Simulation of the movement of a 6-degree freedom Robotic Arm

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1) It is known that $q_m = [x, y, \varphi_z]$ are the dimensions of the platform 0.75 x 1 x 0.5.$Q_{MB} = [1\ 0\ 0\ 0]^T \Rightarrow \cos\left(\frac{\theta}{2}\right) = 1 \Rightarrow \theta = 2\cos^{-1}(n) = 0 \Rightarrow k = [0\ 0\ 0]^T \Rightarrow R_{MB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ *and*

$$g_{MB} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.35 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ Also\ g_{OA} = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From Matlab we calculate for each combination of the angles of the arm $J_e\ \kappa\alpha\iota\ g_{BE} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$. We will consider for the platform that the 1st joint is the prismatic in x , the 2nd the prismatic in y and the 3rd the rotary. We place the boxes according to the modified Craig method.

For transformation 01 we cannot use the matrix from the Craig method . By observation we conclude that

$$g_{01} = \begin{bmatrix} 0 & 0 & 1 & x \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The table for the Craig method for frames 2,3 is:

	$\alpha_{\alpha-1}$	a_{i-1}	d_i	θ_i
2	90 °	0	y	-90 °
3	90 °	0	0	φ_z

It is calculated that $g_{12} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -y \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ *and*

$$g_{23} = \begin{bmatrix} C_\varphi & -S_\varphi & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_\varphi & C_\varphi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.\ Also\ g_{3M} = I_{4x4}$$

For the Jacobean we want

$$g_{3E} = g_{3M}g_{MB}g_{BE} = g_{MB}g_{BE} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y + 0.35 \\ n_z & o_z & a_z & p_z + 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 3 is rotary therefore:

$$J_{3e} = \begin{bmatrix} -n_x P_y - 0.35 n_x + n_y P_x \\ -o_x P_y - 0.35 o_x + o_y P_y \\ -a_x P_y - 0.35 a_x + a_y P_x \\ n_z \\ o_z \\ a_z \\ \end{bmatrix}$$

$$g_{2e} = g_{23}g_{3e}$$

$$= \begin{bmatrix} C_\varphi n_x - n_y S_\varphi & -o_y S_\varphi & a_x C_\varphi - a_y S_\varphi & C_\varphi P_x - S_\varphi (P_y + \frac{7}{20}) \\ -n_z & -o_z & -a_z & -P_z - \frac{1}{2} \\ C_\varphi n_y + n_x S_\varphi & C_\varphi o_y + S_\varphi o_x & a_y C_\varphi + a_x S_\varphi & C_\varphi P_x + C_\varphi (P_y + \frac{7}{20}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 2 is prismatic so:

$$J_{2e} = \begin{bmatrix} C_\varphi n_y + n_x S_\varphi \\ C_\varphi o_y + S_\varphi o_x \\ a_y C_\varphi + a_x S_\varphi \\ 0 \\ 0 \\ 0 \\ \end{bmatrix}$$

$$g_{1e} = g_{12}g_{2e}$$

$$= \begin{bmatrix} n_z & o_z & a_z & P_z + \frac{1}{2} \\ -C_\varphi n_y - n_x S_\varphi & -C_\varphi o_y - S_\varphi o_x & -a_y C_\varphi - a_x S_\varphi & -y - P_x S_\varphi - C_\varphi (P_y + \frac{7}{20}) \\ C_\varphi n_x - n_y S_\varphi & C_\varphi o_\chi - S_\varphi o_y & a_x C_\varphi - a_y S_\varphi & C_\varphi P_x - S_\varphi (P_x + \frac{7}{20}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 1 is prismatic so:

$$J_{1e} = \begin{bmatrix} C_\varphi n_x - n_y S_\varphi \\ C_\varphi o_\chi - S_\varphi o_y \\ a_x C_\varphi - a_y S_\varphi \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So in total:

$$J_e = \begin{bmatrix} J_{e,6DoF} & \begin{matrix} C_\varphi n_x - n_y S_\varphi & C_\varphi n_y + n_x S_\varphi & -n_x P_y - 0.35 n_x + n_y P_x \\ C_\varphi o_\chi - S_\varphi o_y & C_\varphi o_y + S_\varphi o_x & -o_x P_y - 0.35 o_x + o_y P_y \\ a_x C_\varphi - a_y S_\varphi & a_y C_\varphi + a_x S_\varphi & -a_x P_y - 0.35 a_x + a_y P_x \\ 0 & 0 & n_z \\ 0 & 0 & o_z \\ 0 & 0 & a_z \end{matrix} \end{bmatrix}$$

It is observed that:

- Columns 7,8 are not affected by edge position unlike 9
- Columns 7,8 are not affected by the n_z, a_z, o_z
- All 3 columns (7,8,9) are affected by the tip direction on the x, y axes
- None of the 3 columns depends on the position P_z of the tip

2,3) It is desired that the limb's orientation is not changed throughout the movement. This is achieved if we do not move the arm until the platform reaches the desired position next to the table, if we do not change the angle ϕ_z of the platform, and if in the final position of the platform, the angle formed between the line AB joining the 2 frames and x-axis is the same as it was before the platform started moving. This angle is the $\theta_1 = q_1$ and from the initial position of the joints it is equal to 150 °. This is required so that in the final position of the platform the arm just extends and descends (it will descend because $z_{arm,initial} = 0.9049 > z_{cylinder,up} = 0.7$) without having to change the orientation of the end frame. When we park the platform we will leave a gap of 0.01 so that it does not hit. Also we don't move the arm at the same time as the platform because as the arm will go down and the platform will move, it might drop the cylinder (in real life conditions).

In the final position:

$$y_{BE} = 0.15 + 0.01 + 0.175 = 0.335, \quad cos60^o = \frac{y_{BE}}{r} \Rightarrow$$
$$r = 0.67, \qquad |x_{BE}| = 0{,}67 * S_{60} = 0.580237 \; \kappa\alpha\iota$$
$$|z_{BE}| = z_{arm,initial} - z_{cylinder,up} = 0.2049$$

Therefore: $\boldsymbol{x_{BE} = -0.580237, y_{BE} = 0.335, z_{BE} = 0.2049}$

And for the same orientation as the original one from Matlab we calculate:

$$q_{f1,\beta\rho\alpha\chi} = [2.618 \quad -1.0627 \quad 1.1143 \quad 3.1416 \quad 0.9646 \quad -0.9821]$$

$X_{M,f} = 1.5 + |x_{BE}| = 2.080237, Y_{M,f} = 1.5 - y_{BE} - 0.35 = 0.815, X_{M,0} = Y_{M,0} = \varphi_{M0} = \varphi_{Mf} = 0$ and zero initial and final velocities.

We assume that the platform moves for 5 seconds. From the provided formulas for trajectory design (**t_0=0, t_f=5**), we have:

$$x = k_0 + k_1 t + k_2 t^2 + k_3 t^3, k_0 = k_1 = 0,$$
$$k_2 = \frac{3}{5^2} 2.080237 = 0.249628$$
$$k_3 = -\frac{2}{125} 2.080237 = -0.0332838$$

Therefore

$$\boldsymbol{x = 0.249628t^2 - 0.0332838t^3}$$
$$\boldsymbol{\dot{x} = 0.499256t - 0.0998514t^2}$$
$$\boldsymbol{0 \leq t \leq 5}$$

We create a script in Matlab (in the function results . m) that given data q_0, q_f, t_f, it calculates the coefficients k .

So for y we have:

$$\boldsymbol{y = 0.0978t^2 - 0.01304t^3}$$
$$\boldsymbol{\dot{y} = 0.1956t - 0.03912t^2}$$
$$\boldsymbol{0 \leq t \leq 5}$$

$$\boldsymbol{\varphi_M = \dot{\varphi}_M = 0, 0 \leq t \leq 20}$$

In the interval from **5 to 10 sec** the arm moves to catch the cylinder. The initial joint angles are equal to q $_0$ since the angles have not changed

$$q_0 = [2.618 \quad -0.6695 \quad 1.2719 \quad 3.1416 \quad 1.2002 \quad -0.9821]$$

$$q_{f1,\beta\rho\alpha\chi} = [2.618 \quad -1.0627 \quad 1.1143 \quad 3.1416 \quad 0.9646 - 0.9821]$$

Comparing the angles of the initial position with the final position we notice that the joints q_1, q_4, q_6 do not move at all.

