Reconstructing Conformal Field Theoretical Composition with Transformers

Haotian Cao
Department of Physics
Data Science Institute
University of Wisconsin-Madison
hcao39@wisc.edu

Garrett W. Merz
Data Science Institute
University of Wisconsin-Madison
garrett.merz@wisc.edu

Kyle Cranmer
Data Science Institute
University of Wisconsin-Madison
kyle.cranmer@wisc.edu

Gary Shiu
Department of Physics
University of Wisconsin-Madison
gshiu@wisc.edu

Abstract

We study the use of transformers to decompose tensor product of Rational Conformal Field Theories (RCFTs) based on their low-lying spectra. We achieve 98% in-domain performance and show that our method generalizes to RCFTs with larger central charge and unseen classes of RCFTs by adding a small number of out-of-domain examples. Our works shows that transformers can exploit the structure of RCFT data, which has applications from statistical physics to string theory.

1 Introduction

A common theme across the sciences is the task of inferring composition from spectra. In chemistry, nuclear magnetic resonance (NMR) spectroscopy provides structural information by mapping resonance frequencies to atomic environments, enabling the reconstruction of complex molecular architectures from spectral data [9]. Likewise, mass spectrometry identifies unknown compounds by analyzing the pattern in which molecules break into fragments, revealing how these smaller components serve as the fundamental building blocks of the larger molecular structure, a principle that underlies large spectral databases such as METLIN [25]. In this paper, we address a similar inverse problem for conformal field theories.

Conformal Field Theories (CFTs) are Quantum Field Theories (QFTs) with conformal symmetry, which places strong constraints on their dynamics, especially in two dimensions [10, 14]. They appear across physics: in statistical mechanics, CFTs describe scale-invariant systems such as spin lattices at criticality [23], while in string theory, the worldsheet dynamics of strings is governed by a two-dimensional CFT with the appropriate central charge. A particularly tractable subclass is Rational Conformal Field Theories (RCFTs), which possess a finite number of primary operators and allow exact constructions of certain string compactifications [24]. However, worldsheet CFTs are not restricted to be rational. Beyond string compactifications, CFTs also play a central role in the AdS/CFT correspondence [19]. The most celebrated example, 4d $\mathcal{N} = 4$ supersymmetric Yang–Mills theory, conjectured to be dual to type IIB string theory on $AdS_5 \times S^5$, is itself a non-rational CFT.

Even without surveying the full landscape of CFTs, Rational Conformal Field Theories (RCFTs) already provide an infinite family of synthetic models with arbitrarily large central charge. The central charge c measures the effective number of degrees of freedom, especially at high energies. This is reflected in Cardy's formula, $\rho(\Delta) = \exp\left(2\pi\sqrt{c\Delta/6}\right)$ [6] which gives the asymptotic density

Machine Learning and the Physical Sciences Workshop, NeurIPS 2025.

of states $\rho(\Delta)$ at conformal dimension Δ with entropy $S = \ln\rho(\Delta)$. Larger central charges can be engineered by forming tensor products of simpler CFTs, much like molecules assembled from atoms. This principle underlies Gepner models [13, 12], which yield exactly solvable worldsheet constructions of Calabi–Yau compactifications (see [20] for a recent review), and symmetric product orbifolds, which serve as controlled laboratories for holography [2, 11, 3, 16, 17].

In these constructions, the **forward construction** – moving from the seed CFTs to the tensor product – is straightforward: central charges and operator dimensions simply add. The **inverse problem** – moving from the low-lying spectra of tensor product models to the seed CFTs – is much harder, since distinct tensor products can produce nearly identical low-energy behavior. Here the analogy with physical spectroscopy is direct: just as chemists infer molecular composition from spectral lines, we ask whether the "spectral lines" of a tensor product RCFT – its low-lying conformal dimensions – are sufficient to reconstruct its constituents, the central charges and symmetry algebra of the seed CFTs. Progress on this inverse problem would not only enlarge our understanding of the RCFT landscape [22, 21] but also provide sharper tools for reconstructing bulk gravitational spacetimes via holography [18, 1].

