

Introduction

AEP is a free package developed to calculate the distribution function and the Value-at-Risk of the sum $X_1 + \dots + X_d$ of d positive random variables, when the joint distribution H of the vector (X_1, \dots, X_d) is given. For any detail on the theory underlying AEP we refer to the paper

[P. Arbenz, P. Embrechts and G. Puccetti \(2009\): *The AEP algorithm for the fast computation of the distribution of the sum of dependent random variables*, preprint ETH Zurich.](#)

The AEP package contains two softwares: **AEP-Df** evaluates the distribution function $F(s) = P[X_1 + \dots + X_d \leq s]$ at a certain threshold s , while **AEP-VaR** evaluates the quantile (or Value-at-Risk) function $F^{-1}(\alpha)$ at a certain level α . Both programs accept as input the multivariate joint distribution H of the vector (X_1, \dots, X_d) . H has to be provided in terms of the marginal distributions $F_i, i=1, \dots, d$ and copula C of the vector (X_1, \dots, X_d) , i.e. $H = C(F_1, \dots, F_d)$. For any details on the theory of copulas, we refer to: [Nelsen, R.B. \(2006\), *An Introduction to Copulas*, Springer, New York. Second. ed.](#)

Inputs

Both softwares will require:

- 1) The number d of the random variables in the sum $X_1 + \dots + X_d$. It is possible to choose $d=2, 3, 4, 5$.
- 2) The marginal distribution F_i of each of the d random variables (different marginals can be chosen from different families). Possible choices for the softwares are:

Symbol	Distribution	Parameters	Marginal distribution $F_i(x) = P[X_i \leq x]$
p	Pareto	$a > 0, b > 0$	$1 - (1 + x/b)^{-a}, x > 0$
l	Log-Normal	$\mu > 0, \sigma > 0$	$(2\pi\sigma^2)^{-1/2} \exp(-(\ln(x) - \mu)^2 / (2\sigma^2)), x > 0$ (density)
u	Uniform (0,1)	-	$\max(x, 0) - \max(x-1, 0), x \text{ in } (0, 1)$
e	Exponential	$\lambda > 0$	$1 - \exp(-\lambda x), x > 0$
f	Frechét	$a > 0, b > 0$	$\exp(-(x/b)^{-a}), x > 0$

- 3) The multivariate copula C of the vector (X_1, \dots, X_d) . Only so-called Archimedean copulas and reversed Archimedean copulas are possible choices for the softwares. An Archimedean copula C has the form $C(u_1, \dots, u_d) = \psi(\psi^{-1}(u_1) + \dots + \psi^{-1}(u_d))$. Given an archimedean copula C , the corresponding reversed archimedean copula is $C(u_1, \dots, u_d) = C((1-u_1), \dots, (1-u_d))$. All these copulas are given in terms of their generator ψ .

Symbol for the copula	Symbol for the reversed copula	Copula	Parameters	Generator $\psi(t)$
i	i	Independence	-	$-\ln(t)$
c	q	Clayton	$\theta > 0$	$(1+t)^{-1/\theta}$
g	w	Gumbel	$\theta \geq 1$	$\exp(-t^{1/\theta})$
f	e	Frank	$\theta > 0$	$-\ln(\exp(-t)(\exp(-\theta)-1)+1)/\theta$
j	r	Joe	$\theta \geq 1$	$1 - (1 - \exp(-t))^{1/\theta}$
a	t	Ali-Mikhail-Haq	$\theta \text{ in } [0, 1]$	$(1-\theta)/(\exp(t)-\theta)$

- 4) The threshold $s > 0$ (only for AEP-Df) or, respectively, the level α in $(0, 1)$ (only for AEP-VaR) at which to compute the distribution function $P[X_1 + \dots + X_d \leq s]$ or, respectively, the quantile function $F^{-1}(\alpha)$.

- 5) The number of cycles n of the algorithm. Increasing n will increase the precision of the final estimate as well as the memory and the computational time needed to obtain it. We suggest to use the following numbers: $n=12$ ($d=2$), $n=9$ ($d=3$), $n=5$ ($d=4$), $n=4$ ($d=5$); see Fig. 5 in the paper. Note that choosing a larger n gives extra precision but it could

exhaust the memory of your computer.

