

Homework 4

February 5, 2017

Problem 1.

Let $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{3 \times 3}$, find in terms of \mathbf{A} a parametric description for the set of all matrices \mathbf{B} such that

a) $\det(\mathbf{A} + \mathbf{B}) = \det \mathbf{A} \neq 0$. Use in your derivation the family of matrices with determinant equal to 1.

b) $\det \mathbf{A} = \det(\mathbf{A} + \mathbf{B}) = \det \mathbf{B} \neq 0$.

Problem 2.

Prove that the rank of skew symmetric matrices is always even.

Problem 3.

Express

$$\mathbf{A} = \begin{pmatrix} a_{00} & a_{01} & 0 \\ 0 & a_{11} & a_{12} \\ a_{20} & 0 & a_{22} \end{pmatrix},$$

as a product of elementary matrices.

Problem 4.

Let $\mathbf{A} \in \mathbb{C}^{n \times (n-1)}$ and $\mathbf{B} \in \mathbb{C}^{(n-1) \times n}$, show that

$$\det \left\{ (\mathbf{A} \cdot \mathbf{B})^2 \right\} = 0$$

Problem 5.

Express the inverse of

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

as product of elementary matrices.