

Algebraic Topology for the Physical Layer

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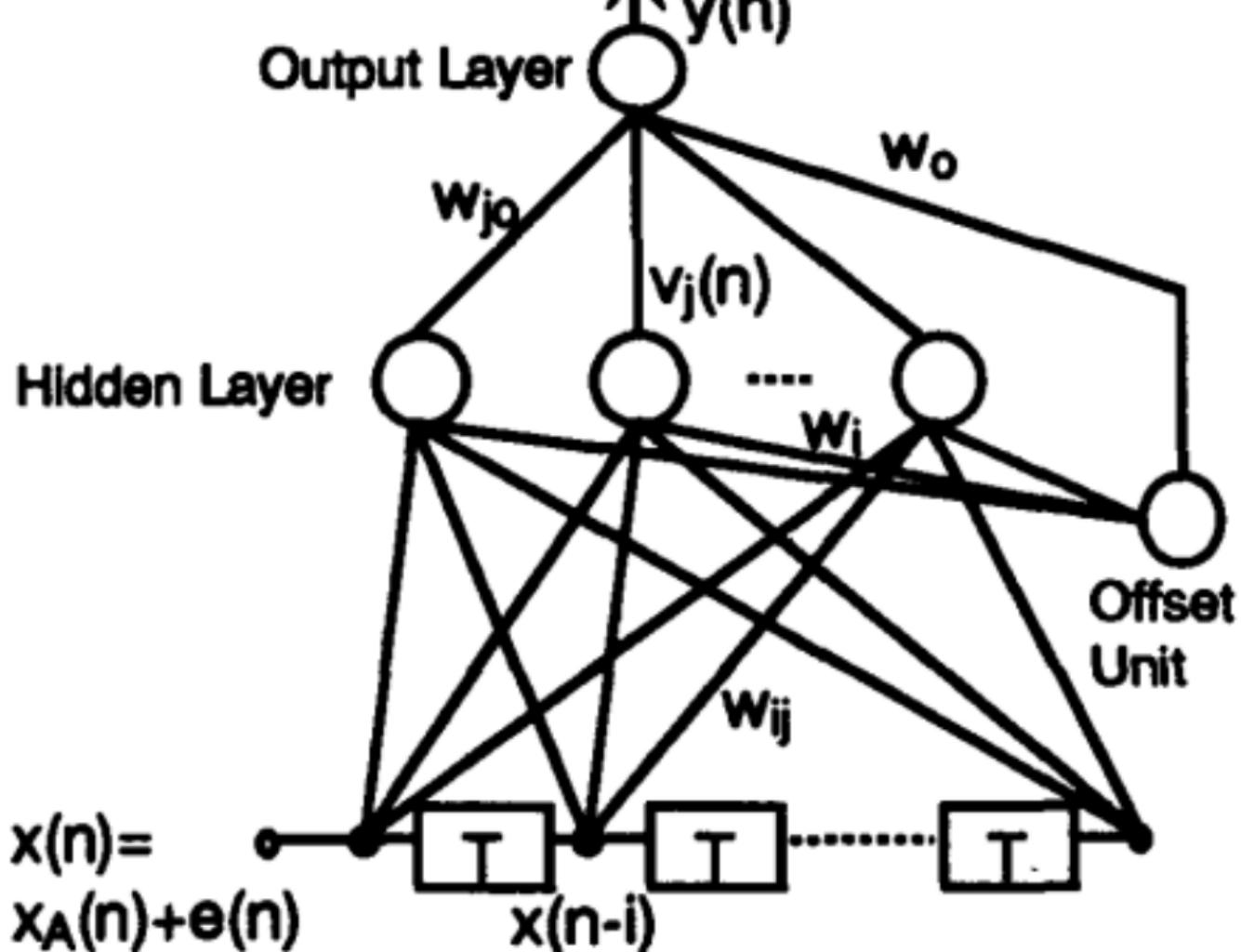
Motivation

The rise of automated methods for synthesizing communication signals may require new tools for analysis in order to keep pace. Algebraic topology offers insights and tools that aid in automatic signal processing and complement traditional methods as well as newer methods such as, deep learning.

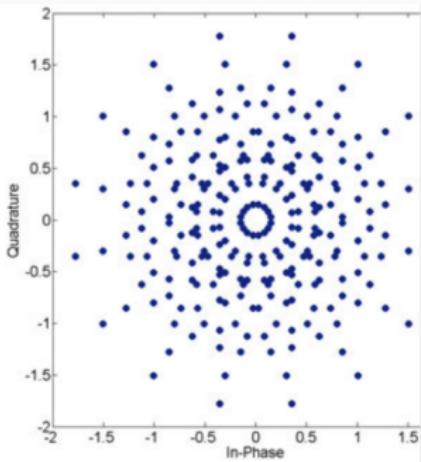
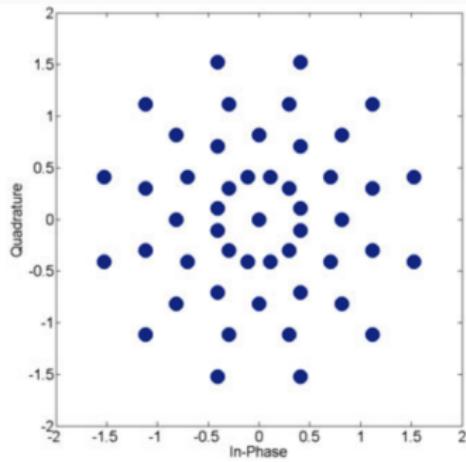
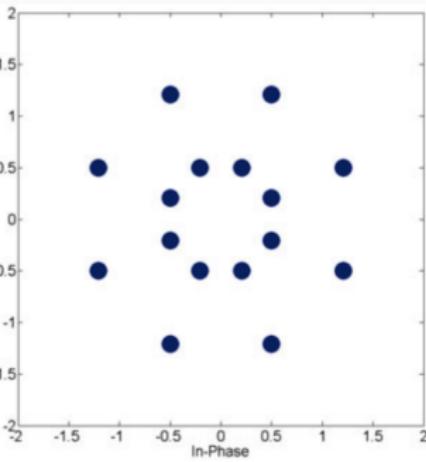
Outline

