Practice Midterm 1, MA 351 Section: Summer 2015

Name:

PUID:

Exam :

No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned
I	50pts	
II	25pts	
III	25pts	
Total Points	100pts	

I

Answer the following questions by clearly circling your answers. No partial credit is awarded for questions in part I.

(1) Which of the expressions below equal $(\mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C})^{-1}$ (circle all that apply)

(a) $\mathbf{A}^{-1} \cdot \mathbf{B}^{-1} \cdot \mathbf{C}^{-1}$
(b) $\mathbf{A}^{-1} \cdot \mathbf{C}^{-1} \cdot \mathbf{B}^{-1}$
(c) $\mathbf{C}^{-1} \cdot \mathbf{A}^{-1} \cdot \mathbf{B}^{-1}$
(d) $\mathbf{C}^{-1} \cdot \mathbf{B}^{-1} \cdot \mathbf{A}^{-1}$

(2) Which of the expression below correspond to the determinant of the matrix

$$7 \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

(a) $-a_{13}a_{22}a_{31} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{11}a_{22}a_{33}$
(b) $7a_{11}a_{22}a_{33}$
(c) $7^2a_{11}a_{22}a_{33}$
(d) $7^3a_{11}a_{22}a_{33}$

(3) Let V denote the vector space of 3×3 matrices endowed with the usual matrix addition and the usual product of matrices with numbers. Which ones of the subsets of V below are not subspaces of V? (circle all that apply)

(a) The set of diagonal 3×3 matrices.
(b) The set of 3×3 matrices for which $\det(\mathbf{A}) \neq 0$.
(c) The set of 3×3 matrices for which $\det(\mathbf{A}) = 0$
(d) The set of 3×3 matrices for which $\mathbf{A}^2 = -\mathbf{I}$
(e) The set of 3×3 matrices for which $\mathbf{A} \cdot \mathbf{A}^T = \mathbf{I}$

(4) Which statements below are true (circle all that apply)

(a) If $\det \mathbf{A} = 0$ then Row Echelon Form of $\mathbf{A}$ corresponds to the identity matrix Matrices of different sizes can sometimes be added to each other
(b) For every 3×3 matrices $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
(c) Let $\mathbf{B}$ denote the matrix which result from interchanging the first and the second row of $\mathbf{A}$. Then it necessary follows that $\det(\mathbf{A}) \neq \det(\mathbf{B})$.
(d) The product of two diagonal matrices always results in a diagonal matrix.

(5) What is the value of the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 3 \\ 2 & 4 & 5 & 1 & 6 \\ 3 & 5 & 1 & 9 & 9 \\ 4 & 4 & 6 & 8 & 12 \\ 5 & 4 & 5 & 6 & 15 \end{pmatrix}$$

(a) 7
(b) -3
(c) 0
(d) 1

II

Use the reduced row echelon method on the augmented matrix to compute the inverse of the matrix $\mathbf{A}$.

$$\mathbf{A} = \begin{pmatrix} 4 & 5 & 1 \\ 1 & 0 & 0 \\ 1 & 3 & 3 \end{pmatrix}$$

Answer:

III(20pts)

Let $\mathbf{A}$ denote an $n \times n$ matrix whose entries are given by

$$\mathbf{A} = \left(\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{32} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{array} \right).$$

Assuming that $a_{11} \neq 0$ give a detail proof of the identity

$$\det \mathbf{A} = \frac{1}{a_{11}^{n-2}} \det \left(\begin{array}{cccc} \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} & \det \begin{pmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{pmatrix} & \cdots & \det \begin{pmatrix} a_{11} & a_{1n} \\ a_{21} & a_{2n} \end{pmatrix} \\ \det \begin{pmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{pmatrix} & \det \begin{pmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{pmatrix} & & \det \begin{pmatrix} a_{11} & a_{1n} \\ a_{31} & a_{3n} \end{pmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \det \begin{pmatrix} a_{11} & a_{12} \\ a_{n1} & a_{n2} \end{pmatrix} & \det \begin{pmatrix} a_{11} & a_{13} \\ a_{n1} & a_{n3} \end{pmatrix} & \cdots & \det \begin{pmatrix} a_{11} & a_{1n} \\ a_{n1} & a_{nn} \end{pmatrix} \end{array} \right).$$

Note that the matrix

$$\left(\begin{array}{cccc} \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} & \det \begin{pmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{pmatrix} & \cdots & \det \begin{pmatrix} a_{11} & a_{1n} \\ a_{21} & a_{2n} \end{pmatrix} \\ \det \begin{pmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{pmatrix} & \det \begin{pmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{pmatrix} & \cdots & \det \begin{pmatrix} a_{11} & a_{1n} \\ a_{31} & a_{3n} \end{pmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \det \begin{pmatrix} a_{11} & a_{12} \\ a_{n1} & a_{n2} \end{pmatrix} & \det \begin{pmatrix} a_{11} & a_{13} \\ a_{n1} & a_{n3} \end{pmatrix} & \cdots & \det \begin{pmatrix} a_{11} & a_{1n} \\ a_{n1} & a_{nn} \end{pmatrix} \end{array} \right)$$

in the equality is $(n-1) \times (n-1)$.