# Homework 8

March 20, 2017

### Problem 1.

Let

$$\det (x \mathbf{I}_n - \mathbf{A}) = x^n + \sum_{0 \le k < n} (-1)^k a_{n-k} x^k$$

decribe how to obtain using the Lapace expansions an  $n \times n$  matrix **A** satisfying the contraint above such that **A** is not a diagonal matrix and **A** has at most 2n-1 non zero entries.

### Problem 2.

Let  $\mathbf{A} \in \mathbb{C}^{n \times n}$  with n distrint eigenvalues and  $\mathbf{D}$  denote the corresponding diagonal matrix of eigenvalues.

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 & \vdots \\ \vdots & & \ddots & & \\ 0 & & \lambda_{n-1} & 0 \\ 0 & & \cdots & 0 & \lambda_n \end{pmatrix},$$

- a) Find a simple expression for the powers of A in terms of D and the matrix
  V obtained by stacking as columns the eigenvectors of A.
- b) Prove that any set n+1 of consecutive powers of **A** must be linearly dependent and determine explicit expressions for the coefficients in the linear combinations first in terms of the eigenvalues of **A** and then dirrectly in terms of entries of **A**.

#### Problem 3.

Prove that eigenvectors of  $\bf A$  with distinct eigenvalues must be linearly independent.

# Problem 4.

Find the standard matrix representation of the linear transformation which perform a rotation in space of  $\frac{\pi}{3}$  around the z axis followed by a rotation  $\frac{\pi}{2}$  around the x axis and a rotation of  $\frac{\pi}{4}$  around the y axis. Does the order of these operation matter? explain why?

## Problem 5.

Let  $\mathbf{A} \in \mathbb{C}^{n \times n}$  with n district eigenvalues and let  $\mathbf{D}$  denote the corresponding diagonal matrix of eigenvalues.

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 & \vdots \\ \vdots & & \ddots & & \\ 0 & & \lambda_{n-1} & 0 \\ 0 & & \cdots & 0 & \lambda_n \end{pmatrix},$$

Let V denote the matrix which stack as columns the eigenvectors of A find the eigenvectors and eigenvalues of

$$\mathbf{M}^{-1}\cdot\mathbf{A}\cdot\mathbf{M}$$

for some specified invertible matrix  $\mathbf{M}$  in terms of eigenvectors and eigenvalues of  $\mathbf{A}$ .