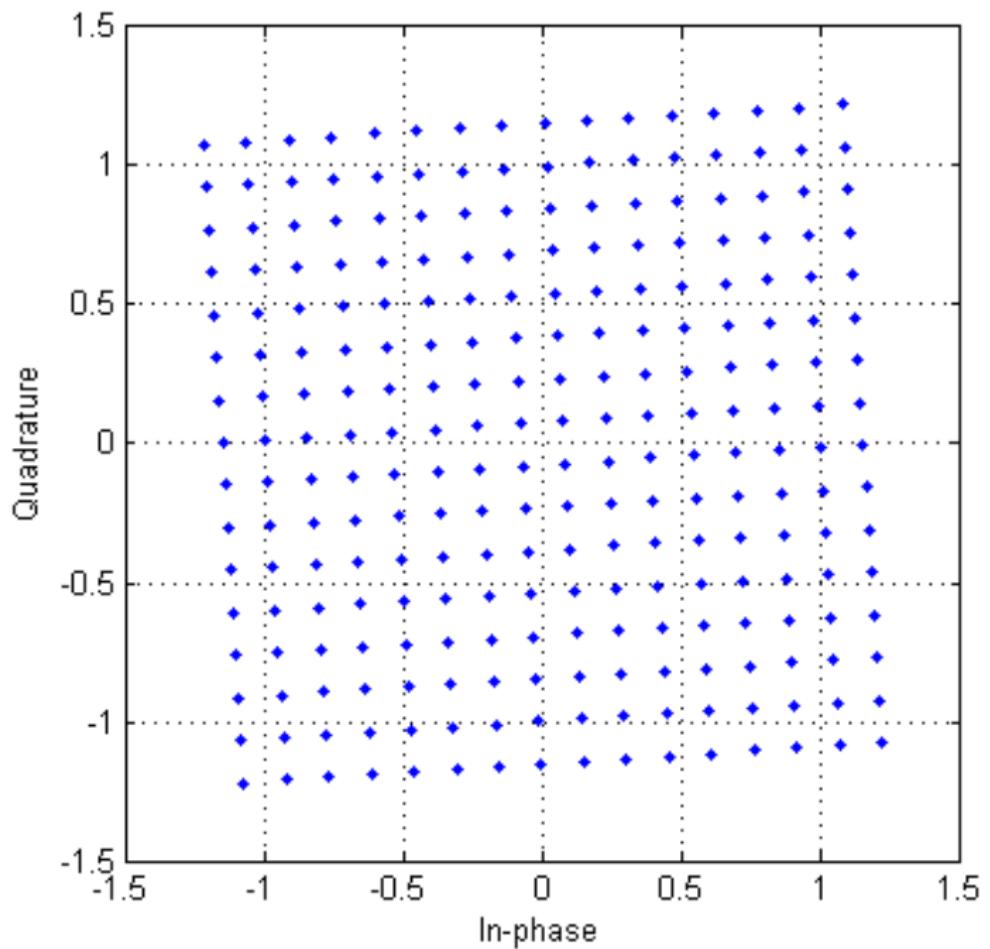
Share how algebraic topology could be used for blind OFDM synchronization and demodulation as well as automatic modulation surveying. Show why unsupervised methods for analyzing physical layers protocols may be required as parts of the PHY-layer become automatically created using machine learning.