2 CFT Basics and Data Generation

Similar to statistical physics, the spectra of CFTs are embedded in the partition functions as conformal "tower" of states. The modular invariant partition function that contain holomorphic (left-moving) and anti-holomorphic (right-moving) sectors takes the general form

$$Z(\tau) = \sum_{j\bar{j}} N_{j\bar{j}} \chi_j(q) \bar{\chi}_{\bar{j}}(\bar{q}) \tag{1}$$

where $q = e^{2\pi i \tau}$ and $\chi_j(q) = \mathrm{Tr}\ q^{L_0 - \frac{c}{24}}$ is the generating function with central charge c. We see that the energy spectra for the left-moving sector are stored in the exponent of the generating function as $E = L_0 - \frac{c}{24}$, where L_0 have eigenvalues h called the conformal weights or conformal dimensions [10] corresponding to the primary fields. There are different classes of RCFTs where the symmetry algebra and the spectra may look different. We include two large classes of RCFTs: affine Kac-Moody algebra and coset models.

Affine Kac-Moody (KM) algebra is an infinite dimensional Lie algebra and it gives rise to the gauge groups similar to what we usually see in high energy particle physics such as $SU(3)$, $SU(2)$, and $U(1)$. The spectra of affine Kac-Moody algebra CFTs can be realized through Sugawara construction of the energy-momentum tensor [10]. Given an algebra A with an arbitrary representation r, the conformal dimension and the central charge are defined as

$$h_r = \frac{C_r/\psi^2}{k+g} \qquad\qquad c_A = \frac{k \dim A}{k+g} \tag{2}$$

where C_r is the quadratic Casimir, k is known as levels or Coxeter numbers, ψ is the highest root and g is the dual Coxeter number [7, 10, 14]. Meanwhile, the coset models are constructed from affine KM algebra by quotienting it with another continuous group. For example, the Virasoro minimal models [5, 10, 14] can be obtained from the coset construction.

To make tensor product models of CFTs, we take the product of the diagonal partition functions 1 when $N_{j\bar{j}} = \delta_{j\bar{j}}$. In this way, the conformal dimensions and the central charges embedded in the exponent of the partition function are additive. We create a library of KM algebra: $su(2)$, $su(3)$, $su(4)$, $su(5)$, $su(6)$, $so(7)$, $so(9)$, $so(11)$, $so(8)$, $so(10)$, $so(12)$, $sp(4)$, $sp(6)$, $sp(8)$, E_6, E_7, E_8, F_4, and G_2. For each algebra type, we include up to $k = 10$ and truncate the number of lowest conformal dimensions that are smaller than 1 up to 6. Then, we add the conformal dimensions from L seed RCFTs in a combinatorial way to generate the spectra of the tensor product models where L is truncated to be 5 at most. In each tensor product theory, we only include conformal dimensions that are smaller than 1 and the maximum number of conformal dimensions is 50. Higher levels, more conformal dimensions, and larger L can be achieved with higher computational power. Since Eq.2 and Eq.1 are closed, our data is exact and discrete, thus demonstrating no noisy structures. The correct model predictions will also be exact as there is no numerical approximation.

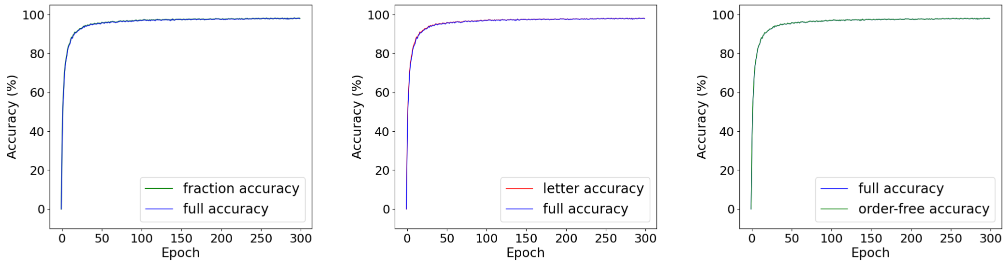

Figure 1: The left, middle, and right figures are comparisons between four main metrics for the base model training.

