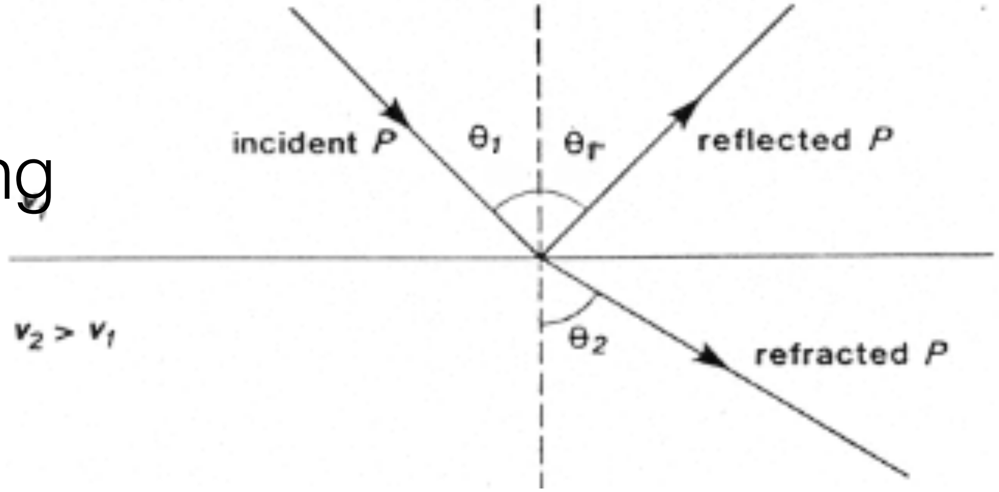
Angles of reflection and refraction

- Now consider an plane wave propagating at an angle θ_1

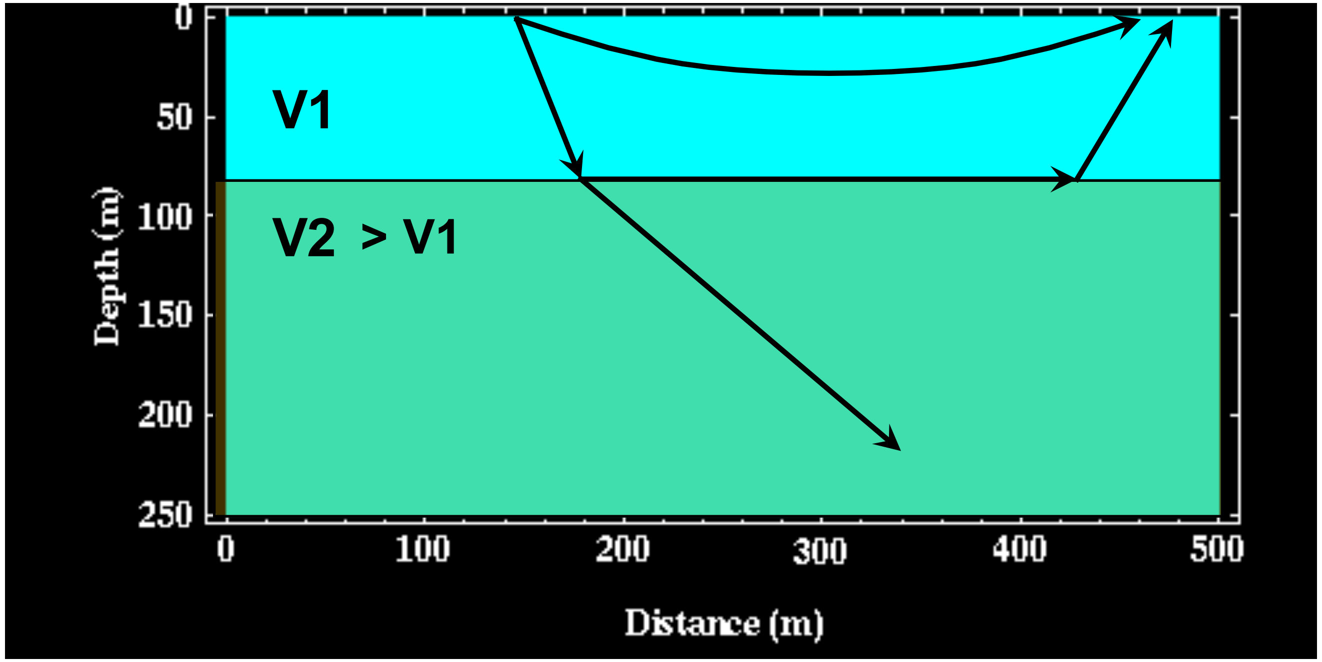


- Law of reflection: reflection angle = incident angle $\boldsymbol{\theta_1 = \theta_r}$

- Law of refraction: refraction angle from Snell's law $\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2}$

Animation of waves

- Slower over faster (most common): v2 > v1



- Notice the relation between wavefronts and rays (arrows)

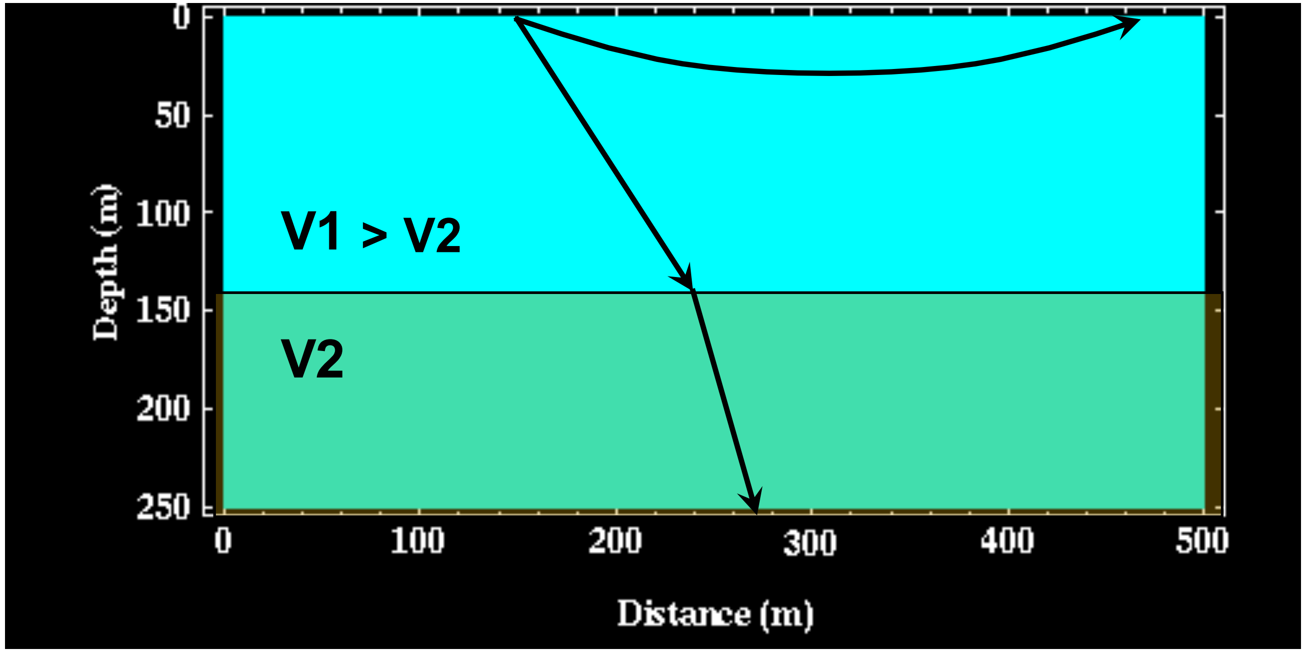
What if…. ?

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

- What if $v_1 > v_2$?
  (ie, faster in top layer)

- Is refraction possible in this situation?

- Implications?



$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

What if…. ?

- What if $\theta_2 = 90°$? This is called the critical angle.

