Post-Quantum Cryptography QIC 891

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CryptoWorks**21**

Agenda

- Preliminaries: security and post-quantum security

- A little history and awareness for post-quantum

- Hash-based cryptography

- Multivariate public-key cryptography

What is security after all?

Computer Security

Computer security is the **protection afforded** to an information system in order to attain the applicable goals of preserving the **integrity/authenticity, availability**, and **confidentiality** of the resources (includes hardware, software, information/data, and telecommunications).

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  - ⇑ security $\implies$ ⇑ cost
  - Larger parameters and bandwidth/energy consumption, less efficiency

Security Goals (CIA, not the agency)

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  - ▶ symmetric ciphers: AES (block cipher), ChaCha20 (stream cipher) . . .

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  - ▶ control messages: pacemakers, drones

  - ▶ Hash functions, MACs (tag): SHA-2, HMAC (keyed hash), Poly1305.

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- **A**uthenticity (asymmetric): check the origin

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  - Public key crypto: RSA, (EC)DSA, . . .

Quantum cryptanalysis effect I

Public-key crypto broken

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*
  - ✗ No more RSA and DSA (discrete log-based)

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- For 128-bit security "ECC easier target than RSA"[b].

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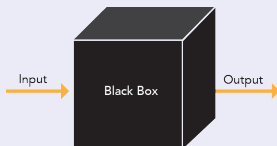
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Quantum killer app II

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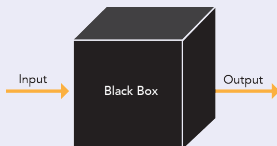# Quantum cryptanalysis effect II

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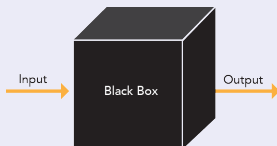
- Complexity is **optimal** Bennett et al. 1997:
  No hope of solving NP-complete problems. $o(\sqrt{N})$ lower bound.
- Partially affects block-ciphers: breaks 128-bit keys in $O(2^{64})$ steps
  Grassl et al. 2016: 2953 qubits required

What are the affected families of cryptosystems?

- **C**onfidentiality, **I**ntegrity (symmetric) ✓

Model of block ciphers

If the encryption key X is chosen at random, then an attacker who does not know the key cannot distinguish between the block cipher and a truly random permutation.

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- **A**uthentication (asymmetric) ✗

Assumptions do not hold in a quantum setting

Mainly based on the hardness of integer factoring or computing discrete logarithm.

But there is hope

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Quantum-safe cryptography includes a broader set of cryprographic algorithms including non-classical assumptions such as laws of quantum physics, e.g. Quantum Key Distribution.

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- Security proofs for classical Full-domain hash RSA or the Fiat-Shamir do not hold for post-quantum security.
    - Do not assume adversary can query an oracle $H(m)$ in superposition.

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One-way functions exist, Lamport-Diffie: 40+ years

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Random-looking multivariate system of non-linear
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$$y_1 = x_1^2 + x_1 x_2 + x_1 x_4 + x_3$$
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Supersingular Isogeny-based crypto:

Computing isogenies between supersingular elliptic curves is hard, CGL hash: 13 years, SIDH kex: 7 years

Post-quantum algorithms aspects

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Supersingular Isogeny-based crypto:

    ★ Pros: Small key sizes (few hundred bytes), many primitives

    ★ Cons: relatively slower

The risk of sticking to pre-quantum crypto

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- 2018, NIST PQC Standardization Conference (submitter's talks)

The risk of sticking to pre-quantum crypto

Mosca's risk analysis formula

Let X be the time to have certain information protected.

Let Y be the time to deploy post-quantum.

Let Z be the Y2Q (countdown of years to quantum).

If $Z < X + Y$: trouble!

What can one expect from Z?

- Huge investments in Quantum Computing research

  NATURE | NEWS

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  - 2017, Google and IBM building general-purpose small prototypes of QCs. Google has no fault-tolerance design plans.
- Experts estimate that large QCs (1k's of qubits) will be around by 2031 with 50% chance.

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  - ▶ **2000**: NIST FIPS 186-2 includes ECDSA and the 15 NIST curves

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- Let's look at the history of ECC (cont.):

2014 Cloudfare's post dedicated to Scott Vanstone

W.r.t. **https** certificates, despite ECDSA being much faster than RSA for TLS handshake/signing, $>$ **90% of the certificates used on the web in 2014 were RSA-based**.