3 Results on the Inverse Problem

We frame the inverse problem as a sequence-to-sequence task. For i-th tensor product theory we provide as input the conformal dimensions $\{h_{i,l}\}$ at energy level l and predict as output its L_i constituent CFTs $A_{i,j}$ and corresponding central charges $c_{i,j}$, with $j = 1 \ldots L_i$. The conformal dimensions and central charges are expressed as reduced fractions, and we only include the $h_{i,l} < 1$. For an example of tensor product of two $su(2)$ theories with central charge equal to 1, the training data looks like

```
0/1, 1/4, 1/2    >    (su2, 1/1), (su2, 1/1)
```

All integers (except those used to label an individual CFT) are expressed in base 1000 and converted to tokens; the CFT labels ($su(2)$, $so(3)$, etc.) and the fraction symbol '/' are tokenized as well.

In the first experiment, we train encoder-decoder transformers [26] built on an existing implementation used for theoretical physics work [4] to predict the $\{(c_{i,j}, A_{i,j})\}_{j=1}^{L_i}$ pairs from the conformal dimensions $\{h_{i,l}\}$ using the cross-entropy loss. We restrict the central charge of the tensor product model to be < 50 in the training data and test on a held-out dataset. The total number of samples in the training set is 3 million whereas the number of samples in the validation set is $10,000$. Our models have 2 encoder-decoder layers, 8 attention heads, and 256 embedding dimensions in most runs. We use Adam optimizer with a learning rate of 10^{-4}. The models are trained on the following GPUs supported by Center for High Throughput Computing (CHTC) [8]: Tesla P100-PCIE-16GB, NVIDIA GeForce GTX 1080 Ti, NVIDIA GeForce GTX 2080 Ti, and NVIDIA A100-SXM4-40GB. Each run takes 61 hours on average for 300 epochs and 20 hours for 100 epochs.

All of the metrics used are recorded here. The full accuracy refers to when the model predicts both $c_{i,j}$ and the algebra letter $A_{i,j}$ correctly. For example, a 90% accuracy on full accuracy means that there are $9,000$ correct predictions compared to $10,000$ target samples, which each pair of $\{c_{i,j}, A_{i,j}\}$ in the predictions matches with that in the target samples. The letter accuracy records the correct predictions of algebra letters while the fraction accuracy corresponds to the correct predictions of central charges $c_{i,j}$. For example, a predicted $\{(su2, 1/1), (su3, 2/1)\}$ will still be marked correctly on letter accuracy if the target sequence is $\{(su2, 1/1), (su3, 3/1)\}$, while this is not the case for fraction accuracy. We also include another separate metric called order-free which counts the number of predictions regardless of the permutations. For example, both $\{(su2, 1/1), (su3, 2/1)\}$ and $\{(su3, 2/1), (su2, 1/1)\}$ will be marked as correct predictions since the permutations don't change the tensor product theory itself.

From Fig.1, we see that the model successfully classifies the algebra types and the central charges with a full accuracy of 98.2%. Also, we see that the letter accuracy and central charge accuracy both align with the full accuracy, meaning that the model is learning the classification and regressing the central charge at the same rate. From the order-free metric, we observe that the model hardly gives predictions with different permutations compared to the target sequences. Based on Eq.2, learning the denominators of central charges is not a trivial memorization task since the conformal dimensions are combinatorial so the denominators of $h_{i,l}$ are different from that of $c_{i,j}$. Specifically, the denominator of $h_{i,l}$ involves terms like $(k_1 + g_1)(k_2 + g_2)...(k_n + g_n)$. Furthermore, learning the numerators is nontrivial as there is no obvious relation between the quadratic Casimir C_r and k levels or $\dim A$. In

