Post-quantum cryptography I

Geovandro C. C. F. Pereira
Institute for Quantum Computing
University of Waterloo

CryptoWorks21

Agenda

- Preliminaries
- A little history and awareness
- Hash-based signatures

What is security after all?

Computer Security

Computer security is the **protection afforded** to an information system in order to attain the applicable goals of preserving the **integrity, availability**, and **confidentiality** of the resources (includes hardware, software, information/data, and telecommunications).
– NIST Computer Security Handbook

Security Goals (CIA, not the agency)

- **C**onfidentiality (symmetric): prevent eavesdropping

Security Goals (CIA, not the agency)

- **C**onfidentiality (symmetric): prevent eavesdropping
  - instant messaging, local storage

Security Goals (CIA, not the agency)

- **C**onfidentiality (symmetric): prevent eavesdropping
  - instant messaging, local storage
  - symmetric ciphers: AES-256, ChaCha20

    ...

Security Goals (CIA, not the agency)

- **I**ntegrity (symmetric): prevent data tampering

Security Goals (CIA, not the agency)

- **I**ntegrity (symmetric): prevent data tampering
  - control messages: pacemakers, drones

Security Goals (CIA, not the agency)

- Integrity (symmetric): prevent data tampering
  - control messages: pacemakers, drones

  - Hash functions, MACs (tag): SHA-2, HMAC (keyed hash), Poly1305.

Security Goals (CIA, not the agency)

- **A**uthenticity (asymmetric): check the origin

Security Goals (CIA, not the agency)

- **A**uthenticity (asymmetric): check the origin
  - web browsing, software updates

Security Goals (CIA, not the agency)

- **A**uthenticity (asymmetric): check the origin
  - web browsing, software updates
  - Public key crypto: RSA, (EC)DSA, ...

Two killer apps on quantum computing

Quantum killer app I

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*

Two killer apps on quantum computing

Quantum killer app I

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*
    - ✗ No more RSA and DSA (discrete log-based)

Two killer apps on quantum computing

Quantum killer app I

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*
  - ✗ No more RSA and DSA (discrete log-based)
- ✗ 2003, extension to ECDSA Proos and Zalka 2003

Two killer apps on quantum computing

Quantum killer app I

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*
  - ✗ No more RSA and DSA (discrete log-based)
- ✗ 2003, extension to ECDSA Proos and Zalka 2003

of qubits required

- **Factoring**: $2n + 2$ logical qubits[a]

[a] Häner, Roetteler, and Svore 2016.
[b] Roetteler et al. 2017.

Two killer apps on quantum computing

Quantum killer app I

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*
  - ✗ No more RSA and DSA (discrete log-based)
- ✗ 2003, extension to ECDSA Proos and Zalka 2003

UNIVERSITY OF WATERLOO

of qubits required

- **Factoring**: $2n + 2$ logical qubits[a]
  - RSA-3072: 6146 qubits

[a] Häner, Roetteler, and Svore 2016.

[b] Roetteler et al. 2017.

Two killer apps on quantum computing

Quantum killer app I

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*
  - ✗ No more RSA and DSA (discrete log-based)
- ✗ 2003, extension to ECDSA Proos and Zalka 2003

of qubits required

- **Factoring**: $2n + 2$ logical qubits[a]
  - RSA-3072: 6146 qubits
- **ECDLP**: $9n + 2\lceil \log n \rceil + 10$ logical qubits[b]

[a] Häner, Roetteler, and Svore 2016.
[b] Roetteler et al. 2017.

Two killer apps on quantum computing

Quantum killer app I

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*
  - ✗ No more RSA and DSA (discrete log-based)
- ✗ 2003, extension to ECDSA Proos and Zalka 2003

UNIVERSITY OF WATERLOO

of qubits required

- **Factoring**: $2n + 2$ logical qubits[a]
  - RSA-3072: 6146 qubits
- **ECDLP**: $9n + 2\lceil \log n \rceil + 10$ logical qubits[b]
  - ECC-256: 2330 qubits

[a] Häner, Roetteler, and Svore 2016.
[b] Roetteler et al. 2017.

Two killer apps on quantum computing

Quantum killer app I

- Shor 1994: *algorithms for quantum computation: discrete logarithms and factoring*
  - ✗ No more RSA and DSA (discrete log-based)
- ✗ 2003, extension to ECDSA Proos and Zalka 2003

UNIVERSITY OF WATERLOO

of qubits required

- **Factoring**: $2n + 2$ logical qubits[a]
  - RSA-3072: 6146 qubits
- **ECDLP**: $9n + 2\lceil \log n \rceil + 10$ logical qubits[b]
  - ECC-256: 2330 qubits
- For 128-bit security "ECC easier target than RSA"[b].

[a] Häner, Roetteler, and Svore 2016.
[b] Roetteler et al. 2017.

Two killer apps on quantum computing

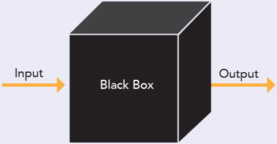
Quantum killer app II

- Grover 1996: *a fast quantum mechanical algorithm for database search*

Two killer apps on quantum computing

Quantum killer app II

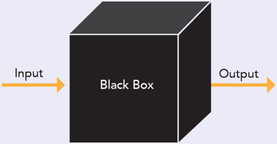
- Grover 1996: *a fast quantum mechanical algorithm for database search*
- Find the unique input to black box function for a given output



Two killer apps on quantum computing

Quantum killer app II

- Grover 1996: *a fast quantum mechanical algorithm for database search*
- Find the unique input to black box function for a given output

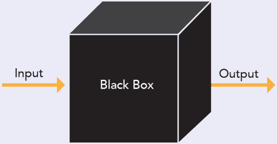


- Quadratic speedup $O(\sqrt{N})$ compared to best classical $O(N)$

Two killer apps on quantum computing

Quantum killer app II

- Grover 1996: *a fast quantum mechanical algorithm for database search*
- Find the unique input to black box function for a given output

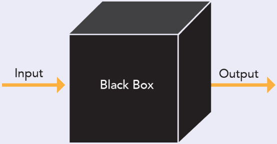


- Quadratic speedup $O(\sqrt{N})$ compared to best classical $O(N)$
- Complexity is optimal Bennett et al. 1997:
  No hope for a quantum solution to solve NP-complete problems (e.g. Unique-SAT)

Two killer apps on quantum computing

Quantum killer app II

- Grover 1996: *a fast quantum mechanical algorithm for database search*
- Find the unique input to black box function for a given output



- Quadratic speedup $O(\sqrt{N})$ compared to best classical $O(N)$
- Complexity is optimal Bennett et al. 1997:
  No hope for a quantum solution to solve NP-complete problems (e.g. Unique-SAT)
- Partially affects block-ciphers: Break 128-bit keys in $O(2^{64})$ steps
  Grassl et al. 2016: 2953 qubits required

What are the affected families of cryptosystems?

- **C**onfidentiality, **I**ntegrity (symmetric) ✓

Model of block ciphers

If the encryption key is chosen at random, then an attacker who does not know the key cannot distinguish between the block cipher and a truly random permutation.

$$f(X) = E_X(I)$$

What are the affected families of cryptosystems?

- **C**onfidentiality, **I**ntegrity (symmetric) ✓

Model of block ciphers

If the encryption key is chosen at random, then an attacker who does not know the key cannot distinguish between the block cipher and a truly random permutation.

$$f(X) = E_X(I)$$

- Moreover, hash functions are modeled as one-way functions.

What are the affected families of cryptosystems?

- **C**onfidentiality, **I**ntegrity (symmetric) ✓

Model of block ciphers

If the encryption key is chosen at random, then an attacker who does not know the key cannot distinguish between the block cipher and a truly random permutation.

$$f(X) = E_X(I)$$

- Moreover, hash functions are modeled as one-way functions.
- **A**uthentication (asymmetric) ✗

Assumptions do not hold in a quantum setting

Mainly based on the hardness of integer factoring or computing discrete logarithm.

But there is a hope

- Fortunately, some old not very interesting algorithms surprisingly resisted to Shor's new attacks.

But there is a hope

- Fortunately, some old not very interesting algorithms surprisingly resisted to Shor's new attacks.

Definition

Post-quantum cryptography consists of classical cryptographic algorithms whose security assumption does not suffer an exponential speedup by quantum attacks are automatically dubbed post-quantum algorithms.

But there is a hope

- Fortunately, some old not very interesting algorithms surprisingly resisted to Shor's new attacks.

