

# gr-doa: Direction Finding in GNU-Radio

GRCon 2017

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Travis F. Collins, PhD  
Srikanth Pagadarai, PhD

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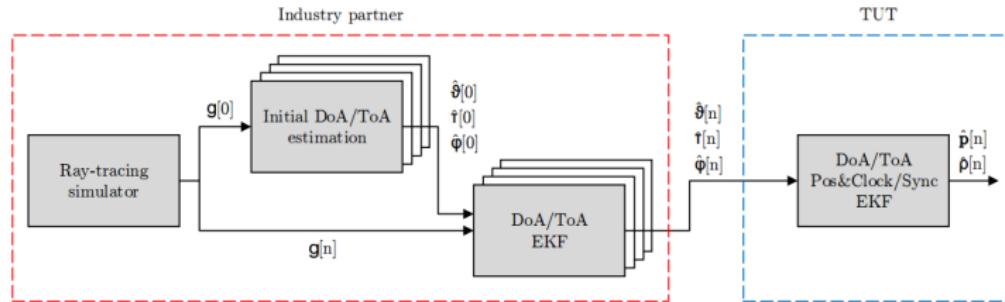
AHEAD OF WHAT'S POSSIBLE™



# Outline

- Project Background
- MUSIC
- Hardware options
  - USRP-N210
  - X300/X310
  - FMComms5
- Antenna Calibration
- Test and Results
- EADF Feasibility
- Test and Results

# WiFiUS Project Collaboration

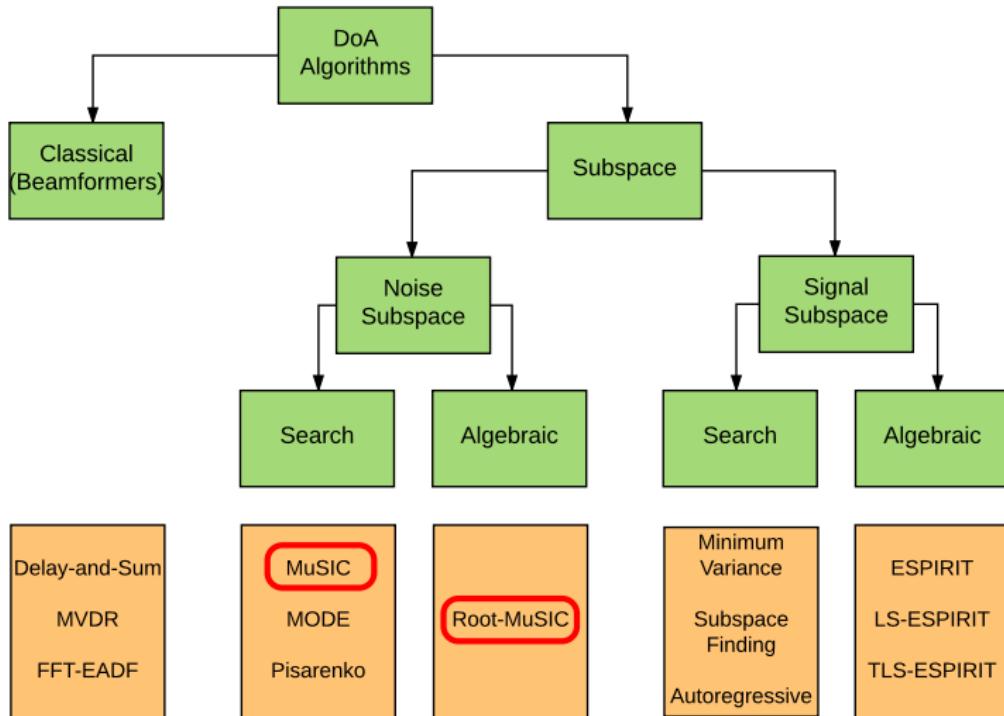


## Collaboration Outline



- Phase 1: Baseline DoA SDR implementation and analysis (gr-doa)
- Phase 2: Link based DoA with TUT DoA modeling specifics (EADF)

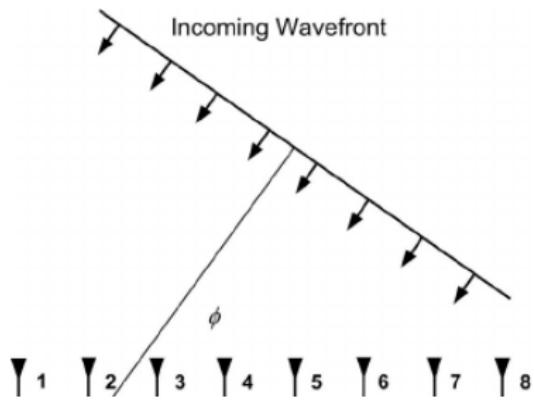
# Algorithms in gr-doa



# Phased Array Direction Finding Basics

## Basics:

- Signal is planar (far field)
- Antenna elements have a fixed and known phase
- Antenna positions are known
- Signal has limited correlation between elements



## ULA: Phased Array Model

Element positioning in reference to array center:

$$p_{z_n} = 0, \quad p_{y_n} = 0, \quad p_{x_n} = \left( n - \frac{N-1}{2} \right) d, \quad n = 0, 1, \dots, N-1,$$

Received signal cascaded delays:

$$u_z = \sin(\theta) \cos(\phi), \quad u_y = \sin(\theta) \sin(\phi), \quad u_x = \cos(\theta),$$

$$\mathbf{u} := [u_z \ u_y \ u_x]^T$$

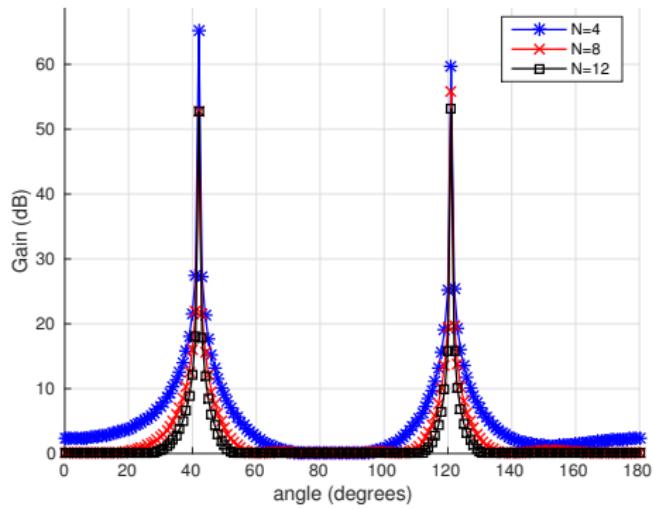
$$\mathbf{k}(\theta, \phi) = -\frac{2\pi}{\lambda} \mathbf{u} = -\frac{2\pi}{\lambda} [\sin(\theta) \cos(\phi) \ \sin(\theta) \sin(\phi) \ \cos(\theta)]^T.$$

Array manifold vector:

$$\mathbf{v}(\mathbf{k}) = \left[ e^{-j\mathbf{k}^T p_0} \quad e^{-j\mathbf{k}^T p_1} \dots \quad e^{-j\mathbf{k}^T p_{N-1}} \right]^T$$

# DoA Baseline: MUSIC

## MUSIC: Multiple Signal Classification



R. Schmidt, "Multiple emitter location and signal parameter estimation," in IEEE Transactions on Antennas and Propagation, vol. 34, no. 3, pp. 276-280, Mar 1986.

