# Learning summary features of time series for likelihood-free inference



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We have a simulator parametrized by  $\theta$ 

Our goal is to determine an approximation

parameters for a given observed time series:

 $p(\boldsymbol{\theta}|\boldsymbol{x}_o) \approx q_{\boldsymbol{\phi}}(\boldsymbol{\theta}|f_{\boldsymbol{\lambda}}(\boldsymbol{x}_o))$ 

 $\phi$  parametrizes the density estimator

 $\lambda$  parametrizes the feature extractor

## CONTEXT



Likelihood-free inference (LFI) methods make **Bayesian inference** on modern physical simulators possible.

Most algorithms use a set of handcrafted summary features to describe data.

We propose a data-driven approach that learns the most appropriate summary features for LFI on time series data generated by linear and non-linear models

# PROBLEM FORMULATION

that generates time series x(t)

to the posterior distribution of the



# **OUR PROPOSAL**



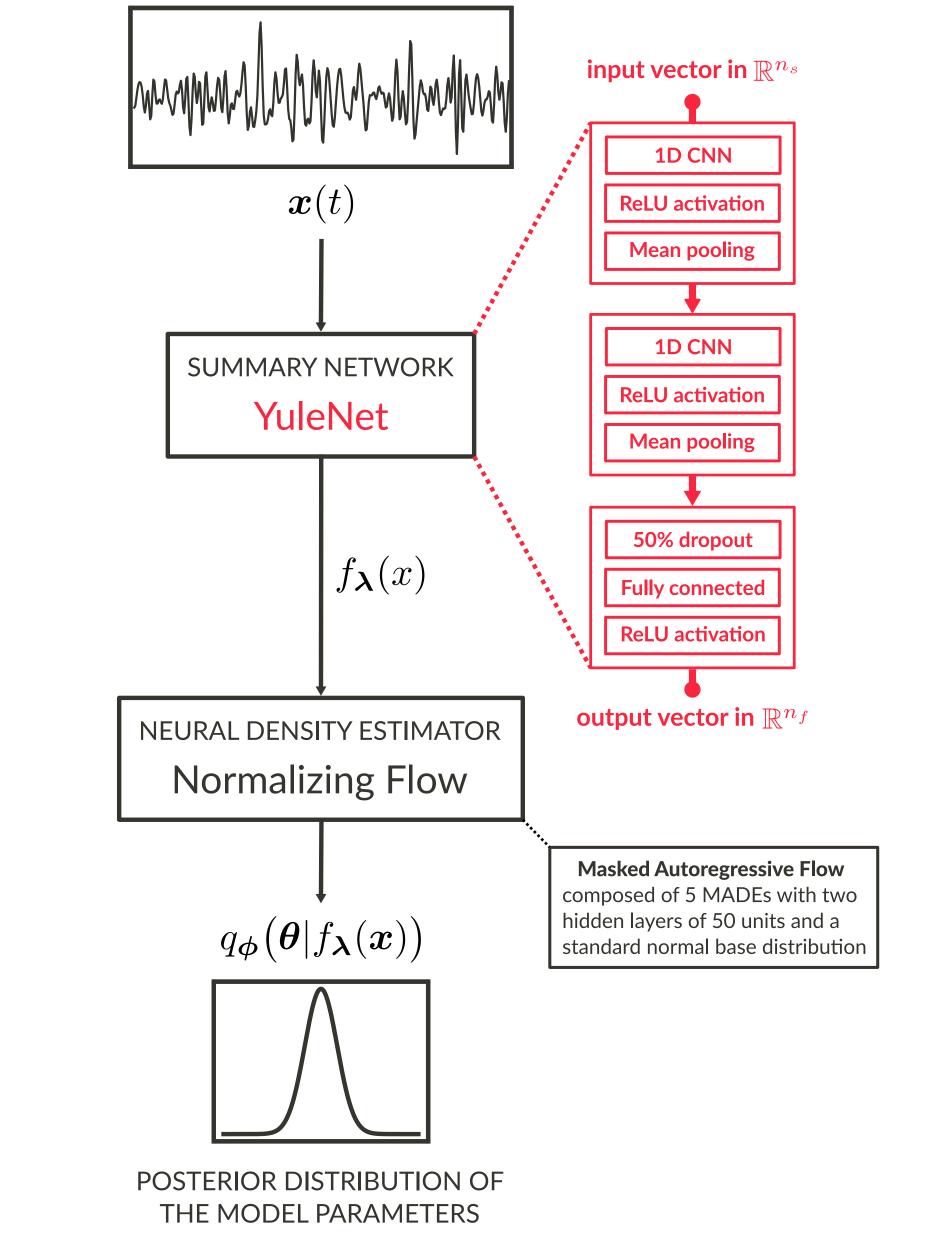
We present the **YuleNet**, a two-layer neural network based on temporal convolutions.