Definition

Post-quantum cryptography consists of classical cryptographic algorithms whose security assumption does not suffer an exponential speedup by quantum attacks are automatically dubbed post-quantum algorithms.

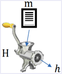
Definition

Quantum-safe cryptography includes a broader set of cryprographic algorithms including non-classical assumptions such as laws of quantum physics, e.g. Quantum Key Distribution.

Post-quantum algorithms and security assumptions

Hash-based crypto:

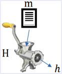
One-way functions exist



Post-quantum algorithms and security assumptions

Hash-based crypto:

One-way functions exist



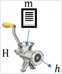
Multivariate-quadratic crypto:

Random-looking multivariate system of non-linear equations is hard

$$y_1 = x_1^2 + x_1x_2 + x_1x_4 + x_3$$
$$y_2 = x_3^2 + x_2x_3 + x_2x_4 + x_1 + 1$$
$$y_3 = \ldots$$

Hash-based crypto:

One-way functions exist



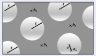
Multivariate-quadratic crypto:

Random-looking multivariate system of non-linear equations is hard

$$y_1 = x_1^2 + x_1x_2 + x_1x_4 + x_3$$
$$y_2 = x_3^2 + x_2x_3 + x_2x_4 + x_1 + 1$$
$$y_3 = \ldots$$

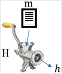
Code-based crypto:

Syndrome decoding for error-correcting codes is hard



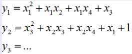
Hash-based crypto:

One-way functions exist



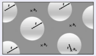
Multivariate-quadratic crypto:

Random-looking multivariate system of non-linear equations is hard



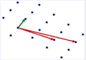
Code-based crypto:

Syndrome decoding for error-correcting codes is hard



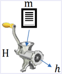
Lattice-based crypto:

Finding short/close vectors on a lattice is hard



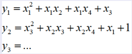
Hash-based crypto:

One-way functions exist



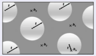
Multivariate-quadratic crypto:

Random-looking multivariate system of non-linear equations is hard



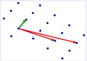
Code-based crypto:

Syndrome decoding for error-correcting codes is hard



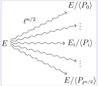
Lattice-based crypto:

Finding short/close vectors on a lattice is hard



Supersingular Isogeny-based crypto:

Computing isogenies between supersingular elliptic curves is hard.



The risk of sticking to pre-quantum crypto

- In August 2015, NSA announces that it plans to replace Suite B with a new cipher suite due to concerns about quantum computing attacks on ECC (even 256-bit curves).

The risk of sticking to pre-quantum crypto

- In August 2015, NSA announces that it plans to replace Suite B with a new cipher suite due to concerns about quantum computing attacks on ECC (even 256-bit curves).
- 2015, Koblitz N., Menezes A. *A riddle wrapped in an enigma*

The risk of sticking to pre-quantum crypto

- In August 2015, NSA announces that it plans to replace Suite B with a new cipher suite due to concerns about quantum computing attacks on ECC (even 256-bit curves).
- 2015, Koblitz N., Menezes A. *A riddle wrapped in an enigma*
- 2016, NIST announces post-quantum standardization
  - Deadline for submission Nov 30th, 2017
  - Focus on digital sigs, encryption and key establishment.

The risk of sticking to pre-quantum crypto

- In August 2015, NSA announces that it plans to replace Suite B with a new cipher suite due to concerns about quantum computing attacks on ECC (even 256-bit curves).
- 2015, Koblitz N., Menezes A. *A riddle wrapped in an enigma*
- 2016, NIST announces post-quantum standardization
  - Deadline for submission Nov 30th, 2017
  - Focus on digital sigs, encryption and key establishment.
- 2018, NIST PQC Standardization Conference (submitter's talks)

NIST
National Institute of
Standards and Technology
U.S. Department of Commerce

The risk of sticking to pre-quantum crypto

Mosca's risk analysis formula

Let X be the time to have certain information protected.
Let Y be the time to deploy post-quantum.
Let Z be the Y2Q (countdown of years to quantum).
If $Z < X + Y$: trouble!

What can one expect from Z?

- Huge investments in Quantum Computing research

NATURE | NEWS

Europe plans giant billion-euro quantum technologies project

Third European Union flagship will be similar in size and ambition to graphene and human brain initiatives.

Elizabeth Gibney

21 April 2016 | Updated: 26 April 2016

What can one expect from Z?

- Huge investments in Quantum Computing research

NATURE | NEWS

Europe plans giant billion-euro quantum technologies project

Third European Union flagship will be similar in size and ambition to graphene and human brain initiatives.

Elizabeth Gibney

21 April 2016 | Updated: 26 April 2016

- Industry competing for quantum supremacy (no crypto purpose).
  - 2017, Google and IBM building general-purpose small prototypes of QCs. Google has no fault-tolerance design plans.

What can one expect from Z?

- Huge investments in Quantum Computing research

  NATURE | NEWS

  Europe plans giant billion-euro quantum technologies project

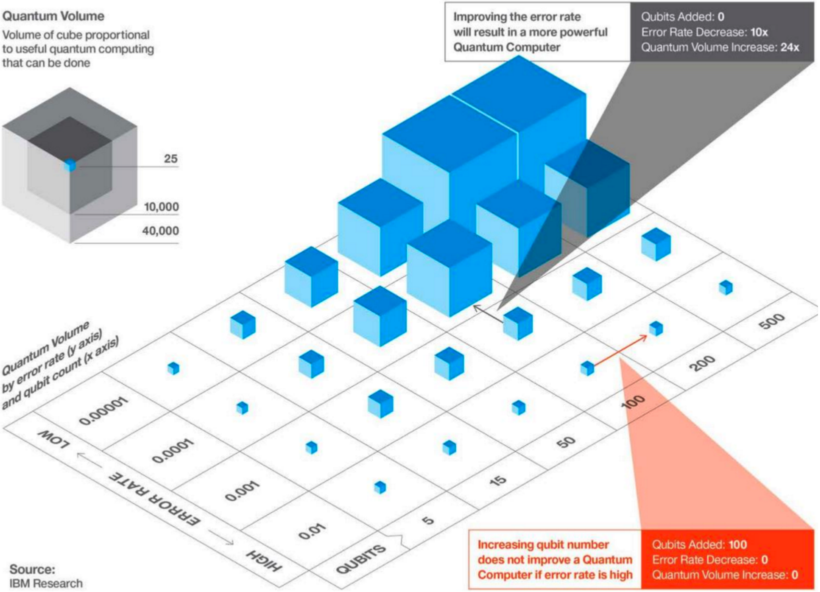
  Third European Union flagship will be similar in size and ambition to graphene and human brain initiatives.

  Elizabeth Gibney

  21 April 2016 | Updated: 26 April 2016

- Industry competing for quantum supremacy (no crypto purpose).
  - 2017, Google and IBM building general-purpose small prototypes of QCs. Google has no fault-tolerance design plans.
- More importantly, there is a steady progress in qubit fidelities. Experts estimate that large QCs (1k's of qubits) will be around by 2031 with 50% chance.

What can one expect from Z?



What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

- Let's look at the history of ECC:

What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

- Let's look at the history of ECC:
  - 1985: Elliptic curves in cryptography by Koblitz and Miller

What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

- Let's look at the history of ECC:
  - 1985: Elliptic curves in cryptography by Koblitz and Miller
  - 1992: ECDSA signature suggested by Scott Vanstone to DSS

What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

- Let's look at the history of ECC:
  - 1985: Elliptic curves in cryptography by Koblitz and Miller
  - 1992: ECDSA signature suggested by Scott Vanstone to DSS
  - 1996: Paul Kotcher introduces side-channel attacks.

What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

- Let's look at the history of ECC:
  - 1985: Elliptic curves in cryptography by Koblitz and Miller
  - 1992: ECDSA signature suggested by Scott Vanstone to DSS
  - 1996: Paul Kotcher introduces side-channel attacks.
    - 1997: Joye-Quisquater introduce fault attacks against ECC

What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

- Let's look at the history of ECC:
  - 1985: Elliptic curves in cryptography by Koblitz and Miller
  - 1992: ECDSA signature suggested by Scott Vanstone to DSS
  - 1996: Paul Kotcher introduces side-channel attacks.
    - 1997: Joye-Quisquater introduce fault attacks against ECC
    - 1998: The ECDLP is actually easy in some (anomalous) curves

What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

- Let's look at the history of ECC:
  - 1985: Elliptic curves in cryptography by Koblitz and Miller
  - 1992: ECDSA signature suggested by Scott Vanstone to DSS
  - 1996: Paul Kotcher introduces side-channel attacks.
    - 1997: Joye-Quisquater introduce fault attacks against ECC
    - 1998: The ECDLP is actually easy in some (anomalous) curves
    - 2000: Biehl-Meyer-Muller devise invalid curve attacks

What can one expect from Y?