Using the script, the position and speed for each joint can be calculated using the formulas below. Any omitted positions or speeds are considered to be constant or zero accordingly for the given time frame.

$$\boldsymbol{q_2 = -0.6695 - 0.047184(t-5)^2 + 0.0062912(t-5)^3}$$
$$\boldsymbol{\dot{q_2} = -0.094368(t-5) + 0.0188736(t-5)^2}$$
$$\boldsymbol{5 \leq t \leq 10}$$

$$\boldsymbol{q_3 = 1.2719 - 0.018912(t-5)^2 + 0.002516(t-5)^3}$$
$$\boldsymbol{\dot{q_3} = -0.037824(t-5) + 0.007548(t-5)^2}$$
$$\boldsymbol{5 \leq t \leq 10}$$

$$\boldsymbol{q_5 = 1.2002 - 0.028272(t-5)^2 + 0.003769(t-5)^3}$$
$$\boldsymbol{\dot{q_5} = -0.056544(t-5) + 0.011309(t-5)^2}$$

$$\mathbf{5 \leq t \leq 10}$$

4) At this poitn the robot has caught the cylinder. To carry it onto the platform we will design a trajectory that will consist of 3 movements

Movement 1 :

It rises by z so that during the next movement there is no friction with the table. We will consider that the coordinates

$\boldsymbol{x_{BE} = -0.580237, y_{BE} = 0.335}$ remain constant and z_{BE}increases to $\boldsymbol{z_{BE} = 0.31.}$

With these coordinates and the same frame orientation as before is obtained from matlab :

$$q_{f2,arm} = [2.618 \quad -0.9278 \quad 1.1007 \quad 3.1416 \quad 1.1131 \ -0.9821]$$

And

$$q_{0'} = [2.618 \quad -1.0627 \quad 1.1143 \quad 3.1416 \quad 0.9646 - 0.9821]$$

Comparing the angles of the initial position with the final position we notice that the joints q_1, q_4, q_6 will not move.

We assume this move will last **3 seconds**

So:

$$\boldsymbol{q_2 = -1.0627 + 0.04496(t-10)^2 - 0.009925(t-10)^3}$$

$$\boldsymbol{\dot{q_2} = 0.089933(t-10) - 0.0299777(t-10)^2}$$

$$\mathbf{10 \leq t \leq 13}$$

$$\boldsymbol{q_3 = 1.1143 - 0.004533(t-10)^2 + 0.001007(t-10)^3}$$

$$\boldsymbol{\dot{q_3} = -0.099(t-10) + 0.0030222(t-10)^2}$$

$$\mathbf{10 \leq t \leq 13}$$

$$\boldsymbol{q_5 = 0.946 + 0.0495(t-10)^2 - 0.011(t-10)^3}$$

$$\dot{q_5} = 0.099(t-10) - 0.033(t-10)^2$$

$$10 \le t \le 13$$

Movement 2 :

The arm will rotate so that the cylinder is over the correct y- axis of the platform. To do this from the angle $q_1 = 150^o$to be -90 ° or 270 ° , that is$q_1 = 4.712389rad$.

We want this movement to last **4 seconds**

Therefore

$$q_0 = [2.618 \quad -0.9278 \quad 1.1007 \quad 3.1416 \quad 1.1131 \; -0.9821]$$

$$q_{f3,arm} = [4.712389 \; -0.9278 \quad 1.1007 \quad 3.1416 \quad 1.1131 \; -0.9821]$$

Comparing the angles of the initial position with the final position we notice that the joints q_2, q_3, q_4, q_5, q_6 will not move.

$$q_1 = 2.618 + 0.39269(t-13)^2 - 0.06544(t-13)^3$$

$$\dot{q_1} = 0.785395(t-13) - 0.196348(t-13)^2$$

$$13 \le t \le 17$$

Movement 3 :

The arm will descend maintaining the same orientation as the position $q_{f3,arm}$ so that the x, y of the end is equal to the x , y of the center of the platform and z is greater than 10 cm so that the cylinder is placed on the platform. So

$$X_{BE} = X_{BM} = 0, \quad Y_{BE} = Y_{BM} = -0.35, \quad Z_{BE} = 0.1$$

$$q_0 = [4.712389\ -0.9278\quad 1.1007\quad 3.1416\quad 1.1131\ -0.9821]$$

$$q_{f3,\beta\rho} = [4.712389\ -0.8788\quad 2.1549\quad 3.1416\quad 0.1079\ -0.97902]$$

Comparing the angles of the initial position with the final position we notice that the joints q_1, q_4 will not move. We consider the movement to last 3 seconds.

$$\boldsymbol{q_2 = -0.9278 + 0.01633(t-17)^2 - 0.00362(t-17)^3}$$

$$\boldsymbol{\dot{q_2} = 0.03266(t-17) - 0.01088(t-17)^2}$$

$$\boldsymbol{17 \leq t \leq 20}$$

$$\boldsymbol{q_3 = 1.1007 + 0.3514(t-17)^2 - 0.078088(t-17)^3}$$

$$\boldsymbol{\dot{q_3} = 0.7028(t-17) - 0.234266(t-17)^2}$$

$$\boldsymbol{17 \leq t \leq 20}$$

$$\boldsymbol{q_5 = 1.1131 - 0.335066(t-17)^2 + 0.074459(t-17)^3}$$

$$\boldsymbol{\dot{q_5} = -0.670133(t-17) + 0.22377(t-17)^2}$$

$$\boldsymbol{17 \leq t \leq 20}$$

$$\boldsymbol{q_6 = -0.9821 + 0.001026(t-17)^2 - 2.28 * 10^{-4}(t-17)^3}$$

$$\boldsymbol{\dot{q_6} = 0.002053(t-17) - 6.84 * 10^{-4}(t-17)^2}$$

$$\boldsymbol{17 \leq t \leq 20}$$

5)

1. **Diagrams of the time response of the joint angles and the position of the limb and the platform with respect to the inertial frame.**

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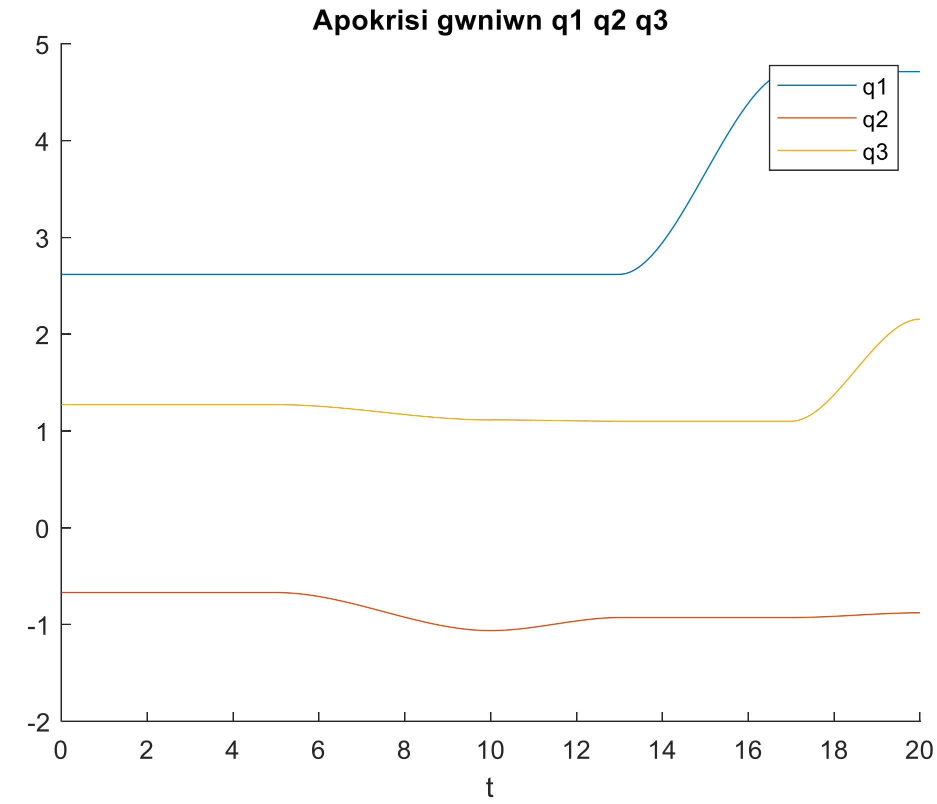