Motivation

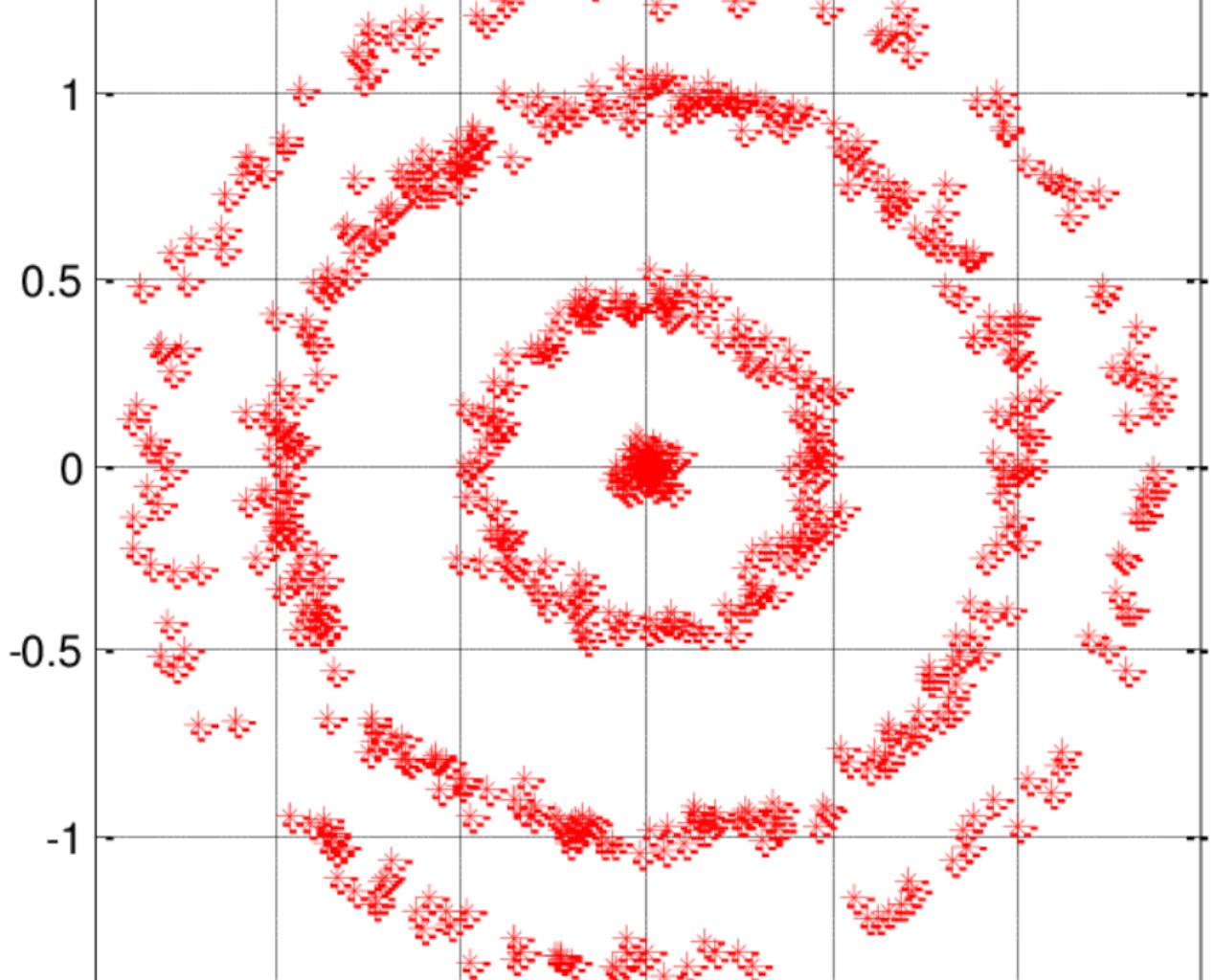


Challenges

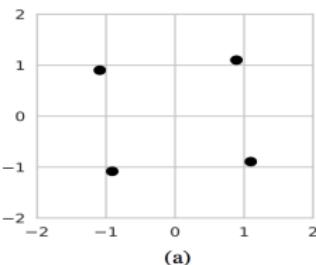




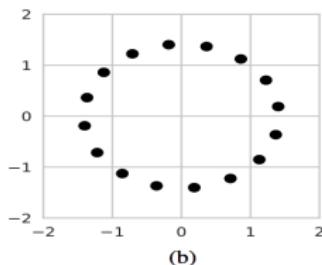
Quadrature



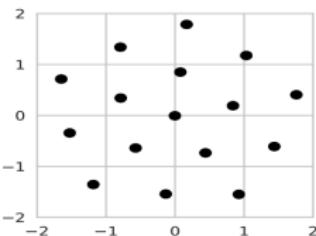
Learning New Physical Layer Waveforms with Autoencoders



(a)

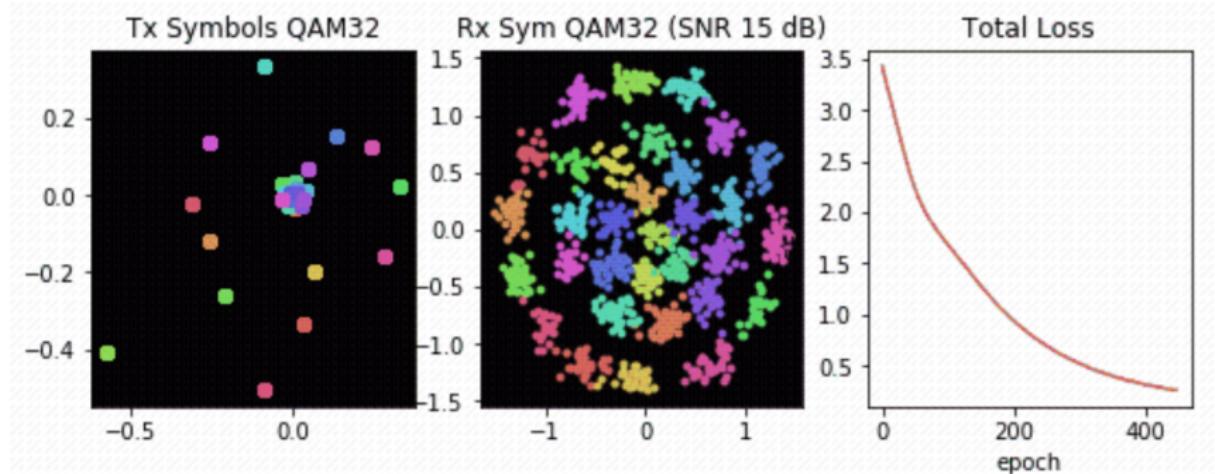


(b)



```
input_signal = Input(shape=(M, ))  
  
transmitter_0 = Dense(M, activation='relu')(input_signal)  
transmitter_1 = Dense(N, activation='linear')(transmitter_0)  
  
channel = NoiseModel(K)(transmitter_2)  
  
receiver_0 = Dense(M, activation='relu')(channel)  
receiver_1 = Dense(M, activation='softmax')(receiver_0)  
  
autoencoder = Model(input_signal, receiver_1)  
autoencoder.compile(loss='categorical_crossentropy')
```