Received signal model:

$$\mathbf{x}(t) = \sum_{d=1}^D u_d(t) \mathbf{v}(\mathbf{k}_d) + \mathbf{n}(t)$$

MUSIC subspace principles:

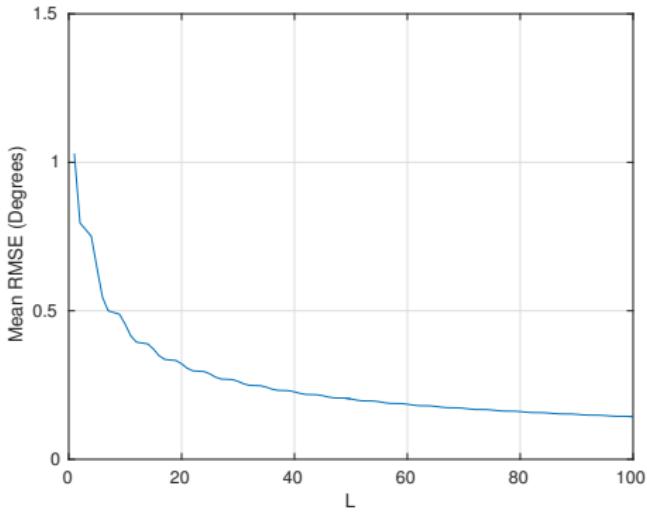
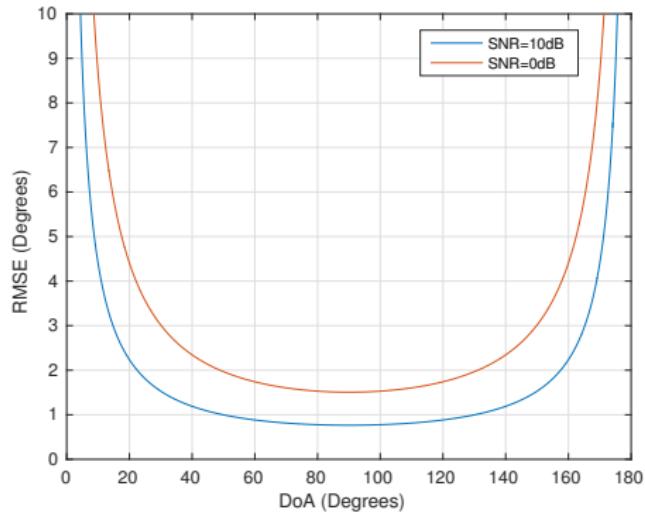
$$\mathbf{R}_{xx} = E[\mathbf{x} \mathbf{x}^H] = \mathbf{V} \mathbf{R}_{uu} \mathbf{V}^H + I\sigma^2$$

$$\mathbf{U}_N \in \mathbb{C}^{(N-D) \times N}, \mathbf{U}_S \in \mathbb{C}^{D \times N}$$

$$\mathbf{v}^H(\mathbf{k}_d) \mathbf{U}_N = 0$$

$$P_{MUSIC} = \frac{1}{\mathbf{v}^H(\mathbf{k}(\theta, \phi)) \mathbf{U}_N \mathbf{U}_N^H \mathbf{v}^H(\mathbf{k}(\theta, \phi))}$$

