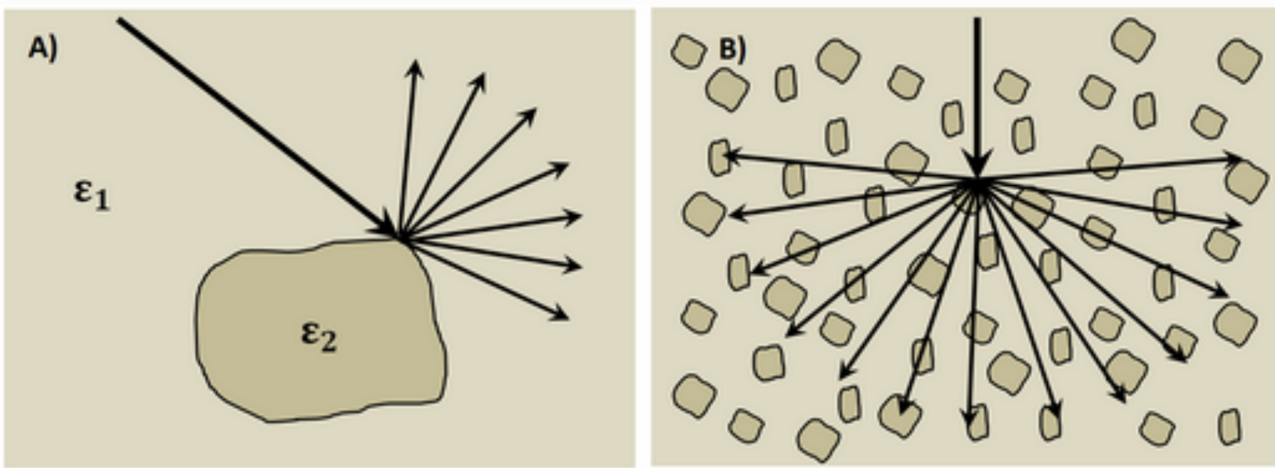


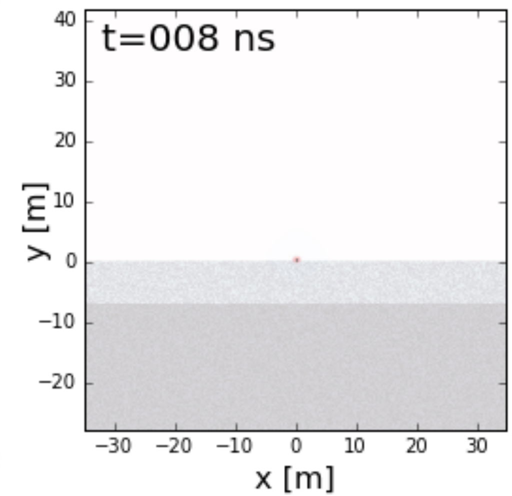
Wire below surface

2 nearby objects

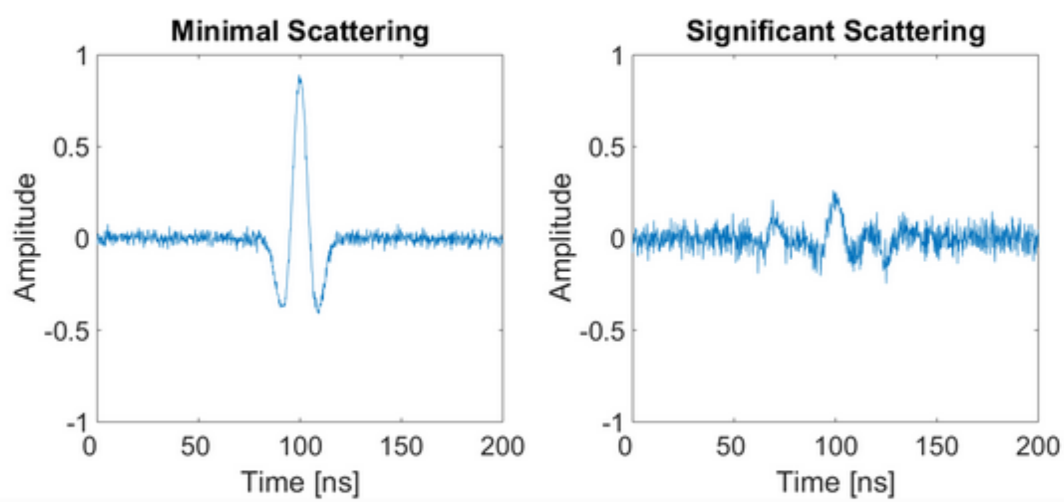
Noise from Scattering

- Deviations in signal path due to localized non-uniformities.





- Reduces amplitude of usable signal and increases noise.



End of GPR lecture 2