Y: Time to deploy new cryptography with wide interoperation

- Let's look at the history of ECC:
  - 1985: Elliptic curves in cryptography by Koblitz and Miller
  - 1992: ECDSA signature suggested by Scott Vanstone to DSS
  - 1996: Paul Kotcher introduces side-channel attacks.
    - 1997: Joye-Quisquater introduce fault attacks against ECC
    - 1998: The ECDLP is actually easy in some (anomalous) curves
    - 2000: Biehl-Meyer-Muller devise invalid curve attacks
  - 2000: NIST FIPS 186-2 includes ECDSA and the 15 NIST curves

What can one expect for Y?

- Let's look at the history of ECC (cont.):

2014 Cloudfare's post dedicated to Scott Vanstone

W.r.t. **https** certificates, despite ECDSA being much faster than RSA for TLS handshake/signing, $>$ **90% of the certificates used on the web in 2014 were RSA-based**.
Even as late as 2012, out of 13 million TLS certificates found in a scan of the internet, fewer than 50 use an ECDSA key pair.

What can one expect for Y?

- Let's look at the history of ECC (cont.):

2014 Cloudfare's post dedicated to Scott Vanstone

W.r.t. **https** certificates, despite ECDSA being much faster than RSA for TLS handshake/signing, $>$ **90% of the certificates used on the web in 2014 were RSA-based**.
Even as late as 2012, out of 13 million TLS certificates found in a scan of the internet, fewer than 50 use an ECDSA key pair.

Main reason

Web sites owners slow to adopt new certificates due to maintainance of compataibility with legacy browsers that do not support the new algorithms. – Sullivan, N. 2014

What can one expect for Y?

The good news to ECC

In Apr'17, ECDSA finally surpassed RSA with 60% of all TLS connections. Recall that ECDSA was proposed in 1992 for DSS. It took 25 years for ECDSA to become widely deployed. – Cloudfare report.

What can one expect for Y?

The good news to ECC

In Apr'17, ECDSA finally surpassed RSA with 60% of all TLS connections. Recall that ECDSA was proposed in 1992 for DSS. It took 25 years for ECDSA to become widely deployed. – Cloudfare report.

The bad news to ECC

"For those partners and vendors that have not yet made the transition to Suite B algorithms, we recommend not making a significant expenditure to do so at this point but instead to prepare for the upcoming quantum resistant algorithm transition." – NSA 2015 announcement

What can one expect for Y?

Jul'17, Pereira's suggested bound:

Conjectured bound $Y \geq 25$

If $Z < X + 25 \Rightarrow$ trouble!

What can one expect for Y?

Jul'17, Pereira's suggested bound:

Conjectured bound $Y \geq 25$

If $Z < X + 25 \Rightarrow$ trouble!

- It is true that the more robust TLS infrastructure and experienced commmunity will be faster at deploying implementations.

What can one expect for Y?

Jul'17, Pereira's suggested bound:

Conjectured bound $Y \geq 25$
If $Z < X + 25 \Rightarrow$ trouble!

- It is true that the more robust TLS infrastructure and experienced commmunity will be faster at deploying implementations.
- On the other hand:
  - NIST standardization analysis phase will take 5 years and 2 more for the drafts – D. Moody
  - The field of quantum cryptanalysis has only just begun (recent attacks against NTRU and binary MQ)
  - Two lines of attacks imply higher chances to break post-quantum assumptions.

Hash-based Signatures (HBS)

1976, Diffie-Hellman in "New directions in cryptography" introduce HBS:

One-way message authentication has a partial solution suggested to the authors by Leslie Lamport of Massachusetts Computer Associates. This technique employs a one-way function f mapping k-dimensional binary space into itself for k on the order of 100. If the transmitter wishes to send an N bit message he generates $2N$, randomly chosen, k-dimensional binary vectors $x_1, X_1, x_2, X_2, \cdots, x_N, X_N$ which he keeps secret. The receiver is given the corresponding images under f, namely $y_1, Y_1, y_2, Y_2, \cdots, y_N, Y_N$. Later, when the message $\boldsymbol{m} = (m_1, m_2, \cdots, m_N)$ is to be sent, the transmitter sends x_1 or X_1 depending on whether $m_1 = 0$ or 1. He sends x_2 or X_2 depending on whether $m_2 = 0$ or 1, etc. The receiver operates with f on the first received block and sees whether it yields y_1 or Y_1 as its image and thus learns whether it was x_1 or X_1, and whether $m_1 = 0$ or 1. In a similar manner the receiver is able to determine $m_2, m_3, \cdots, m_N$. But the receiver is incapable of forging a change in even one bit of $\boldsymbol{m}$.

1976, Lamport's signature: sign a n-bit message

Let $f : \{0,1\}^k \to \{0,1\}^k$ be a public one-way function

1976, Lamport's signature: sign a n-bit message

Let $f : \{0,1\}^k \to \{0,1\}^k$ be a public one-way function

Signer generates n pairs of k-bit strings $(x_i, X_i)_{i=1}^n \in_R \{0,1\}^k$:

$$sk = \{(x_1, X_1), (x_2, X_2), \cdots, (x_n, X_n)\}$$

1976, Lamport's signature: sign a n-bit message

Let $f : \{0,1\}^k \rightarrow \{0,1\}^k$ be a public one-way function

Signer generates n pairs of k-bit strings $(x_i, X_i)_{i=1}^n \in_R \{0,1\}^k$:

$$sk = \{(x_1, X_1), (x_2, X_2), \cdots, (x_n, X_n)\}$$

Evaluate f on them: $y_i, Y_i \leftarrow f(x_i), f(X_i)$ and publishes:

$$pk = \{(y_1, Y_1), (y_2, Y_2), \cdots, (y_n, Y_n)\}$$

1976, Lamport's signature: sign a n-bit message

Let $f : \{0,1\}^k \to \{0,1\}^k$ be a public one-way function

Signer generates n pairs of k-bit strings $(x_i, X_i)_{i=1}^n \in_R \{0,1\}^k$:

$$sk = \{(x_1, X_1), (x_2, X_2), \cdots, (x_n, X_n)\}$$

Evaluate f on them: $y_i, Y_i \leftarrow f(x_i), f(X_i)$ and publishes:

$$pk = \{(y_1, Y_1), (y_2, Y_2), \cdots, (y_n, Y_n)\}$$

To sign $m \in \{0,1\}^n$ reveal

$$\sigma_i \leftarrow \overline{m_i} x_i + m_i X_i \quad // m_i \text{ selects the } i\text{-th element}$$

1976, Lamport's signature: sign a n-bit message

Let $f : \{0,1\}^k \to \{0,1\}^k$ be a public one-way function

Signer generates n pairs of k-bit strings $(x_i, X_i)_{i=1}^n \in_R \{0,1\}^k$:

$$sk = \{(x_1, X_1), (x_2, X_2), \cdots, (x_n, X_n)\}$$

Evaluate f on them: $y_i, Y_i \leftarrow f(x_i), f(X_i)$ and publishes:

$$pk = \{(y_1, Y_1), (y_2, Y_2), \cdots, (y_n, Y_n)\}$$

To sign $m \in \{0,1\}^n$ reveal

$$\sigma_i \leftarrow \overline{m_i} x_i + m_i X_i \quad // m_i \text{ selects the } i\text{-th element}$$

To verify $\sigma = (\sigma_1, \ldots, \sigma_n)$ check if

$$f(\sigma_i) \stackrel{?}{=} \begin{cases} y_i \Rightarrow m_i = 0 \\ Y_i \Rightarrow m_i = 1 \end{cases}$$

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

  Imagine the message to be signed is $N = 1Mbit$.

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

  Imagine the message to be signed is $N = 1Mbit$.

  The size of sk, pk would be $2kN \approx 2 \cdot 100 \cdot 10^6 = 200M$ bits

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

  Imagine the message to be signed is $N = 1Mbit$.