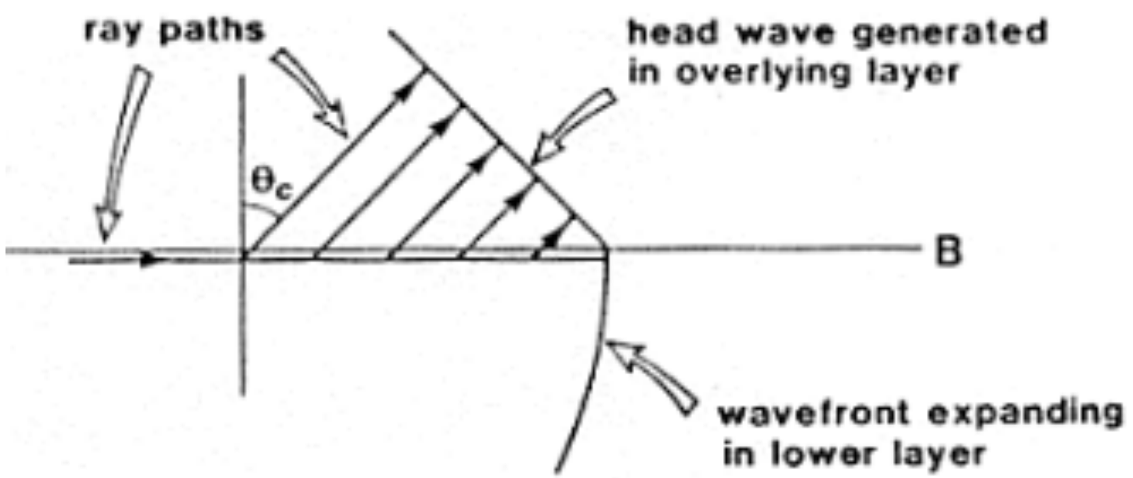
- $\sin\theta_1 = v_1/v_2$

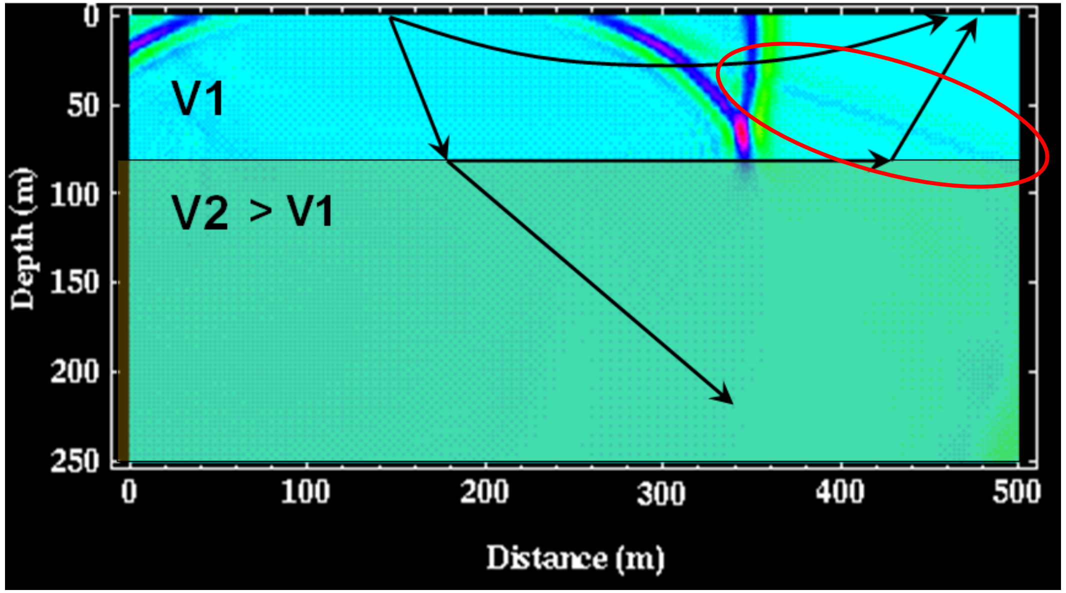
- The refracted ray travels horizontally along the interface

What if…. ?

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

- What if $\theta_2 = 90°$? This is called the critical angle.
- $\sin\theta_1 = v_1/v_2$
- "Head waves" or critically refracted rays send energy back to the surface.





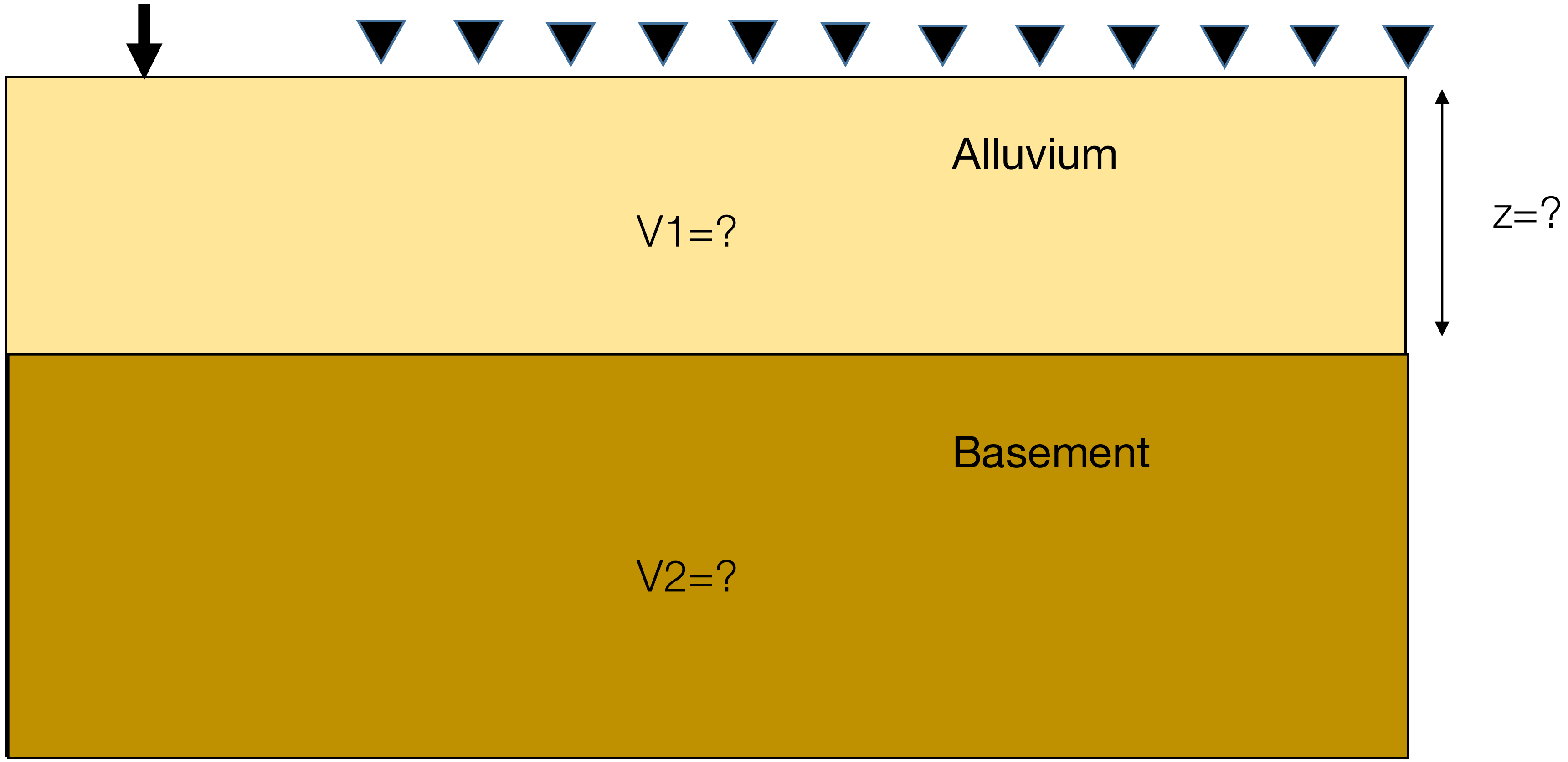
Apps on the web

- http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=16
    - Illustration of reflection and refracted wavefronts using Fresnel-Huygens principles.

- http://staff.washington.edu/aganse/raydemo/RayDemo2.med.html
    - Ray paths in arbitrary 1D earth. Generate the velocity model and observe first arrivals and curved ray paths. (Visualizing bending rays in linearly increasing velocities)

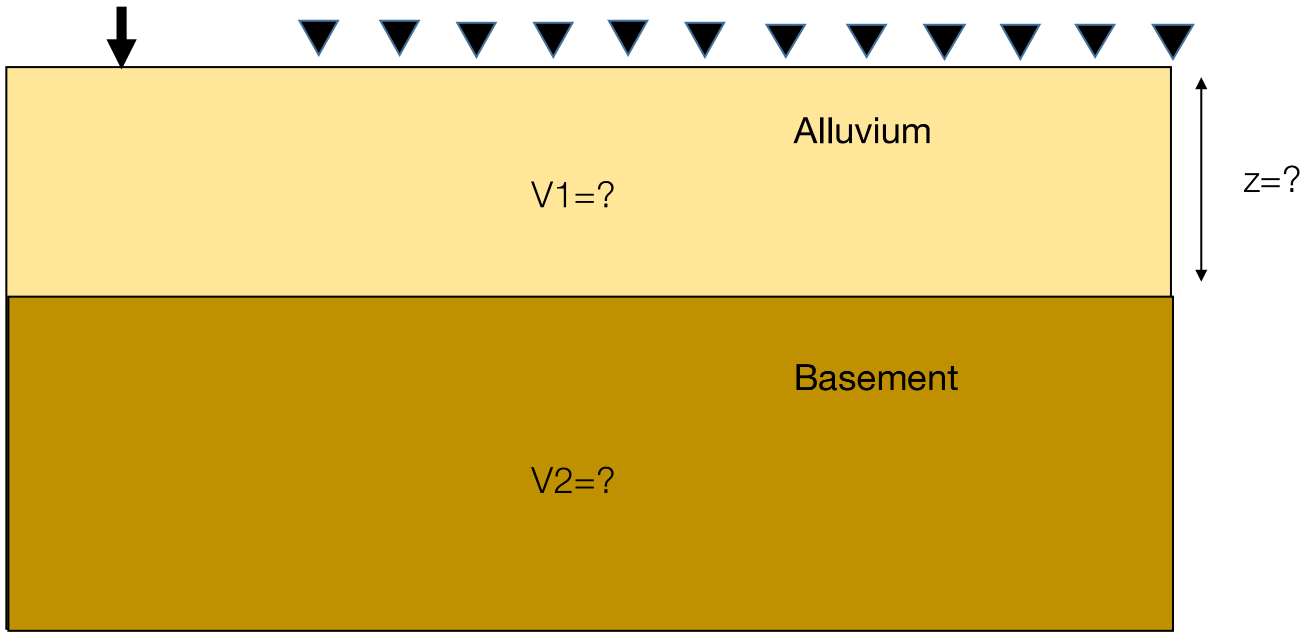
- http://www.iris.edu/hq/programs/education_and_outreach/visualizations
    - Global Earthquakes recorded by US seismometer arrays. Learn about particle motions, wave .

