Homework 3

June 20, 2016

Problem 1.

Using the permutation expansion of the determinant of an $n \times n$ matrix $\mathbf{A}$ expressed by :

$$\sum_{p \in \text{Permutations of } n \text{ elements}} (-1)^{\# \text{ of inversions in } p} \prod_{1 \leq i \leq n} a_{p(i)i}$$

Show that

$$\det \mathbf{B} = \det \mathbf{A},$$

if the $n \times n$ matrix $\mathbf{B}$ is obtained from $\mathbf{A}$ via the row linear combination operation described by

$$k\,\mathrm{R}_{n-1} + \mathrm{R}_n \to \mathrm{R}_n.$$

You may use without proof all other properties of the determinant showed in class.

Problem 2.

Compute the determinants, show your computation in detail

a)

$$\det \begin{pmatrix} 1 & 2 & 3 \\ 8 & 9 & 4 \\ 7 & 6 & 5 \end{pmatrix}$$

b)

$$\det \begin{pmatrix} x+1 & x+2 & x+3 \\ x+8 & x+9 & x+4 \\ x+7 & x+6 & x+5 \end{pmatrix}$$

c)

$$\det \begin{pmatrix} x^1 & x^2 & x^3 \\ x^8 & x^9 & x^4 \\ x^7 & x^6 & x^5 \end{pmatrix}$$

Problem 3.

Using properties of the determinant. Explain without computation why the determinant of

$$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ b_1 & b_2 & b_3 & b_4 & b_5 \\ c_1 & c_2 & 0 & 0 & 0 \\ d_1 & d_2 & 0 & 0 & 0 \\ e_1 & e_2 & 0 & 0 & 0 \end{pmatrix}$$

is zero.

Problem 4.

Let $\mathbf{A}$ be a $n \times n$ matrix for which the entries are specified for all $1 \leq i, j \leq n$ by

$$a_{ij} = j^{i-1}.$$

Determine and prove a simple formula for the determinant of such family of matrices.

Problem 5.

Compute the determinant of the folloiwng matrices

$$\begin{pmatrix} 0 & 0 & a_1 & b_1 \\ 0 & 0 & a_2 & b_2 \\ a_3 & b_3 & 0 & 0 \\ a_4 & b_4 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} a_1 & 0 & 0 & b_1 \\ 0 & a_2 & b_2 & 0 \\ 0 & b_3 & a_3 & 0 \\ b_4 & 0 & 0 & a_4 \end{pmatrix}$$