Homework 2

January 22, 2017

Bonus Problem.

Find the simplest possible formula in terms of positive integers $m \geq 1$, $n \geq 1$ and $p > 1$ which counts the number of matrices :
a) of size $m \times n$ in REF and whose entries are taken from the set $\{0, 1, \cdots, (p-1)\}$.
b) of size $m \times n$ in RREF and whose entries are taken from the set $\{0, 1, \cdots, (p-1)\}$.
c) of size $n \times n$ in REF, invertible and whose entries are taken from the set $\{0, 1, \cdots, (p-1)\}$.

Updated Problem 1.

Find the simplest possible formula in terms of positive integers $1 < m \leq n$, which counts the number of matrices :
a) of size $m \times n$ in REF and whose entries are taken from the set $\{0, 1, \cdots, (p-1)\}$. Assume that each one of the first m rows has a pivot and each one of the first m columns has a pivot.
b) of size $m \times n$ in RREF and whose entries are taken from the set $\{0, 1, \cdots, (p-1)\}$. Assume that each one of the first m rows has a pivot and each one of the first m columns has a pivot.
c) of size $n \times n$ in REF, invertible and whose entries are taken from the set $\{0, 1, \cdots, (p-1)\}$.

Problem 2.

Show that for any nonzero matrix $\mathbf{A} \in \mathbb{C}^{m \times n}$

$$0 < \mathrm{Rank}\,(\mathbf{A}) \leq \min\{m, n\}\,.$$

Problem 3.

Using the permutation expansion of the determinant of an $n \times n$ matrix $\mathbf{A}$ expressed by :

$$\sum_{\sigma \in \text{ permutations of } 1,\cdots,n.} (-1)^{\#\text{ of inversions in } \sigma} \prod_{1 \le i \le n} \mathbf{A}\left[\sigma\left(i\right), i\right].$$

Show that $\det \mathbf{B} = \det \mathbf{A}$, if the $n \times n$ matrix $\mathbf{B}$ is obtained from $\mathbf{A}$ via the row linear combination operation described by

$$k \cdot \mathrm{R}_{n-1} + \mathrm{R}_n \to \mathrm{R}_n.$$

Problem 4.

Given data points $\{(x_1 = 1, y_1 = -5), (x_2 = -1, y_2 = 1), (x_3 = 2, y_3 = 7)\}$ set up a linear system of equation to solve for the variables a_0, a_1, a_2 such that for all $1 \le i \le 3$ we have

$$y_i = a_2 \cdot (x_i)^2 + a_1 \cdot (x_i)^1 + a_0 \cdot (x_i)^0$$

Problem 5.

Set up the following chemical balance equation as a system of linear constraints in order to find the coefficients $\{x_i\}_{1 \le i \le 4}$ via Gauss-Jordan elimination

$$x_1\,\mathrm{NaOH} + x_2\,\mathrm{H_2SO_4} \to x_3\,\mathrm{Na_2SO_4} + x_4\,\mathrm{H_2O}$$