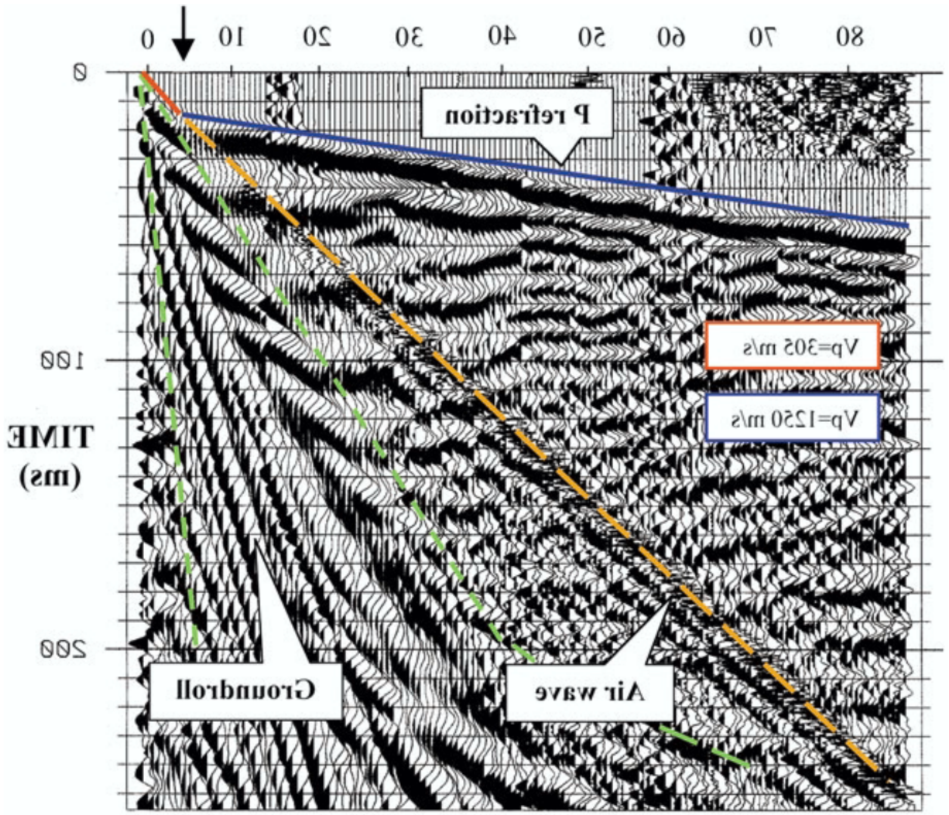
Motivation: travel time



$$t = \frac{d}{v}$$

Real seismic data?





$$t = \frac{d}{v}$$

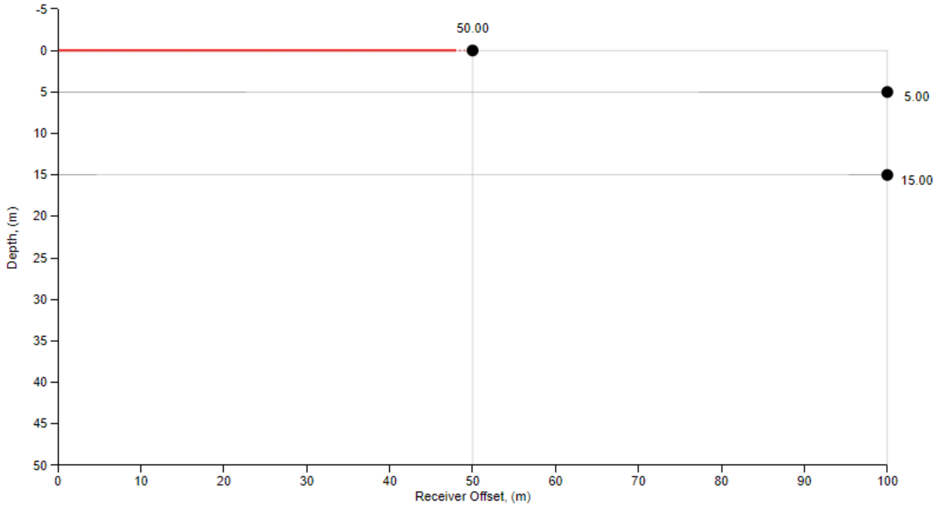
Travel times

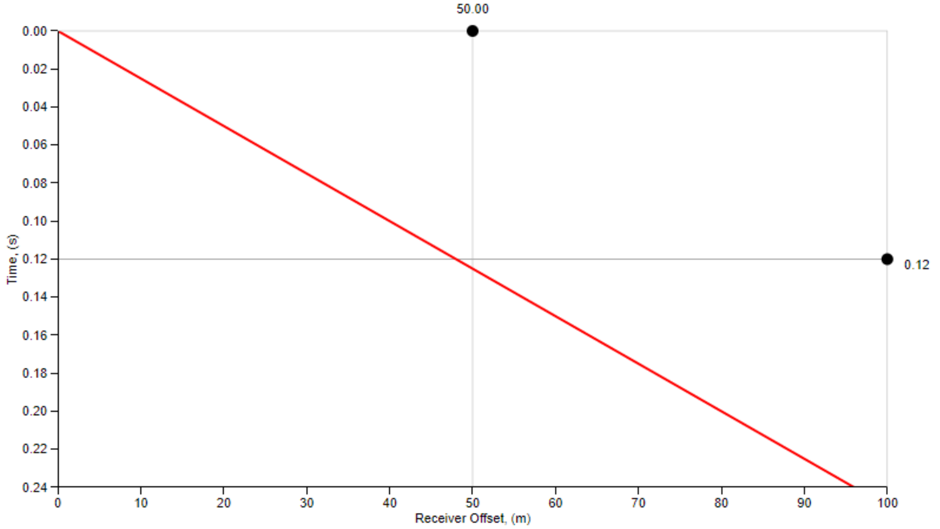
- Time required for a seismic wave to travel from source to receiver in a homogeneous earth layer:

$$t = \frac{d}{v}$$

- Seismic survey: measure time (=data)

- Then estimate subsurface properties

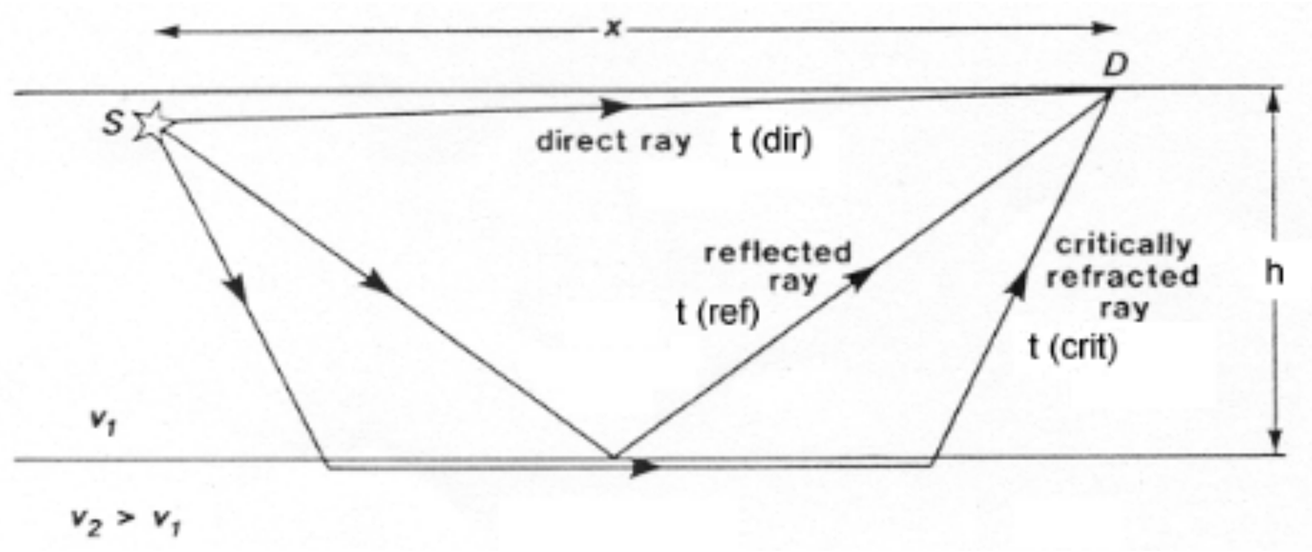




3 principal waves

- Interested in: arrival times at the receiver location

- Direct waves
- Reflected waves
- Critically refracted waves (head waves)