The architecture comes from [1] and has been applied for classifying EEG sleep signals.

We jointly estimate  $\phi$  and  $\lambda$  by minimizing

$$\mathcal{L}(\boldsymbol{\phi}, \boldsymbol{\lambda}) = \mathbb{E}_{(\boldsymbol{x}, \boldsymbol{\theta}) \sim p(\boldsymbol{x}, \boldsymbol{\theta})} \left[ -\log(q_{\boldsymbol{\phi}}(\boldsymbol{\theta} | f_{\boldsymbol{\lambda}}(\boldsymbol{x}))) \right]$$

with a multiround procedure (SNPE-C) [2].





(1) Validation on simulations with linear Gaussian time series models and comparison with inference based on autocorrelations as summary features.

Uniformly better results with data driven features than handcrafted ones

(2) Comparison of YuleNet to Partially Exchangeable Networks (PEN) from [3].

Equivalent results on the Gaussian example but YuleNet has less parameters and demands less computations

(3) Evaluation on a non-linear dynamical system and comparison of autocorrelations to YuleNet

Autocorr gives poor results whereas YuleNet finds the ground truth parameters on every situation

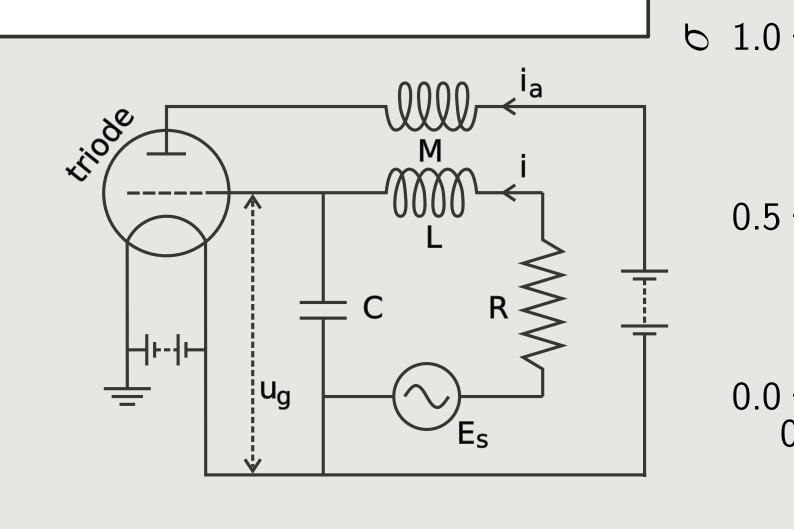
### The stochastic Van der Pol Oscillator

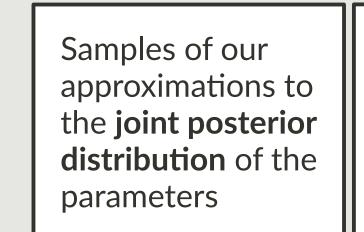
$$\ddot{x} = \varepsilon (1 - x^2)\dot{x} - x + \sigma \dot{w}$$

Observed time series: x(t)(  $\dot{w}(t)$  is an input Gaussian white noise)

### Parameters to infer:

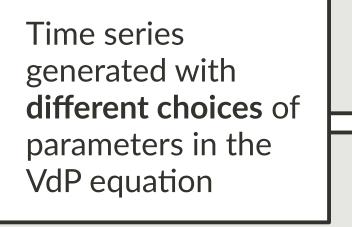
- $\bullet$   $\sigma$  is the variance of the stochastic input
- $\varepsilon$  is the degree of non-linearity of the VdP model