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Main reason

Web sites owners slow to adopt new certificates due to maintainance of **compatibility with legacy browsers** that do not support the new algorithms. – Sullivan, N. 2014

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The good news to ECC

In **Apr'17**, ECDSA finally **surpassed RSA** with 60% of all TLS connections. Recall that ECDSA was proposed in 1992 for DSS. It took 25 years for ECDSA to become widely deployed from its conception and **17** years from its standardization. – Cloudfare report.

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The bad news to ECC

"For those partners and vendors that have not yet made the transition to Suite B algorithms, we recommend **not** making a significant expenditure to do so at this point but instead to prepare for the upcoming quantum resistant algorithm transition." – NSA 2015 announcement

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Jul'18, Pereira's suggested bound:

Conjectured bound $Y \geq 20$

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> $$\text{Conjectured bound } Y \geq 20$$
> $$\text{If } Z < X + 20 \Rightarrow \text{trouble!}$$

- It is true that the more robust TLS infrastructure and experienced community will be faster at deploying implementations.
- On the other hand:
  - NIST standardization analysis phase will take 5 years and 2 more for the drafts – D. Moody
  - The field of quantum cryptanalysis has only just begun (recent attacks against NTRU and binary MQ)
  - Two lines of attacks imply higher chances to break post-quantum assumptions.

Agenda

- Preliminaries: security and post-quantum security

- A little history and awareness for post-quantum

- Hash-based cryptography

- Multivariate public-key cryptography

Hash-based Signatures (HBS)

1976, Diffie-Hellman in "New directions in cryptography" introduce HBS:

One-way message authentication has a partial solution suggested to the authors by Leslie Lamport of Massachusetts Computer Associates. This technique employs a one-way function f mapping k-dimensional binary space into itself for k on the order of 100. If the transmitter wishes to send an N bit message he generates $2N$, randomly chosen, k-dimensional binary vectors $x_1, X_1, x_2, X_2, \cdots, x_N, X_N$ which he keeps secret. The receiver is given the corresponding images under f, namely $y_1, Y_1, y_2, Y_2, \cdots, y_N, Y_N$. Later, when the message $\boldsymbol{m} = (m_1, m_2, \cdots, m_N)$ is to be sent, the transmitter sends x_1 or X_1 depending on whether $m_1 = 0$ or 1. He sends x_2 or X_2 depending on whether $m_2 = 0$ or 1, etc. The receiver operates with f on the first received block and sees whether it yields y_1 or Y_1 as its image and thus learns whether it was x_1 or X_1, and whether $m_1 = 0$ or 1. In a similar manner the receiver is able to determine $m_2, m_3, \cdots, m_N$. But the receiver is incapable of forging a change in even one bit of $\boldsymbol{m}$.

1976, Lamport-Diffie signature: sign a n-bit message

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$$sk = \{(x_1, X_1), (x_2, X_2), \cdots, (x_n, X_n)\}$$

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Evaluate f on them: $y_i, Y_i \leftarrow f(x_i), f(X_i)$ and publishes:

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To verify $\sigma = (\sigma_1, \ldots, \sigma_n)$ check if

$$f(\sigma_i) \stackrel{?}{=} \begin{cases} y_i \Rightarrow m_i = 0 \\ Y_i \Rightarrow m_i = 1 \end{cases}$$

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  The size of sk, pk would be $2kN \approx 2 \cdot 100 \cdot 10^6 = 200M$ bits

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    Therefore, g should have stronger properties than f (collision resistance).

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Remark: Lamport-Diffie is a one-time signature (OTS)!

1. Assume a message $m_1 = (\mathbf{0}, 1, 1) \in \{0,1\}^3$ is signed. Thus

$$\sigma = \{\overline{\mathbf{0}}x_1 + \mathbf{0}X_1, \overline{1}x_2 + 1X_2, \overline{1}x_3 + 1X_3\}$$
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4. Then it's easy to forge a signature of $m_3 = (1, 1, 1)$ for example. Thus, Lamport-Diffie signature is OTS and each key pair can be only used once.