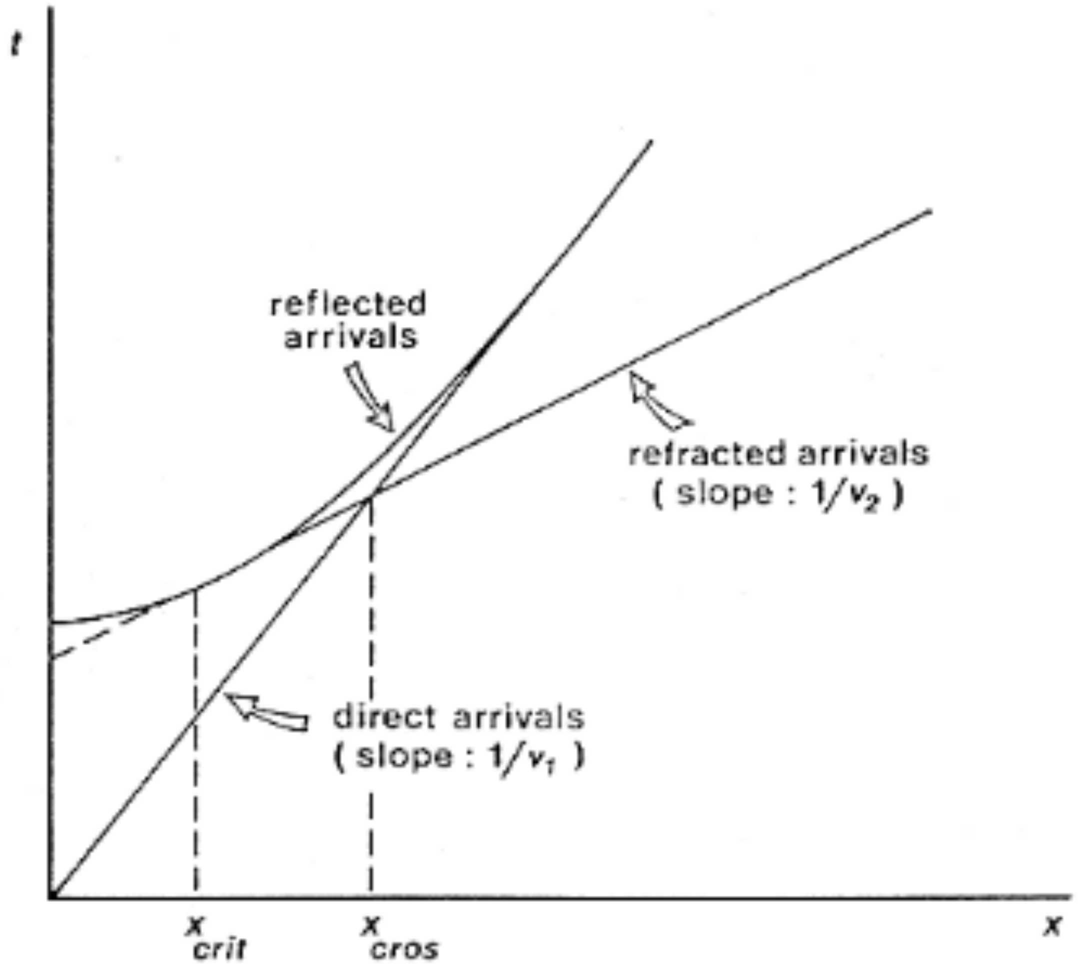


3 principal waves

- Travel time curves

- Critical distance x_{crit}
  - Closed surface point to the source at which the refracted ray can be observed
- Crossover distance x_{cros}
  - Surface point at which the direct and refracted rays arrive at the same time

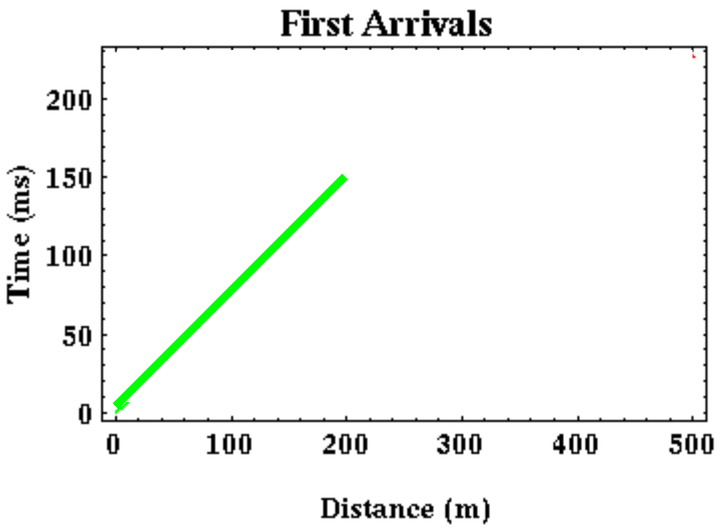
- Note: when the offset from the source is greater than x_{cros}, the refracted ray will be the first signal to arrive.
  - Why is that?

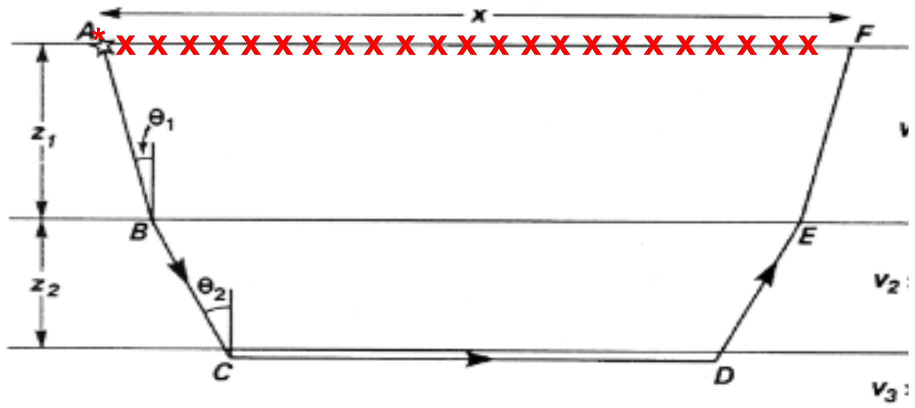


Travel times

- Travel times depend on distance between source and receiver, layer thickness, and wave velocities
- Direct wave

$$t_{dir} = \frac{x}{v_1}$$





Travel times

- Travel times depend on distance between source and receiver, layer thickness, and wave velocities
- Direct wave

$$t_{dir} = \frac{x}{v_1}$$

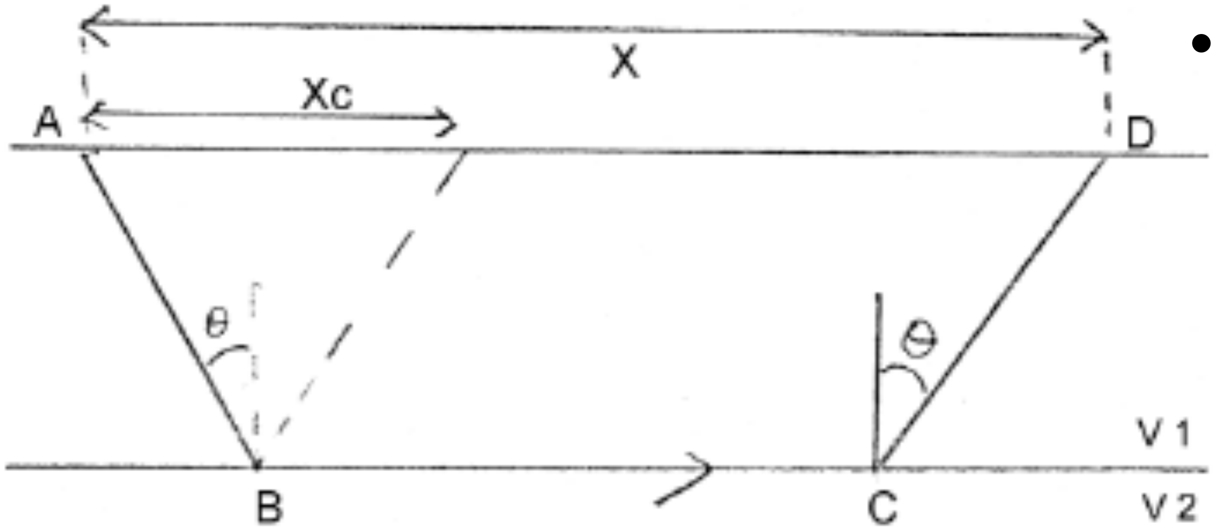
- Reflected wave

$$t_{refl} = \frac{\sqrt{x^2 + 4h^2}}{v_1}$$

- Critically refracted wave

$$t_{refr} = \frac{x}{v_2} + \frac{2h\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

Refracted ray in 2-layer earth



- Critical distance

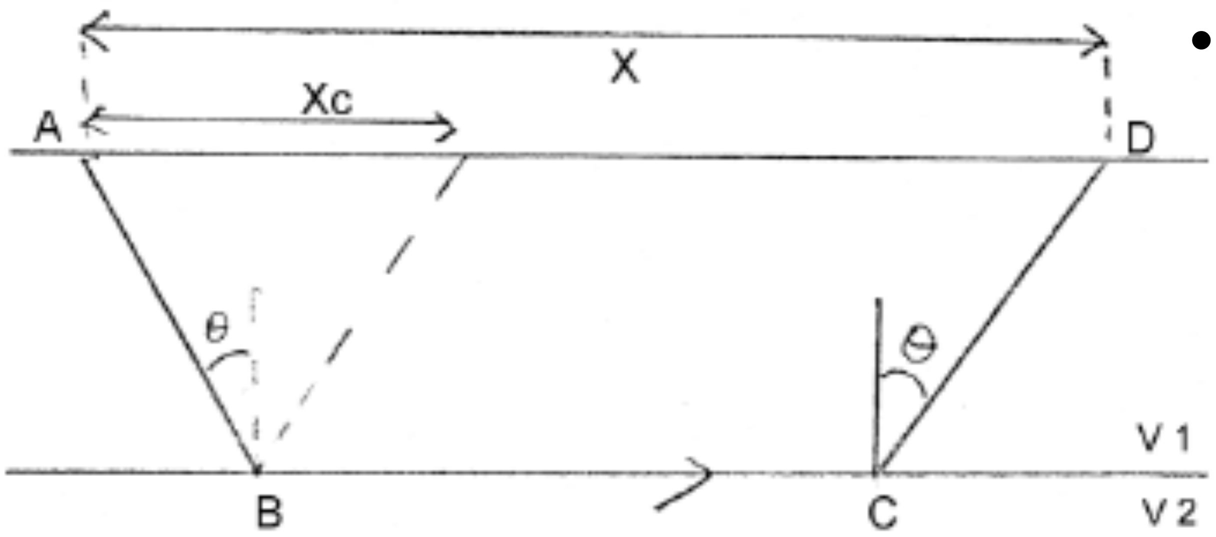
$$x_c = 2z\tan\theta \quad l = \frac{z}{\cos\theta}$$

$$\tan\theta = \frac{x_c}{2z} \quad \cos\theta = \frac{z}{l}$$

- Total travel time is cumulative time for wave to traverse the path ABCD

$$t = t_{AB} + t_{BC} + t_{CD} = \frac{2z}{v_1\cos\theta} + \frac{x - 2z\tan\theta}{v_2}$$

Refracted ray in 2-layer earth



- Total travel time
  - Do some trigonometry to get the following useful relations