  The size of sk, pk would be $2kN \approx 2 \cdot 100 \cdot 10^6 = 200M$ bits
- An improvement would be:

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

  Imagine the message to be signed is $N = 1Mbit$.

  The size of sk, pk would be $2kN \approx 2 \cdot 100 \cdot 10^6 = 200M$ bits

- An improvement would be:

  Define a new one-way function $g : \{0,1\}^* \to \{0,1\}^n$.

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

  Imagine the message to be signed is $N = 1Mbit$.

  The size of sk, pk would be $2kN \approx 2 \cdot 100 \cdot 10^6 = 200M$ bits

- An improvement would be:

  Define a new one-way function $g : \{0,1\}^* \to \{0,1\}^n$.

  Under message $m \in \{0,1\}^N$, compute $m' = g(m) \in \{0,1\}^n$

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

  Imagine the message to be signed is $N = 1Mbit$.

  The size of sk, pk would be $2kN \approx 2 \cdot 100 \cdot 10^6 = 200M$ bits

- An improvement would be:

  Define a new one-way function $g : \{0,1\}^* \to \{0,1\}^n$.

  Under message $m \in \{0,1\}^N$, compute $m' = g(m) \in \{0,1\}^n$

  Generate sk and pk using $f : \{0,1\}^k \to \{0,1\}^k$ for each bit $1, \ldots, n$.

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

  Imagine the message to be signed is $N = 1Mbit$.

  The size of sk, pk would be $2kN \approx 2 \cdot 100 \cdot 10^6 = 200M$ bits

- An improvement would be:

  Define a new one-way function $g : \{0,1\}^* \rightarrow \{0,1\}^n$.

  Under message $m \in \{0,1\}^N$, compute $m' = g(m) \in \{0,1\}^n$

  Generate sk and pk using $f : \{0,1\}^k \rightarrow \{0,1\}^k$ for each bit $1, \ldots, n$.

  Note that g can have many inputs mapped to a same output.

Hash-based Signatures (HBS)

1976, Lamport's digital signature idea

- A problem pointed by Diffie and Hellman:

  Imagine the message to be signed is $N = 1Mbit$.

  The size of sk, pk would be $2kN \approx 2 \cdot 100 \cdot 10^6 = 200M$ bits

- An improvement would be:

  Define a new one-way function $g : \{0,1\}^* \to \{0,1\}^n$.

  Under message $m \in \{0,1\}^N$, compute $m' = g(m) \in \{0,1\}^n$

  Generate sk and pk using $f : \{0,1\}^k \to \{0,1\}^k$ for each bit $1, \ldots, n$.

  Note that g can have many inputs mapped to a same output.

  Therefore, g should have stronger properties than f (collision resistance).

Hash-based Signatures (HBS)

Remark: Lamport-Diffie is a one-time signature (OTS)!

1. Assume a message $m_1 = (\mathbf{0}, 1, 1) \in \{0,1\}^3$ is signed. Thus

$$\begin{aligned} \sigma &= \{\overline{\mathbf{0}} x_1 + \mathbf{0} X_1, \overline{1} x_2 + 1 X_2, \overline{1} x_3 + 1 X_3\} \\ &= \{x_1, X_2, X_3\} \end{aligned}$$

Hash-based Signatures (HBS)

Remark: Lamport-Diffie is a one-time signature (OTS)!

1. Assume a message $m_1 = (\mathbf{0}, 1, 1) \in \{0,1\}^3$ is signed. Thus

$$\sigma = \{\bar{\mathbf{0}} x_1 + \mathbf{0} X_1, \bar{1} x_2 + 1 X_2, \bar{1} x_3 + 1 X_3\}$$
$$= \{x_1, X_2, X_3\}$$

2. Let the signer produce a new signature of $m_2 = (1,1,0)$ under sk:

$$\sigma' = \{X_1, X_2, x_3\}$$

Hash-based Signatures (HBS)

Remark: Lamport-Diffie is a one-time signature (OTS)!

1. Assume a message $m_1 = (\mathbf{0}, 1, 1) \in \{0,1\}^3$ is signed. Thus

$$\begin{aligned}\sigma &= \{\bar{\mathbf{0}}x_1 + \mathbf{0}X_1, \bar{1}x_2 + 1X_2, \bar{1}x_3 + 1X_3\} \\ &= \{x_1, X_2, X_3\}\end{aligned}$$

2. Let the signer produce a new signature of $m_2 = (1,1,0)$ under sk:

$$\sigma' = \{X_1, X_2, x_3\}$$

3. Notice that $\{x_1, X_1, X_2, x_3, X_3\}$ are now public.

Hash-based Signatures (HBS)

Remark: Lamport-Diffie is a one-time signature (OTS)!

1. Assume a message $m_1 = (\mathbf{0}, 1, 1) \in \{0,1\}^3$ is signed. Thus

$$\begin{aligned}\sigma &= \{\bar{\mathbf{0}}x_1 + \mathbf{0}X_1, \bar{1}x_2 + 1X_2, \bar{1}x_3 + 1X_3\} \\ &= \{x_1, X_2, X_3\}\end{aligned}$$

2. Let the signer produce a new signature of $m_2 = (1, 1, 0)$ under sk:

$$\sigma' = \{X_1, X_2, x_3\}$$

3. Notice that $\{x_1, X_1, X_2, x_3, X_3\}$ are now public.
4. Then it's easy to forge a signature of $m_3 = (1, 1, 1)$ for example. Thus, Lamport-Diffie signature is OTS and each key pair can be only used once.

Hash-based Signatures (HBS)

Security assumption of Lamport-Diffie (LD)

1. one-way function f is hard to invert and
2. it is hard to find different input values that map to a same output

Put the above together, HBS rely on the existence of modern **cryptographically secure hash functions**.

Hash-based Signatures (HBS)

- Recall that LD's sk, pk keys consist of n pairs of k-bit strings.
- Can we do better in terms of space?

Hash-based Signatures (HBS)

- Recall that LD's sk, pk keys consist of n pairs of k-bit strings.
- Can we do better in terms of space?

Merkle 1979, an optimization of LD due to Winternitz

Idea: instead of processing $m \in \{0,1\}^n$ bit-by-bit, use w-bit chunks.

$$m = (m_1 || \cdots || m_{\lceil n/w \rceil})$$

where $m_i \in \{0,1\}^w$ can be viewed as w-bit integers.

Hash-based Signatures (HBS)

- Recall that LD's sk, pk keys consist of n pairs of k-bit strings.
- Can we do better in terms of space?

Merkle 1979, an optimization of LD due to Winternitz

Idea: instead of processing $m \in \{0,1\}^n$ bit-by-bit, use w-bit chunks.

$$m = (m_1 || \cdots || m_{\lceil n/w \rceil})$$

where $m_i \in \{0,1\}^w$ can be viewed as w-bit integers.

- KeyGen: Precompute $x_i, y_i = f^{2^w-1}(x_i)$ for $i = 1, \cdots, \lceil n/w \rceil$ where $f^t(x) = f(\cdots f(x) \cdots)$ means t applications of f

Hash-based Signatures (HBS)

- Recall that LD's sk, pk keys consist of n pairs of k-bit strings.
- Can we do better in terms of space?

Merkle 1979, an optimization of LD due to Winternitz

Idea: instead of processing $m \in \{0,1\}^n$ bit-by-bit, use w-bit chunks.

$$m = (m_1 || \cdots || m_{\lceil n/w \rceil})$$

where $m_i \in \{0,1\}^w$ can be viewed as w-bit integers.

- KeyGen: Precompute $x_i, y_i = f^{2^w-1}(x_i)$ for $i = 1, \cdots, \lceil n/w \rceil$ where $f^t(x) = f(\cdots f(x) \cdots)$ means t applications of f
- Actually, it is possible to do better:

$$pk = y = g(y_1 \parallel \cdots \parallel y_{\lceil n/w \rceil})$$

Hash-based Signatures (HBS)

- Recall that LD's sk, pk keys consist of n pairs of k-bit strings.
- Can we do better in terms of space?

Merkle 1979, an optimization of LD due to Winternitz

Idea: instead of processing $m \in \{0,1\}^n$ bit-by-bit, use w-bit chunks.

$$m = (m_1 || \cdots || m_{\lceil n/w \rceil})$$

where $m_i \in \{0,1\}^w$ can be viewed as w-bit integers.

