
Surrogate-Assisted PINNs with Hard Constraints for Heterogeneous Catalytic Reactor Modeling

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Abstract

1 We propose a hard-constrained PINN framework for efficient catalytic reactor
2 modeling that guarantees atom conservation through a dedicated neural
3 network layer. By choosing the weights of this layer based on the concept
4 of key species, we replace multiple output-nodes with physical constraints,
5 while simultaneously preserving positivity and improving training stability.
6 Further, we include a detailed micro kinetic description of the surface
7 chemistry through a physically plausible Global Reaction Neural Network
8 surrogate. Applied to a CO₂ methanation reactor, our approach achieves
9 1000× speed-up over conventional solvers while maintaining physical fidelity.

10 1 Introduction

11 Heterogeneous catalysts are directly involved in 85% of the large-scale chemical processes and
12 play a major role for climate protection through the production of renewable energies and
13 the removal of harmful pollutants [1]. Detailed numerical modeling lays the foundation for
14 the rational design and on-line control of these devices [2]. In particular, multiscale modeling
15 has shown great potential due to its ability to describe the physical phenomena across all
16 relevant time and length scales [3]. However, this level of detail comes at the expense of
17 high computational demand, often resulting in simulation times that are unbearable for
18 industrial applications. Here, we propose hard constrained physics-informed neural networks
19 (PINNs) [4, 5] as efficient and physically consistent surrogates of catalytic reactors with a
20 detailed description of the surface chemistry.

21 In our atom conserving catalytic reactor PINN framework we, leverage the recently developed
22 Global Reaction Neural Network (GRNN) [6] as a differentiable and physically plausible
23 surrogate for detailed surface kinetics to re-enable stable gradient propagation via automatic
24 differentiation. Otherwise, detailed surface kinetics would require solving a set of implicit
25 functions which is not natively compatible with gradient computation via automatic dif-
26 ferentiation. Further, we overcome the drawbacks of soft-constraining atom conservation,
27 which can lead to residual violations that accumulate and destabilize simulations [7] as
28 well as downstream analyses. To do so, we generalize recently developed linear equality
29 constraints [8, 9] into an atom conservation layer and identify the most effective choice of
30 physically interpretable weights to enforce atom conservation.

31 **2 Related work**

32 While PINNs have been used for catalytic reactor modeling with simplified descriptions
 33 of the surface chemistry [10–12], there is a lack of solutions considering detailed surface
 34 chemistry through micro kinetic models. In contrast to the simplified kinetics, they require
 35 finding the steady state solution to systems of famously stiff ODEs. Their computational
 36 cost is so high, that it sparked a long tradition of kinetic surrogate modeling [6, 13–32]. The
 37 required stiff ODE solvers are not natively compatible with automatic differentiation, but
 38 even the usage of differentiable solver implementations [33] or implicit differentiation [34]
 39 would be computationally too demanding as the ODE system has to be solved for every
 40 collocation point in every PINN training epoch. To reduce the computational demand and
 41 simultaneously re-enable automatic differentiation, instead we employ the state-of-the-art
 42 physically plausible kinetic surrogate [6] to provide efficient and differentiable solutions to
 43 the detailed surface chemistry during the training of the AC-CatalyticReactor-PINN.
 44 Recently, PINNs have been hard-constrain to follow the fundamental law of atom conservation
 45 through orthogonal projection [9] or projection in scaled space [7, 35]. We generalize the
 46 projection step by introducing physically inspired weights and embed it as a neural network
 47 layer.