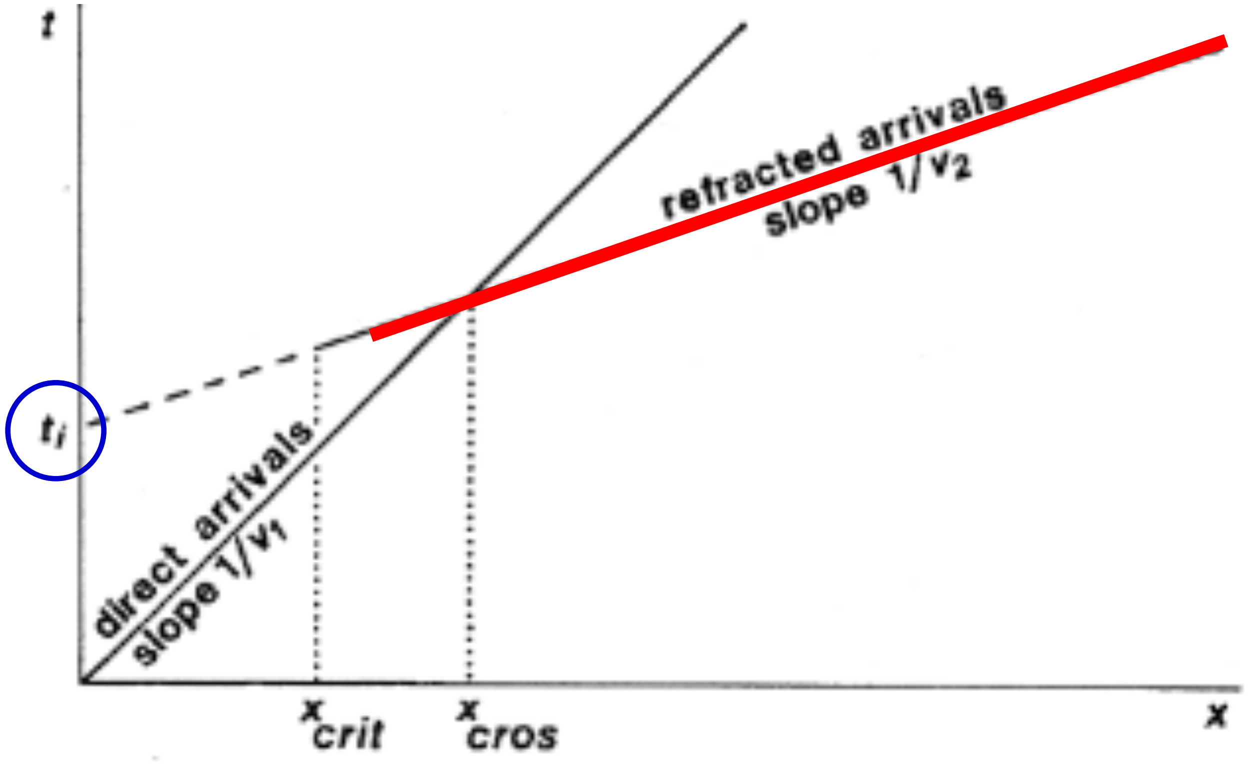
$$
\begin{aligned}
t &= \frac{x}{v_2} + \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2} \\
&= \frac{x}{v_2} + t_i
\end{aligned}
$$

- This says that the travel time curve is a straight line with a slope of $\frac{1}{v_2}$ and an intercept time of t_i

What this looks like on a graph

$$t = \frac{x}{v_2} + \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$
$$= \frac{x}{v_2} + t_i$$

- The slope of the refracted arrival can tell us the velocity of the second layer!
- We can calculate the velocity of the first layer from the direct arrival!
- Note: intercept time t_i is not a “real” time – it is derived from the graph



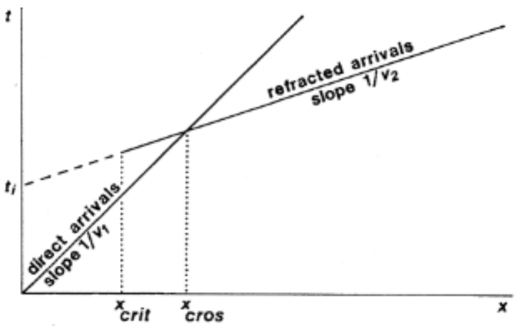
Ok, so…

1. Plot the times of first arrivals on a time-offset plot
2. The direct arrival are observed along a straight line from the origin
   - It's slope is $1/v_1$, giving the velocity of the upper layer
3. The refracted arrival appears as a straight line
   - It has a smaller slope equal to $1/v_2$, giving the velocity of the second layer
4. For the refracted wave, the intercept time equation can be converted to give us the layer thickness!

$$t_i = \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2} \longrightarrow z = \frac{t_i v_1 v_2}{2\sqrt{v_2^2 - v_1^2}}$$

Ok, so…

Now we can compute: v1, v2, and layer thickness



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4. For the refracted wave, the intercept time equation can be converted to give us the layer thickness!

$$t_i = \frac{2z\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

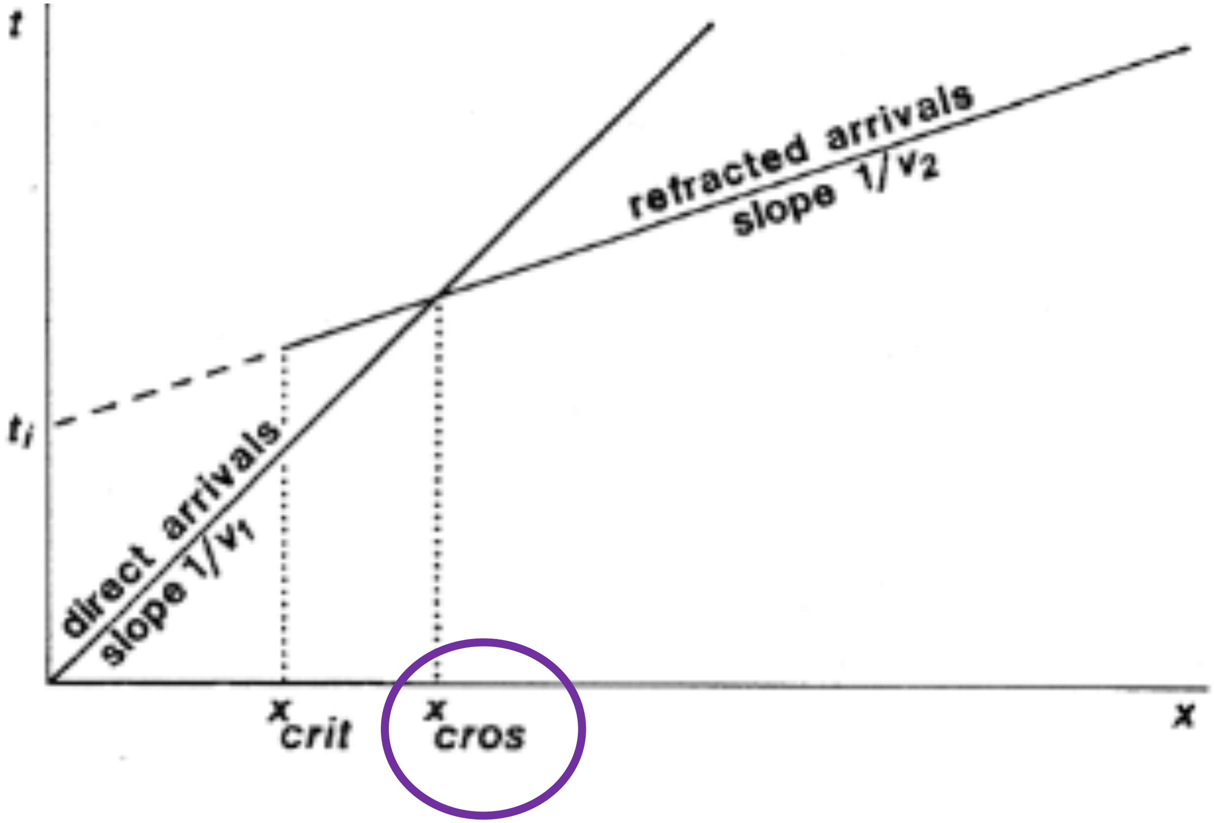
$$z = \frac{t_i v_1 v_2}{2\sqrt{v_2^2 - v_1^2}}$$

One more thing

- Cross-over distance: where the direct wave and the refracted wave arrive at the same time

$$\boxed{\frac{x_{\text{cross}}}{v_1}} = \boxed{\frac{x_{\text{cross}}}{v_2} + t_i}$$

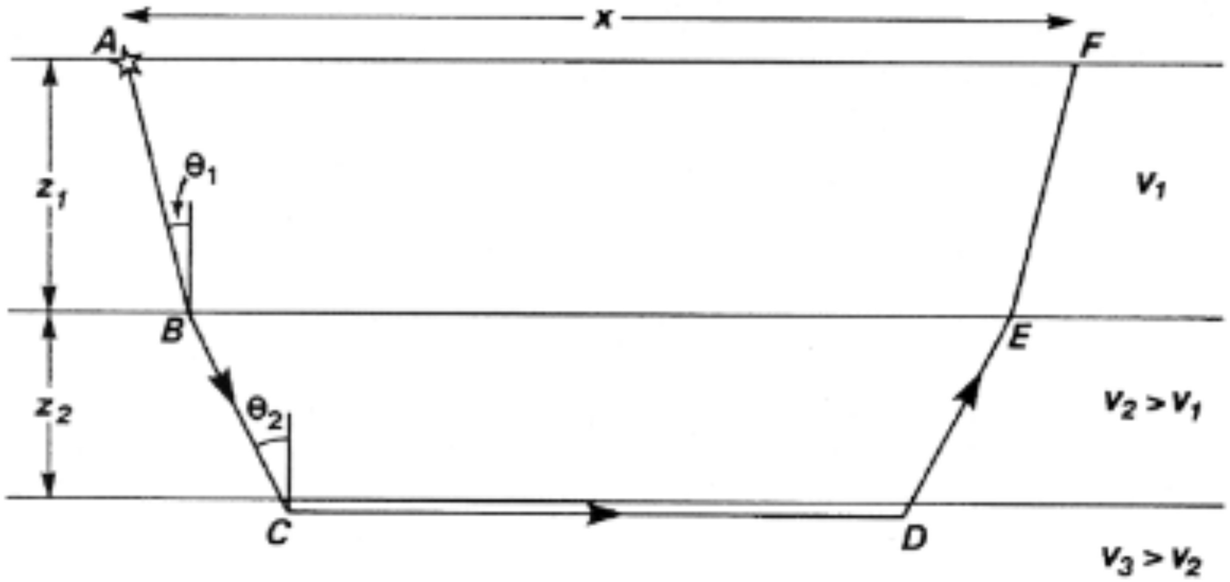
$$x_{\text{cross}} = \left(\frac{v_1 v_2}{v_2 - v_1}\right) t_i$$
$$= 2z\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}$$