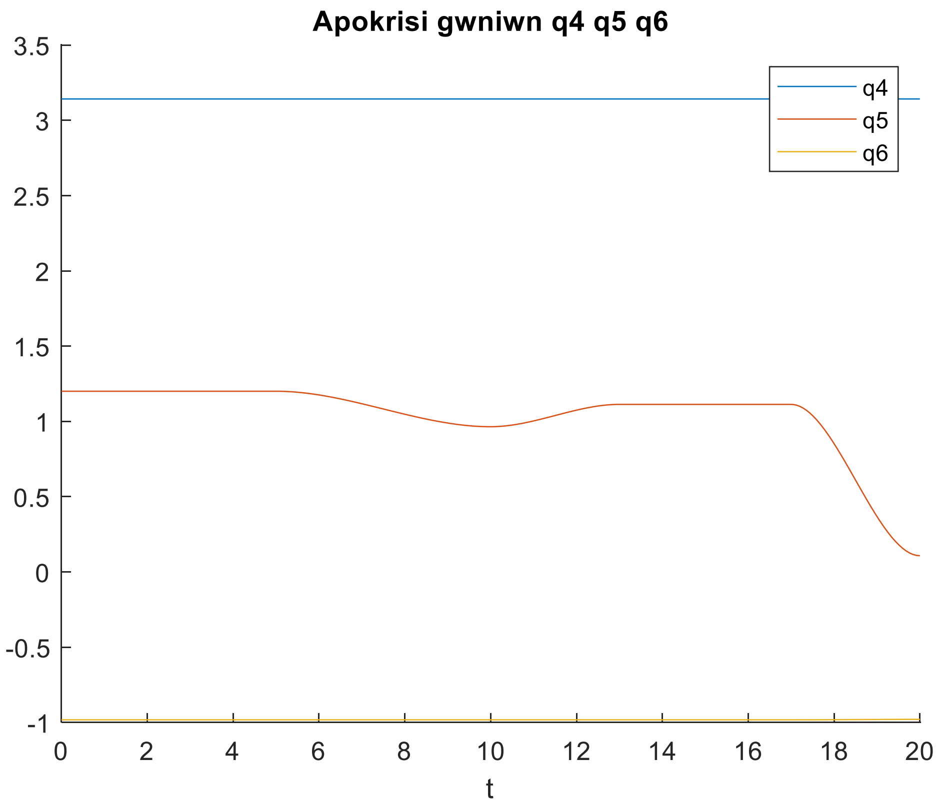

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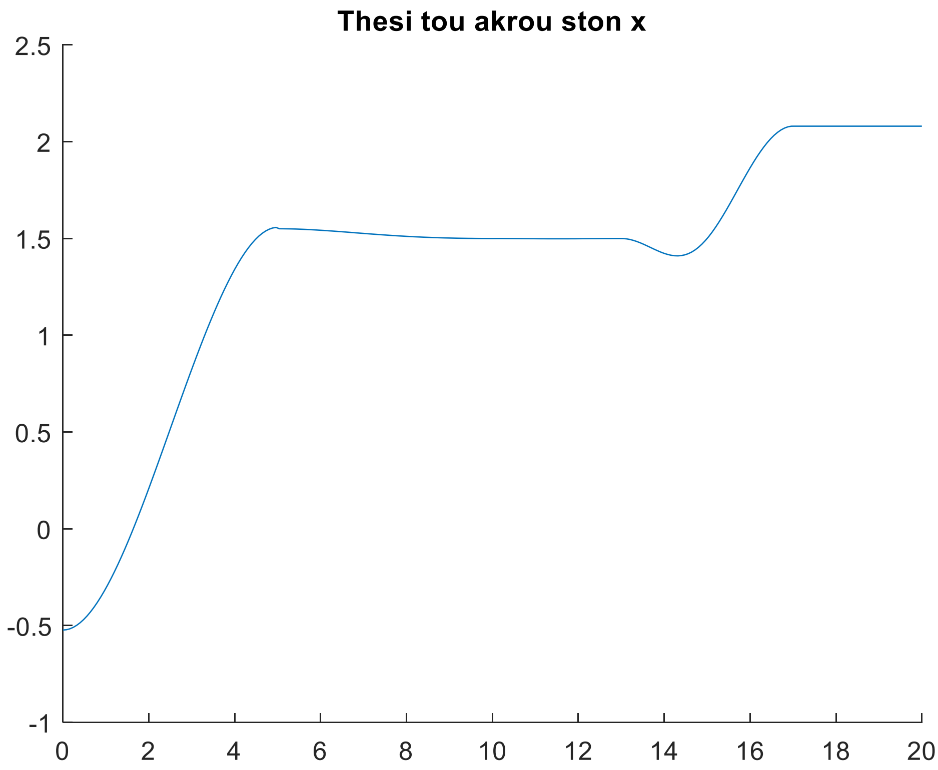
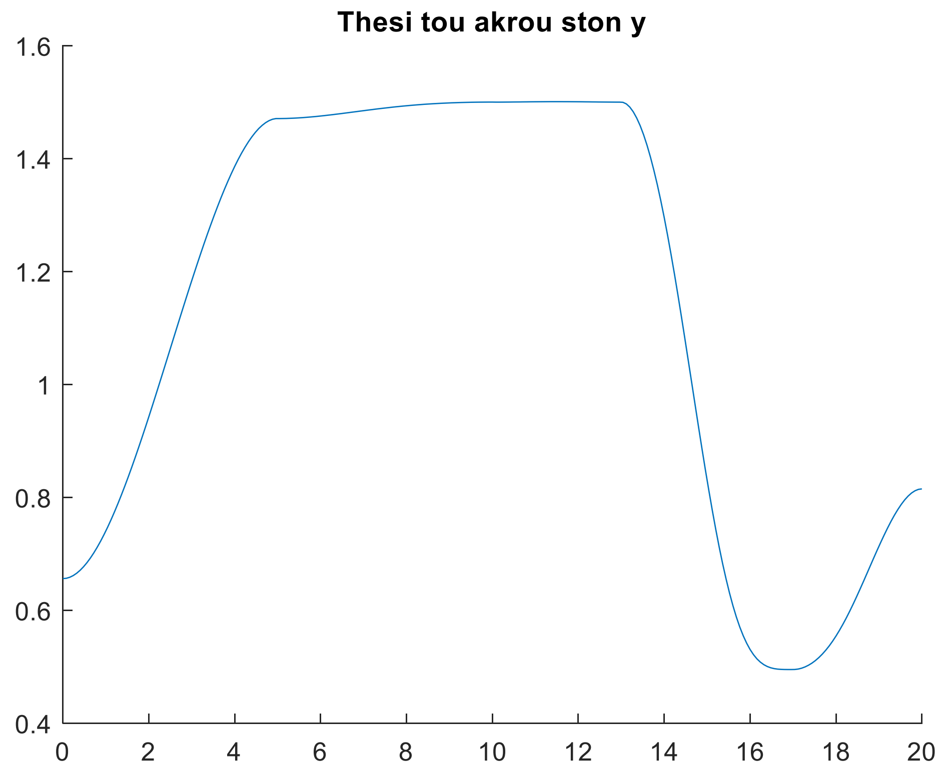
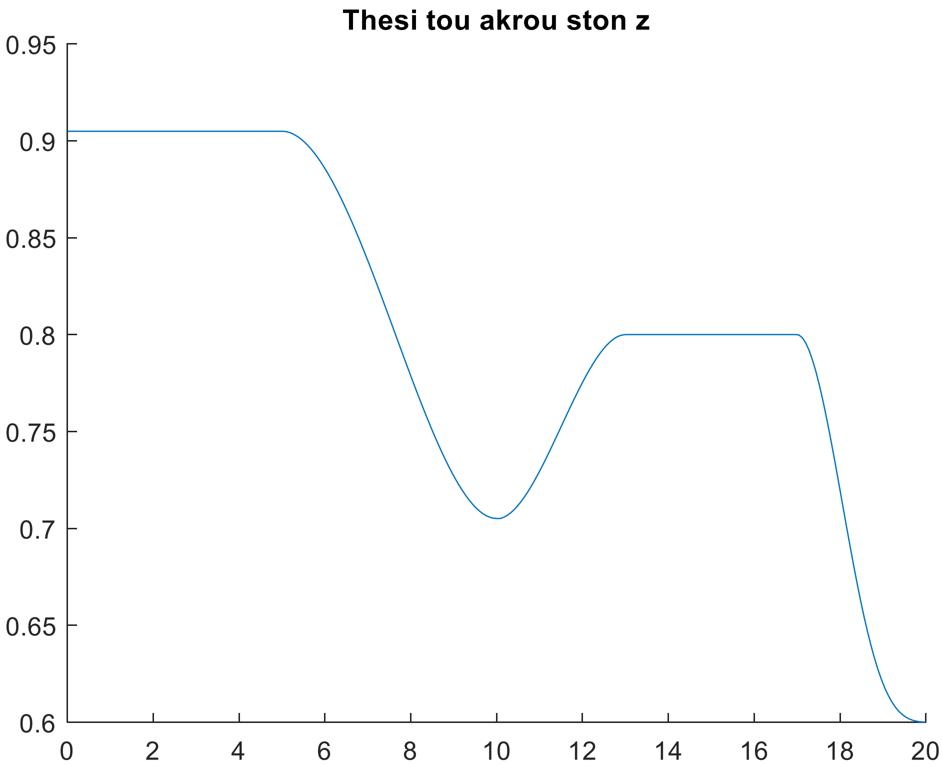
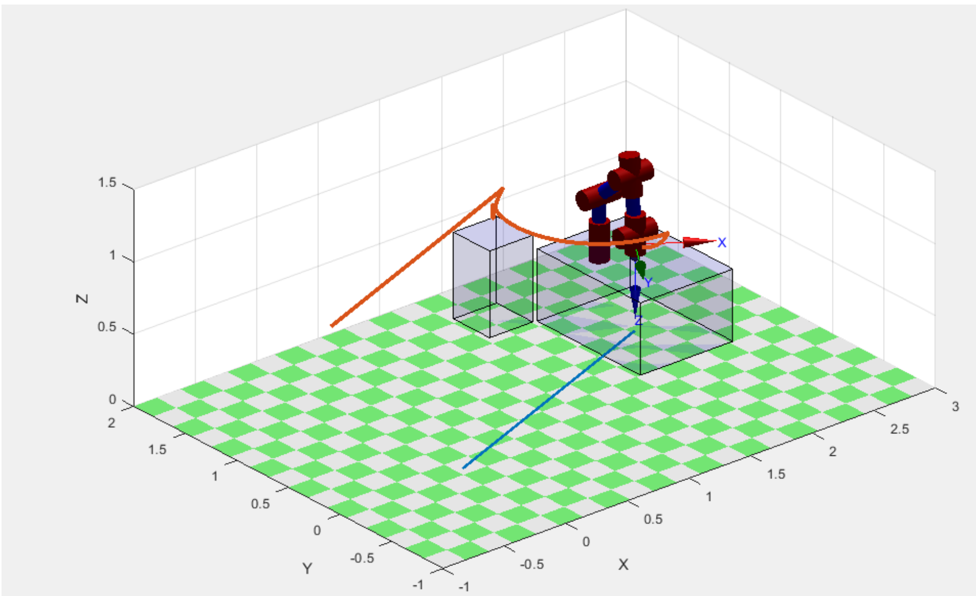