- KeyGen: Precompute $x_i, y_i = f^{2^w-1}(x_i)$ for $i = 1, \cdots, \lceil n/w \rceil$ where $f^t(x) = f(\cdots f(x) \cdots)$ means t applications of f
- Actually, it is possible to do better:

$$pk = y = g(y_1 \parallel \cdots \parallel y_{\lceil n/w \rceil})$$

- Public key pk boils down to one hash value (instead of $2n$).

Hash-based Signatures (HBS)

(cont. $\cdots$)

Winternitz OTS

- Sign: compute

$$\sigma = (f^{m_1}(x_1), \cdots, f^{m_{\lceil n/w \rceil}}(x_{\lceil n/w \rceil}))$$

Hash-based Signatures (HBS)

(cont. $\cdots$)

Winternitz OTS

- Sign: compute

$$\sigma = (f^{m_1}(x_1), \cdots, f^{m_{\lceil n/w \rceil}}(x_{\lceil n/w \rceil}))$$

- Verify: compute

$$y'_i = f^{2^w - 1 - m_i}(\sigma_i), \text{ for all } i$$
$$y' = g(y'_1 \parallel \cdots \parallel y'_{\lceil n/w \rceil})$$

Check

$$y' \stackrel{?}{=} y$$

Hash-based Signatures (HBS)

- **Problem**: Winternitz defined exactly as previously is **insecure**!

Hash-based Signatures (HBS)

- **Problem**: Winternitz defined exactly as previously is **insecure**!

  Assume a message $m = (m_1, \cdots, \mathbf{m_i}, \cdots, m_{\lceil n/w \rceil})$ with signature

$$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}}(\mathbf{x_i}), \cdots))$$

Hash-based Signatures (HBS)

- **Problem**: Winternitz defined exactly as previously is **insecure**!

  Assume a message $m = (m_1, \cdots, \mathbf{m_i}, \cdots, m_{\lceil n/w \rceil})$ with signature

  $$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}}(\mathbf{x_i}), \cdots))$$

- Assume that for some i we have $m_i < 2^w - 1$

Hash-based Signatures (HBS)

- **Problem**: Winternitz defined exactly as previously is **insecure**!
  Assume a message $m = (m_1, \cdots, \mathbf{m_i}, \cdots, m_{\lceil n/w \rceil})$ with signature

$$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}}(\mathbf{x_i}), \cdots))$$

- Assume that for some i we have $m_i < 2^w - 1$
- Then, an adversary can produce a valid signature for the message

$$m' = (m_1, \cdots, m_i + 1, \cdots, m_{\lceil n/w \rceil})$$

Hash-based Signatures (HBS)

- **Problem**: Winternitz defined exactly as previously is **insecure**!

  Assume a message $m = (m_1, \cdots, \mathbf{m_i}, \cdots, m_{\lceil n/w \rceil})$ with signature

$$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}(x_i)}, \cdots))$$

- Assume that for some i we have $m_i < 2^w - 1$
- Then, an adversary can produce a valid signature for the message

$$m' = (m_1, \cdots, m_i + 1, \cdots, m_{\lceil n/w \rceil})$$

given by

$$\begin{aligned}\sigma' &= (f^{m_1}(x_1), \cdots, f(\mathbf{f^{m_i}(x_i)}), \cdots)) \\ &= (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i+1}(x_i)}, \cdots)) \\ &= \sigma(m')\end{aligned}$$

Hash-based Signatures (HBS)

- **Problem**: Winternitz defined exactly as previously is **insecure**!

  Assume a message $m = (m_1, \cdots, \mathbf{m_i}, \cdots, m_{\lceil n/w \rceil})$ with signature

$$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}(x_i)}, \cdots))$$

- Assume that for some i we have $m_i < 2^w - 1$
- Then, an adversary can produce a valid signature for the message

$$m' = (m_1, \cdots, m_i + 1, \cdots, m_{\lceil n/w \rceil})$$

  given by

$$\begin{aligned} \sigma' &= (f^{m_1}(x_1), \cdots, f(\mathbf{f^{m_i}(x_i)}), \cdots)) \\ &= (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i+1}(x_i)}, \cdots)) \\ &= \sigma(m') \end{aligned}$$

- Violates the notion of existential unforgeability!

Hash-based Signatures (HBS)

- **Solution:** 1979, Merkle introduces a checksum.

Hash-based Signatures (HBS)

- **Solution:** 1979, Merkle introduces a checksum.
- From $m = (m_1, \cdots, m_{\lceil n/w \rceil})$, compute

$$CS = \sum_{i=1}^{\lceil n/w \rceil} 2^w - 1 - m_i$$

which can be stored in $t = log_2(\lceil n/w \rceil \cdot (2^w - 1))$ bits.

Hash-based Signatures (HBS)

- **Solution:** 1979, Merkle introduces a checksum.
- From $m = (m_1, \cdots, m_{\lceil n/w \rceil})$, compute

$$CS = \sum_{i=1}^{\lceil n/w \rceil} 2^w - 1 - m_i$$

  which can be stored in $t = log_2(\lceil n/w \rceil \cdot (2^w - 1))$ bits.
- **Idea**: represent CS as w-bit chunks as well:

$$CS = (m_{\lceil n/w \rceil + 1}, \cdots, m_{\lceil n/w \rceil + \lceil t/w \rceil})$$

Hash-based Signatures (HBS)

- **Solution:** 1979, Merkle introduces a checksum.
- From $m = (m_1, \cdots, m_{\lceil n/w \rceil})$, compute

$$CS = \sum_{i=1}^{\lceil n/w \rceil} 2^w - 1 - m_i$$

which can be stored in $t = log_2(\lceil n/w \rceil \cdot (2^w - 1))$ bits.

- **Idea**: represent CS as w-bit chunks as well:

$$CS = (m_{\lceil n/w \rceil + 1}, \cdots, m_{\lceil n/w \rceil + \lceil t/w \rceil})$$

- Extend the message to be $m || CS = (m_1, \cdots, m_{\lceil n/w \rceil + \lceil t/w \rceil})$

Hash-based Signatures (HBS)

(checksum: cont . . .)

- Given a signature

$$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}(x_i)}, \cdots))$$

Hash-based Signatures (HBS)

(checksum: cont ...)

- Given a signature

$$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}(x_i)}, \cdots))$$

- Let $i \leq \lceil n/w \rceil$. If the adversary tries to go forward on $\mathbf{f^{m_i}(x_i)}$:

$$\begin{aligned} \sigma' &= (f^{m_1}(x_1), \cdots, f(\mathbf{f^{m_i}(x_i)}), \cdots)) \\ &= (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i+1}(x_i)}, \cdots)) \end{aligned}$$

Hash-based Signatures (HBS)

(checksum: cont ...)

- Given a signature

$$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}(x_i)}, \cdots))$$

- Let $i \leq \lceil n/w \rceil$. If the adversary tries to go forward on $\mathbf{f^{m_i}(x_i)}$:

$$\begin{aligned}\sigma' &= (f^{m_1}(x_1), \cdots, f(\mathbf{f^{m_i}(x_i)}), \cdots)) \\ &= (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i+1}(x_i)}, \cdots))\end{aligned}$$

- Then the new checksum is $CS' = CS - 1$ which implies an inversion $f^{-1}(f^{m_j}(x_j))$ for some $m_j \in CS$.

Hash-based Signatures (HBS)

(checksum: cont ...)

- Given a signature

$$\sigma = (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i}(x_i)}, \cdots))$$

- Let $i \leq \lceil n/w \rceil$. If the adversary tries to go forward on $\mathbf{f^{m_i}(x_i)}$:

$$\begin{aligned} \sigma' &= (f^{m_1}(x_1), \cdots, f(\mathbf{f^{m_i}(x_i)}), \cdots)) \\ &= (f^{m_1}(x_1), \cdots, \mathbf{f^{m_i+1}(x_i)}, \cdots)) \end{aligned}$$

- Then the new checksum is $CS' = CS - 1$ which implies an inversion $f^{-1}(f^{m_j}(x_j))$ for some $m_j \in CS$.
- If a forger targets $m_j \in CS$, an inversion is also implied on m_i. Thus, the checksum protects against such attacks.

Winternitz OTS example

- Let $w = 2$ and one wants to sign the $(n = 4)$-bit message

$$m = (1011)$$

with $\lceil n/w \rceil = 2$ and $m = (m_1 = 2, m_2 = 3)$.

Winternitz OTS example

- Let $w = 2$ and one wants to sign the $(n = 4)$-bit message

$$m = (1011)$$

  with $\lceil n/w \rceil = 2$ and $m = (m_1 = 2, m_2 = 3)$.