48 **3 Atom Conserving Catalytic Reactor-PINN framework**

49 We present the AC-CatalyticReactor-PINN framework to accelerate the solution of catalytic
 50 reactor models (Fig. 1). It uses the mass fractions ω_0 of each chemical species in the reactor
 51 inlet (feed) and the feed temperature T_0 to predict the composition ω at any other position
 52 z inside the reactor. The hidden layers use \tanh activation while the output layer uses
 53 exponential activation to guarantee positive outputs while effectively capturing the widely
 54 different orders of magnitude typically present in ω [10]. The subsequent AC-layer enforces
 55 atom conservation through a linear projection step mapping inconsistent outputs $\tilde{\omega}$ onto
 56 consistent $\tilde{\omega}$ with the pre-computed weights $I - C$ and bias $\omega_0 C$ [9], where I is the unit
 57 matrix and C is computed from the elemental mass fractions F that are known *a priori*.¹
 58 To estimate the training data ω over all relevant orders of magnitude, the data loss $\mathcal{L}_{\text{data}}$ is
 59 given as the mean squared asinh distance, a generalized version of the thresholded relative
 60 error [14, 26]. The threshold value $r_{\text{ref}}_{\text{data}}$ modulates between relevant $\omega > r_{\text{ref}}_{\text{data}}$ and
 61 irrelevant $\omega < r_{\text{ref}}_{\text{data}}$ values.

$$\mathcal{L}_{\text{data}} = \frac{1}{N_{\text{train}}} \cdot \sum_{n=1}^{N_{\text{train}}} \left(\operatorname{asinh} \left(\frac{\omega_{i,n}}{r_{\text{ref}}_{\text{data}}} \right) - \operatorname{asinh} \left(\frac{\tilde{\omega}_{i,n}}{r_{\text{ref}}_{\text{data}}} \right) \right)^2 \quad (1)$$

62 The physics loss $\mathcal{L}_{\text{physics}}$ applies the same distance measure to the normalized reaction rates
 63 \dot{s}/ω given by the detailed surface chemistry. To enable automatic differentiation through the
 64 detailed surface chemistry, we employ a physically plausible surrogate [6] $\text{GRNN} \approx \dot{s}(\omega, T)$.
 65 The derivative $\dot{\tilde{\omega}}$ of $\tilde{\omega}$ with respect to z is obtained through automatic differentiation.

$$\mathcal{L}_{\text{physics}} = \frac{1}{N_{\text{physics}}} \cdot \sum_{n=1}^{N_{\text{physics}}} \left(\operatorname{asinh} \left(\frac{\dot{s}(\tilde{\omega}_{i,n}, T)/\tilde{\omega}_{i,n}}{r_{\text{ref}}_{\text{physics}}} \right) - \operatorname{asinh} \left(\frac{\dot{\tilde{\omega}}_{i,n}/\tilde{\omega}_{i,n}}{r_{\text{ref}}_{\text{physics}}} \right) \right)^2 \quad (2)$$

66 The total loss $\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{data}} + \lambda \mathcal{L}_{\text{physics}}$ includes the weight λ to balance the typically unequal
 67 contributions of physics and data. While we used an empirical schedule for λ , this can easily
 68 be replaced by adaptive loss weighting algorithms [36].

¹ $C = (WW^T)^{-1} F^T \left(F (WW^T)^{-1} F^T \right)^{-1} F$ with weights $W = I$ for orthogonal projection,
 $W = \operatorname{diag}(\tilde{\omega}^{-1})$ for weighted projection, and $W_{i,i} = \begin{cases} 1 & \text{if } i \text{ is key species,} \\ 0 & \text{if } i \text{ is dependent species.} \end{cases}$ for completion.

