Exercise

Given an image Im and a filter f, compute the convolution and correlation in coordinates (3,3)

$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 1 & 2 & 2 & 0 \\
2 & 3 & \boxed{2} & 1 & 6 \\
3 & 1 & 0 & 5 & 2 \\
4 & 2 & 1 & 0 & 6
\end{array}
\qquad
\begin{bmatrix} 2 & 3 & -2 \\ 0 & -2 & -1 \\ 0 & -1 & 0 \end{bmatrix}
$$

-1,-1

0,0

Image 3,3

Kernel

$(Im * f)(x,y) = ?$

col index x-axis

row index y-axis

$(Im \otimes f)(x,y) = ?$

flipping the kernel

convolution : $(I * K)(x,y) = \sum_i \sum_j I(x,y) K(x-i, y-j)$

correlation : $(I \otimes K)(x,y) = \sum_i \sum_j I(x,y) K(x+i, y+i)$

maintaining the kernel

Compute the convolution :

$$
\begin{matrix} 2 & 3 & -2 \\ 0 & -2 & -1 \\ 0 & -1 & 0 \end{matrix}
\qquad
\begin{matrix} 0 & -1 & 0 \\ 0 & -2 & -1 \\ 2 & 3 & -2 \end{matrix}
\qquad
\begin{matrix} 0 & -1 & 0 \\ -1 & 2 & 0 \\ -2 & 3 & 2 \end{matrix}
$$

original kernel horizontal flip vertical flip

we can now overlap the kernel to the image and sum all the products we get

$$
\begin{matrix} 2_{0} & 1_{-1} & 6_{0} \\ 0_{-1} & 5_{2} & 2_{0} \\ 1_{-2} & 0_{3} & 6_{2} \end{matrix}
$$

$$2 \cdot 0 + 1 \cdot (-1) + 6 \cdot 0 + 0 \cdot (-1) + 5 \cdot 2 + 2 \cdot 0 + 1 \cdot (-2) + 0 \cdot 3 + 6 \cdot 2 = -1$$

To compute the correlation we just need to overlap the original kernel

$$
\begin{matrix} 2_{2} & 1_{3} & 6_{-2} \\ 0_{0} & 5_{-2} & 2_{-1} \\ 1_{0} & 0_{-1} & 6_{0} \end{matrix}
$$

$$2 \cdot 2 + 1 \cdot 3 + 6 \cdot (-2) + 0 \cdot 0 + 5(-2) + 2(-1) + 1 \cdot 0 + 0(-1) + 6 \cdot 0 = -17$$