Practice Midterm 2, MA 351 Section: Spring 2017

Name:

PUID:

Exam

No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned
I	50pts	
II	30pts	
III	20pts	
Total Points	100pts	

I (50pts)

Answer the following questions by clearly circling your answers. No partial credit is awarded for the questions in Part I

(1) Consider the subspace of $\mathbb{C}^{4\times 1}$:

$$W = \text{Span of } \left\{ \begin{pmatrix} i & i & 1 & i \end{pmatrix}^\top, \begin{pmatrix} 0 & 0 & 3 & 1+i \end{pmatrix}^\top, \begin{pmatrix} -i & -1 & 2 & 0 \end{pmatrix}^\top, \begin{pmatrix} i & 1 & 4 & 2i \end{pmatrix}^\top \right\}$$

What is the dimensions of $W^\perp$?
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

(2) Suppose $\mathbf{A}$ is 3×5 matrix such that Rank$\mathbf{A} = 3$. Which of the following statement is TRUE? (circle all that applies)

(a) The rank of $\mathbf{A}^\top$ is 5
(b) The dimensions of the null space of $\mathbf{A}^\top$ is 2
(c) $\mathbf{A} \cdot \mathbf{x} = \mathbf{0}$ only admit as a solution the trivial solution
(d) The rows of $\mathbf{A}$ are linearly dependent
(e) The columns of $\mathbf{A}$ are linearly dependent

(3) Let W be the vector space spanned by the vectors:

$$\mathbf{u}_1 = \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 0 & 1 & 1 & 0 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 1 & 1 & 0 & 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}.$$

Apply the Gram-Schmidt process to the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, $\mathbf{u}_4$ (in the order specified) to derive an orthonormal basis $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$ of W. What is $\mathbf{v}_3$.

(a) $\mathbf{v}_3 = \begin{pmatrix} 0 & 1 & -1 & 0 \end{pmatrix}$
(b) $\mathbf{v}_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$
(c) $\mathbf{v}_3 = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}$
(d) $\mathbf{v}_3 = \begin{pmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{pmatrix}$

(e) $\mathbf{v}_3 = \left(\begin{array}{cccc} (1-\sqrt{2}) & {}^{-1}/_{\sqrt{2}} & {}^{-1}/_{\sqrt{2}} & (1-\sqrt{2}) \end{array} \right)$

(4) Let $\mathbf{A}$ be a 7×3 matrix such that its null space is spanned by the vectors

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

consequently the rank of $\mathbf{A}$ is

(a) 1
(b) 2
(c) 3
(d) 4
(e) 6

(5) Consider the linear system

$$\begin{pmatrix} 1 & a \\ 2 & a^2-3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{a+5}{2} \end{pmatrix}.$$

Determine all values of the parameter a for which the linear system above has NO solutions.
(a) $a = \pm\sqrt{3}$
(b) $a = -5$
(c) $a = -3$
(d) $a = 3$
(e) $a \neq 3$
(f) none of the above

II (30pts)

a) Find the least square solution

$$\begin{pmatrix} 3i+1 & -3 & 5 \\ 2 & 0 & 4 \\ -i+3 & 1 & 5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} i \\ 4i \\ 9i \end{pmatrix}.$$

Show all your work in detail and clarity.

Answer:

b) Construct a 5×5 matrix $\mathbf{A}$ of rank 3 whose Nullspace is given by

$$\text{Span}\left\{ \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \right\}$$

Show all your work in detail and clarity.

Answer:

III (20pts)

Let $\mathbf{A} \in \mathbb{C}^{n\times n}$ having distinct eigenvalues, give a detail proof of the fact that eigenvectors with distinct eigenvalues must be linearly independent.