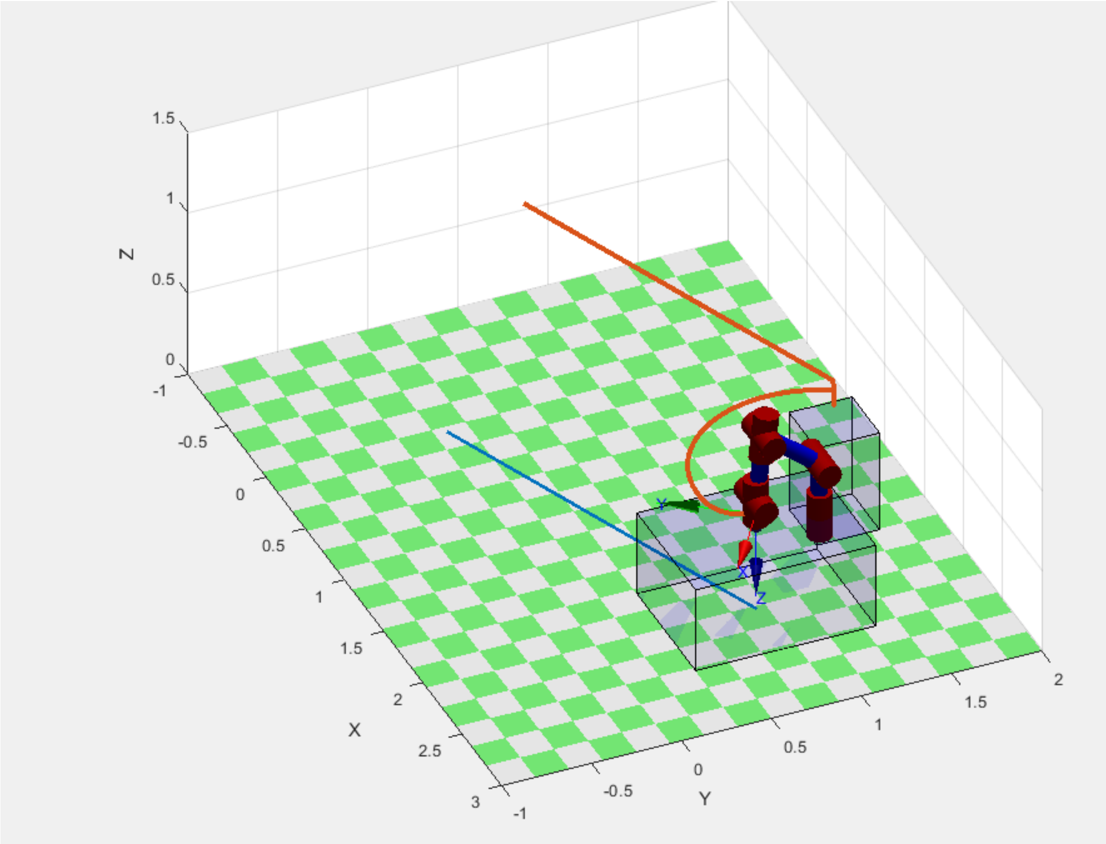


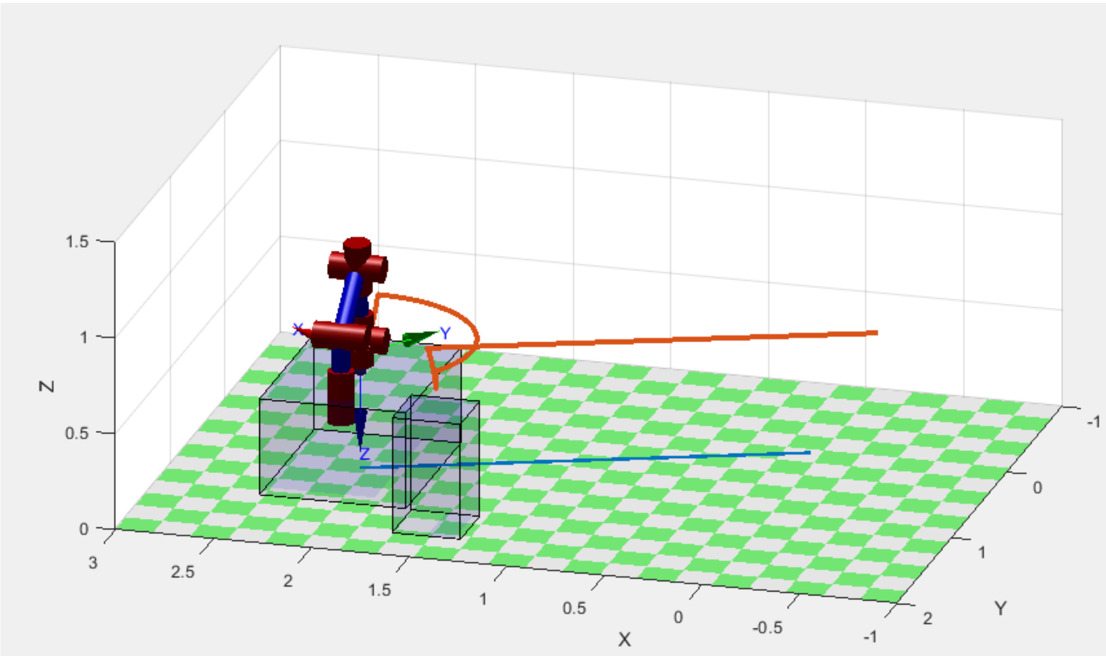


We notice that angle 4 does not change at all throughout the motion.

2. Diagrams of the path of the arm and the platform in 3D space

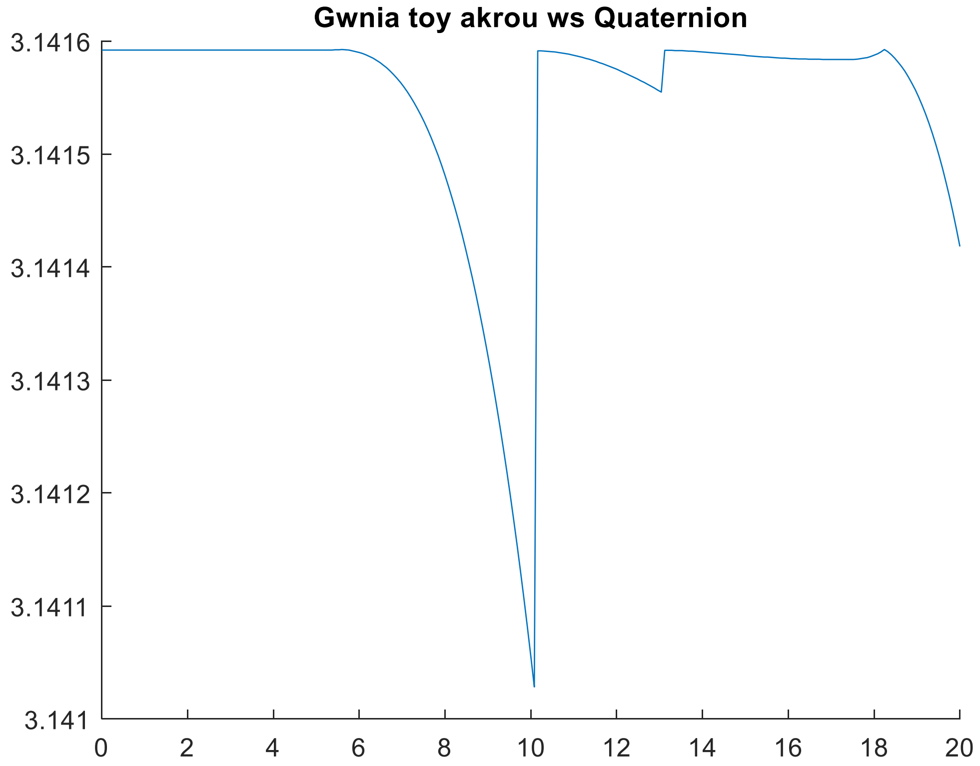






The path of the center of the platform is shown in blue and the edge in orange. The path is printed in figure 1 which is simulated after the end of the simulation.