Hash-based Signatures (HBS)

Security assumption of Lamport-Diffie (LD)

1. one-way function f is hard to invert and

2. it is hard to find different input values that map to a same output

Put the above together, HBS rely on the existence of modern **cryptographically secure hash functions**.

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$$m = (m_1 || \cdots || m_{\lceil n/w \rceil})$$

where $m_i \in \{0,1\}^w$ can be viewed as w-bit integers.

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▸ Actually, it is possible to do better:

$$pk = y = g(y_1 \| \cdots \| y_{\lceil n/w \rceil})$$

▸ Public key pk boils down to one hash value (instead of $2n$).

Hash-based Signatures (HBS)

(cont. $\cdots$)

Winternitz OTS

- Sign: compute

$$\sigma = (f^{m_1}(x_1), \cdots, f^{m_{\lceil n/w \rceil}}(x_{\lceil n/w \rceil}))$$

Hash-based Signatures (HBS)

(cont. $\cdots$)

Winternitz OTS

- Sign: compute

$$\sigma = (f^{m_1}(x_1), \cdots, f^{m_{\lceil n/w \rceil}}(x_{\lceil n/w \rceil}))$$

- Verify: compute

$$y_i' = f^{2^w - 1 - m_i}(\sigma_i), \text{ for all } i$$
$$y' = g(y_1' \parallel \cdots \parallel y_{\lceil n/w \rceil}')$$

Check

$$y' \overset{?}{=} y$$

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- Violates the notion of existential unforgeability!

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which can be stored in $t = log_2(\lceil n/w \rceil \cdot (2^w - 1))$ bits.

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- Extend the message to be $m || CS = (m_1, \cdots, m_{\lceil n/w \rceil + \lceil t/w \rceil})$

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(checksum: cont ...)

- Given a signature

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- If a forger targets $m_j \in CS$, an inversion is also implied on m_i.
  Thus, the checksum protects against such attacks.

Winternitz OTS example

- Let $w = 2$ and one wants to sign the ($n = 4$)-bit message

$$m = (1011)$$

with $\lceil n/w \rceil = 2$ and $m = (m_1 = 2, m_2 = 3)$.

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Thus, $CS = (0001) = (m_3 = 0, m_4 = 1)$ and the actual message is

$$m||CS = (2, 3, 1, 0)$$

(cont . . .) Winternitz OTS: example

- Key generation including CS:

$$sk = (x_1, \cdots, x_4)$$
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  - ▸ Compute and check

$$(y_1', y_2', y_3', y_4') = (f^{3-2}(\sigma_1), f^{3-3}(\sigma_2), f^{3-1}(\sigma_3), f^{3-0}(\sigma_4))$$

$$g(y_1' \parallel y_2' \parallel y_3' \parallel y_4') \stackrel{?}{=} y$$

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- Since hash evaluations can be very fast, it is a reasonable tradeoff

Hash-based Signatures (HBS)

Scheme	$n = k$	$|PrivKey|$	$|PubKey|$	$|Sig|$
LD OTS	256	16	16	16
WOTS (w=2)	256	4.2	32 bytes	4.2
WOTS (w=8)	256	1.1	32 bytes	1.1
WOTS (w=16)	256	0.6	32 bytes	0.6

