Homework 6

February 27, 2017

Problem 1.

Prove that for $\mathbf{A} \in \mathbb{C}^{n\times n}$ such that NullSpace($\mathbf{A}$) has dimensions $> k$ then there is a nonzero vector $\mathbf{v}$ in the nullspace of $\mathbf{A}$ vanishing at any k specified entries of $\mathbf{v}$.

Problem 2.

Let $\mathbf{x}$, $\mathbf{y} \in \mathbb{C}^{5\times 1}$ show that

$$\det\left(\mathbf{I}_n - \mathbf{x}\cdot\mathbf{y}^\top\right) = 1 - \mathbf{y}^\top\cdot\mathbf{x}$$

Problem 3.

Let $\mathbf{A} \in \mathbb{C}^{n\times n}$, $\mathbf{B} \in \mathbb{C}^{n\times m}$, $\mathbf{C} \in \mathbb{C}^{m\times n}$ and $\mathbf{D} \in \mathbb{C}^{m\times m}$, prove the following block matrix identity

$$\det\left\{\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}\right\} = \det\left(\mathbf{A}\right)\cdot\det\left(\mathbf{D} - \mathbf{C}\cdot\mathbf{A}^{-1}\cdot\mathbf{B}\right)$$

Problem 4.

Let $\mathbf{A}$ denote the 4×4 matrix whose entries are given by

$$\forall\, 1 \le u, v \le 4, \quad \mathbf{A}\left[u, v\right] = \left(\sqrt{-1}\, u\right)^v.$$

Use the Gram-Schmidt process to expresse $\mathbf{A}$ as the product of a unitary matrix $\mathbf{Q}$ with an upper triangular matrix $\mathbf{R}$.

Problem 5.

Construct a 5×5 matrix $\mathbf{A}$ of rank 3 whose Nullspace is equal to

$$\operatorname{Span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 1^2 \\ 2^2 \\ 3^2 \\ 4^2 \\ 5^2 \end{pmatrix} \right\}$$