Homework 8

March 20, 2017

Problem 1.

Let

$$\det\left(x\,\mathbf{I}_n - \mathbf{A}\right) = x^n + \sum_{0\le k<n} (-1)^k\, a_{n-k}\, x^k$$

describe how to obtain using the Laplace expansions an $n\times n$ matrix $\mathbf{A}$ satisfying the constraint above such that $\mathbf{A}$ is not a diagonal matrix and $\mathbf{A}$ has at most $2n-1$ non zero entries.

Problem 2.

Let $\mathbf{A} \in \mathbb{C}^{n\times n}$ with n distinct eigenvalues and $\mathbf{D}$ denote the corresponding diagonal matrix of eigenvalues.

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 & \cdots & & 0 \\ 0 & \lambda_2 & & 0 & \vdots \\ \vdots & & \ddots & & \\ & 0 & & \lambda_{n-1} & 0 \\ 0 & & \cdots & 0 & \lambda_n \end{pmatrix},$$

a) Find a simple expression for the powers of $\mathbf{A}$ in terms of $\mathbf{D}$ and the matrix $\mathbf{V}$ obtained by stacking as columns the eigenvectors of $\mathbf{A}$.
b) Prove that any set $n+1$ of consecutive powers of $\mathbf{A}$ must be linearly dependent and determine explicit expressions for the coefficients in the linear combinations first in terms of the eigenvalues of $\mathbf{A}$ and then directly in terms of entries of $\mathbf{A}$.

Problem 3.

Prove that eigenvectors of $\mathbf{A}$ with distinct eigenvalues must be linearly independent.

Problem 4.

Find the standard matrix representation of the linear transformation which perform a rotation in $\mathbb{R}^{3\times 1}$ space of $\frac{\pi}{3}$ around the z axis followed by a rotation $\frac{\pi}{2}$ around the x axis and a rotation of $\frac{\pi}{4}$ around the y axis. Does the order of these operation matter ? explain why ?

Problem 5.

Let $\mathbf{A} \in \mathbb{C}^{n\times n}$ with n distinct eigenvalues and let $\mathbf{D}$ denote the corresponding diagonal matrix of eigenvalues.

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 & \cdots & & 0 \\ 0 & \lambda_2 & & 0 & \vdots \\ \vdots & & \ddots & & \\ & 0 & & \lambda_{n-1} & 0 \\ 0 & & \cdots & 0 & \lambda_n \end{pmatrix},$$

Let $\mathbf{V}$ denote the matrix which stack as columns the eigenvectors of $\mathbf{A}$ find the eigenvectors and eigenvalues of

$$\mathbf{M}^{-1} \cdot \mathbf{A} \cdot \mathbf{M}$$

for some specified invertible matrix $\mathbf{M}$ in terms of eigenvectors and eigenvalues of $\mathbf{A}$.