Homework 1

January 13, 2017

Problem 1.

Let $z_1 = a_1 + b_1\sqrt{-1}$ and $z_2 = a_2 + b_2\sqrt{-1}$ denote arbitrary complex numbers where $a_1, a_2, b_1, b_2 \in \mathbb{R}$. Quaternions can be represented as 2×2 complex matrices as follows

$$q(z_1, z_2) := \begin{pmatrix} z_1 & z_2 \\ -\overline{z_2} & \overline{z_1} \end{pmatrix}.$$

a) Use the 2×2 determinant formula introduced in class to derive an expression for the norm of $q(z_1, z_2)$ in terms of the real part and imaginary parts of z_1 and z_2.

b) Use the matrix representation of complex numbers discussed in class to obtain a 4×4 real matrix representation of $q(z_1, z_2)$.

c) Use the Cayley diagram discussed in class to illustrate the product of two quaternions and $q(z_1, z_2) \times q(z_3, z_4)$, where $z_3 = a_3 + b_3\sqrt{-1}$ and $z_4 = a_4 + b_4\sqrt{-1}$. (use the 4×4 real matrix representation of quaternions)

d) Find the 4×4 matrix representation of

$$\frac{1}{q(z_1, z_2)}$$

e) Determine the coefficients a_0, a_1, a_2, a_3 in terms of z_1 and z_2 such that

$$0 = (q(z_1, z_2))^4 - a_3\,(q(z_1, z_2))^3 + a_2\,(q(z_1, z_2))^2 - a_1\,(q(z_1, z_2))^1 + a_0\,(q(z_1, z_2))^0$$

Problem 2.

Exercise 1 page 58 from the Book

Problem 3.

Exercise 2 page 59 from the Book

Problem 4.

Exercise 3 page 59-60 from the Book

Problem 5.

Exercise 4 page 60 from the Book