Automatically Learned PHY-layer for NASA's TDRS by DeepSig

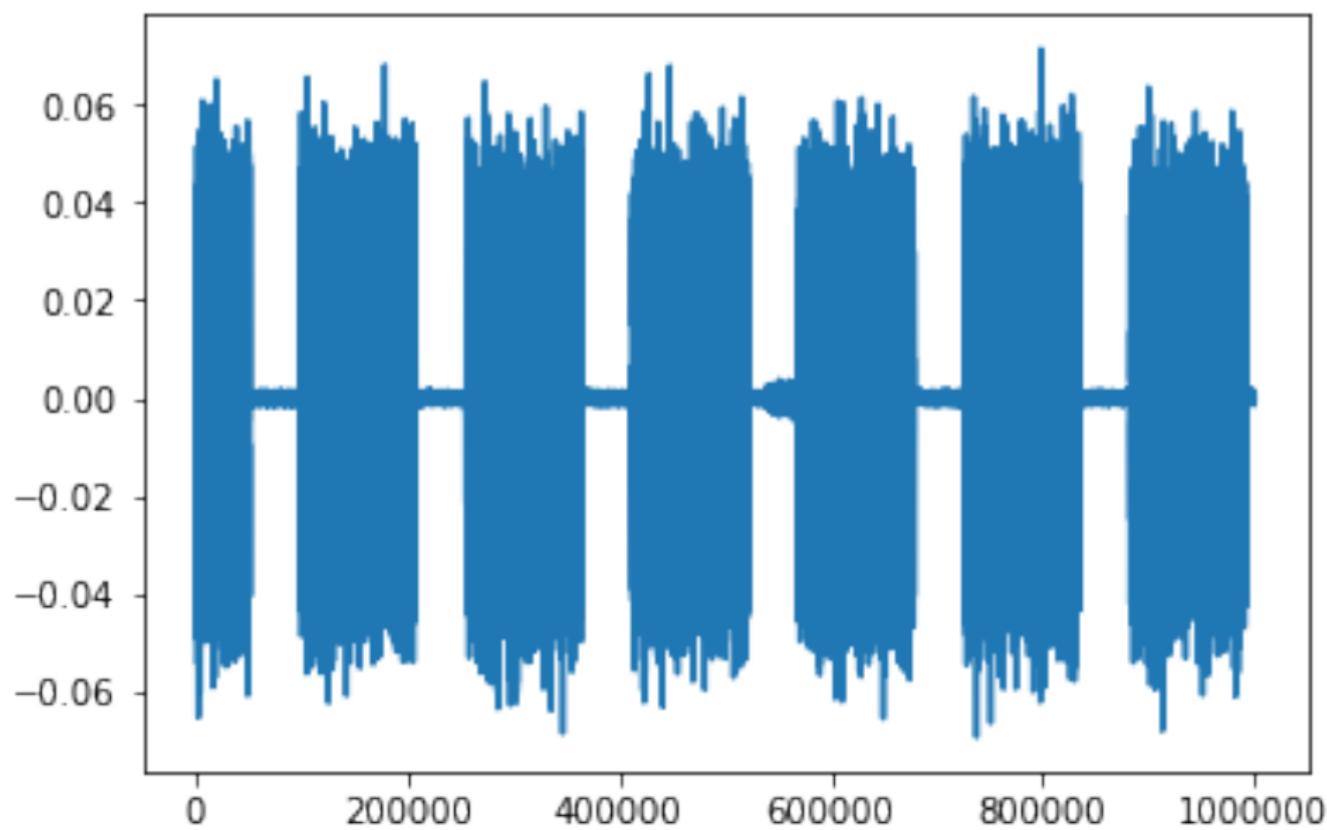


Non-cooperative Synchronization and Demodulation for OFDM

Create a computer program to automatically demodulate preamble waveforms without making too many assumptions and indirectly creating a DSP expert system.

Assumptions

- Sampling Rate
- Use of N-point FFT/IFFT-based modulation



Some Synchronization Methods from Least to Most Assumptions

- Energy Detection
- Delayed Conjugate Multiply
- Standard Deviation of fixed-sized FFT
- Cyclostationarity
- Schmidl-Cox
- Cross Correlation

Some Challenges in Non-cooperative OFDM Synchronization

- Unknown data
- Unknown preamble structure
- Unknown subcarrier modulation
- Unknown symbol to bit mapping
- Unknown OFDM parameters
- Unknown Channel

A Possible Paradox in Blind Reverse Engineering

Need to clean up the signal to recover parameters for demodulation, but to remove noise we did to know the structure of the signal.