- Compute the checksum:

$$\begin{aligned} CS &= \sum_{i=1}^{2} 2^2 - 1 - m_i \\ &= 3 - 2 + 3 - 3 = 1 \end{aligned}$$

Winternitz OTS example

- Let $w = 2$ and one wants to sign the $(n = 4)$-bit message

$$m = (1011)$$

  with $\lceil n/w \rceil = 2$ and $m = (m_1 = 2, m_2 = 3)$.

- Compute the checksum:

$$\begin{aligned} CS &= \sum_{i=1}^{2} 2^2 - 1 - m_i \\ &= 3 - 2 + 3 - 3 = 1 \end{aligned}$$

  Thus, $CS = (0001) = (m_3 = 0, m_4 = 1)$ and the actual message is

$$m||CS = (2, 3, 1, 0)$$

(*cont ...*) Winternitz OTS: example

- Key generation including CS:

$$sk = (x_1, \cdots, x_4)$$
$$pk = y = g(f^3(x_1) \parallel \cdots \parallel f^3(x_4))$$

(*cont ...*) Winternitz OTS: example

- Key generation including CS:

$$sk = (x_1, \cdots, x_4)$$
$$pk = y = g(f^3(x_1) \parallel \cdots \parallel f^3(x_4))$$

- The signature of $m||CS = (2, 3, 1, 0)$ will be

$$\sigma = (f^2(x_1), f^3(x_2), f(x_3), x_4) = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$$

(*cont ...*) Winternitz OTS: example

- Key generation including CS:

$$sk = (x_1, \cdots, x_4)$$
$$pk = y = g(f^3(x_1) \parallel \cdots \parallel f^3(x_4))$$

- The signature of $m||CS = (2, 3, 1, 0)$ will be

$$\sigma = (f^2(x_1), f^3(x_2), f(x_3), x_4) = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$$

- Verification of σ will be

(*cont ...*) Winternitz OTS: example

- Key generation including CS:

$$sk = (x_1, \cdots, x_4)$$
$$pk = y = g(f^3(x_1) \parallel \cdots \parallel f^3(x_4))$$

- The signature of $m||CS = (2, 3, 1, 0)$ will be

$$\sigma = (f^2(x_1), f^3(x_2), f(x_3), x_4) = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$$

- Verification of σ will be
  - Recompute CS from m getting $m||CS = (2, 3, 1, 0)$

(*cont . . .*) Winternitz OTS: example

- Key generation including CS:

$$sk = (x_1, \cdots, x_4)$$
$$pk = y = g(f^3(x_1) \parallel \cdots \parallel f^3(x_4))$$

- The signature of $m||CS = (2, 3, 1, 0)$ will be

$$\sigma = (f^2(x_1), f^3(x_2), f(x_3), x_4) = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$$

- Verification of σ will be
  - Recompute CS from m getting $m||CS = (2, 3, 1, 0)$
  - Compute and check

$$(y_1', y_2', y_3', y_4') = (f^{3-\mathbf{2}}(\sigma_1), f^{3-\mathbf{3}}(\sigma_2), f^{3-\mathbf{1}}(\sigma_3), f^{3-\mathbf{0}}(\sigma_4))$$

$$g(y_1' \parallel y_2' \parallel y_3' \parallel y_4') \stackrel{?}{=} y$$

Hash-based Signatures (HBS)

Note on the efficiency of Winternitz OTS

- $|sk| = |\sigma| \approx (\mathbf{n}/\mathbf{w})\mathbf{k}$ (ignoring the checksum)

Hash-based Signatures (HBS)

Note on the efficiency of Winternitz OTS

- $|sk| = |\sigma| \approx (\mathbf{n}/\mathbf{w})\mathbf{k}$ (ignoring the checksum)
  - Compares well with the **2nk** bits in $|sk|, |\sigma|$ for LD
  - A **2w** reduction factor for sk
  - A **w** reduction factor for σ

Hash-based Signatures (HBS)

Note on the efficiency of Winternitz OTS

- $|sk| = |\sigma| \approx (\mathbf{n}/\mathbf{w})\mathbf{k}$ (ignoring the checksum)
  - Compares well with the $\mathbf{2nk}$ bits in $|sk|, |\sigma|$ for LD
  - A $\mathbf{2w}$ reduction factor for sk
  - A $\mathbf{w}$ reduction factor for σ
- $|pk| = \mathbf{n}$

Hash-based Signatures (HBS)

Note on the efficiency of Winternitz OTS

- $|sk| = |\sigma| \approx (\mathbf{n}/\mathbf{w})\mathbf{k}$ (ignoring the checksum)
  - Compares well with the $\mathbf{2nk}$ bits in $|sk|, |\sigma|$ for LD
  - A $\mathbf{2w}$ reduction factor for sk
  - A $\mathbf{w}$ reduction factor for σ
- $|pk| = \mathbf{n}$
- But, WOTS requires $(\mathbf{2^w} - \mathbf{1})/\mathbf{w}$ hash evaluations per bit
  - While LD requires $\mathbf{2}$ evaluations per bit
  - Notice that for $w = 1$ we get exactly Lamport-Diffie

Hash-based Signatures (HBS)

Note on the efficiency of Winternitz OTS

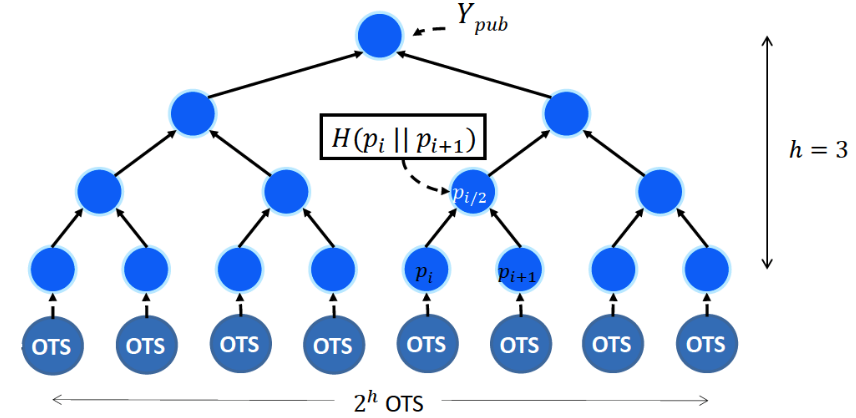
- $|sk| = |\sigma| \approx (\mathbf{n}/\mathbf{w})\mathbf{k}$ (ignoring the checksum)
  - Compares well with the **2nk** bits in $|sk|, |\sigma|$ for LD
  - A **2w** reduction factor for *sk*
  - A **w** reduction factor for σ
- $|pk| = \mathbf{n}$
- But, WOTS requires $(\mathbf{2^w} - \mathbf{1})/\mathbf{w}$ hash evaluations per bit
  - While LD requires **2** evaluations per bit
  - Notice that for $w = 1$ we get exactly Lamport-Diffie
- Since hash evaluations can be very fast, it is a reasonable tradeoff

Hash-based Signatures (HBS)

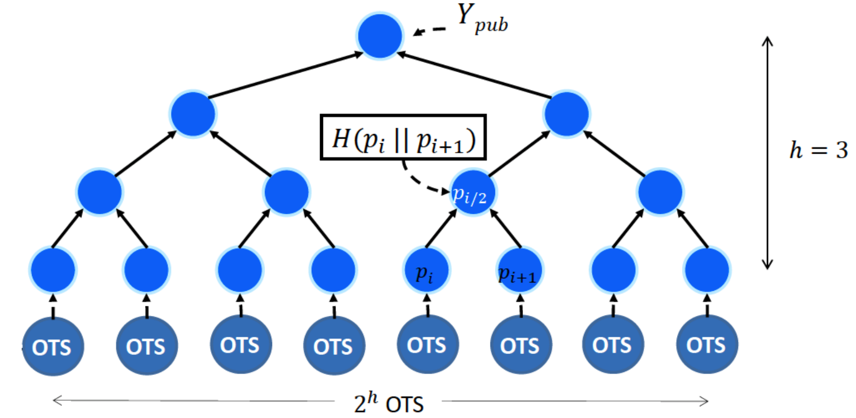
Scheme	$n = k$	$\|PrivKey\|$	$\|PubKey\|$	$\|Sig\|$
LD OTS	256	16	16	16
WOTS (w=2)	256	4.2	32 bytes	4.2
WOTS (w=8)	256	1.1	32 bytes	1.1
WOTS (w=16)	256	0.6	32 bytes	0.6