# MUSIC: Estimation Performance

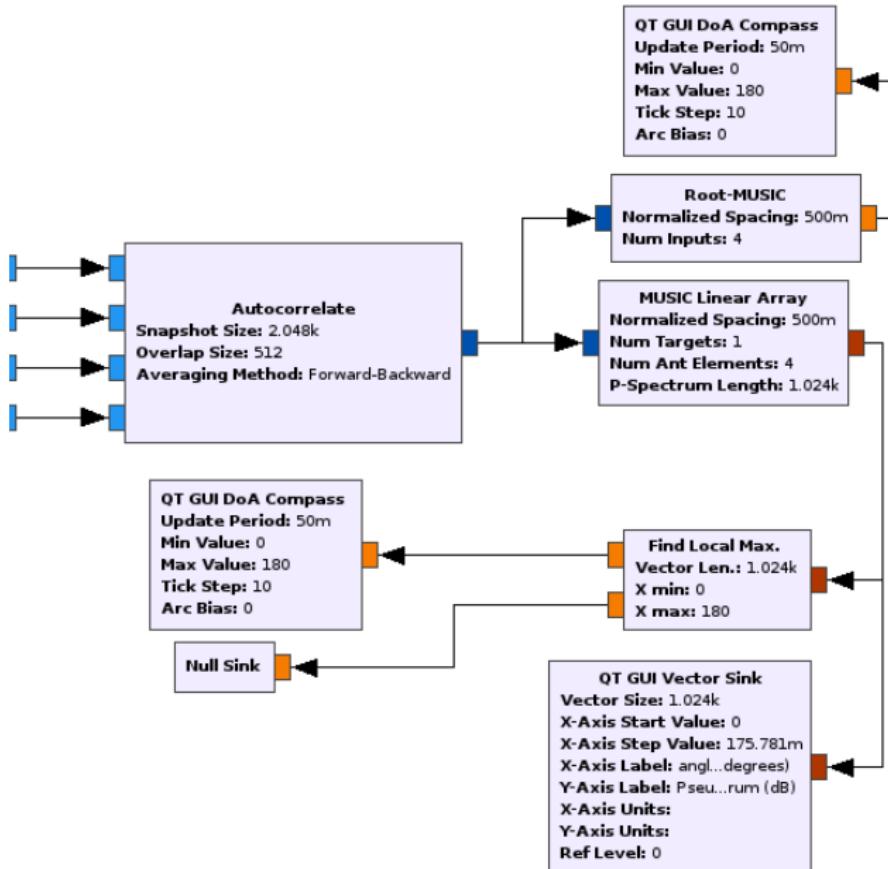


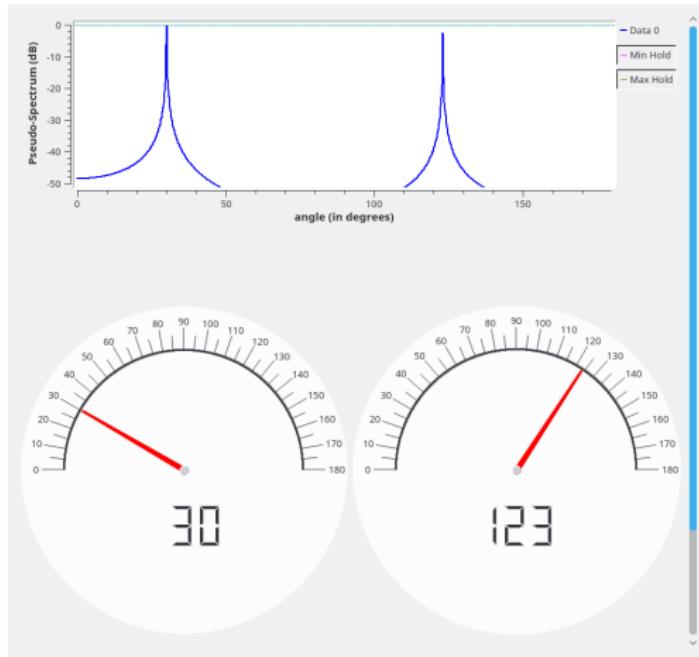
$$var(\theta) \geq \frac{\sigma^2}{2} \left\{ \sum_{k=1}^L \operatorname{Re} \left[ u^*(k) \frac{\partial \mathbf{v}^H}{\partial \theta} \cdot \left[ \mathbf{I}_N - \mathbf{v}(\mathbf{v}^H \mathbf{v})^{-1} \mathbf{v}^H \right] \frac{\partial \mathbf{v}}{\partial \theta} u(k) \right] \right\}^{-1}$$

$L = \text{snapshot length}$

P. Stoica and A. Nehorai, "MUSIC, maximum likelihood, and Cramer-Rao bound," in IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 37, no. 5, pp. 720-741, May 1989.

# Estimation Blocks

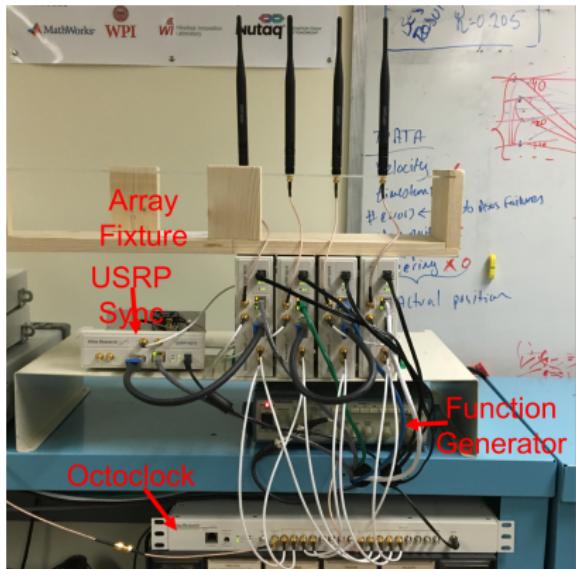




## Real-time visuals

- QT Compass
- Pseudo spectrum (MUSIC block only)

# N210 Based Array Construction (First Try)



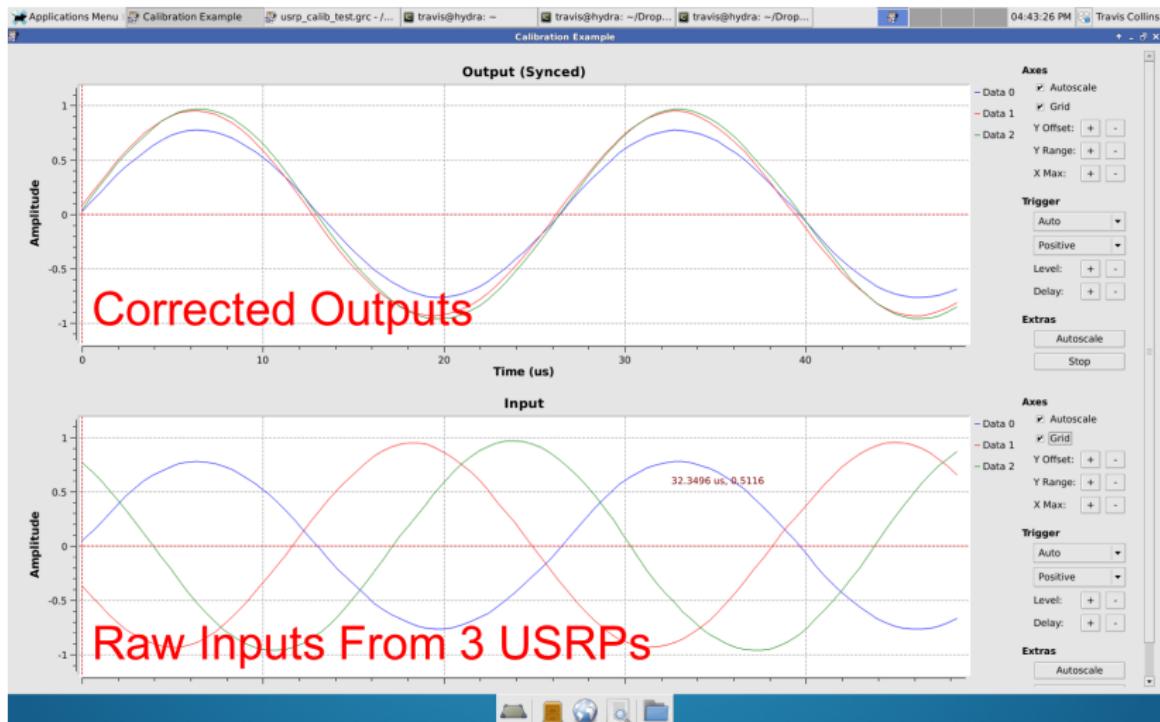
## Array Fixture

- $d = \frac{\lambda}{2}$  for  $f_c = 2.45$  GHz
- Same length cabling between components

## Radios and Software

- Utilizing 4xUSRP N210 radios
- Octoclock provides MB frequency and PPS sampling signals
- Not phased aligned
- GNURadio prototyping platform