Topology Background

The Infinite vs. The Finite

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

$$R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

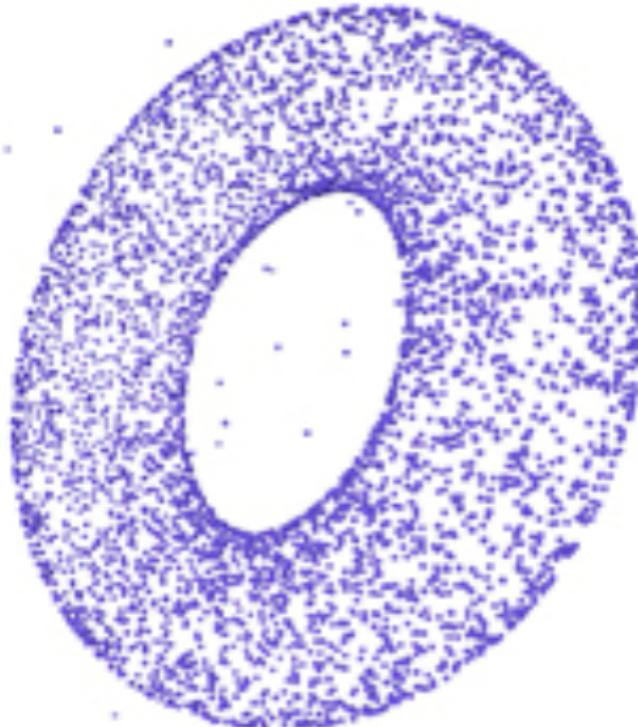
$$R = 1$$

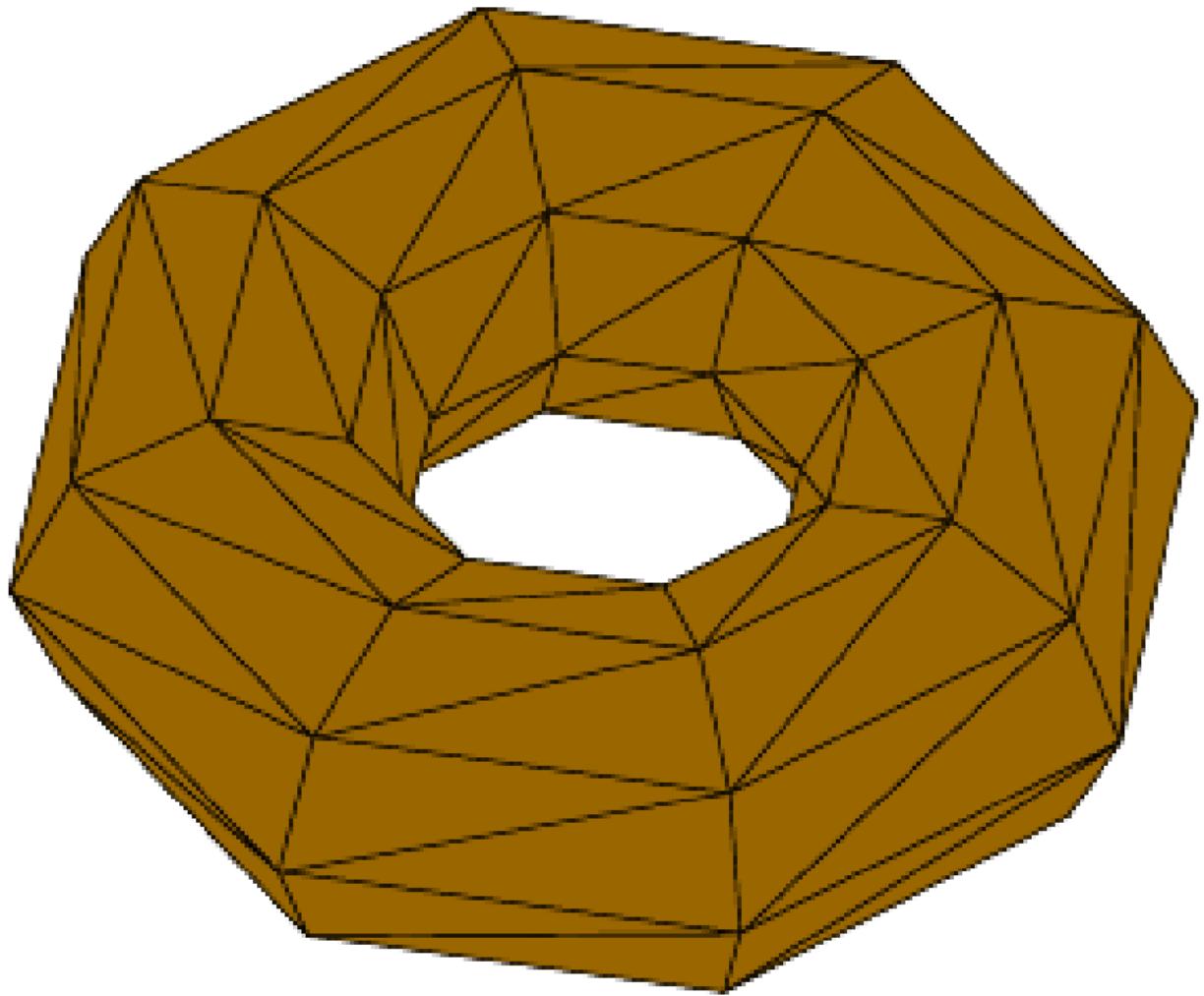
$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \dots$$

$$R = 1$$





Simplices

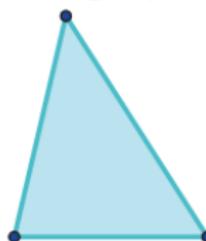
0-simplex
(vertex)



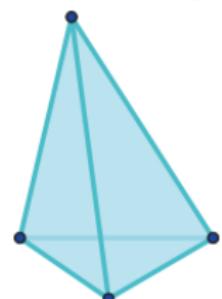
1-simplex
(edge)