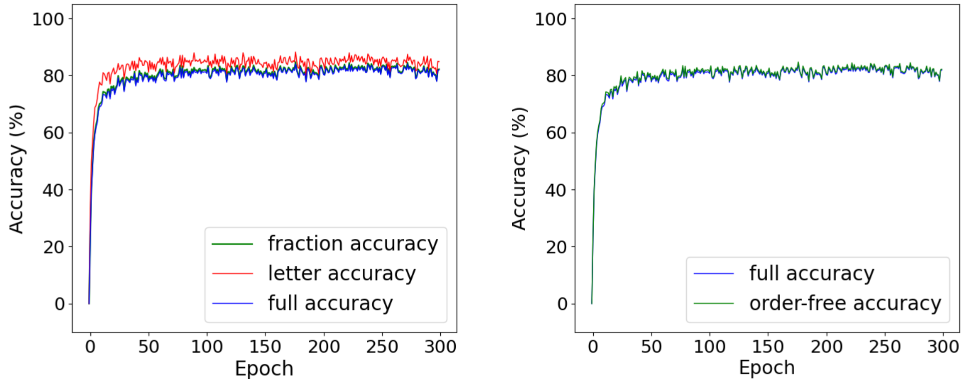

Figure 2: Both figures show the comparisons between four main metrics used in higher central charge generalizations.

Fig 1, our model achieves around 90% accuracy score at epoch 16 and is able to scale up to around 98%. These results show that Transformers can extract the features from the low-lying conformal dimensions from tensor product models. These features include rich information about the symmetry and the central charges of the tensor product theories.

4 Generalization to Higher Central Charges

Train on $0 < c < 50$		**Train on** $0 < c < 100$	
Test Data	**Accuracy (%)**	**Test Data**	**Accuracy (%)**
$50 \leq c \leq 75$	**91.29**	$0 \leq c \leq 50$	**96.48**
$75 \leq c \leq 100$	47.20	$50 \leq c \leq 100$	**99.37**
$50 \leq c \leq 100$	84.23		

Table 1: Accuracies for two central charges ranges in the training data shown side-by-side.

A natural extension of our task is to scale up central charges as large central charges are interesting in various contexts mentioned in the introduction section. We train encoder-decoder transformers on the same dataset in the previous section with the same model setup. We test on tensor product theories from KM algebra CFTs with central charge $50 \leq c \leq 100$. The test data follows a different distribution as the training data: while the algebra letters remain fixed, higher central charges imply more seed CFTs with larger central charge and more factors in the tensor products. On the right of Fig 2, the accuracy score gives around 80% even if the test data is out-of-domain. This indicates that transformers are able to extract out-of-domain data features. Again, the order-free metric indicates that the model hardly generates predictions with different permutation of the central charge-algebra letter pairs.

We further study the learning behaviors across different ranges of central charges. In Table 1, we see that on the left, the accuracy score is dependent of the range of central charges. The closer range of central charges in test data to that in training data, the higher accuracy we can achieve. This training behavior is consistent with extrapolation decay in distribution-shift settings. Following from this, we can add a small number of examples that follow the test data distribution to the training data. This is known as the priming method. In Table 2, we add $50 \leq c \leq 100$ datasets unseen by the test data to the training data. The model accuracy increases monotonically as the number of added examples rises. We achieve 97.01% accuracy score when the number of added examples is only 1% of the number of samples in training data. Our result strongly indicates that when the model is able to learn out-domain features, we can anchor it towards the true distribution by adding a tiny number of examples. This is similar to language tasks where Large Language Models (LLM) adapts to new domains with just a few examples.

$50 \leq c \leq 100$		KM + coset	
Examples added	Accuracy (%)	Examples added	Accuracy (%)
0	82.9	0	0
30	84.71	30	1.16
300	85.41	300	17.47
3000	93.33	3000	52.4
30000	**97.01**	30000	**83.02**

Table 2: Priming results for c-generalization (left) and KM + coset (right).

5 Generalization to Unseen Classes

In the final experiment, we add coset models into our current KM algebra CFT library. The coset models include the Virasoro minimal models, the parafermionic theory, $\mathcal{N} = 1$ SCFTs, and $\mathcal{N} = 2$ SCFTs. A detailed explanation of generating the spectra of these theories can be found in [14, 10, 5, 13, 12, 15]. The key difference between KM algebra and coset models is that they follow distinct algebraic construction. Adding coset models to the CFT library will significantly enrich the symmetry structure. The tensor product models built from this library will bring us to a new data distribution, potentially harder to learn compared to the higher central charge generalization.

