

# Homework 4

January 30, 2017

## Problem 1.

Let  $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{3 \times 3}$ , find in terms of  $\mathbf{A}$  a parametric description for the set of all matrices  $\mathbf{B}$  such that

- a )  $\det(\mathbf{A} + \mathbf{B}) = \det \mathbf{A}$ .
- b )  $\det \mathbf{A} = \det(\mathbf{A} + \mathbf{B}) = \det \mathbf{B}$ .
- c )  $\det(2\mathbf{A}) = \det(\mathbf{A} + \mathbf{B}) = \det(2\mathbf{B})$ .

## Problem 2.

Prove that the rank of skew symmetric matrices is always even.

## Problem 3.

Express

$$\mathbf{A} = \begin{pmatrix} a_{00} & a_{01} & 0 \\ 0 & a_{11} & a_{12} \\ a_{20} & 0 & a_{22} \end{pmatrix},$$

as a product of elementary matrices.

## Problem 4.

Let  $\mathbf{A} \in \mathbb{C}^{n \times (n-1)}$  and  $\mathbf{B} \in \mathbb{C}^{(n-1) \times n}$ , show that

$$\det \left\{ (\mathbf{A} \cdot \mathbf{B})^2 \right\} = 0$$

## Problem 5.

Express the inverse of

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

as product of elementary matrices.