Table: Parameter sizes for one-time signatures in KiB

1979, Merkle turns OTS into multi-time signatures

Y_{pub}

$H(p_i \,||\, p_{i+1})$

$p_{i/2}$

p_i p_{i+1}

$h = 3$

2^h OTS

OTS OTS OTS OTS OTS OTS OTS OTS
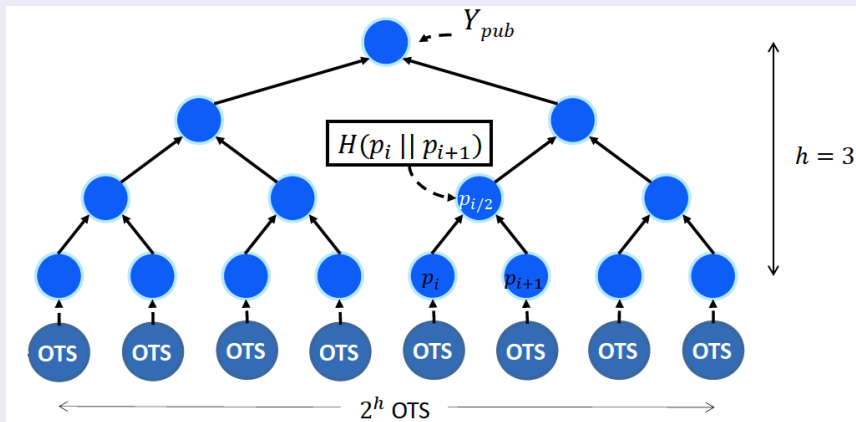
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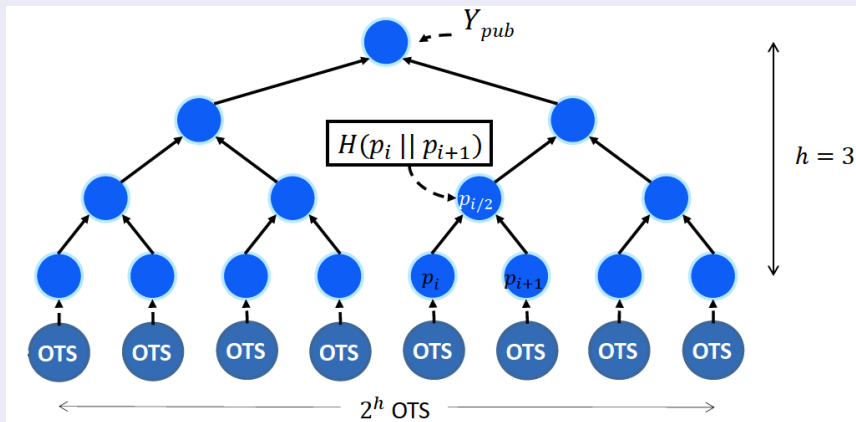
- OTS can be **any** one-time signature scheme.
- Y_{pub} authenticates 2^h OTS key pairs.

Hash-based Signatures (HBS)

1979, Merkle signature: **Key Generation**
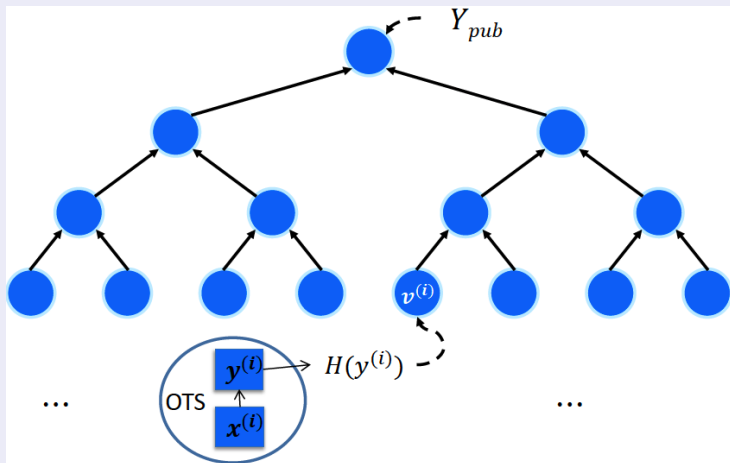
Hash-based Signatures (HBS)

1979, Merkle signature: **Sign**
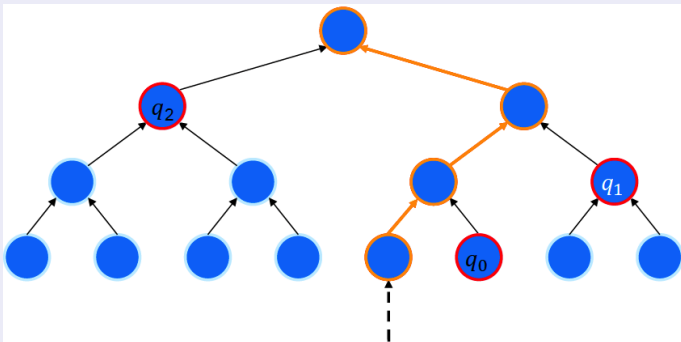
$$\sigma(data) = \big(i, Sign_{OTS}(x^{(i)}, data), v^{(i)}, (q_0, ..., q_{h-1})\big)$$

