Homework 11

April 16, 2017

Problem 1.

a) Give a detail proof that all $\mathbf{A} \in \mathbb{C}^{n\times n}$ admit a Singular Value Decomposition or SVD. Your proof should be along the lines of the proof presented in class. (Note: Be sure to include the argument omitted at the end of the proof discussed in class)
b) Let $\mathbf{A} \in \mathbb{C}^{n\times n}$ with eigenvalues $\{\lambda_k\}_{1\leq k\leq n}$ such that $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3| \geq \cdots \geq |\lambda_{n-1}| \geq |\lambda_n| > 0$. Using the SVD describe how to find matrix $\mathbf{M}$ of rank $r < n$ which minimizes $\|\mathbf{A} - \mathbf{M}\|$.

Problem 2.

a) Let $\mathbf{A} \in \mathbb{C}^{n\times n}$, prove that if for all $1 \leq i, j \leq n$

$$|\mathbf{A}[i,j]| \leq 1$$

then we have that

$$|\det \mathbf{A}|^2 \leq n^n$$

b) Show that the inequality is tight over $\mathbb{C}^{n\times n}$ for all n by identifying a family of $n \times n$ matrices for which

$$|\det \mathbf{A}|^2 = n^n$$

Problem 3.

a) Given $\mathbf{A} \in \mathbb{C}^{n\times n}$ find a set of mutually orthogonal solutions matrix $\mathbf{M}$ of rank 1 to the equation

$$\det(\mathbf{A} - \mathbf{M}) = 0$$

b) Does the solution set change if we remove the mutually orthogonal condition among the solutions.

Problem 4.

Let $\mathbf{A} \in \mathbb{C}^{n\times n}$ be a unitary matrix show that the singular values of $\mathbf{A}$ must all have absolute value equal to 1.

Problem 5.

Compute the SVD of the following matrices

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$