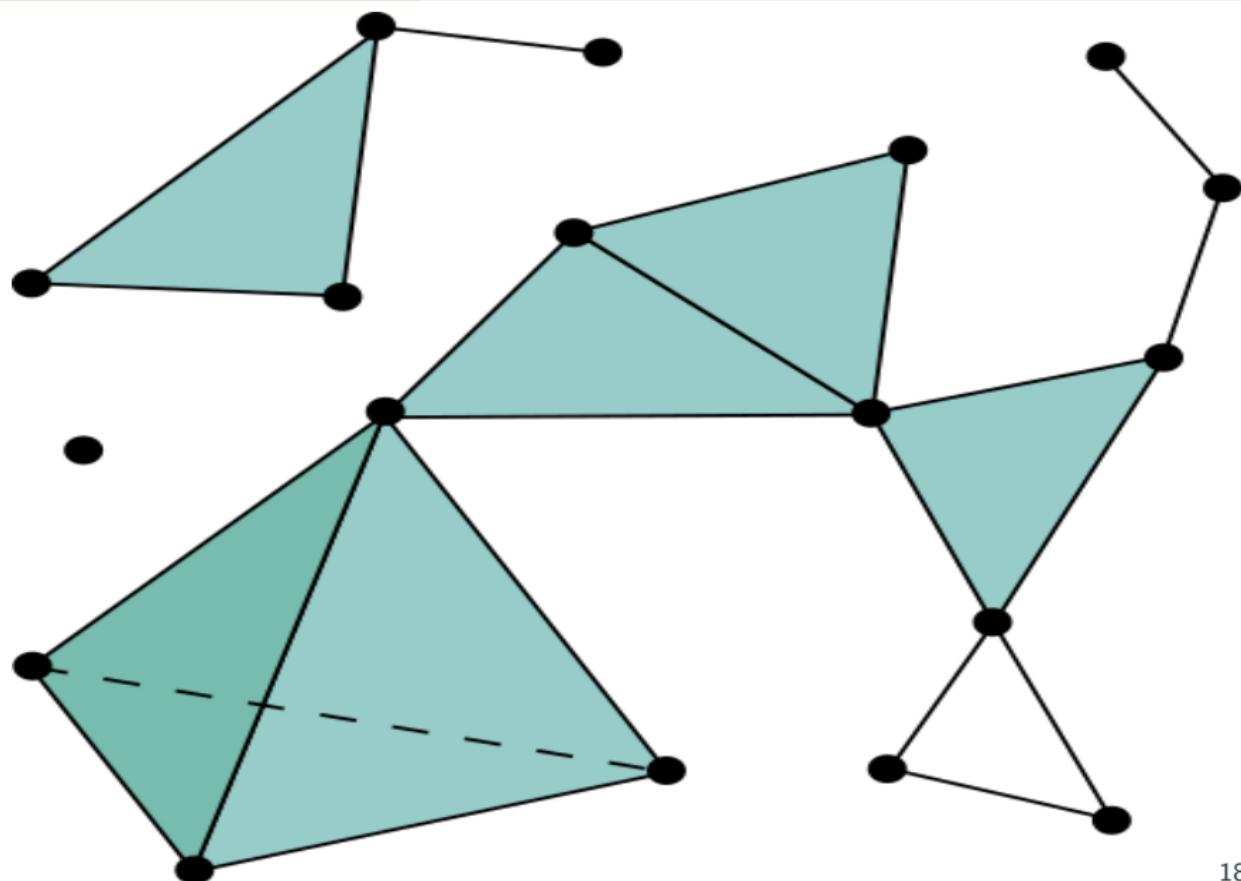
2-simplex
(triangle)