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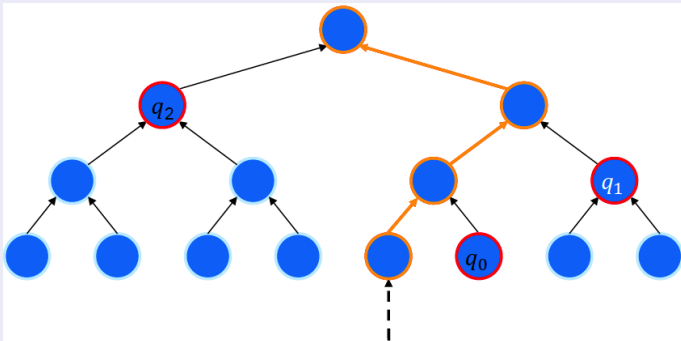
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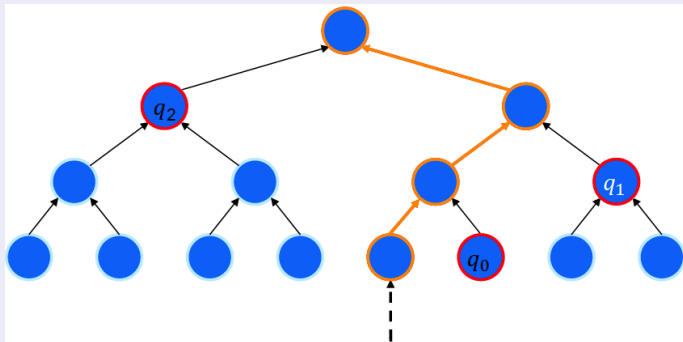
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- Stateful: susceptible to some attacks, e.g. 'restart attacks'

Time efficiency of the Merkle signature
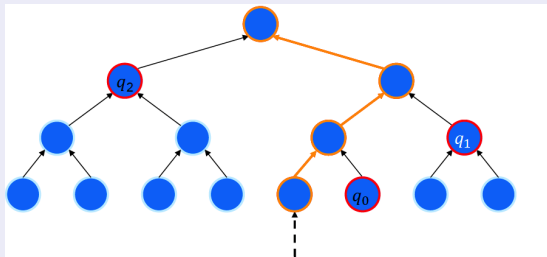
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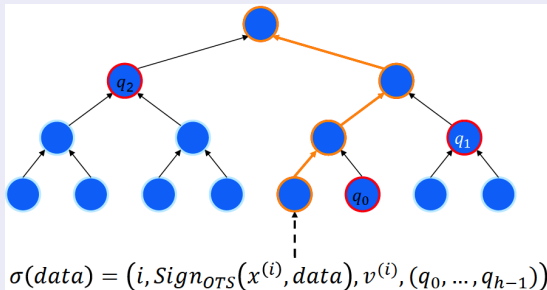
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- Improvement by BDS'08.
  - Store strategic (higher) nodes on a state during KeyGen.
  - Allows for a tradeoff between size of the state *vs* # leaf computations at each signature.

Hash-based Signatures (HBS)

1979, Merkle signature: **Verify**

$\sigma(data) = (i, \sigma_{OTS}, v^{(i)}, (q_0, \dots, q_{h-1}))$

$Y'_{pub} \overset{?}{=} Y_{pub}$

$H(v^{(i)} \| q_0)$

1. $\boldsymbol{y'^{(i)}} \leftarrow Ver_{OTS}(\sigma_{OTS}, data)$,
2. $\boldsymbol{v'^{(i)}} \leftarrow H(\boldsymbol{y'^{(i)}})$,
3. $\boldsymbol{v'^{(i)}} \overset{?}{=} v^{(i)}$

ok?