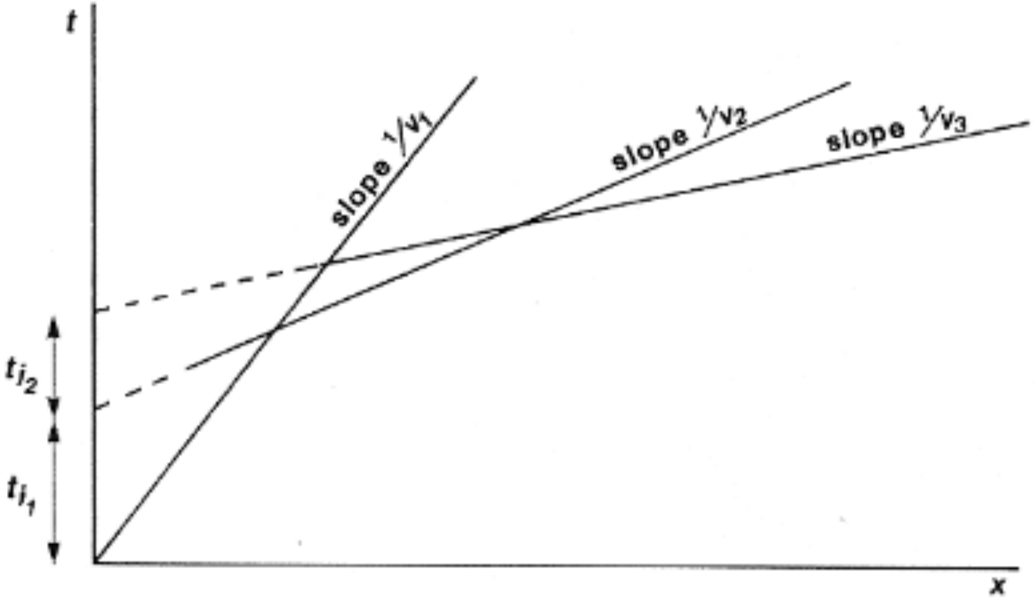


- Another way to calculate one of the variables given values for the others.

What about three layers?

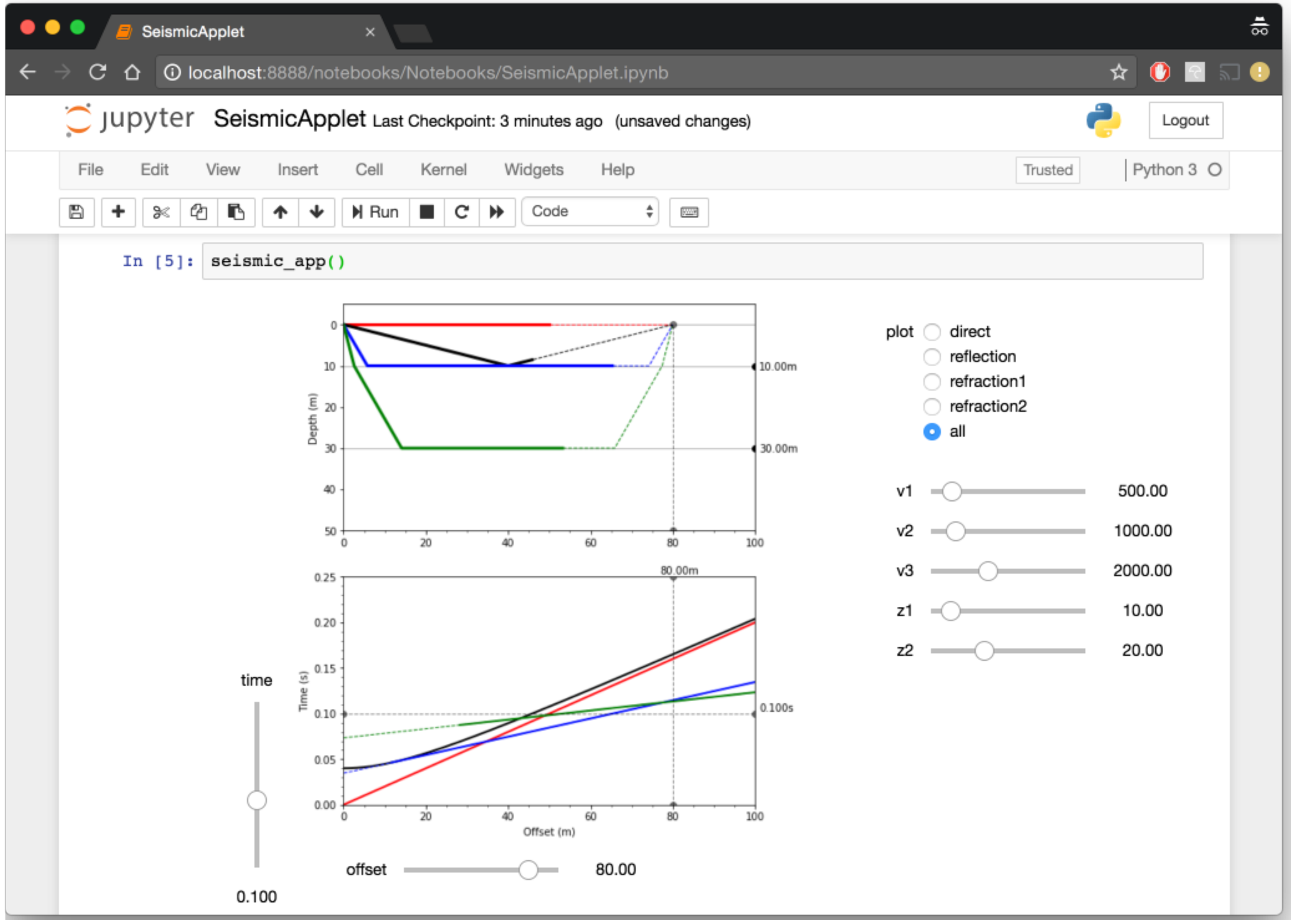
- Snell's law holds: $$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} = \ldots$$





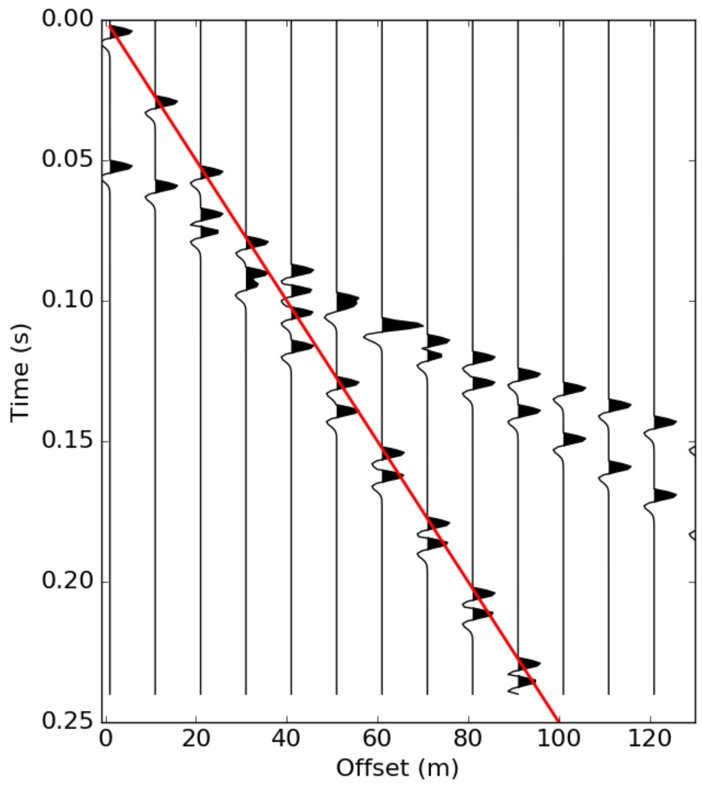
What about three layers?

