Homework 6

July 9, 2016

Problem 1.

Consider the procedure which takes as input a matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ and alternatively repeats the following two sub-routines one after the other

- Gram-Schmidt process on the columns of $\mathbf{A}$ followed by the normalization of the vectors to obtain an orthonormal basis.
- Squaring all the entries of the resulting matrix.

Show that under conditions to be determined on the input matrix $\mathbf{A}$ the procedure converges to a special type of unitary matrix discussed in class.

Problem 2.

Consider the following data showing the atmospheric pollutants y_i at half hour intervals t_i :

t_i	1	1.5	2	2.5	3	3.5	4.5
y_i	-0.15	0.24	0.68	1.04	1.21	1.15	0.86

Use the least square solution to solve for x_0, x_1, x_2 which achieves the best fit for

$$y(t) = x_2 \cdot t^2 + x_1 \cdot t^1 + x_0 \cdot t^0.$$

Problem 3.

We discussed in class that the least square solution to a system of equation

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$$

where $\mathbf{A} \in \mathbb{C}^{m \times n}$, $\mathbf{b} \in \mathbb{C}^{m \times 1}$ is found by solving

$$\mathbf{A} \cdot \mathbf{x} = \text{Proj}_{\text{Column Space of } \mathbf{A}}(\mathbf{b})$$

Show using properties of the orthogonal complements that the least square solution can also be obtained by solving

$$\overline{\mathbf{A}^\top} \cdot \mathbf{A} \cdot \mathbf{x} = \overline{\mathbf{A}^\top} \cdot \mathbf{b}$$

Problem 4.

Let $\mathbf{u}$, $\mathbf{v}$ denote two linear independent vectors in $\mathbb{C}^{n\times 1}$. Express in terms of $\mathbf{v}$ the $n \times n$ matrix $\mathbf{P_v}$ subject to the equality

$$\mathrm{Proj}_{\mathrm{Span}\{\mathbf{v}\}}(\mathbf{u}) = \mathbf{P_v} \cdot \mathbf{u}$$

Problem 5.

Let $\mathbf{A} \in \mathbb{C}^{m\times m}$ and $\mathbf{B} \in \mathbb{C}^{n\times n}$ be unitary matrices. Let the matrix $\mathbf{C} \in \mathbb{C}^{m\cdot n\times m\cdot n}$ have entries given by

$$c_{k+n\cdot(i-1),\, n\cdot(j-1)+l} = a_{i,j} \cdot b_{k,l}.$$

Show that the matrix $\mathbf{C}$ is unitary.