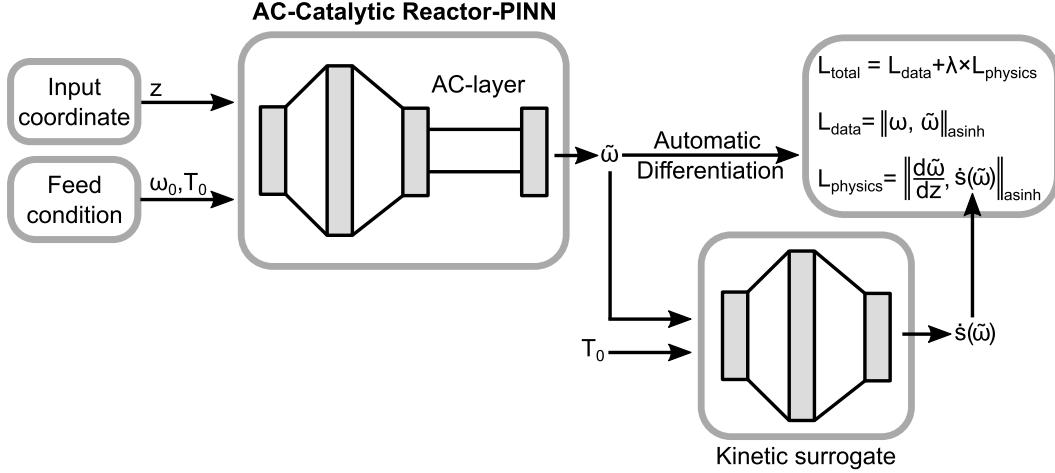


Figure 1: Schematic representation of the AC-CatalyticReactor-PINN. The parameters of the atom conservation layer depend on the feed conditions. The kinetic surrogate supports automatic differentiation and efficiently provides physically consistent steady state source terms of a detailed surface kinetic scheme as a function of the reaction conditions at the input coordinate as predicted by the AC-CatalyticReactor-PINN.

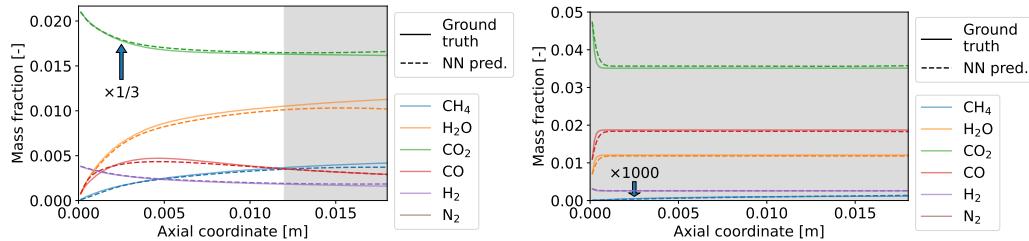
69 4 Experiments

70 The AC-CatalyticReactor-PINN is trained with a synthetic dataset consisting of inlet and
 71 outlet compositions of a 1D pseudo-homogeneous CO_2 methanation reactor model. The
 72 reactor is assumed isothermal and simulated with a molar inlet composition of 4 % CO_2 , 5.3 %
 73 H_2 , and 90.7 % N_2 at 40 equally spaced temperatures in the range of 550 K to 850 K. The
 74 detailed surface kinetic scheme by SCHMIDER *et al.* [37] is employed and solved with IDAS [38].
 75 4000 collocation points are placed at uniformly random T and z values inside the training
 76 range.² Threshold values are chosen as $r_{\text{ref, data}} = 10^{-6}$ and $r_{\text{ref, physics}} = 5 \times 10^{-2} \text{ m}^{-1}$. PINN
 77 training is performed for 1000 epochs with the L-BFGS optimizer [39] using strong Wolfe
 78 line-search and a learning rate of 10^{-6} . Through preliminary experiments we found good
 79 convergence with $\lambda = 10^\alpha$ with α following a linear ramp of -8 to -2 from epoch 15 to 50.

80 5 Results and Discussion

81 We evaluate the AC-CatalyticReactor-PINN performance by employing it as a surrogate for
 82 a methanation reactor model with detailed surface kinetics. The AC-CatalyticReactor-PINN
 83 accurately captures the spatial concentration profiles of all species when queried for unseen
 84 initial conditions within (Fig. 2a) or outside (Fig. 2b) the training range. Remarkably, it even
 85 captures the correct equilibrium composition (Fig. 2b). For a more general assessment, we
 86 consider 100 trajectories with unseen initial conditions and split them into two test sets. One
 87 with z values within the training region, and the other for up to 50% extrapolation. Here,
 88 the AC-CatalyticReactor-PINN achieves a relative prediction error of 3.8 % for interpolation
 89 and 2.8 % for extrapolation while atom balance is strictly enforced for any condition. These
 90 results are particularly promising given the compact training setup. The amount of training
 91 data is experimentally well accessible and with fewer than 1000 parameters the model remains
 92 lightweight and suitable for embedded or resource-constrained deployment scenarios. For
 93 example, a computational speed-up of 1000 is reached compared to the established Cantera
 94 solver with a spatial resolution of 100 axial positions (21 μs vs 28 ms on average).³

