Homework 4

June 22, 2016

Problem 1.

For some $n \times n$ matrix $\mathbf{A}$ let $\mathrm{REF}_\star(\mathbf{A})$ denote the Row Echelon Form matrix obtained by performing a sequence of row linear combination operations on matrix $\mathbf{A}$. Show that

$$\det(\mathbf{A}) = \det\left\{\mathrm{REF}_\star(\mathbf{A})^\top\right\}$$

Problem 2.

a) Assuming that a_{12}, a_{23} and a_{31} are non-zero parameters express the matrix

$$\begin{pmatrix} 0 & a_{12} & 0 \\ 0 & 0 & a_{23} \\ a_{31} & 0 & 0 \end{pmatrix}$$

as a sum of 3 outer products.
b) Show that any $n \times n$ matrix must have rank less or equal to n.

Problem 3.

a) Describe how to obtain the $n \times n$ elementary matrices associated with each of the column operations bellow

$$\mathrm{C}_i \leftrightarrow \mathrm{C}_j,$$

$$k\,\mathrm{C}_i \to \mathrm{C}_i,$$

$$k\,\mathrm{C}_i + \mathrm{C}_j \to \mathrm{C}_j.$$

b) Show that if an $n \times n$ matrix $\mathbf{B}$ is obtained from $\mathbf{A}$ via the column operation $k\,\mathrm{C}_i + \mathrm{C}_j \to \mathrm{C}_j$ then

$$\det(\mathbf{B}) = \det(\mathbf{A}).$$

Problem 4.

Show that for any $n \times 1$ matrix $\mathbf{A}$ and any $1 \times n$ matrix $\mathbf{B}$ we have

$$\det(\mathbf{A} \cdot \mathbf{B}) = 0$$

Problem 5.

Express the inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

as product of elementary matrices. It suffices for this problem to exhibits the elementary matrix factors of $\mathbf{A}^{-1}$ in the order in which they are meant to be multiplied.