# Remaining Phase Offset

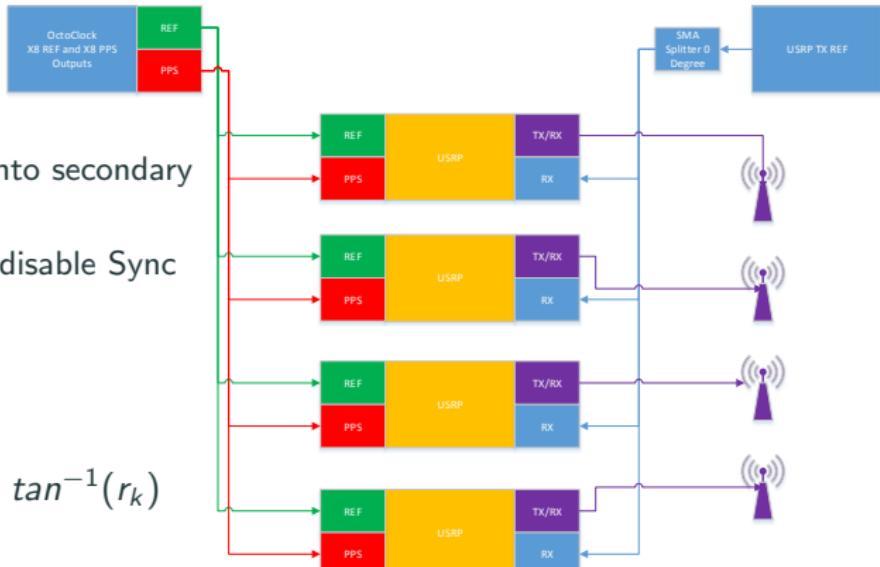


- 10 kHz tone sent into 3xUSRPs through splitter and matched cabling

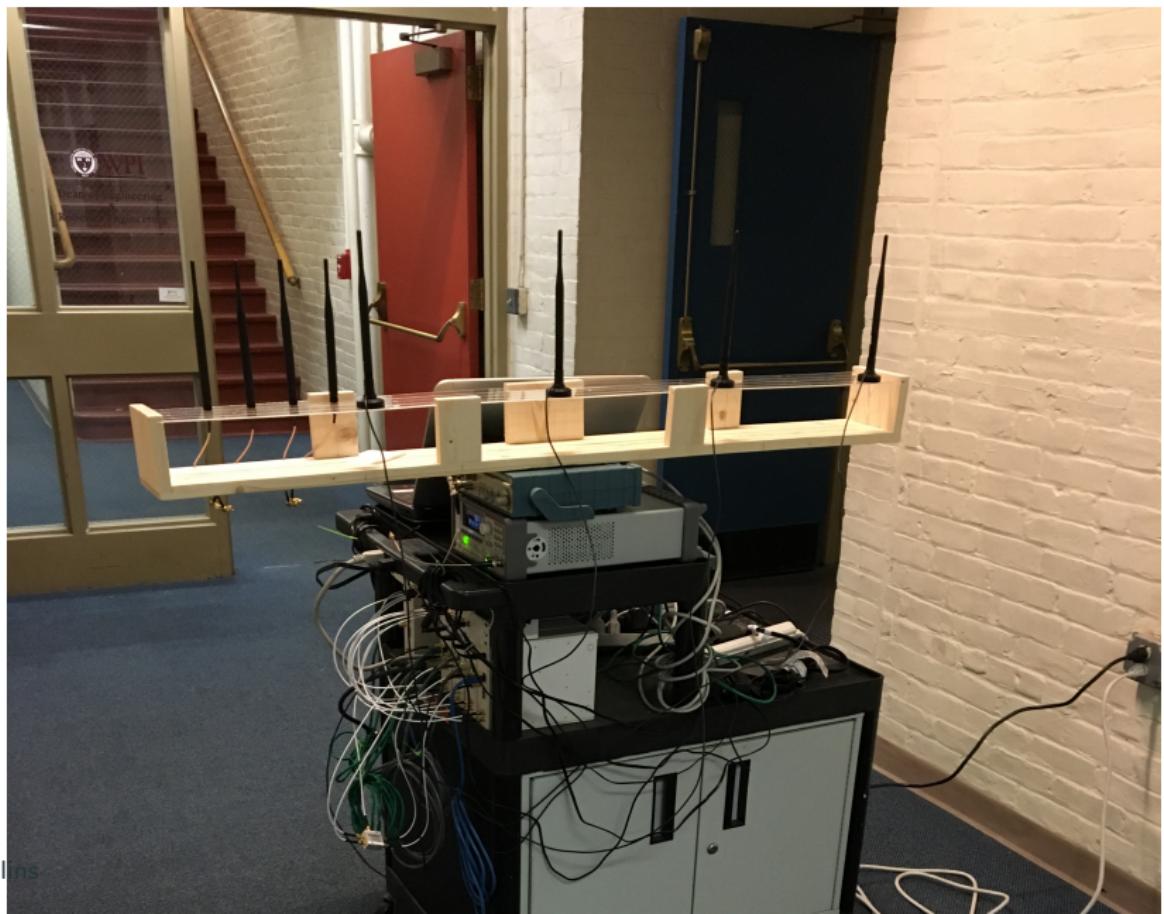
# USRP Phase Correction Harness

## System Details

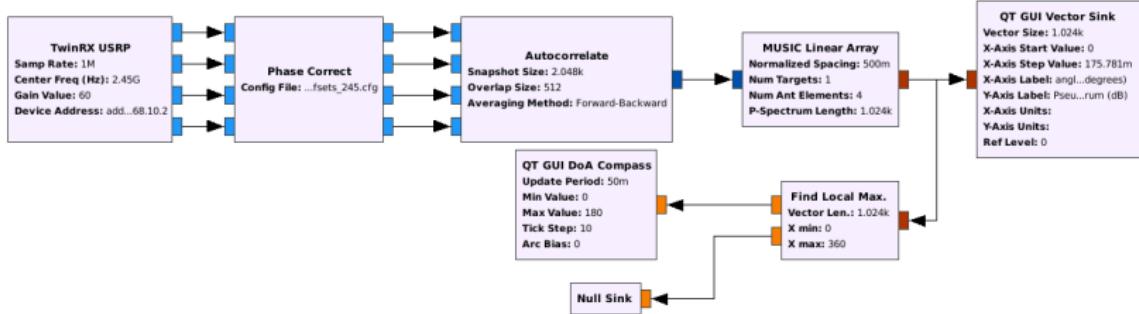
- Two Phase Start:
  1. Transmit tone into secondary port (TX/RX)
  2. Once corrected disable Sync TX
- Can calibrate any daughterboard(s)
- $\Delta\phi_n = \tan^{-1}(r_n) - \tan^{-1}(r_k)$
- $\hat{r}_n = r_n \exp(j\Delta\phi_n)$
- Open and closed loop versions