²The rates obtained from every tenth IDAS solver step are saved and split into training/validation/test = 50/25/25 for training a GRNN with three hidden layers of 20 \tanh nodes. Methane and hydrogen are assumed as key species. L-BFGS [39] with strong-wolfe line-search is used to reduce the thresholded relative error [14, 25] with $r_{\text{ref}} = 10^{-8} \text{ m}^{-1}$ until the validation error did not improve for 1000 consecutive epochs. The test error at unseen conditions is 0.25 %.



(a) $T = 655\text{ K}$ (within training range).

(b) $T = 910\text{ K}$ (outside of training range).

Figure 2: Composition trajectories predicted at unseen conditions (dashed) vs. ground truth (solid lines). Gray areas indicate extrapolation conditions.

The atom conservation layer is strongly contributing to the accuracy and robustness of the AC-CatalyticReactor-PINN. Omitting it leads to physically inconsistent results, with atom balance errors of up to 10 %, which is unacceptable for practical applications. Within the AC-layer, we explored different sets of weights, each corresponding to a different direction of the performed projection step. The orthogonal projection [9] maintains the atom balance but predicts negative concentrations for 1 % and 21.6 % for interpolation and extrapolation conditions, respectively. Such negative values are not only physically implausible but also incompatible with the kinetic surrogate, requiring cutoffs that undermine model convergence. The weighted projection aims to apply changes of equal relative magnitude for all chemical species. Thereby, it preserves positivity but its performance becomes depend on the prediction accuracy. In early training stages, inaccurate predictions lead to impractical projection directions, amplifying deviations from the ground truth. Even a pre-training phase does not fully resolve the convergence issue. In contrast, completion [40] is less sensitive to inaccurate predictions and therefore improves convergence from the first epoch. Further, it reduces the number of output nodes by the amount of chemical elements in the system [41, 42] (here, it's four) from six to two. This reduction in unphysical degrees of freedom prevents overfitting and ensures physically plausible inputs to the kinetic surrogate during PINN training.

During the PINN training, the kinetic surrogate (GRNN) is queried $>10^8$ times. Calling the full surface kinetics instead, training would take ~ 37 days. With the GRNN, training completes in 191 s, corresponding to a speed-up of $\sim 16\,500$.³ This acceleration is crucial: it turns previously infeasible training into a practical workflow on consumer-grade hardware.

6 Conclusions and Future Work

We proposed the AC-CatalyticReactor-PINN framework for efficient catalytic reactor modeling with full physical fidelity. In particular, we were able to employ detailed surface kinetics through surrogate-assisted training. Not only did this re-enable gradient flow through automatic differentiation, but also vastly reduced the computational demand of the physics loss, accelerating model training by four orders of magnitude. Comparing different atom conservation hard-constraint implementations, we identified the completion strategy as the most promising. With this, positivity is preserved and unphysical degrees of freedom are removed. We found stable training and a 4-fold increase in accuracy.

Using an isothermal CO_2 methanation showcase with only 40 training data, we achieved a $1000\times$ speed-up with relative errors below 4 %. Importantly, the framework is readily extendable to non-isothermal systems by including the energy balance in the physics loss.

Overall, the AC-CatalyticReactor-PINN paves the way for efficient yet accurate industrial-scale catalytic reactor modeling, enabling real-time control and on-line process optimization. Moreover, we anticipate that the surrogate-assisted training strategy could also benefit other domains considering multi-scale systems involving complex surface kinetics, including atmospheric chemistry [43, 44], battery research [45–47], and more [48, 49].

³Data collected using 32 GB of RAM @ 6000 MHz and a Ryzen 7 9800X3D processor @ 5.2 GHz.

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