Homework 12

April 23, 2017

Problem 1.

Let

$$f(x) = a_m \prod_{0 \le i \le m} (x - \alpha_i), \quad \text{such that} \quad a_m \neq 0,$$

$$g(x) = b_n \prod_{0 \le j \le n} (x - \beta_j), \quad \text{such that} \quad b_n \neq 0.$$

Prove that

$$\left(\mathrm{Res}_x \left(f(x), g(x)\right)\right)^2 = (a_m)^{2n} (b_n)^{2m} \prod_{\substack{1 \le i \le m \\ 1 \le j \le n}} (\alpha_i - \beta_j)^2.$$

Problem 2.

Use the Sylvesterian elimination to determine the existence of a matrix $\mathbf{A} \in \mathbb{C}^{3\times 3}$ subject to

$$\mathbf{A} \cdot \mathbf{A}^{\top} = 3\,\mathbf{I}_n, \ \forall 1 \le i, j \le 3, \quad \left(\mathbf{A}[i,j]\right)^2 = 1.$$