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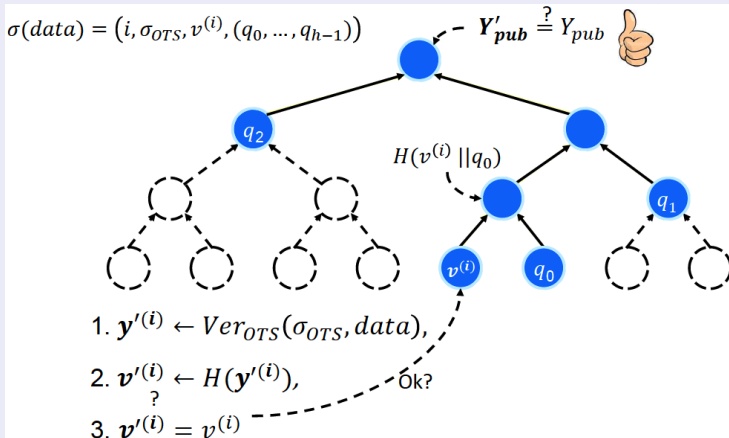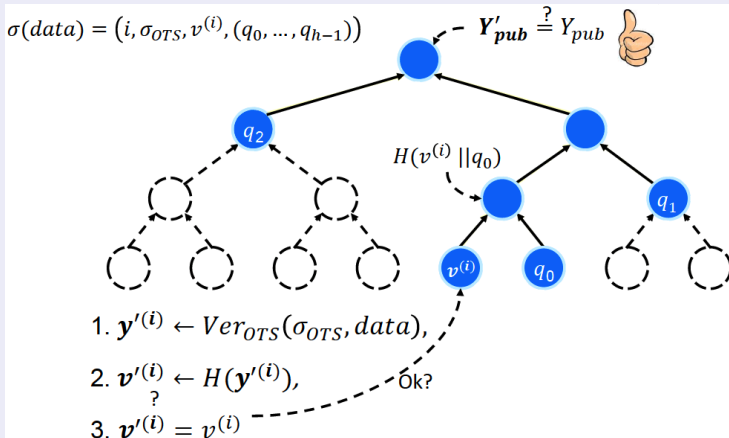
- An obvious optimization is not sending $v^{(i)}$. Verifier only checks the root.

Space efficiency of the Merkle signature

- Private key size: $2^h \cdot |sk_{OTS}|$

- Public key size: size of hash H, e.g. 256 bits.

- Signature size: $|\sigma| = |i| + |\sigma_{OTS}| + |v^{(i)}| + |(q_0, \cdots, q_{h-1})|$

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Merkle parameter sizes example

- $|f| = |H| = n = 256, h = 10$

Scheme	$|PrivKey|$	$|PubKey|$	$|Sig|$
Merkle+LD	16 MiB	32 bytes	16.4 KiB
Merkle+WOTS (w=2)	4.2 MiB	32 bytes	4.5 KiB
Merkle+WOTS (w=16)	0.6 MiB	32 bytes	0.9 KiB

Table: Parameter sizes for Merkle multi-time signature (**1024 signatures**)

Merkle signature: XMSS'11 introduces additional properties
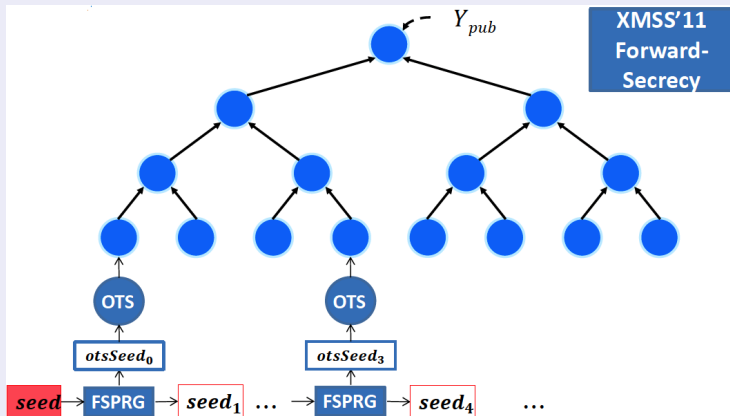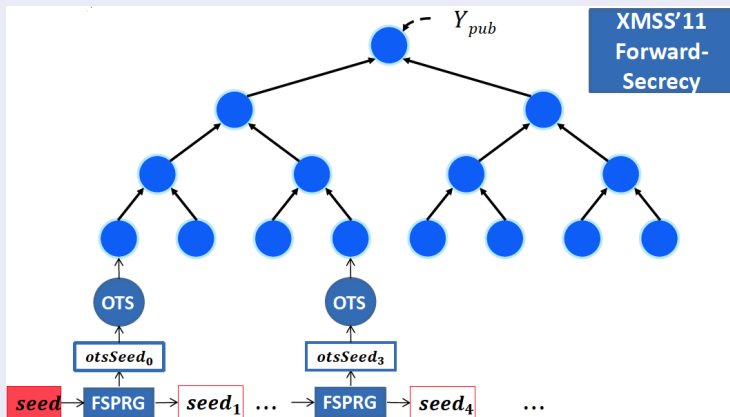
- XMSS uses the variant WOTS$^+$. Collision-resistance unecessary.