Table: Parameter sizes for one-time signatures in KiB

1979, Merkle turns OTS into multi-time signatures



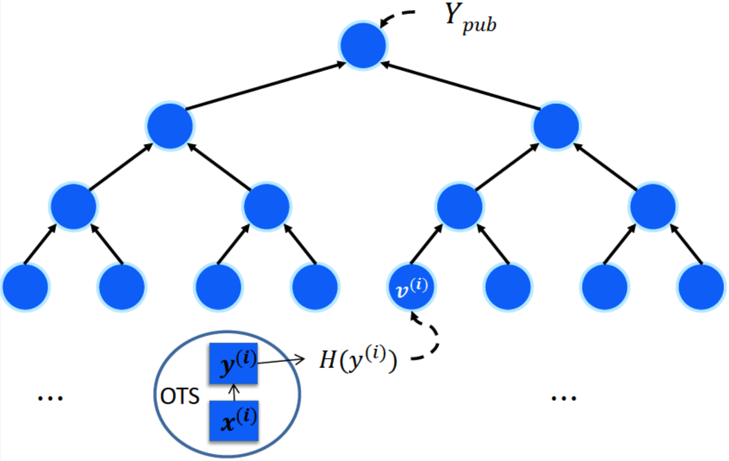
1979, Merkle turns OTS into multi-time signatures



- OTS can be **any** one-time signature scheme.
- Y_{pub} authenticates 2^h OTS key pairs.

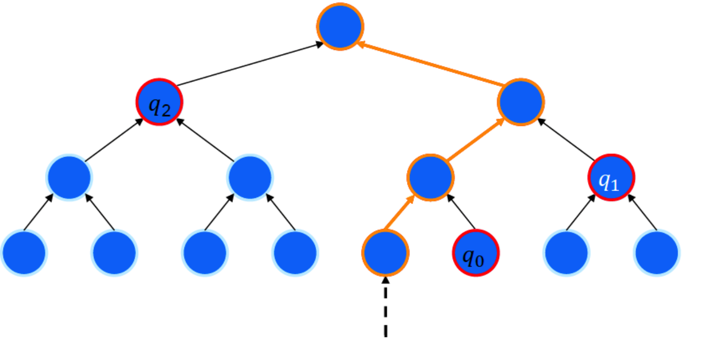
Hash-based Signatures (HBS)

1979, Merkle signature: **Key Generation**



Hash-based Signatures (HBS)

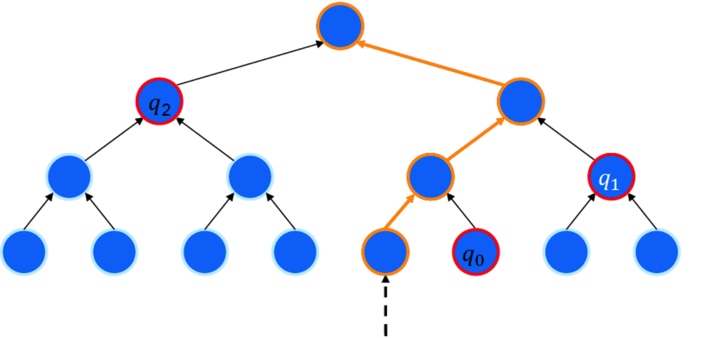
1979, Merkle signature: **Sign**



$$\sigma(data) = \left(i, Sign_{OTS}\left(x^{(i)}, data\right), v^{(i)}, (q_0, \dots, q_{h-1})\right)$$

Hash-based Signatures (HBS)

1979, Merkle signature: **Sign**

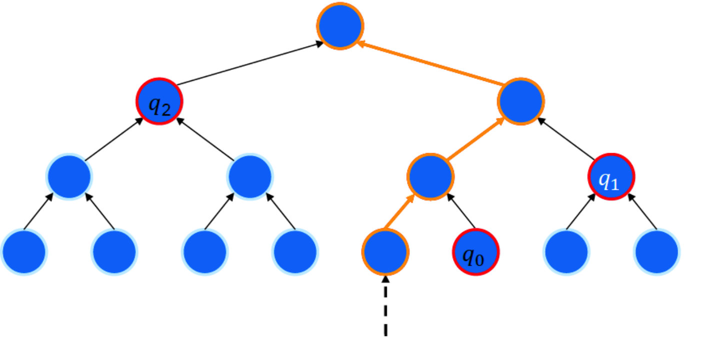


$$\sigma(data) = \left(i, Sign_{OTS}\left(x^{(i)}, data\right), v^{(i)}, (q_0, \ldots, q_{h-1})\right)$$

- Nodes q_i are called the **authentication path** of i-th signature

Hash-based Signatures (HBS)

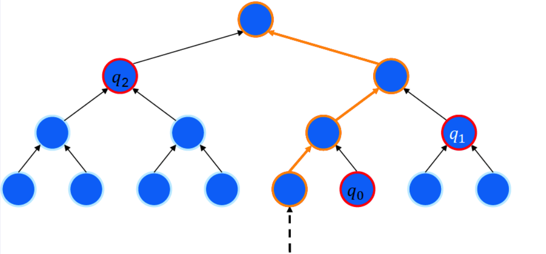
1979, Merkle signature: **Sign**



$$\sigma(data) = \left(i, Sign_{OTS}\left(x^{(i)}, data\right), v^{(i)}, (q_0, \dots, q_{h-1})\right)$$

- Nodes q_i are called the **authentication path** of i-th signature
- Stateful: susceptible to some attacks, e.g. 'restart attacks'

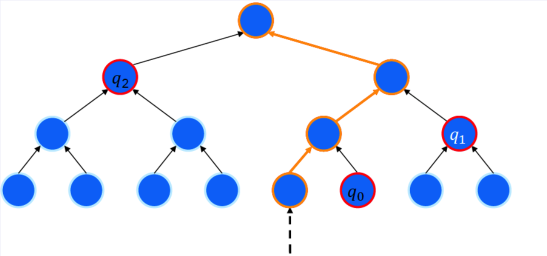
Time efficiency of the Merkle signature



- $\sigma(data) = \big(i, Sign_{OTS}(x^{(i)}, data), v^{(i)}, (q_0, \ldots, q_{h-1})\big)$

- Requires $O(2^h)$ hash evaluations per signature

Time efficiency of the Merkle signature

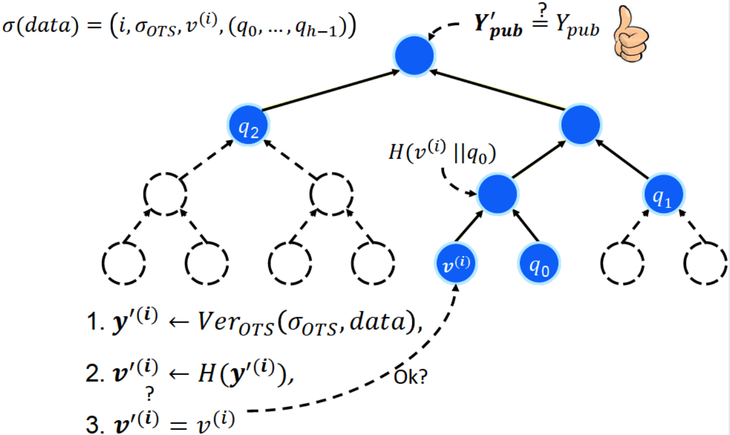


- $$\sigma(data) = \left(i, Sign_{OTS}\left(x^{(i)}, data\right), v^{(i)}, (q_0, \ldots, q_{h-1})\right)$$

- Requires $O(2^h)$ hash evaluations per signature
- Improvement by BDS'08.
  - Store strategic (higher) nodes on a state during KeyGen.
  - Allows for a tradeoff between size of the state *vs* # leaf computations at each signature.