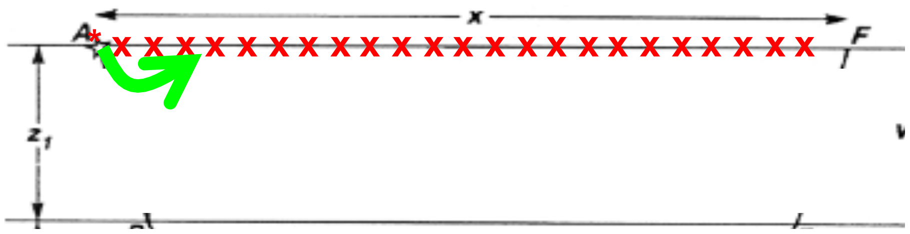
- Explore using the seismic app

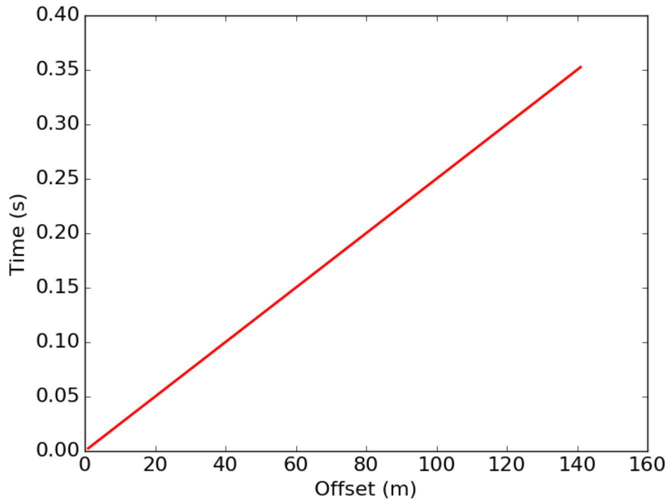


Raw data

- Direct arrivals
- The T-X plot

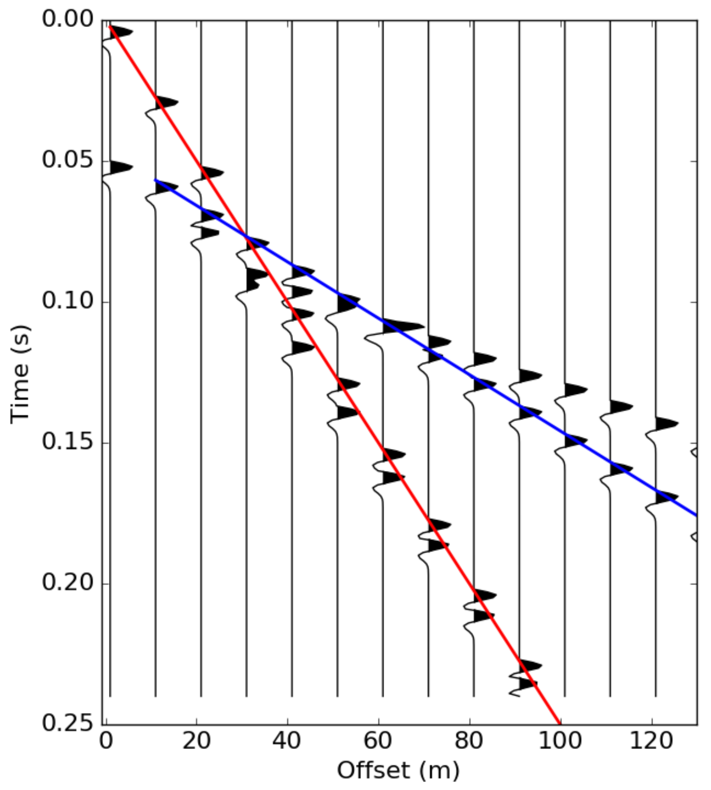


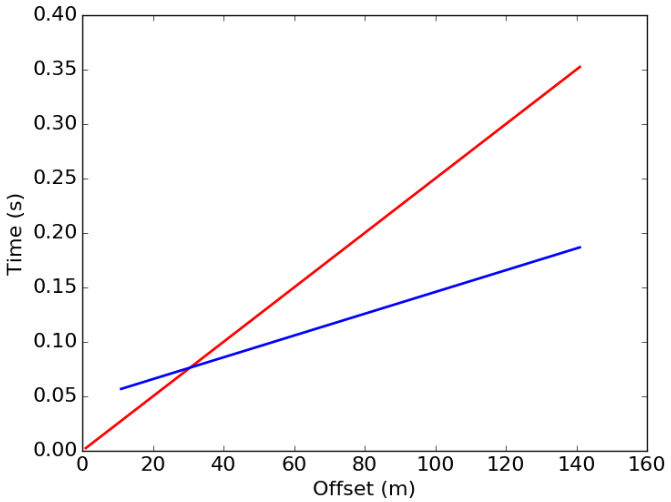


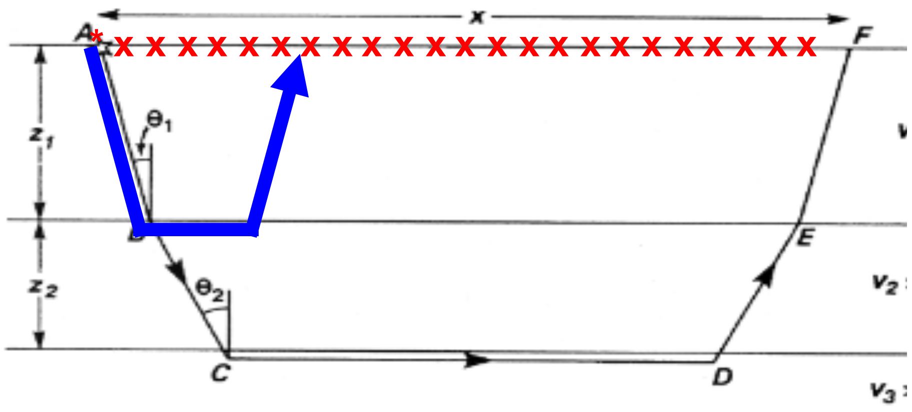


Raw data

- First refractions
- The T-X plot

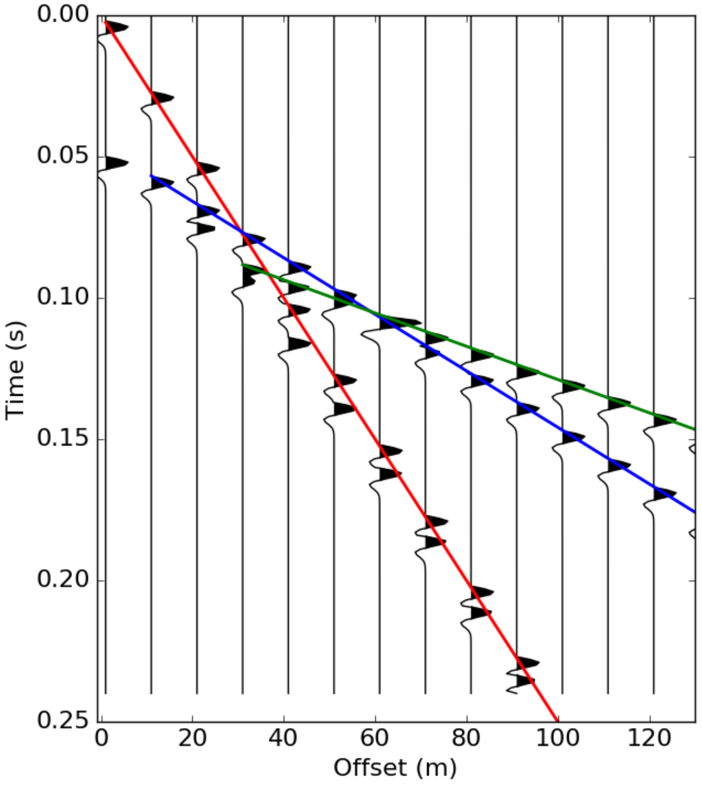


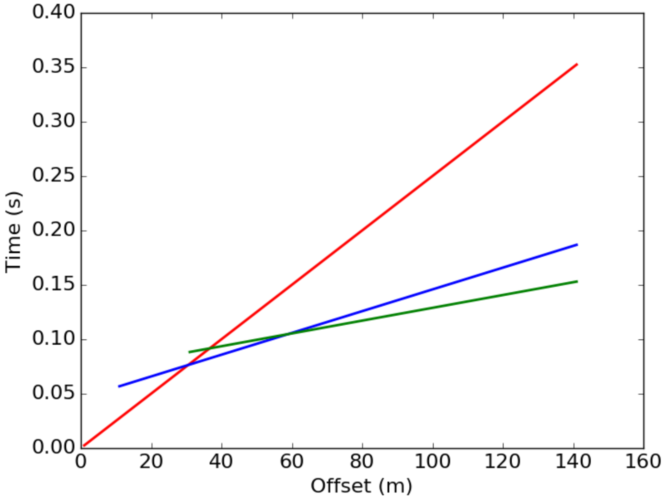


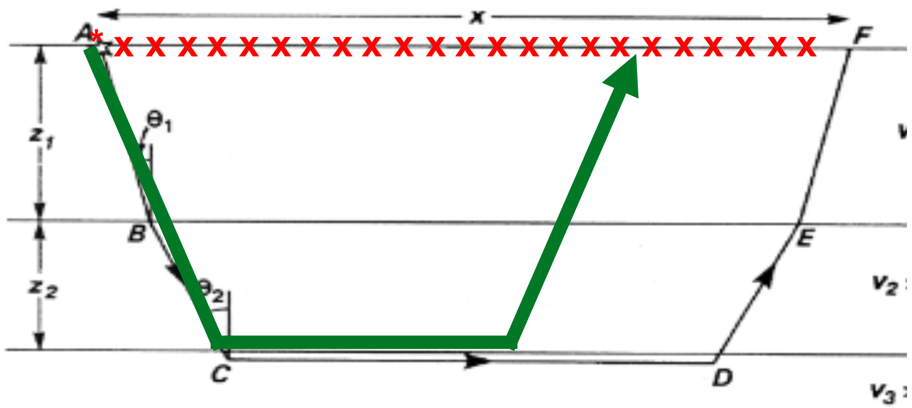


Raw data

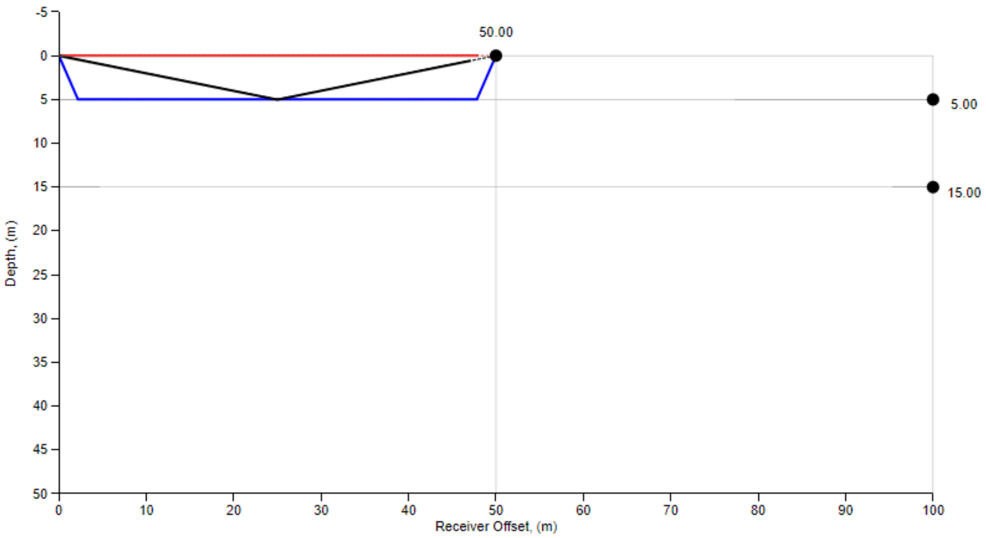
- Second refractions
- The T-X plot

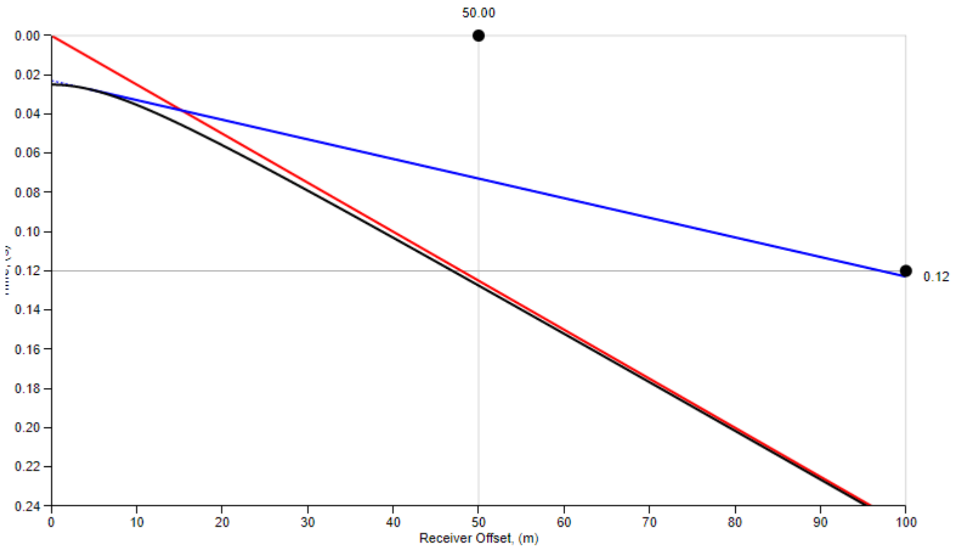