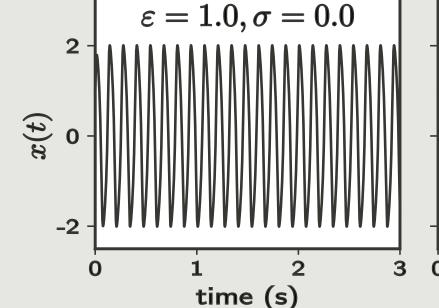


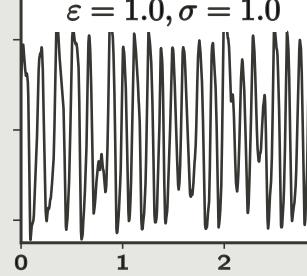


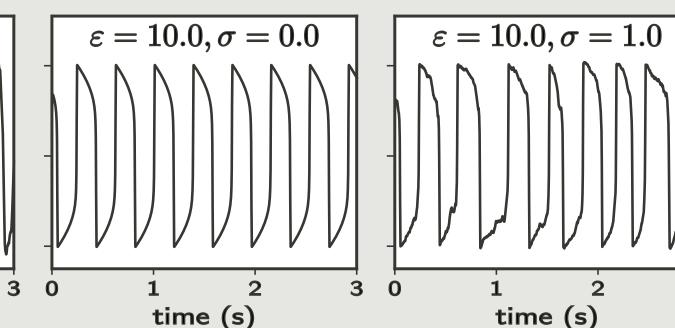
autocorr

 $m{ heta}_0^{(1)}$ 

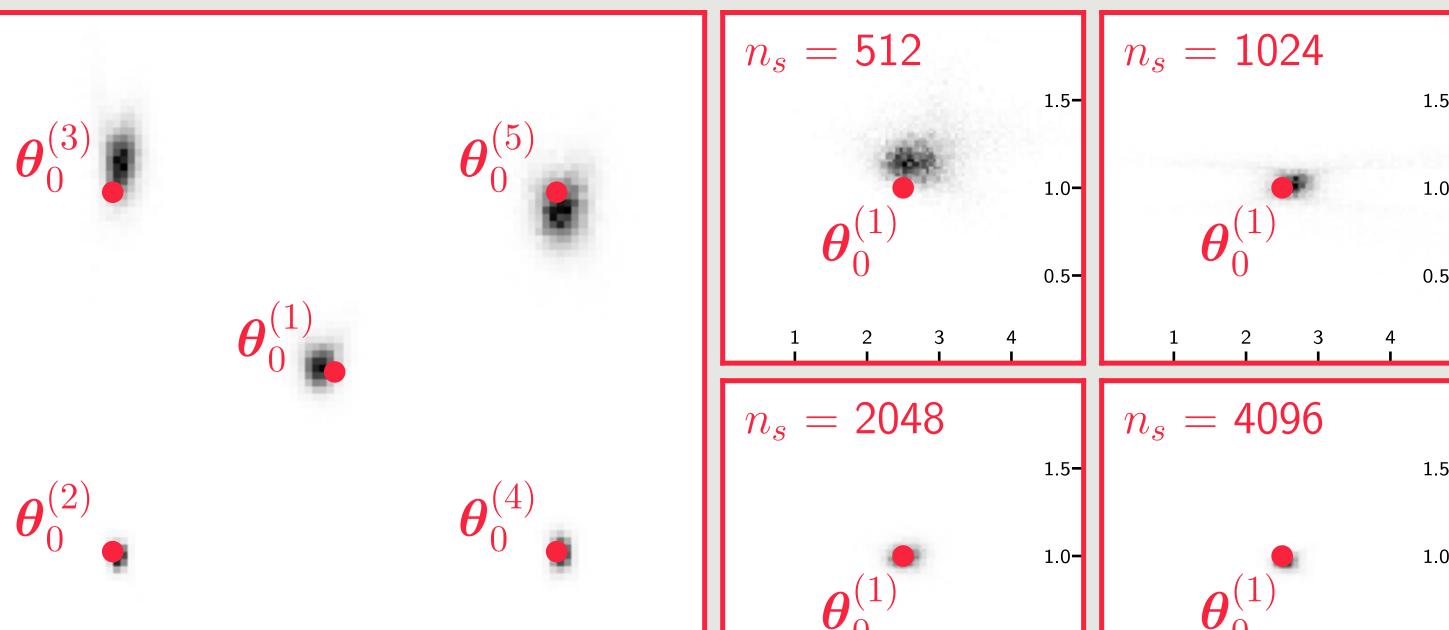












# DISCUSSION AND CONCLUSION

- YuleNet has good results on linear and non-linear stochastic models. Better think twice before using autocorrelations.
- YuleNet requires less parameters to train than PEN and is computationally efficient.
- YuleNet is a promising tool for studying non-linear dynamical systems such as those used in computational neuroscience.

# REFERENCES /

2.0

1.0

5.0

4.0

- [1] Chambon et al. (2017) arxiv.org/abs/1707.03321
- [2] Greenberg et al. (2019) arxiv.org/abs/1905.07488
- [3] Wiqvist et al. (2019) arxiv.org/abs/1901.10230