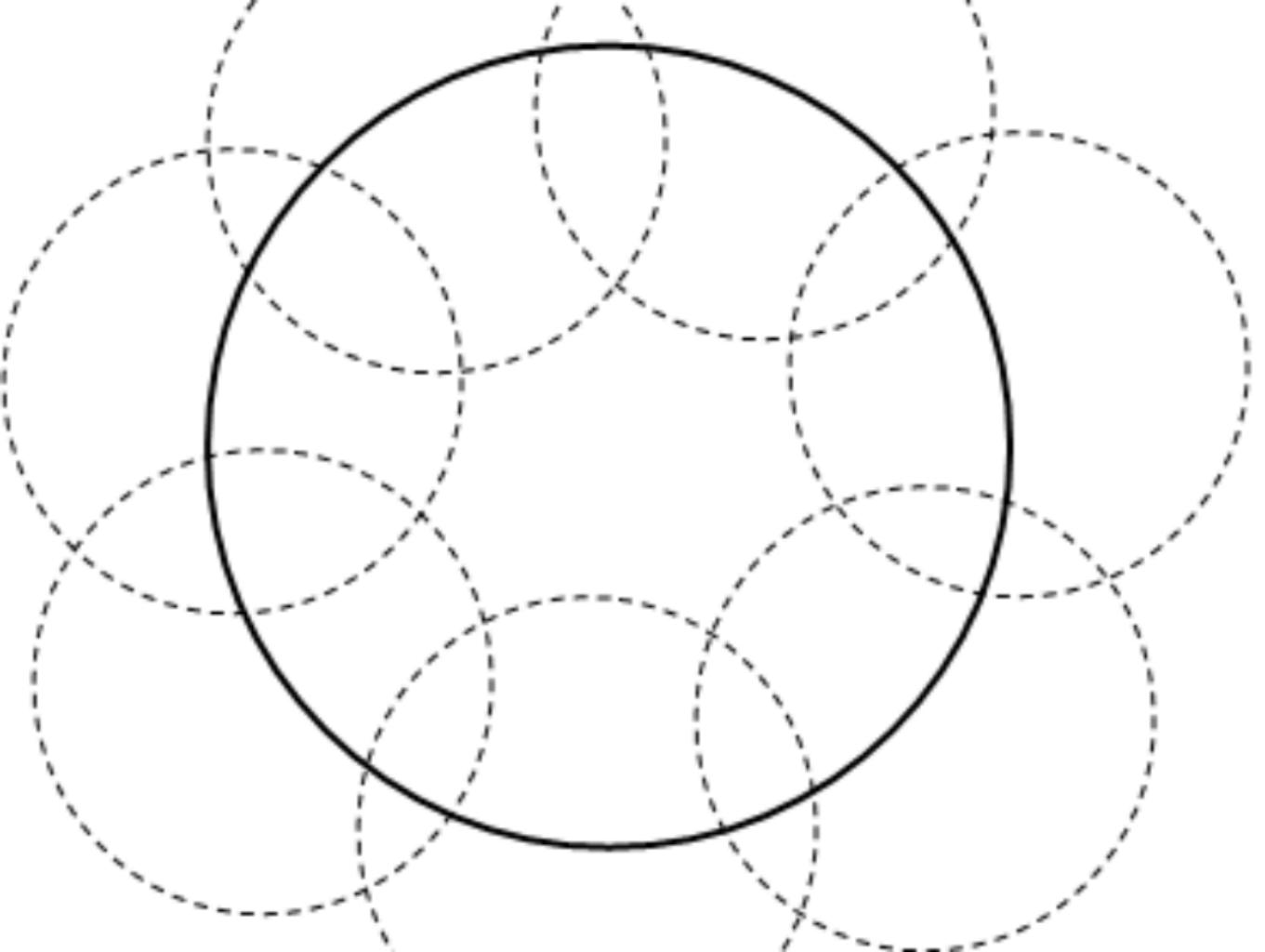


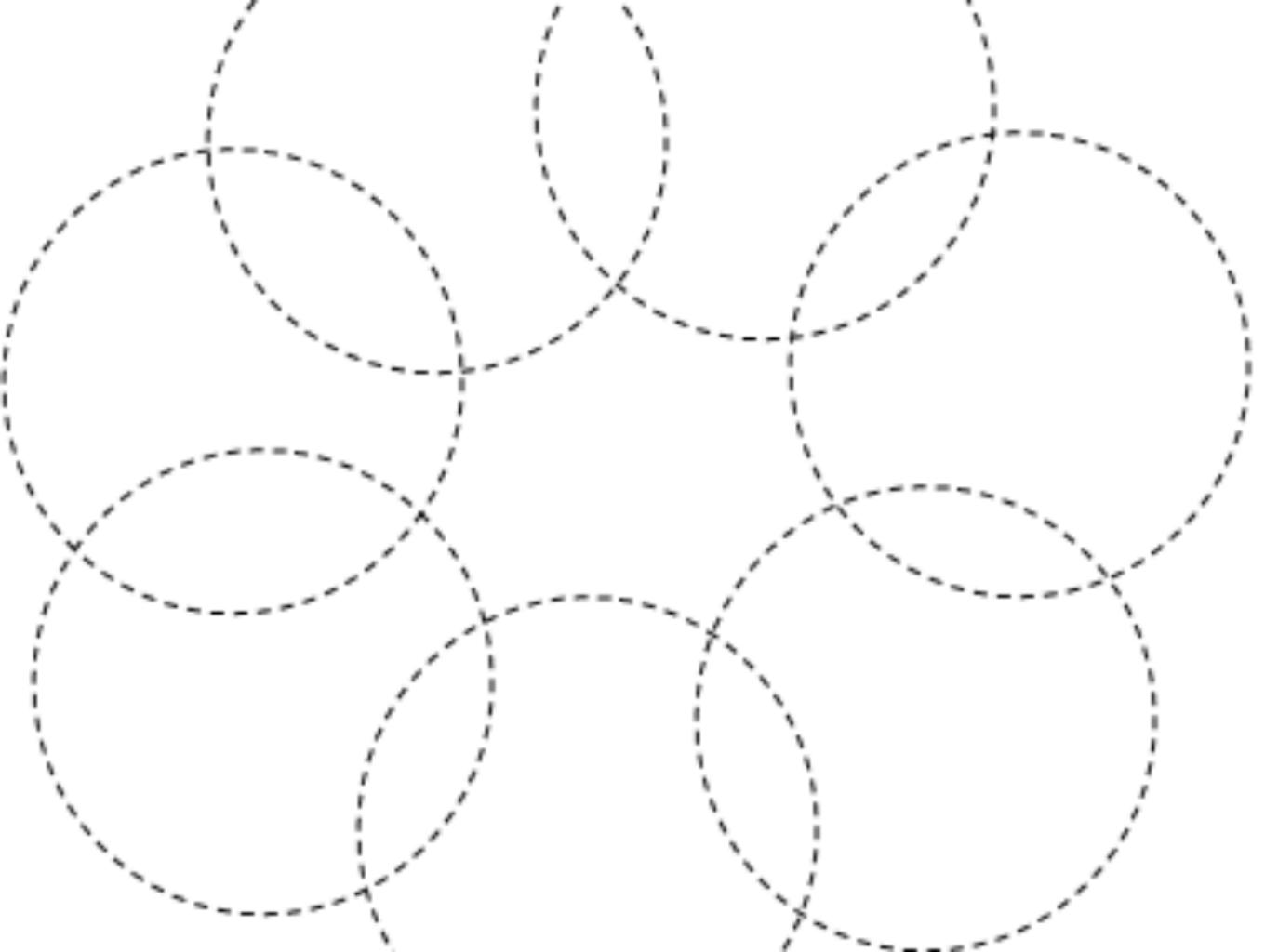
3-simplex
(tetrahedron)