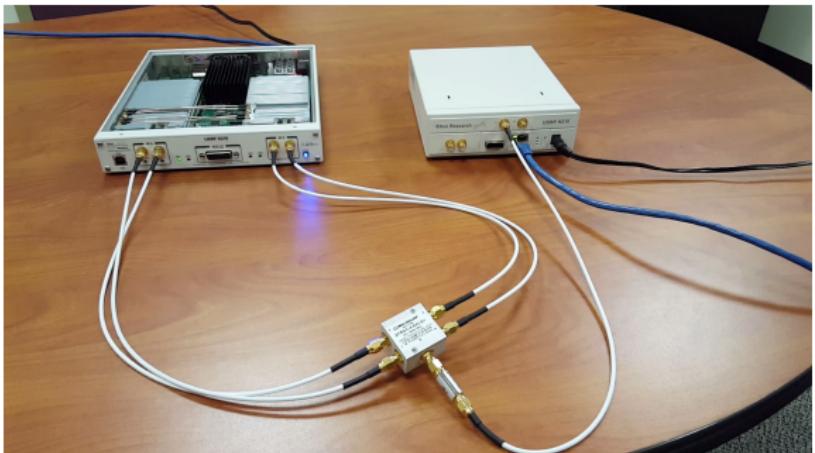
# Array Cart



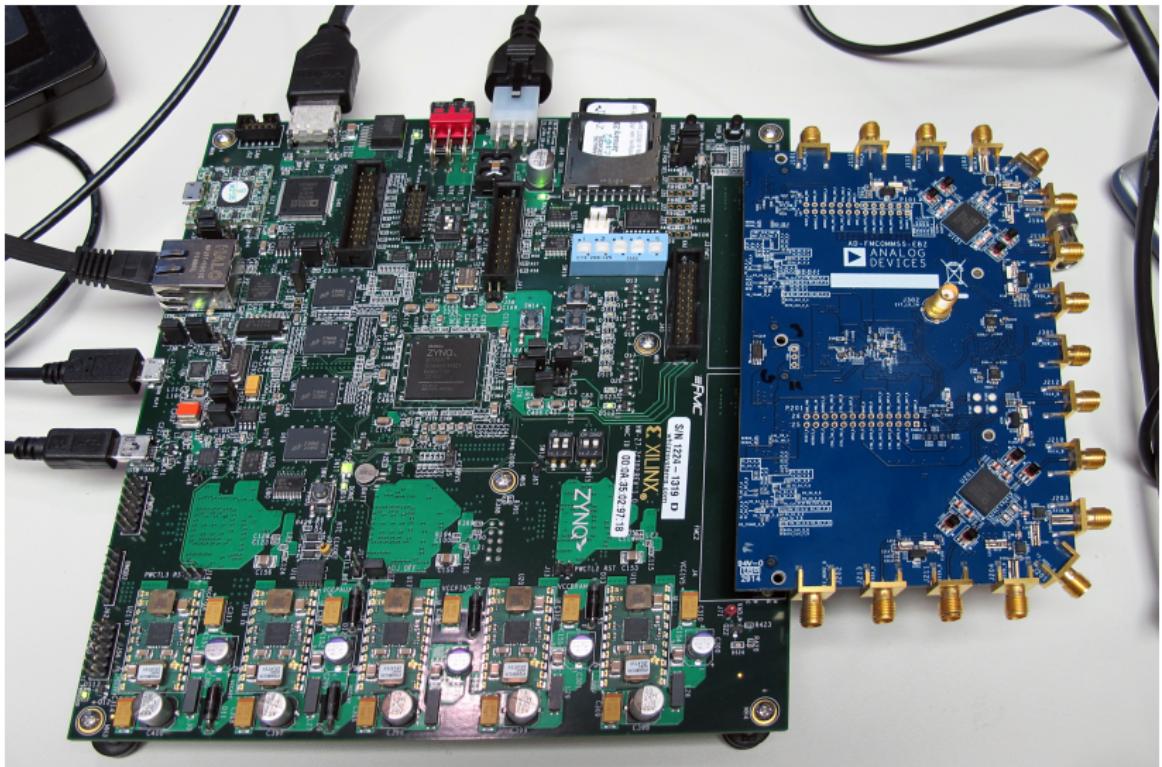
# X300/X310 TwinRx Array



- Still needs calibration
- Better phase stability
- Limited to 4-RX
- Easier to carry around!



# FMComms5 Array



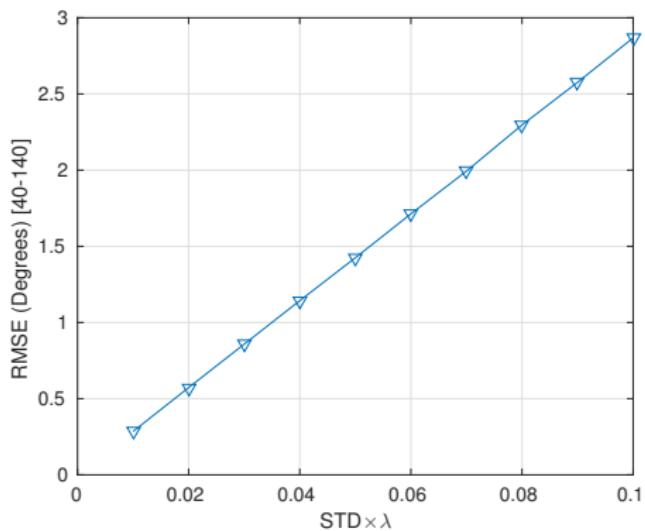
- Self phase calibrating!