In Table 2, we report our findings on model's capability of generalizing to mixing of KM algebra and coset tensor product theories. Again, we keep the training data fixed while testing on new data. The initial training with no examples added has only zero accuracy score. However, as we add more examples, the model performance scales up to 83.02%, which is an incredable jump compared to the initial run. This is a similar learning behavior as in the previous section. The model requires a stronger anchor that leads the learned distribution towards the true distribution.

6 Conclusion / Future Work

We have shown that Transformers are able to learn the central charges and symmetry algebra of composite theories in tensor product models based on low-lying conformal dimensions. This indicates that the inverse problem can be formulated as a discrete symbolic task where the model is able to extract the rich information contained in low-lying spectra. We further show that the model is able to learn out-of-domain data features. In future work, we plan to gain more interpretations of our results by looking at how models are learning the algebraic structures and their learning dynamics across training epochs. In this work, we observe that additional examples that follow can strongly constrain model's learning dynamics towards the true distribution. Two extrapolation tasks are performed: larger central charge generalization and unseen CFT generalization. We observe that the latter task is harder to generalize due to the increased diversity in algebraic structures in the test data and we aim to touch more on this by looking at the domain shifts of tasks in future work. Our limitation is that this work so far only focuses on RCFTs in two dimensions. Also, our computation resources restricted us from generating tensor products with larger parameters such as k, L, and N.

Our work provides insights to understanding the RCFT landscape and bulk reconstruction in the context of AdS/CFT, thus we comment that detailed downstream tasks along either one of these tracks will be interesting future directions.

Acknowledgement and Disclosure of Funding

This work is partly supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics under Award Numbers DE-SC-0023719 and DE-SC-0017647. KC and HC are partially supported through the university of Wisconsin Madison, and the Wisconsin Alumni Research Foundation. GM and KC are supported by the U.S. Department of Energy (DOE) under Award No. DE-FOA-0002705, KA/OR55/22 (AIHEP).

References

[1] Tetsuya Akutagawa, Koji Hashimoto, and Takayuki Sumimoto. "Deep learning and AdS/QCD". In: *Physical Review D* 102.2 (July 2020). ISSN: 2470-0029. DOI: 10.1103/physrevd.102.026020. URL: http://dx.doi.org/10.1103/PhysRevD.102.026020.

[2] P. Bantay. *Symmetric products, permutation orbifolds and discrete torsion*. 2000. arXiv: hep-th/0004025 [hep-th]. URL: https://arxiv.org/abs/hep-th/0004025.

[3] Alexandre Belin et al. "Symmetric product orbifold universality and the mirage of an emergent spacetime". In: *JHEP* 05 (2025), p. 190. DOI: 10.1007/JHEP05(2025)190. arXiv: 2502.01734 [hep-th].

[4] Tianji Cai et al. "Transforming the bootstrap: using transformers to compute scattering amplitudes in planar $\mathcal{N} = 4$ super Yang–Mills theory". In: *Mach. Learn. Sci. Tech.* 5.3 (2024), p. 035073. DOI: 10.1088/2632-2153/ad743e. arXiv: 2405.06107 [cs.LG].

[5] Andrea Cappelli and Jean-Bernard Zuber. *A-D-E Classification of Conformal Field Theories*. 2009. arXiv: 0911.3242 [hep-th]. URL: https://arxiv.org/abs/0911.3242.

[6] John L. Cardy. "Operator Content of Two-Dimensional Conformally Invariant Theories". In: *Nucl. Phys. B* 270 (1986), pp. 186–204. DOI: 10.1016/0550-3213(86)90552-3.

[7] Roger W Carter. *Lie algebras of finite and affine type*. Cambridge studies in advanced mathematics 96. Cambridge University Press, 2005.

[8] Center for High Throughput Computing. *Center for High Throughput Computing*. 2006. DOI: 10.21231/GNT1-HW21. URL: https://chtc.cs.wisc.edu/.

[9] T.D.W. Claridge. *High-Resolution NMR Techniques in Organic Chemistry*. Elsevier, 2016.

[10] P. Di Francesco, P. Mathieu, and D. Senechal. *Conformal Field Theory*. Graduate Texts in Contemporary Physics. New York: Springer-Verlag, 1997. ISBN: 978-0-387-94785-3, 978-1-4612-7475-9. DOI: 10.1007/978-1-4612-2256-9.