Outputs

The software will give n estimates (one for each cycle of the algorithm) of $F(s)=P[X_1+\dots X_d \leq s]$ (only for AEP-Df) or $F^{(-1)}(\alpha)$ (only for AEP-VaR). For any issues concerning the precision of these estimates, we refer to the paper.

Error Handling

No error handling has been implemented in this version of AEP, so be extremely careful when inserting inputs. This means: if you are asked a character, simply type that character and enter. Any number has to be typed in decimal format, i.e. 12.3456 or 1.234e3.

Requirements

AEP runs on any Windows, MAC or Linux machine. Just double-click the version with the name corresponding to your operating system (i.e. AEP-DfMac and AEP-VaRMac identify the MAC-friendly versions of the softwares) and execute the program. The linux files have to be executed by typing in a terminal `./AEP-DfLin` or `./AEPVaRLin` in the directory containing the file.

Licence

The softwares in this version are intended only for the personal use of the editors and the referees of *Bernoulli*. We kindly request not to copy or distribute them.

Support

The following two examples show the correct use of the softwares.

AEP-Df

Please enter the dimension (≤ 5): 3

Please enter the distribution of the marginals. For a Pareto distribution, enter "p". Lognormal = "l". Exponential = "e". Uniform = "u", Frechet = "f":

First marginal? p

Pareto: $F(x) = 1 - (1 + x/b)^{-a}$. Please set $a (> 0)$: 0.9

Please set $b (> 0)$: 1

Second marginal? u

Uniform: $F(x) = \max(x, 0) - \max(x + 1, 0)$.

Third marginal? e

Exponential: $F(x) = 1 - \exp(-a \cdot x)$. Please set a : 2

*Available copulas are Independence (enter "i"), Clayton ("c"), Frank ("f"), Gumbel ("g"), Joe ("j"), Ali-Mikhail-Haq ("a"), reversed Clayton ("q"), reversed Frank ("w"), reversed Gumbel ("e"), reversed Joe ("r"), reversed Ali-Mikhail-Haq ("t"). Please enter the desired copula.*f

Frank: Please set $\theta > 0$: 2

Please enter the threshold s : 10

Number of iterations? 12

*estimate $P1 = 0.80059999898251$ Extrapolated estimate $P^*1 = 1.06746666531$
estimate $P2 = 0.85367972500268$ Extrapolated estimate $P^*2 = 0.8713729670094$
estimate $P3 = 0.86856862359423$ Extrapolated estimate $P^*3 = 0.87353158979142$
estimate $P4 = 0.86922669678187$ Extrapolated estimate $P^*4 = 0.86944605451108$*

estimate P5 = 0.8698980927183 Extrapolated estimate P*5 = 0.87012189136378
estimate P6 = 0.86996399391454 Extrapolated estimate P*6 = 0.86998596097995
estimate P7 = 0.8700083023897 Extrapolated estimate P*7 = 0.87002307188142
estimate P8 = 0.87001182256128 Extrapolated estimate P*8 = 0.87001299595181
estimate P9 = 0.87001454229651 Extrapolated estimate P*9 = 0.87001544887493
estimate P10 = 0.87001477104056 Extrapolated estimate P*10 = 0.87001484728857
estimate P11 = 0.87001494175006 Extrapolated estimate P*11 = 0.87001499865323
estimate P12 = 0.87001495590837 Extrapolated estimate P*12 = 0.87001496062781 logout

AEP-VaR

Please enter the dimension (≤ 5): 2

Please enter the distribution of the marginals. For a Pareto distribution, enter "p". Lognormal = "l". Exponential = "e". Uniform = "u", Frechet = "f":

First marginal? e

Exponential: $F(x) = 1 - \exp(-a \cdot x)$. Please set $a (> 0)$: 2

Second marginal? p

Pareto: $F(x) = 1 - (1 + x/b)^{-a}$. Please set $a (> 0)$: 2

Please set $b (> 0)$: 2

Available copulas are Independence (enter "i"), Clayton ("c"), Frank ("f"), Gumbel ("g"), Joe ("j"), Ali-Mikhail-Haq ("a"), reversed Clayton ("q"), reversed Frank ("w"), reversed Gumbel ("e"), reversed Joe ("r"), reversed Ali-Mikhail-Haq ("t"). Please enter the desired copula: i

Please enter the threshold α : 0.99

Please enter the maximal number of iterations the VaR search can employ to evaluate $P_n(s)$: 12

final VaR-estimate: 18.5202logout