# DOA: Error Sources

## Remaining Possible Sources of Error

- Phase drift between radios over time (Physical)
- Positioning error (Physical)
- Antenna gain and phase missmatches (Physical)
- Reflections in environment (Physical)
- Possible errors in software implementation (Software)

# MUSIC: Positioning Error



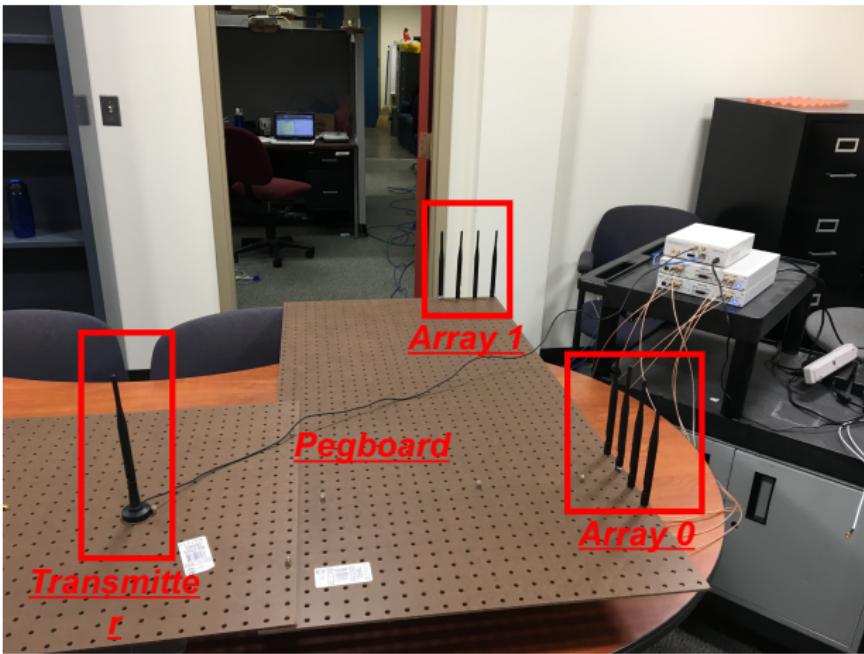
Modified signal model

$$\mathbf{x}(t) = \sum_{d=1}^D u_d(t) \hat{\mathbf{v}}(\mathbf{k}_d) + \mathbf{n}(t)$$

Position Error  $\mathbf{s}_k = N(0, \sigma^2)$

$$\hat{\mathbf{v}}(\mathbf{k}_d) = \mathbf{v}(\mathbf{k}_d) + \left[ \mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_D \right]^T$$

# MUSIC: Testbed Refinement



## Positioning

- Moved to pegboard for precise positioning
- $+/-0.125"$  tolerance  $\rightarrow 0.028\lambda$  (0.7 Degree RMSE)

# MUSIC: Antenna Calibration

- Implemented subspace processing method for antenna calibration
- Only phase mismatches between elements cause estimation error
- Utilize a target placed at a known position

$$\mathbf{R}_{xx} = \boldsymbol{\Gamma} \mathbf{V} \mathbf{R}_{uu} \mathbf{V}^H \boldsymbol{\Gamma}^H + I\sigma^2.$$

$$\mathbf{C}_{xx,eig} = \mathbf{E}_s \Lambda_s \mathbf{E}_s^H \boldsymbol{\Gamma}^H + I\sigma^2 \mathbf{E}_N \mathbf{E}_N^H,$$

$$\mathbf{E}_s \mathbf{E}_s^H \boldsymbol{\Gamma} \mathbf{V} = \boldsymbol{\Gamma} \mathbf{V}.$$

$\mathbf{V}_d$  true response of target at known position

$$diag\{\boldsymbol{\Gamma}\} = \gamma$$

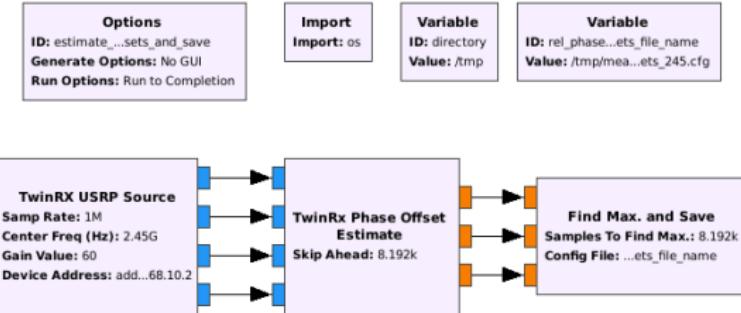
$$\mathbf{E}_s \mathbf{E}_s^H \mathbf{V}_d \gamma = \mathbf{V}_d \gamma$$

$$\mathbf{V}_d^H \mathbf{E}_s \mathbf{E}_s^H \mathbf{V}_d \gamma = \gamma,$$

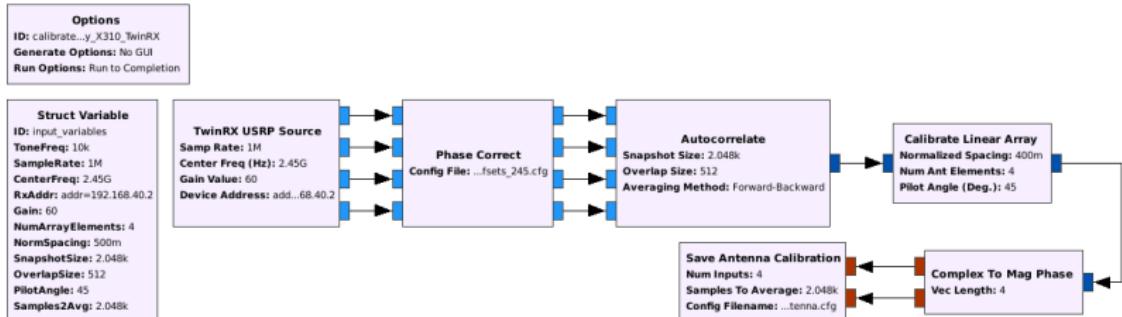
V. C. Soon, L. Tong, Y. F. Huang and R. Liu, "A Subspace Method for Estimating Sensor Gains and Phases," in IEEE Transactions on Signal Processing, vol. 42, no. 4, pp. 973-976, Apr 1994.