3. Diagrams of the time response of the tip orientation angle as Quaternion .



We observe that the angle throughout the motion is almost constant and the small fluctuations are due to rounding errors.

4. Diagrams of the time response of the limb, platform and joint velocities.

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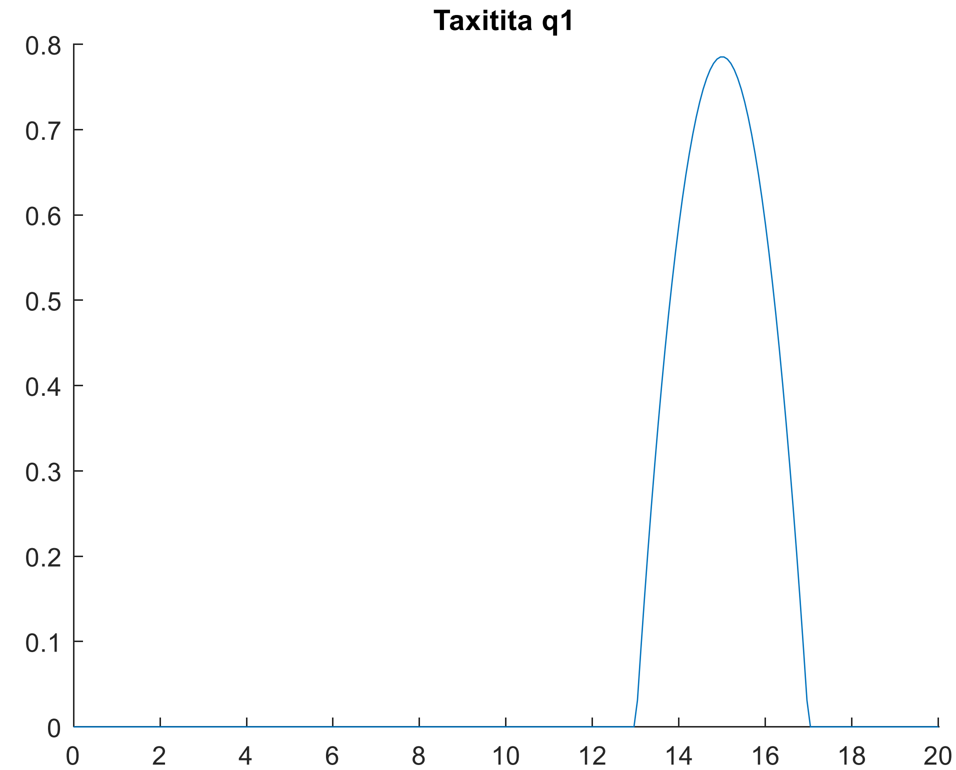

