

Homework 12

April 23, 2017

Problem 1.

Let

$$f(x) = a_m \prod_{0 \leq i \leq m} (x - \alpha_i), \quad \text{such that } a_m \neq 0,$$

$$g(x) = b_n \prod_{0 \leq j \leq n} (x - \beta_j), \quad \text{such that } b_n \neq 0.$$

Prove that

$$(\text{Res}_x(f(x), g(x)))^2 = (a_m)^{2n} (b_n)^{2m} \prod_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} (\alpha_i - \beta_j)^2.$$

Problem 2.

Use the Sylvesterian elimination to determine the existence of a matrix $\mathbf{A} \in \mathbb{C}^{3 \times 3}$ subject to

$$\mathbf{A} \cdot \mathbf{A}^\top = 3\mathbf{I}_n, \quad \forall 1 \leq i, j \leq 3, \quad (\mathbf{A}[i, j])^2 = 1.$$

You may use a computer algebra system but write the down the intermediate degree matrices you get along the way.