Homework 5

July 5, 2016

Problem 1.

Let $\mathbf{A}$ denote an $(n-1)\times n$ matrix with complex number entries for which the RREF($\mathbf{A}$) has $n-1$ pivots show that we can prepend $\mathbf{A}$ with an additional row so as to ensure that

$$\det(\mathbf{A}) \in \mathbb{R} \quad \text{and} \quad \det(\mathbf{A}) > 0.$$

Describe how to obtain that vector using the Laplace expansion of the determinant.

Problem 2.

Construct a 5×5 matrix of rank 4 whose Nullspace is spanned by the vector

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

Problem 3.

Let $\mathbf{A}$ and $\mathbf{B}$ denote a $m\times n$ and $p\times n$ matrices where $n > m > p > 2$. Describe how to find a basis for the subspaces :

a)

$$\text{NullSpace}(\mathbf{A}) \cup \text{NullSpace}(\mathbf{B}).$$

b)

$$\text{NullSpace}(\mathbf{A}) \cap \text{NullSpace}(\mathbf{B}).$$

Problem 4.

a) Find the rank of an $n \times n$ matrix $\mathbf{A}$ whose entries are given by

$$a_{ij} = 1 + n \cdot (i-1) + (j-1)\,,$$

for all $1 \leq i, j \leq n$, where $n > 2$.
b) What can you conclude about the dimension of the Nullspace of $\mathbf{A}$ for all $n > 1$.
c) For what value of n is the matrix $\mathbf{A}$ invertible ?

Problem 5.

Let $\mathbf{A}$ denote the $n \times n$ matrix whose entries are given by

$$a_{uv} = \exp\left\{ i \cdot \frac{2\pi}{n} \cdot (u-1) \cdot (v-1) \right\}$$

for all $1 \leq i, j \leq n$, where $n \geq 2$. Using the standard inner-product discussed in class for elements of the vector space $\mathbb{C}^{n \times 1}$, Find the angle between any two columns of $\mathbf{A}$.