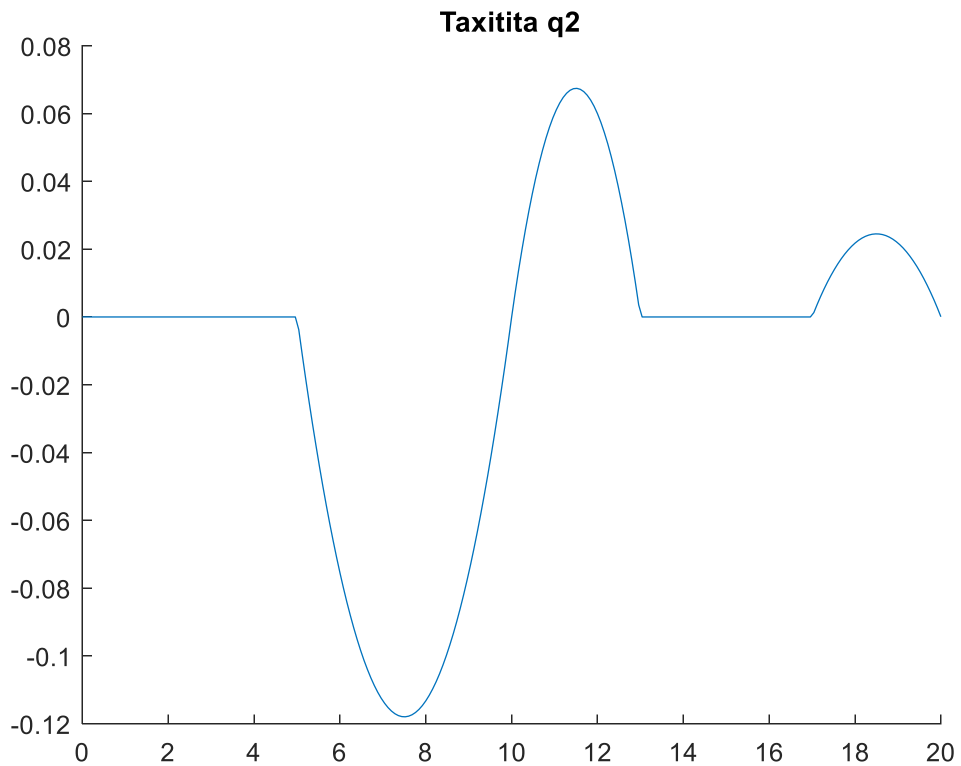

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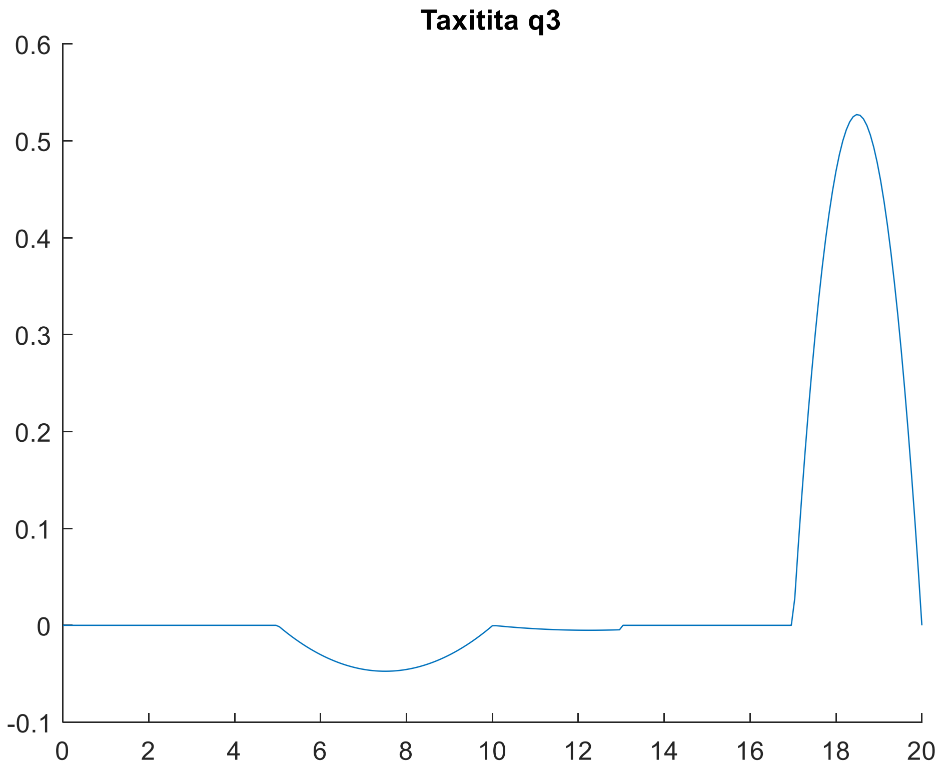
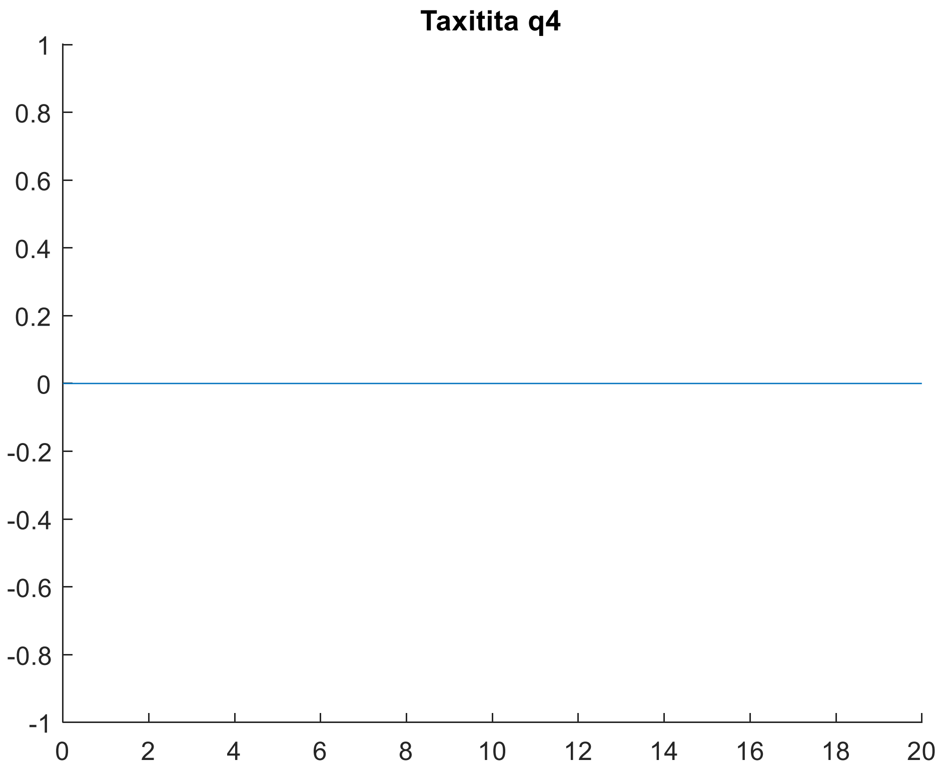
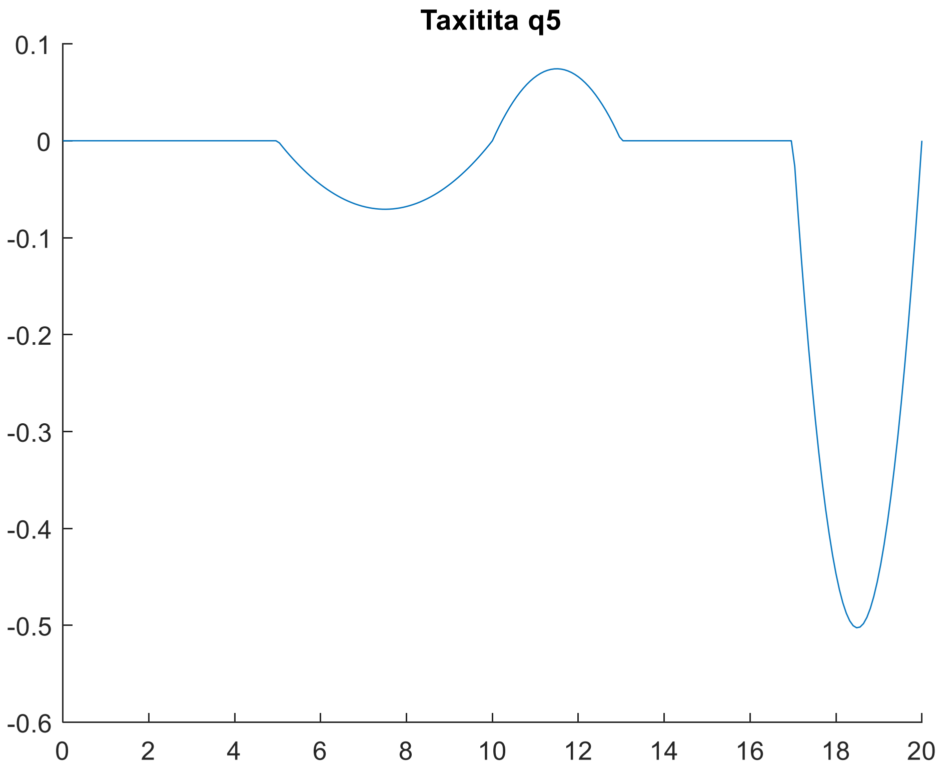
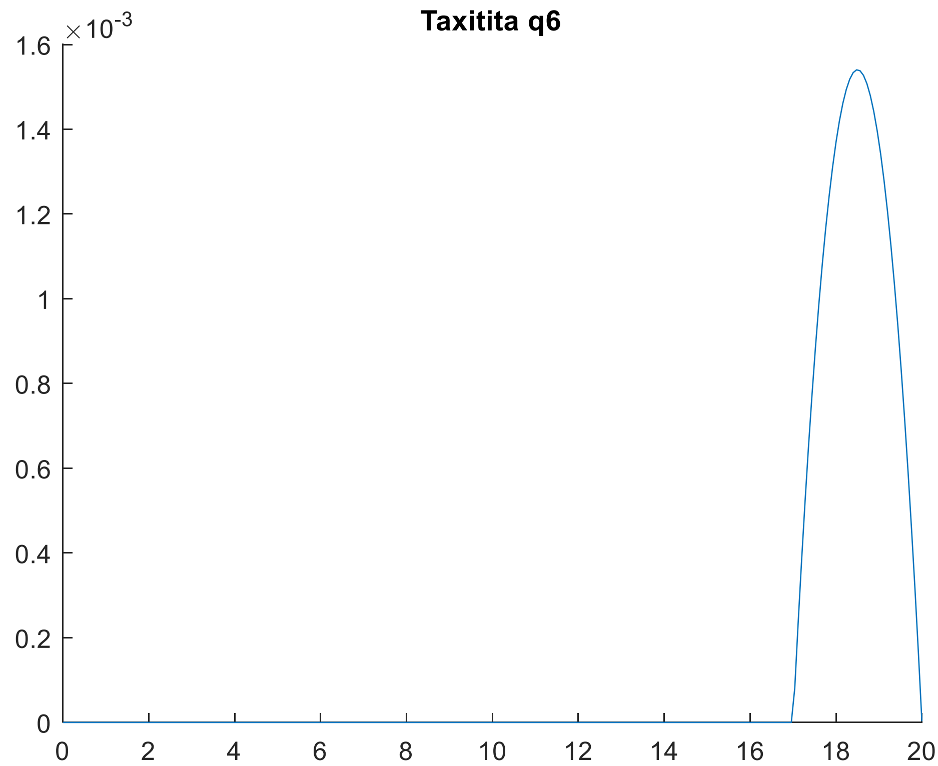





Taxitita q5
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
2
4
6
8
10
12
14
16
18
20


Taxitita q6
×10-3
1.6
1.4
1.2
1
0.8
0.6
0.4
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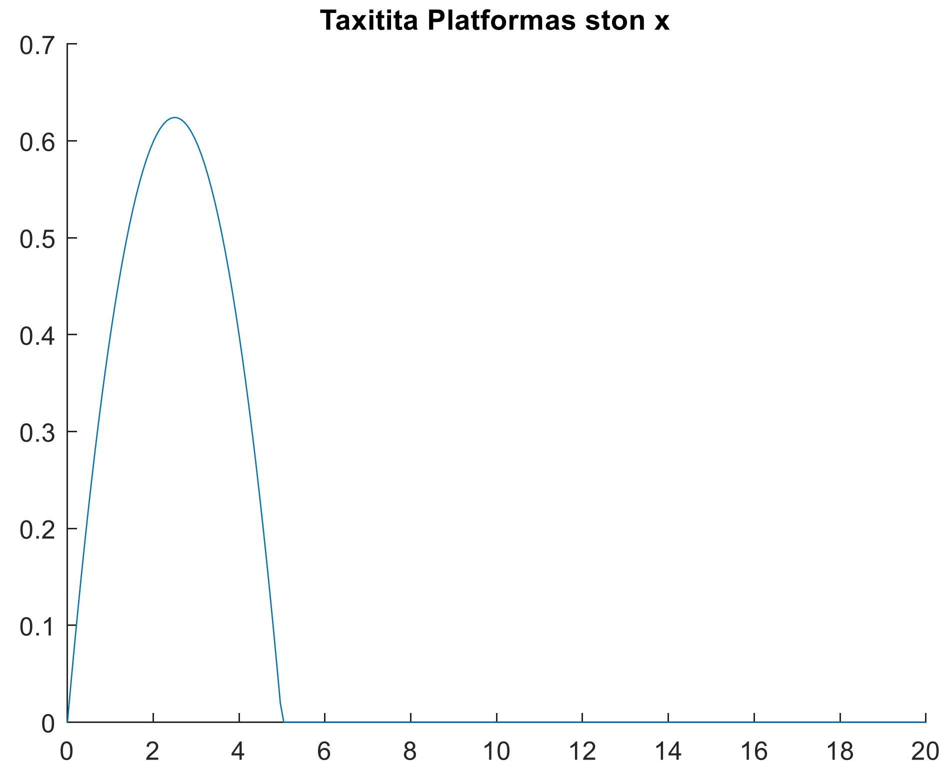