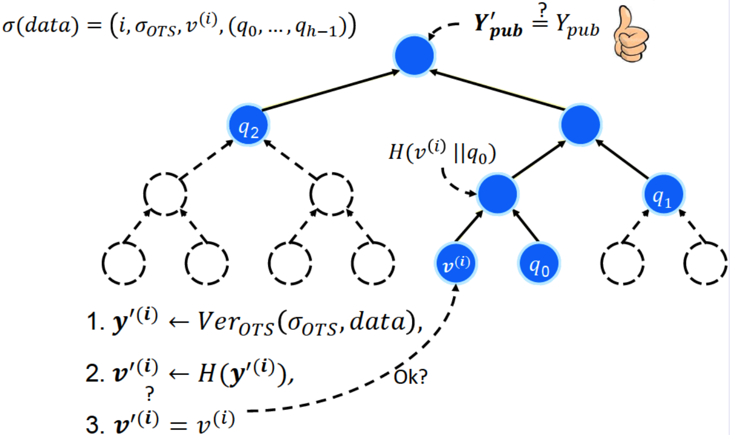
Hash-based Signatures (HBS)

1979, Merkle signature: **Verify**



Hash-based Signatures (HBS)

1979, Merkle signature: **Verify**



- An obvious optimization is not sending $v^{(i)}$. Verifier only checks the root.

Space efficiency of the Merkle signature

- Private key size: $2^h \cdot |sk_{OTS}|$
- Public key size: size of hash H, e.g. 256 bits.
- Signature size: $|\sigma| = |i| + |\sigma_{OTS}| + |v(i)| + |(q_0, \cdots, q_{h-1})|$

Space efficiency of the Merkle signature

- Private key size: $2^h \cdot |sk_{OTS}|$
- Public key size: size of hash H, e.g. 256 bits.
- Signature size: $|\sigma| = |i| + |\sigma_{OTS}| + |v(i)| + |(q_0, \cdots, q_{h-1})|$

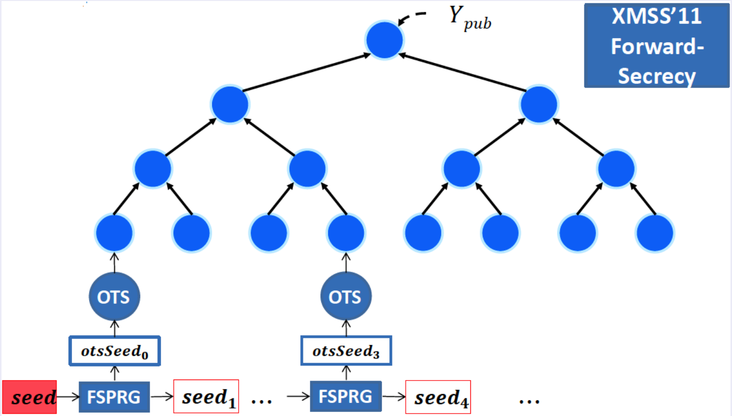
Merkle parameter sizes example

- $|f| = |H| = n = 256, h = 10$

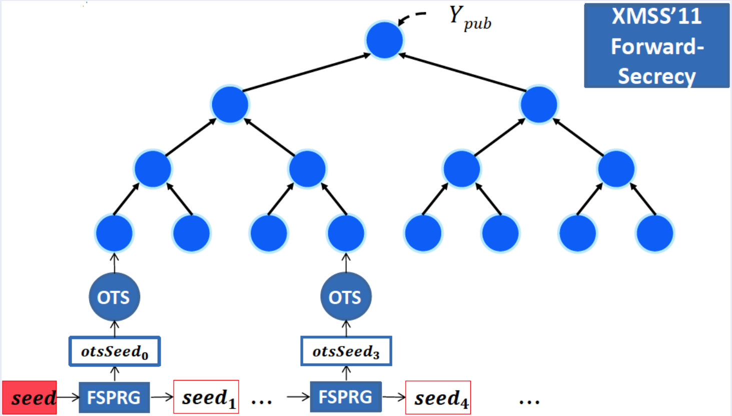
Scheme	$\lvert PrivKey \rvert$	$\lvert PubKey \rvert$	$\lvert Sig \rvert$
Merkle+LD	16 MiB	32 bytes	16.4 KiB
Merkle+WOTS (w=2)	4.2 MiB	32 bytes	4.5 KiB
Merkle+WOTS (w=16)	0.6 MiB	32 bytes	0.9 KiB

Table: Parameter sizes for Merkle multi-time signature (1024 signatures)

Merkle signature: XMSS'11 introduces additional properties

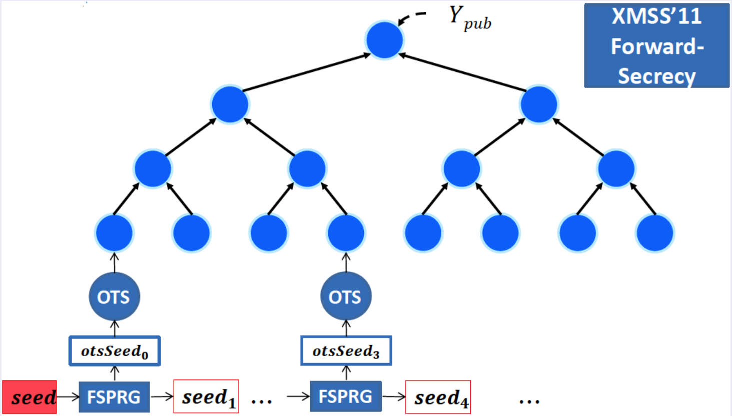


Merkle signature: XMSS'11 introduces additional properties



- XMSS uses the variant WOTS$^+$. Collision-resistance unecessary.

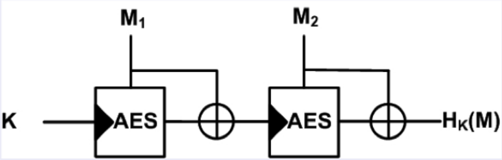
Merkle signature: XMSS'11 introduces additional properties



- XMSS uses the variant WOTS$^+$. Collision-resistance unecessary.
- Implication: half-size hashes can be used safely.

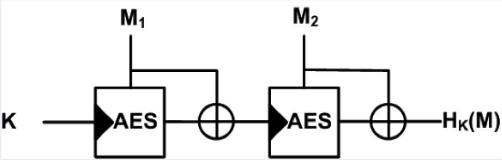
Merkle signature: implemention of PRNG and hash function

- Matyas-Meyer-Oseas: block-cipher-based hash function



Merkle signature: implemention of PRNG and hash function

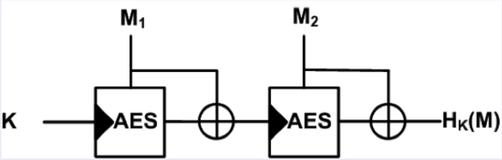
- Matyas-Meyer-Oseas: block-cipher-based hash function



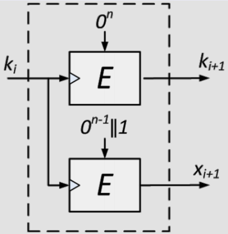
- Fast optimized (hw/sw) block-ciphers available in many platforms

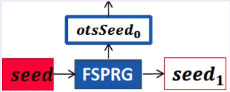
Merkle signature: implemention of PRNG and hash function

- Matyas-Meyer-Oseas: block-cipher-based hash function



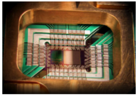
- Fast optimized (hw/sw) block-ciphers available in many platforms
- FSPRG by Standaert et al. 2010:



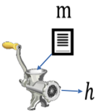


Hash-based Signatures (HBS) – A holistic view

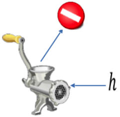
Post-quantum security



Only require hash functions
(efficient/minimal security assumption)



No reliance on trapdoors



Robust security (1976)
(cryptanalysis with little progress)

www.ieee.org/about/awards/bios/hamming_recipients

Larger signatures

References I

Shor, Peter W (1994).
"Algorithms for quantum computation: Discrete logarithms and factoring".

Proos, John and Christof Zalka (2003).
"Shor's discrete logarithm quantum algorithm for elliptic curves".

Häner, Thomas, Martin Roetteler, and Krysta M Svore (2016).
"Factoring using 2n+2 qubits with Toffoli based modular multiplication".

Roetteler, Martin et al. (2017).
"Quantum resource estimates for computing elliptic curve discrete logarithms".

Grover, Lov K (1996).
"A fast quantum mechanical algorithm for database search".

Bennett, Charles H et al. (1997).
"Strengths and weaknesses of quantum computing".

Grassl, Markus et al. (2016).
"Applying Grovers algorithm to AES: quantum resource estimates".

References II

Merkle, Ralph Charles (1979).
"Secrecy, authentication, and public key systems".

Standaert, FranCcois-Xavier et al. (2010).
"Leakage resilient cryptography in practice".