[11] Robbert Dijkgraaf et al. "Elliptic Genera of Symmetric Products and Second Quantized Strings". In: *Communications in Mathematical Physics* 185.1 (Apr. 1997), pp. 197–209. ISSN: 1432-0916. DOI: 10.1007/s002200050087. URL: http://dx.doi.org/10.1007/s002200050087.

[12] Doron Gepner. "Exactly Solvable String Compactifications on Manifolds of SU(N) Holonomy". In: *Phys. Lett. B* 199 (1987), pp. 380–388. DOI: 10.1016/0370-2693(87)90938-5.

[13] Doron Gepner. "Space-Time Supersymmetry in Compactified String Theory and Superconformal Models". In: *Nucl. Phys. B* 296 (1988). Ed. by B. Schellekens, p. 757. DOI: 10.1016/0550-3213(88)90397-5.

[14] Paul Ginsparg. *Applied Conformal Field Theory*. 1988. arXiv: hep-th/9108028 [hep-th].

[15] Brian R. Greene. "String theory on Calabi-Yau manifolds". In: *Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 96): Fields, Strings, and Duality*. June 1996, pp. 543–726. arXiv: hep-th/9702155.

[16] Michael Gutperle et al. "Non-invertible symmetries in S_N orbifold CFTs and holography". In: *JHEP* 09 (2024), p. 110. DOI: 10.1007/JHEP09(2024)110. arXiv: 2405.15693 [hep-th].

[17] Sebastian Harris et al. "Holographic Interfaces in Symmetric Product Orbifolds". In: (Mar. 2025). arXiv: 2504.00078 [hep-th].

[18] Koji Hashimoto et al. "Machine-learning emergent spacetime from linear response in future tabletop quantum gravity experiments". In: *Mach. Learn. Sci. Tech.* 6.1 (2025), p. 015030. DOI: 10.1088/2632-2153/adb09f. arXiv: 2411.16052 [hep-th].

[19] Juan Maldacena. "The Large-N Limit of Superconformal Field Theories and Supergravity". In: *International Journal of Theoretical Physics* 38.4 (Apr. 1999), pp. 1113–1133. ISSN: 1572-9575. DOI: 10.1023/a:1026654312961. URL: http://dx.doi.org/10.1023/A:1026654312961.

[20] Fernando Marchesano, Gary Shiu, and Timo Weigand. "The Standard Model from String Theory: What Have We Learned?" In: *Ann. Rev. Nucl. Part. Sci.* 74 (2024), pp. 113–140. DOI: 10.1146/annurev-nucl-102622-012235. arXiv: 2401.01939 [hep-th].

[21] Samir D. Mathur, Sunil Mukhi, and Ashoke Sen. "On the Classification of Rational Conformal Field Theories". In: *Phys. Lett. B* 213 (1988), pp. 303–308. DOI: 10.1016/0370-2693(88)91765-0.

[22] Sunil Mukhi and Brandon C. Rayhaun. “Classification of Unitary RCFTs with Two Primaries and Central Charge Less Than 25”. In: *Communications in Mathematical Physics* 401.2 (Mar. 2023), pp. 1899–1949. ISSN: 1432-0916. DOI: 10.1007/s00220-023-04681-1. URL: http://dx.doi.org/10.1007/s00220-023-04681-1.

[23] A.Z. Patashinskii and V.L. Pokrovskii. *Fluctuation Theory of Phase Transitions*. International Series in Natural Philosophy. Oxford: Pergamon Press, 1979. URL: https://books.google.com/books?id=GOIuywEACAAJ.

[24] J. Polchinski. *String theory. Vol. 2: Superstring theory and beyond*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Dec. 2007. ISBN: 978-0-511-25228-0, 978-0-521-63304-8, 978-0-521-67228-3. DOI: 10.1017/CBO9780511618123.

[25] Christopher A. Smith et al. “METLIN: a metabolite mass spectral database”. In: *Nucleic Acids Research* 33.suppl_1 (2005), pp. D275–D278.

[26] Ashish Vaswani et al. *Attention Is All You Need*. 2023. arXiv: 1706.03762 [cs.CL]. URL: https://arxiv.org/abs/1706.03762.