

Homework 3

January 22, 2017

Problem 1.

Let $\mathbf{X} \in \mathbb{C}^{m \times \ell}$ and $\mathbf{Y} \in \mathbb{C}^{\ell \times n}$. Let $\{\mathbf{y}_k\}_{1 \leq k \leq n}$ denote the rows of \mathbf{Y} and assume that

$$\mathbf{y}_\ell = \sum_{1 \leq t \leq \ell-1} \alpha_t \mathbf{y}_t$$

then show that $(\mathbf{X} \cdot \mathbf{Y})$ has rank $< \ell$.

Problem 2.

Let \mathbf{A} be a $n \times n$ matrix for which the entries are specified for all $1 \leq i, j \leq n$ by

$$a_{ij} = j^{i-1}.$$

Determine and prove a simple formula for the determinant of this family of matrices.

Problem 3.

Compute the determinants, show your computation in detail

a)

$$\det \begin{pmatrix} 1 & 2 & 3 \\ 8 & 9 & 4 \\ 7 & 6 & 5 \end{pmatrix}$$

b)

$$\det \begin{pmatrix} x+1 & x+2 & x+3 \\ x+8 & x+9 & x+4 \\ x+7 & x+6 & x+5 \end{pmatrix}$$

c)

$$\det \begin{pmatrix} x^1 & x^2 & x^3 \\ x^8 & x^9 & x^4 \\ x^7 & x^6 & x^5 \end{pmatrix}$$

Problem 4.

Using properties of the determinant. Explain without computation why the determinant of

$$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ b_1 & b_2 & b_3 & b_4 & b_5 \\ c_1 & c_2 & 0 & 0 & 0 \\ d_1 & d_2 & 0 & 0 & 0 \\ e_1 & e_2 & 0 & 0 & 0 \end{pmatrix}$$

is zero.

Problem 5.

Compute the determinant of the following matrices

$$\begin{pmatrix} 0 & 0 & a_1 & b_1 \\ 0 & 0 & a_2 & b_2 \\ a_3 & b_3 & 0 & 0 \\ a_4 & b_4 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} a_1 & 0 & 0 & b_1 \\ 0 & a_2 & b_2 & 0 \\ 0 & b_3 & a_3 & 0 \\ b_4 & 0 & 0 & a_4 \end{pmatrix}$$