Homework 9

July 26, 2016

Problem 1.

a) Express the spectral decomposition of the Hermitian matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1+i & 0 & 0 \\ 1-i & 4 & 0 & 0 \\ 0 & 0 & 2 & i \\ 0 & 0 & -i & 7 \end{pmatrix}$$

in terms of a unitary matrix.

Problem 2.

a) Use the Gram-Schmidt process to find an orthonormal basis for

$$\operatorname{Span}\left\{ \mathbf{u}_1 = \begin{pmatrix} 1 \\ i \\ -i \\ 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 1 \\ i \\ i \\ 1 \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} 0 \\ i \\ 0 \\ 1 \end{pmatrix}, \mathbf{u}_4 = \begin{pmatrix} 1 \\ 0 \\ i \\ 0 \end{pmatrix} \right\}.$$

b) Use the resulting basis to find the projection of

$$\mathbf{b} = \begin{pmatrix} 1 \\ 2i \\ 3 \\ 4i \end{pmatrix}$$

onto $\operatorname{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.

Problem 3.

Find the maximum and minimum value that the function

$$f(x_0, x_1, x_2, x_3) = \frac{6\,x_0^2 - 4\,x_0x_1 + 6\,x_1^2 - 8\,x_0x_2 + 6\,x_2^2 - 8\,x_1x_3 - 4\,x_2x_3 + 6\,x_3^2}{x_0^2 + x_1^2 + x_2^2 + x_3^2}$$

can take over real value assignments to the variables when the variables are not identically zero.

Problem 4.

Find a basis for $W^\perp$ for

$$W = \text{Span}\left\{ \begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix}, \begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix} \right\}$$

Problem 5.

Let $G_1 := (V, E_1 \subset V \times V)$ and $G_2 := (V, E_2 \subset V \times V)$ denote two graphs. Let $\mathbf{A}$, $\mathbf{B}$ denote their respective $n \times n$ adjacency matrices with entries determined by

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E_1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and

$$b_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E_2 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Prove or provide counter example to the following statement . Two graphs are isomorphic if and only if their adjacency matrices have the same set of eigenvalues (i.e. cospectral).