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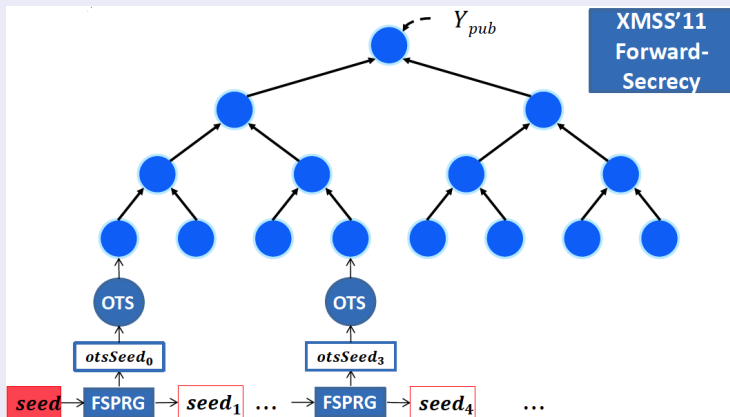
- XMSS uses the variant WOTS$^+$. Collision-resistance unecessary.
- Implication: half-size hashes can be used safely.

Merkle signature: implemention of PRNG and hash function

- Matyas-Meyer-Oseas: block-cipher-based hash function
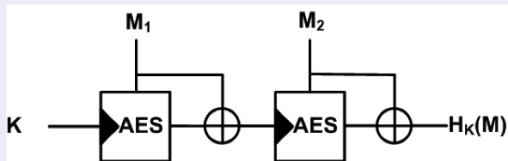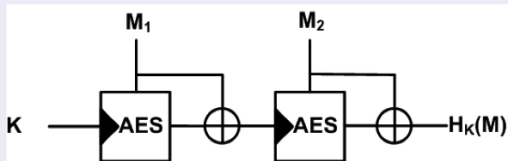
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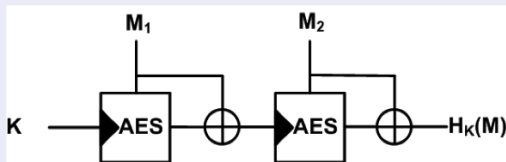
- Fast optimized (hw/sw) block-ciphers available in many platforms

Merkle signature: implemention of PRNG and hash function

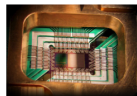- Matyas-Meyer-Oseas: block-cipher-based hash function

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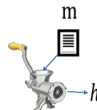- FSPRG by Standaert et al. 2010:
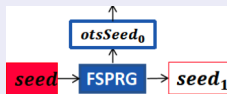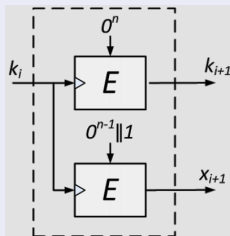
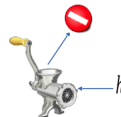# Hash-based Signatures (HBS) – A holistic view

Post-quantum security

Only require hash functions
(efficient/minimal security assumption)

m

h

No reliance on trapdoors

h

Robust security (1976)
(cryptanalysis with little progress)

www.ieee.org/about/awards/bios/hamming_recipients

Larger signatures

References I

Shor, Peter W (1994).
"Algorithms for quantum computation: Discrete logarithms and factoring".

Proos, John and Christof Zalka (2003).
"Shor's discrete logarithm quantum algorithm for elliptic curves".

Häner, Thomas, Martin Roetteler, and Krysta M Svore (2016).
"Factoring using 2n+2 qubits with Toffoli based modular multiplication".

Roetteler, Martin et al. (2017).
"Quantum resource estimates for computing elliptic curve discrete logarithms".

Grover, Lov K (1996).
"A fast quantum mechanical algorithm for database search".

Bennett, Charles H et al. (1997).
"Strengths and weaknesses of quantum computing".

Grassl, Markus et al. (2016).
"Applying Grover's algorithm to AES: quantum resource estimates".

References II

Merkle, Ralph Charles (1979).
"Secrecy, authentication, and public key systems".

Standaert, FranCcois-Xavier et al. (2010).
"Leakage resilient cryptography in practice".