Reflected waves



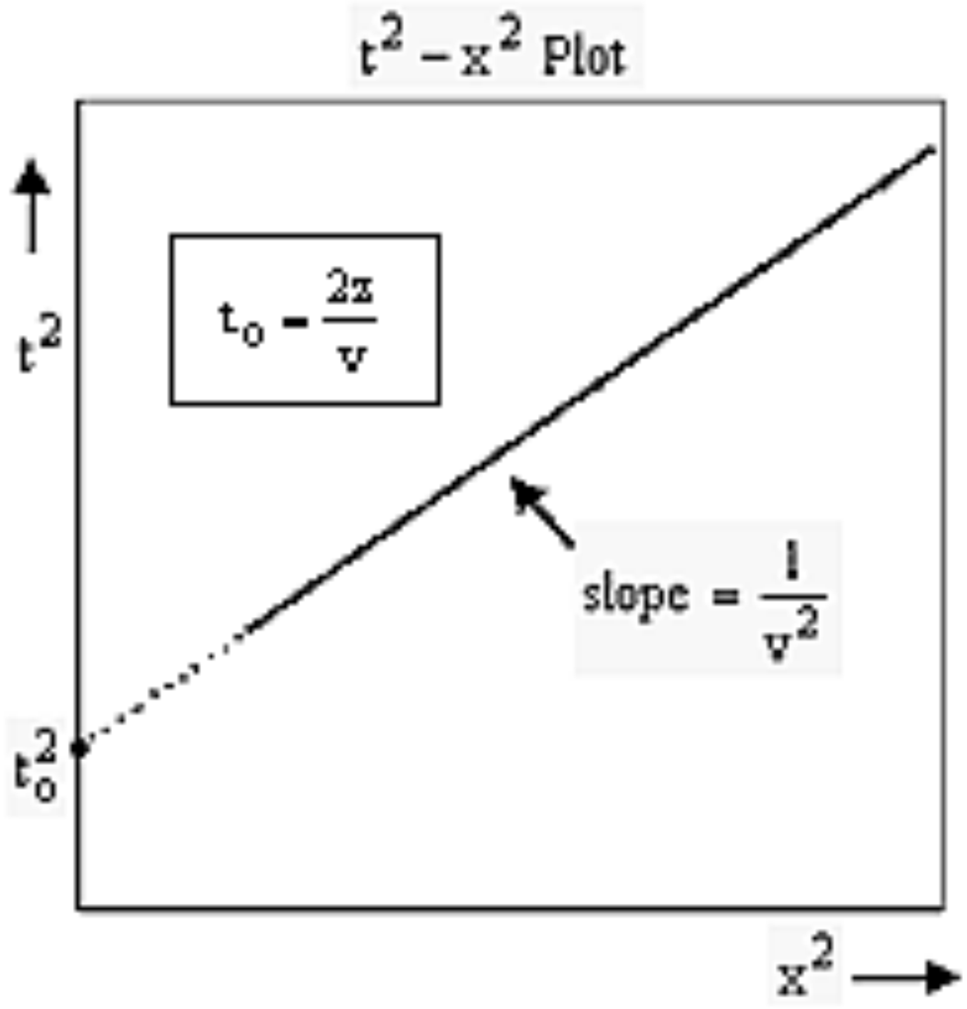


- Reflection occurs if there is a change in the acoustic impedance at the boundary.

- Acoustic impedance: $Z = \rho V$
  - The product of density and velocity (either for P or S waves)

- Travel time: $$t = \frac{(x^2 + 4z^2)^{\frac{1}{2}}}{v}$$

t^2-x^2 plot

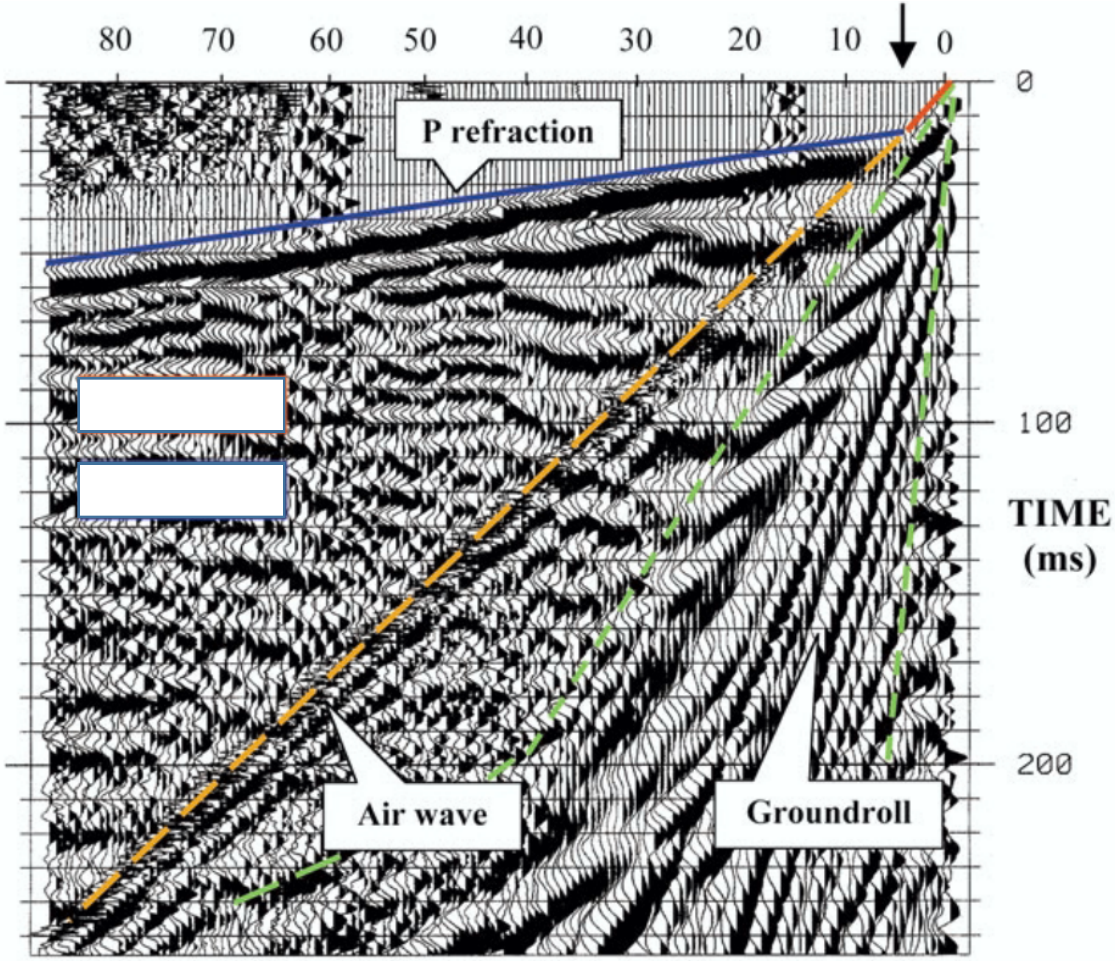


$$t = \frac{(x^2 + 4z^2)^{\frac{1}{2}}}{v}$$

$$t^2 = \frac{x^2}{v^2} + \underbrace{\frac{4z^2}{v^2}}_{t_0^2}$$

Can estimate velocity and thickness of the layer

What can we estimate from data?



End of Seismic lecture 2