# Calibration Flowgraphs: Phase and Antennas

## Phase

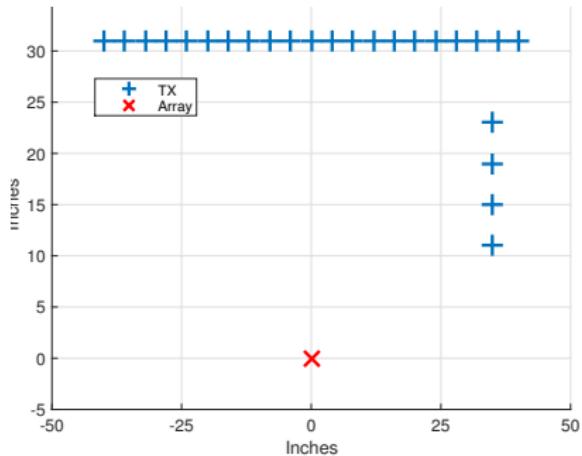


## Antennas



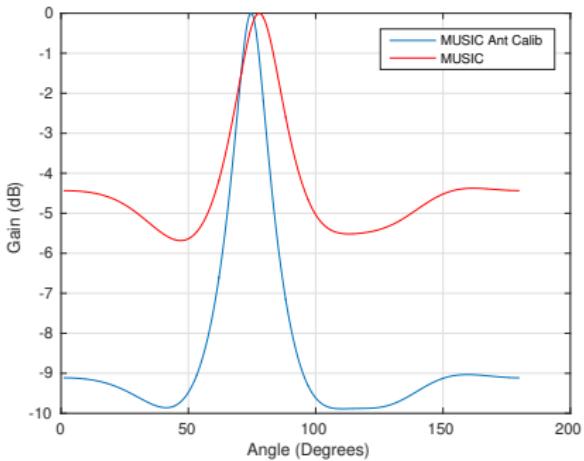
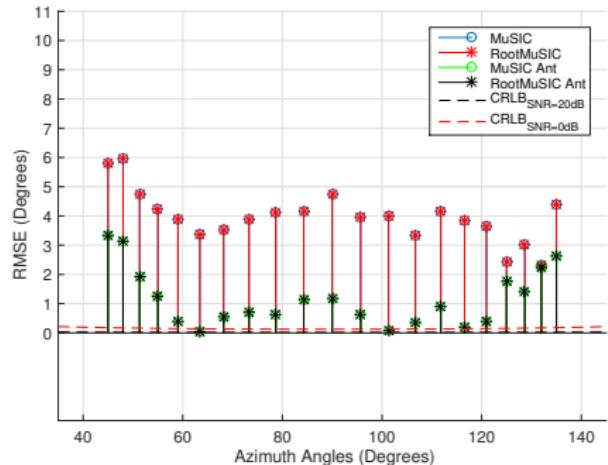
input\_variables:  
Contains all the user-defined variables.

# MUSIC: Anechoic Chamber Testing



- Testing in 4x4x4m (W,L,H) room with a single target
- Array positioned away from walls
- X310 dual TwinRX (4 receivers)
- Measurements include MUSIC and Root-MUSIC

# MUSIC: Chamber Results



- Antenna calibration uses target at known position
- Increase P-Spectrum SNR and DoA estimate

DEMO

# Repository Apps

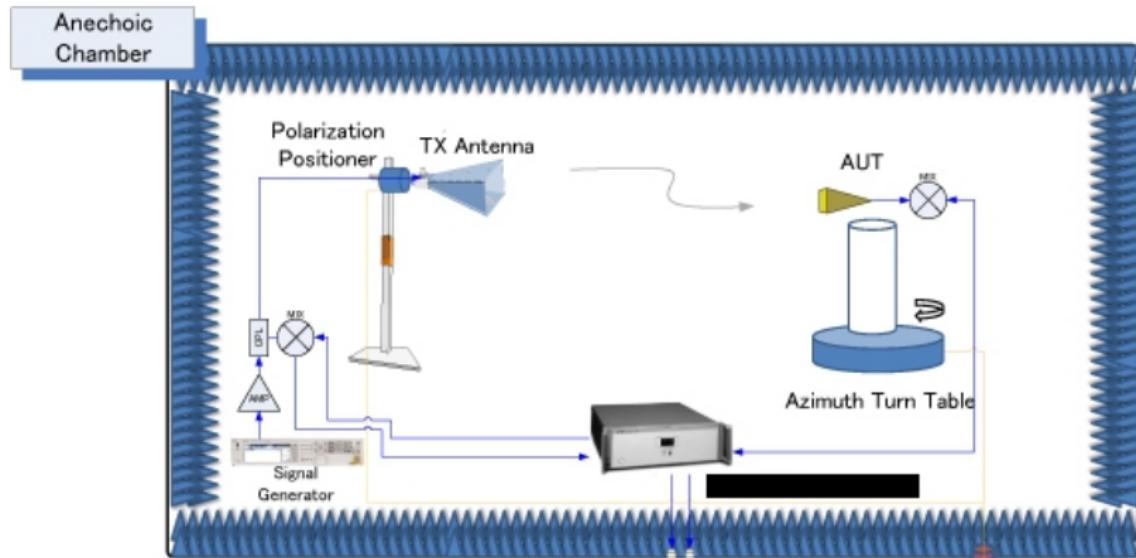
## Hardware Specific gr-doa/apps

- `estimate_X310_TwinRX_constant_phase_offsets_and_save.grc`: TwinRX phase correction
- `run_MUSIC_calib_lin_array_X310_TwinRX.grc`: TwinRX antenna calibration
- Run estimation flowgraphs
  - `run_MUSIC_lin_array_X310_TwinRX.grc`: MUSIC
  - `run_RootMUSIC_lin_array_X310_TwinRX.grc`: Root-MUSIC

## Octave Simulations and QA gr-doa/examples

- Algorithmic examples for MUSIC, Root-MUSIC, and calibrations

# DOA Research: Effective Aperture Distribution Function



## Effective Aperture Distribution Function (EADF)

- Alternative and compact representation of antenna angular response
- Applicable to generic array geometrics
- Computationally convenient form (CRLB,DoA)

# EADF Antenna Model

Antenna response matrices and EADF representation

$$\mathbf{B}_p = \begin{bmatrix} \mathbf{B} \\ -\mathbf{B} \end{bmatrix}, \mathbf{G} = FFT_{2D}(\mathbf{B}_p)$$

$\mathbf{G}^{N \times M_A M_E}$ ,  $M_A$  and  $M_E$  are the number of azimuth and elevations modes

Mapping vectors are:

$$\mathbf{d}(\theta) = \left[ \exp(-j\theta(M_A - 1)/2), \dots, \exp(j\theta(M_A - 1)/2) \right]$$

