

Practice Midterm 2, MA 351      Section:      Summer 2016

Name:

PUID:

Exam

No calculators may be used on this exam.

| Problem<br>Number | Possible<br>Points | Points<br>Earned |
|-------------------|--------------------|------------------|
| I                 | 50pts              |                  |
| II                | 25pts              |                  |
| III               | 25pts              |                  |
| Total Points      | 100pts             |                  |

## I (50pts)

Answer the following questions by clearly circling your answers. No partial credit is awarded for the questions in Part I

(1) Consider the subspace of  $\mathbb{C}^{4 \times 1}$  :

$$W = \text{Span of } \left\{ \begin{pmatrix} i & i & 1 & i \end{pmatrix}^\top, \begin{pmatrix} 0 & 0 & 3 & 1+i \end{pmatrix}^\top, \begin{pmatrix} -i & -1 & 2 & 0 \end{pmatrix}^\top, \begin{pmatrix} i & 1 & 4 & 2i \end{pmatrix}^\top \right\}$$

What is the dimensions of  $W^\perp$  ?

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 4

(2) Suppose  $\mathbf{A}$  is  $3 \times 5$  matrix such that  $\text{Rank}\mathbf{A} = 3$ . Which of the following statement is TRUE? (circle all that applies)

- (a) The rank of  $\mathbf{A}^\top$  is 5
- (b) The dimensions of the null space of  $\mathbf{A}^\top$  is 2
- (c)  $\mathbf{A} \cdot \mathbf{x} = \mathbf{0}$  only admit as a solution the trivial solution
- (d) The rows of  $\mathbf{A}$  are linearly dependent
- (e) The columns of  $\mathbf{A}$  are linearly dependent

(3) The dimension of the vector space of all  $4 \times 4$  Hermitian matrices is

- (a) 12
- (b) 16
- (c) 10
- (d) 8
- (e) 6

(4) Let  $\mathbf{A}$  be a  $7 \times 3$  matrix such that its null space is spanned by the

vectors

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

consequently the rank of  $\mathbf{A}$  is

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 6

(5) Consider the linear system

$$\begin{pmatrix} 1 & a \\ 2 & a^2 - 3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{a+5}{2} \end{pmatrix}.$$

Determine all values of the parameter  $a$  for which the linear system above has NO solutions.

- (a)  $a = \pm\sqrt{3}$
- (b)  $a = -5$
- (c)  $a = -3$
- (d)  $a = 3$
- (e)  $a \neq 3$
- (f) none of the above

## II (25pts)

Find the least square solution

$$\begin{pmatrix} 1 & -3 & 5 \\ 2 & 0 & 4 \\ 3 & 1 & 5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}.$$

Show all your work in detail and clarity.

Answer:



### III (25pts)

Let  $\mathbf{A} \in \mathbb{C}^{n \times n}$  having distinct eigenvalues, give a detail of proof of the fact that the eigenvector must be linearly independent.



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