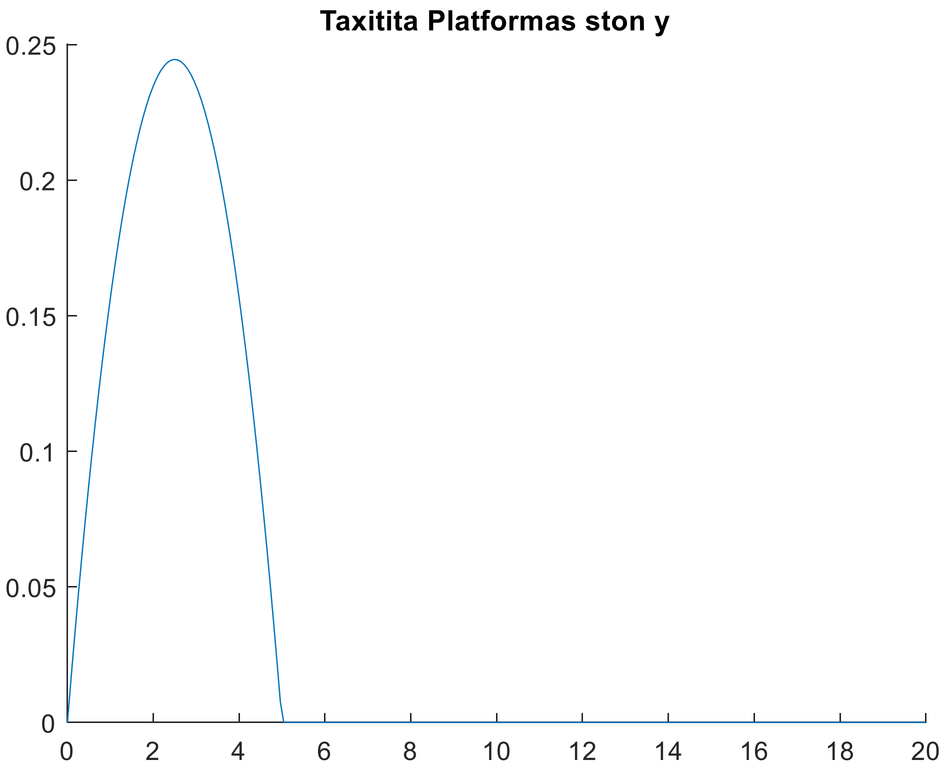

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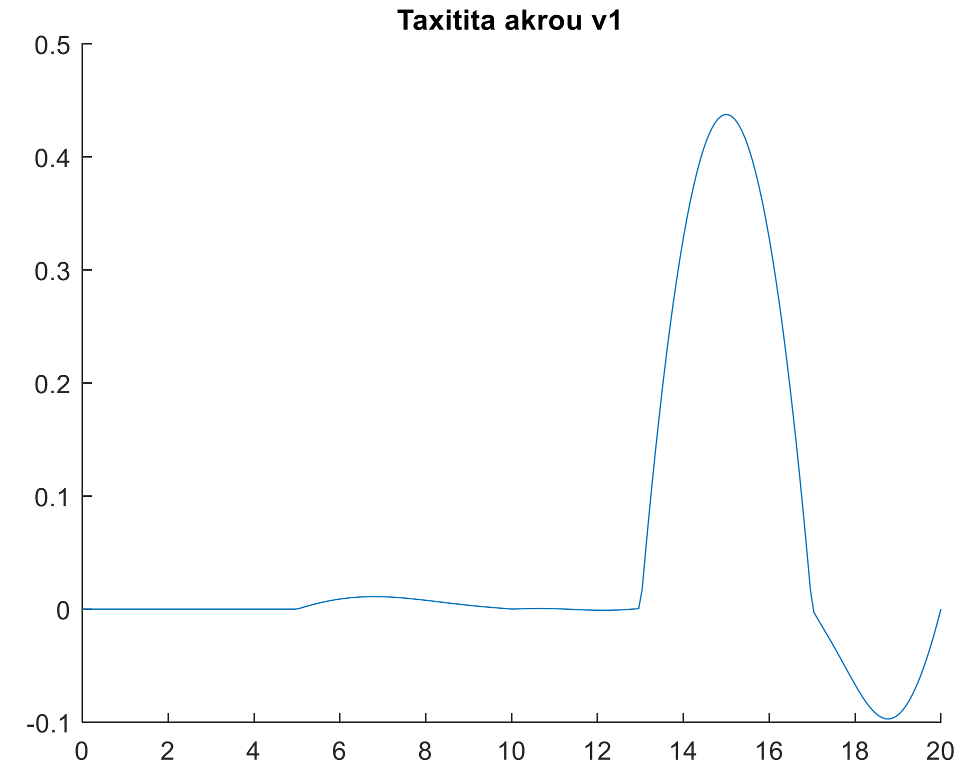

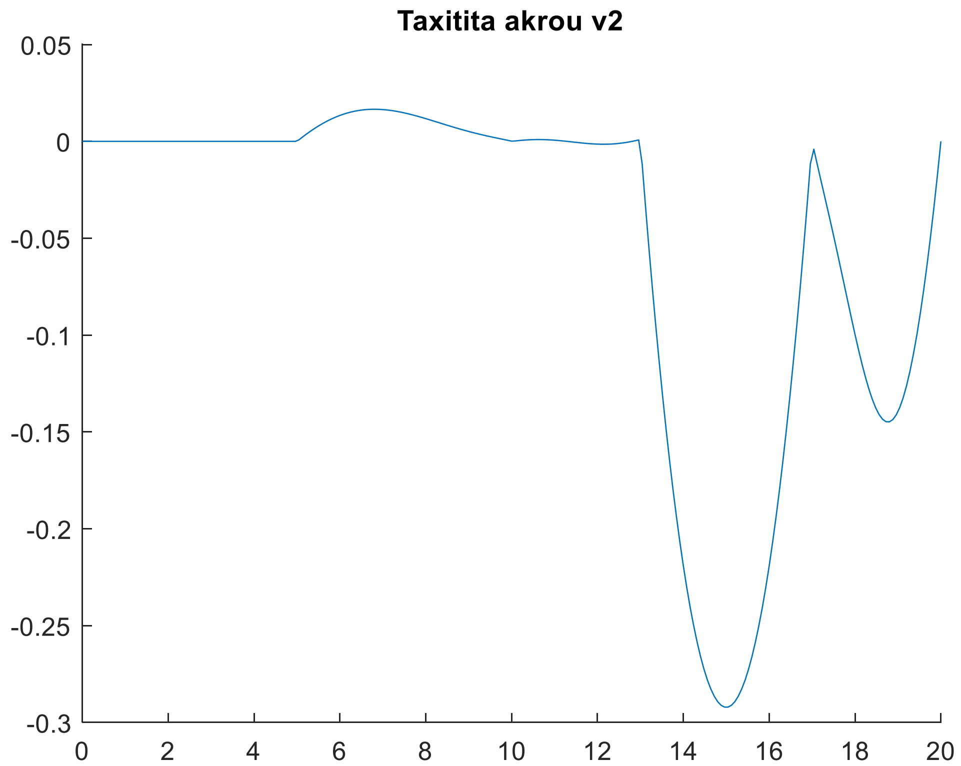

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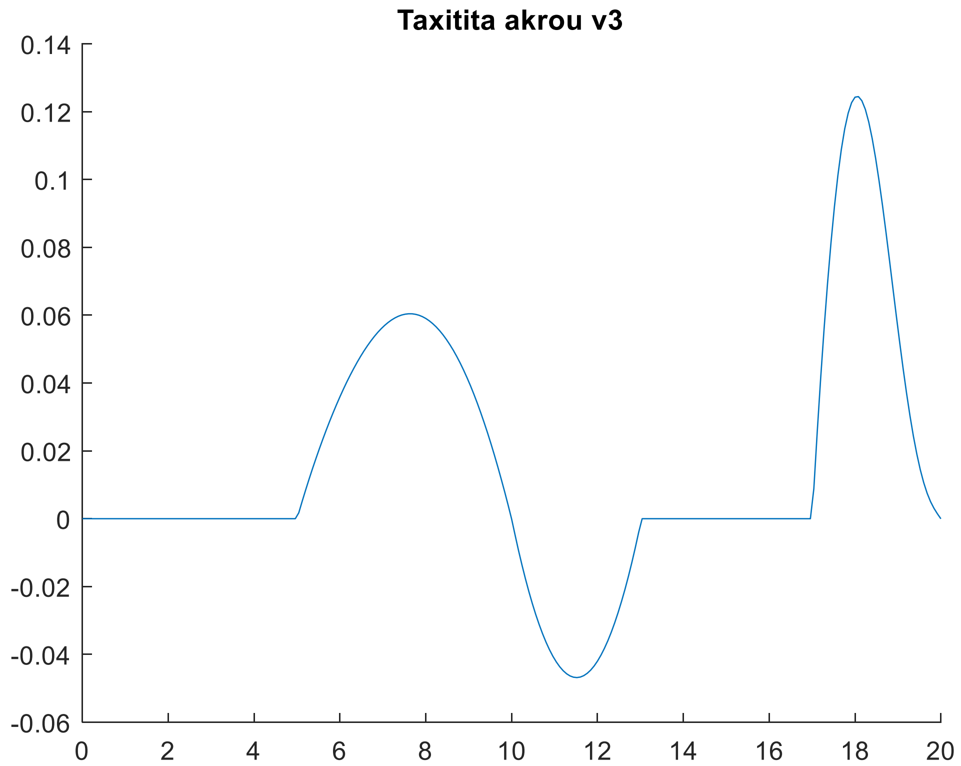