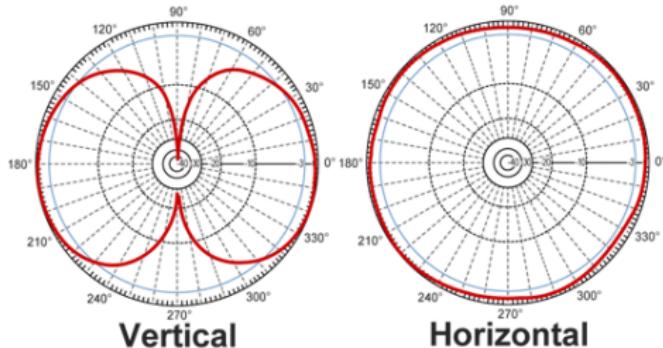
$$\mathbf{d}(\phi) = \left[ \exp(-j\phi(M_E - 1)/2), \dots, \exp(j\phi(M_E - 1)/2) \right]$$

$$\mathbf{d}(\theta, \phi) = \mathbf{d}(\theta) \otimes \mathbf{d}(\phi)$$

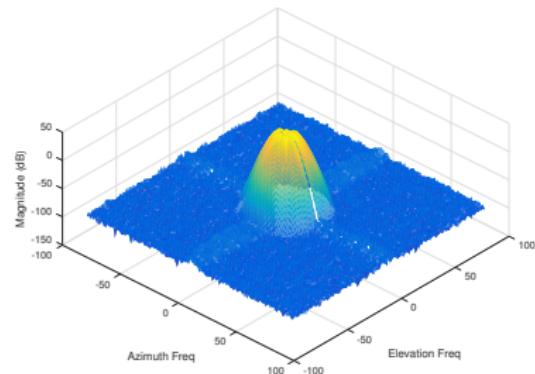
Polarimetric array response:

$$\mathbf{C}(\theta, \phi) = \begin{bmatrix} \mathbf{G}_H \mathbf{d}(\theta, \phi) \\ \mathbf{G}_V \mathbf{d}(\theta, \phi) \end{bmatrix}$$

# EADF Synthetic Model



[LComm Inc.]



## EADF Realization

- Monopoles well estimated in azimuth
- Synthetic model as replacement for  $\mathbf{B}_p$

# EADF-FFT DoA (EFD)

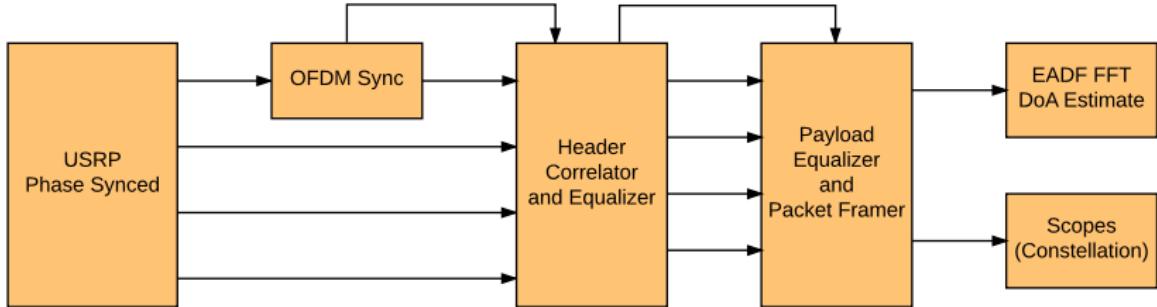
The channel between the transmitting node and receiving array in terms of the polarimetric response is:

$$\mathbf{h} = \mathbf{C}(\theta, \phi) \boldsymbol{\gamma} + \mathbf{n}$$

## DoA Algorithm

1. Perform channel estimate:  $\hat{\mathbf{h}}$
2. Correlate:  $\mathbf{A}_H = \hat{\mathbf{h}}\mathbf{G}_H^*$ ,  $\mathbf{A}_V = \hat{\mathbf{h}}\mathbf{G}_V^*$
3. Convert coordinates:  $\mathbf{B}_H = |FFT_{3D}(\mathbf{A}_H)|^2$ ,  $\mathbf{B}_V = |FFT_{3D}(\mathbf{A}_V)|^2$
4. Search Max:  $[\theta, \phi]^T = argmax(\mathbf{B}_H + \mathbf{B}_V)$

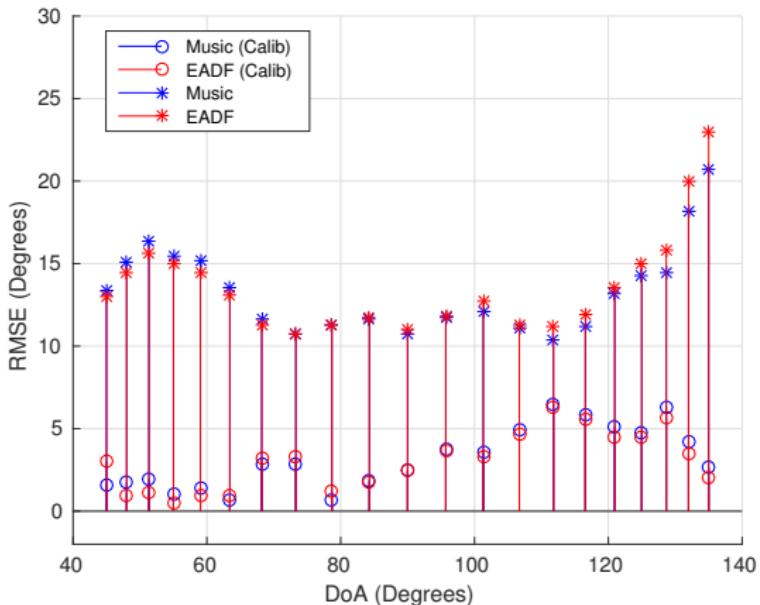
# Channel Estimation Platform



## OFDM System

- Parameterized system (2 Arrays at 5 MHz each with TUT Config)
- N supported coherent receive channels (4x per radio)
- One directional link
- Successful packets (Header and CRC Check) produce channel estimates
- Verified through simulation and cabling

# EFD vs. MUSIC



## Results

- Measurements taken in lab environment
- 1000 packets collected minimal variance among measurements
- Adopted correction algorithms from MUSIC implementation

# Questions?

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**Repo:** <https://github.com/EttusResearch/gr-doa>

**Whitepaper:** [https://github.com/EttusResearch/gr-doa/blob/master/docs/whitepaper/doa\\_whitepaper.pdf](https://github.com/EttusResearch/gr-doa/blob/master/docs/whitepaper/doa_whitepaper.pdf)

**GitHub:** [@tfcollins](#)

# References I

## Images

<http://www.tut.fi/5G/positioning/>

[http://www.toyo.co.jp/files/user/img/english/images/  
Antenna\\_Receiver.jpg](http://www.toyo.co.jp/files/user/img/english/images/Antenna_Receiver.jpg)

[https:](https://www.ettus.com/content/images/X300_w_twinRX_Large.png)

[//www.ettus.com/content/images/X300\\_w\\_twinRX\\_Large.png](https://www.ettus.com/content/images/X300_w_twinRX_Large.png)

[http://blogs-images.forbes.com/scottdavis/files/2016/01/  
IoT1-1200x900.jpg](http://blogs-images.forbes.com/scottdavis/files/2016/01/IoT1-1200x900.jpg)

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Get the source of this theme and the demo presentation from

[github.com/matze/mtheme](https://github.com/matze/mtheme)

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