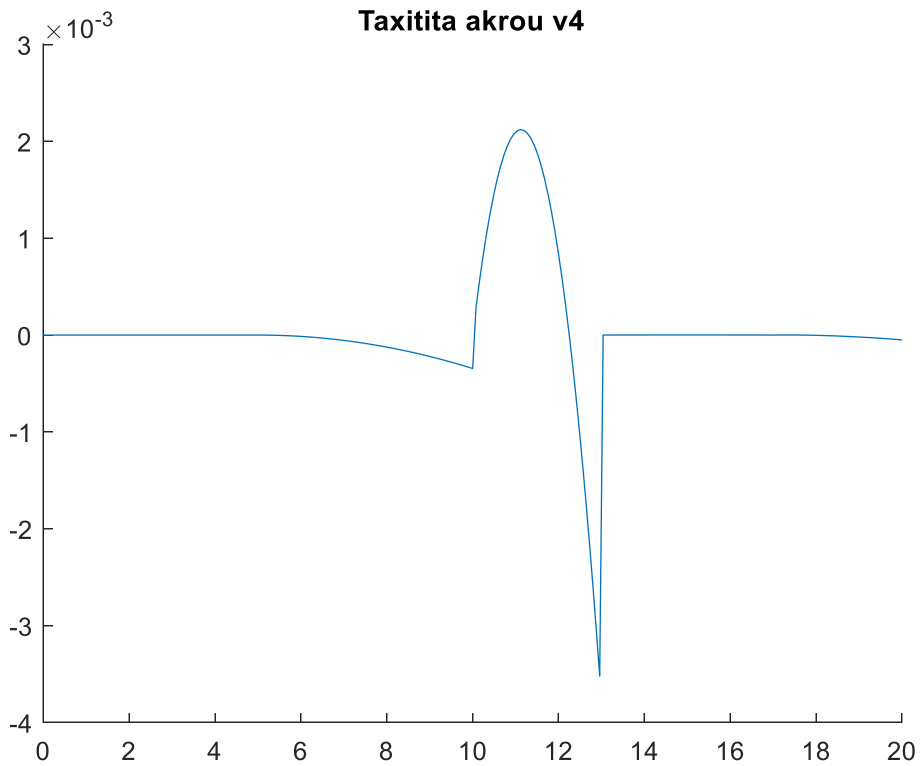

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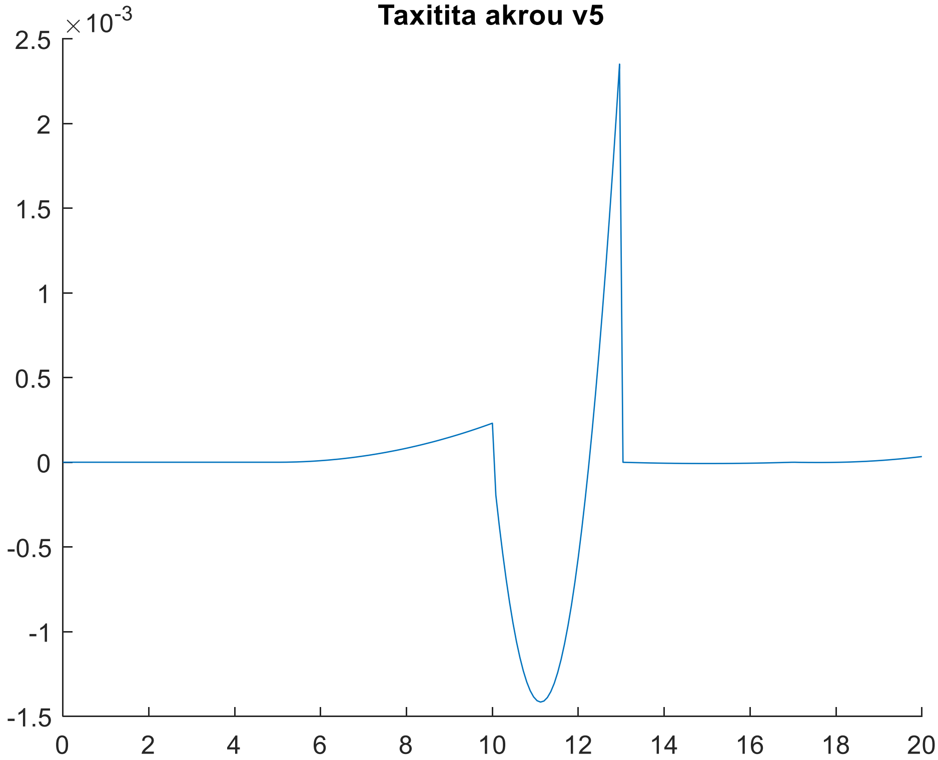

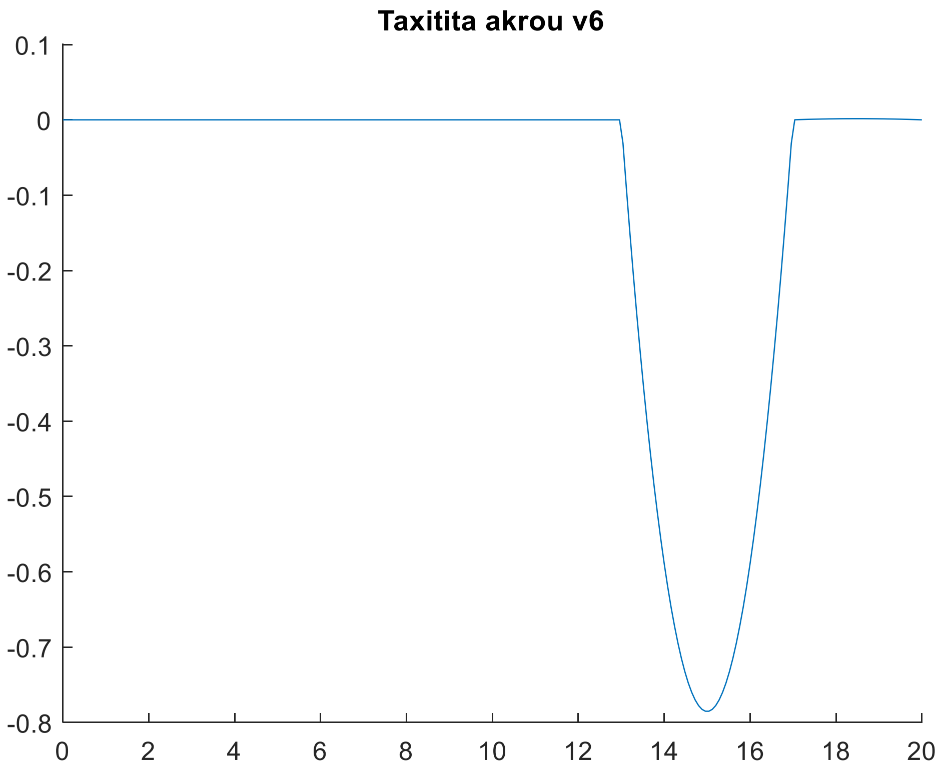

Some observations are the following:

- Matlab files contain some files .mat . These are transformation matrices in SE 3 format that were created from an original SE 3 and used in the results function.m to find the arm angles in the desired positions.
- plotcube function was found on the Internet and plots a rectangle with specific dimensions at some desired location.
- The simulation and plots are produced by running main .m
- During the simulation it was chosen not to draw the cylinder because it makes the simulation too slow. That is why it is only visible in its initial and final position
- Although the trajectory was designed to last 20 seconds, the simulation does not last exactly 20 seconds, and each individual movement does not take exactly the correct amount of time in the simulation. This is due to delays introduced by the plot function