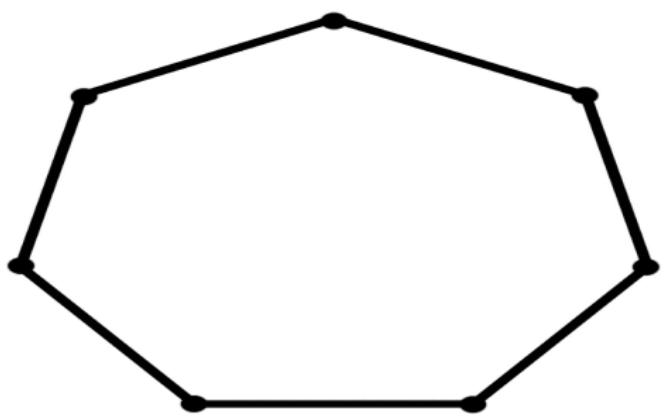


Simplicial Complex





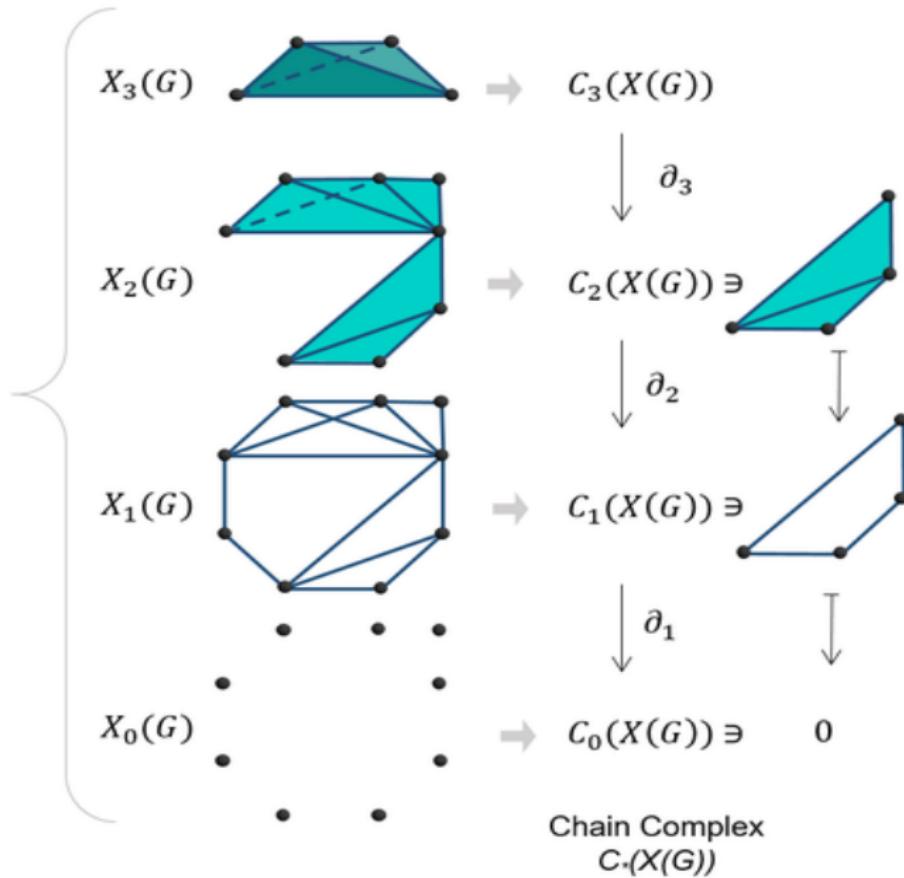




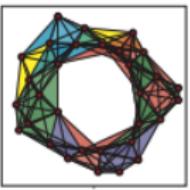
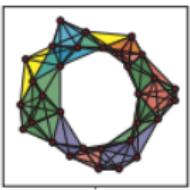
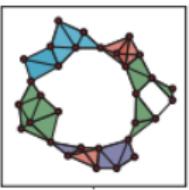
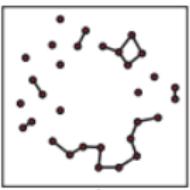
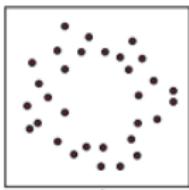
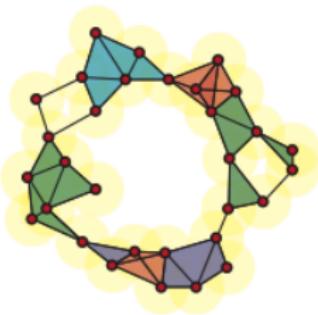
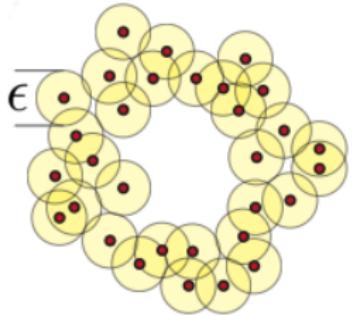
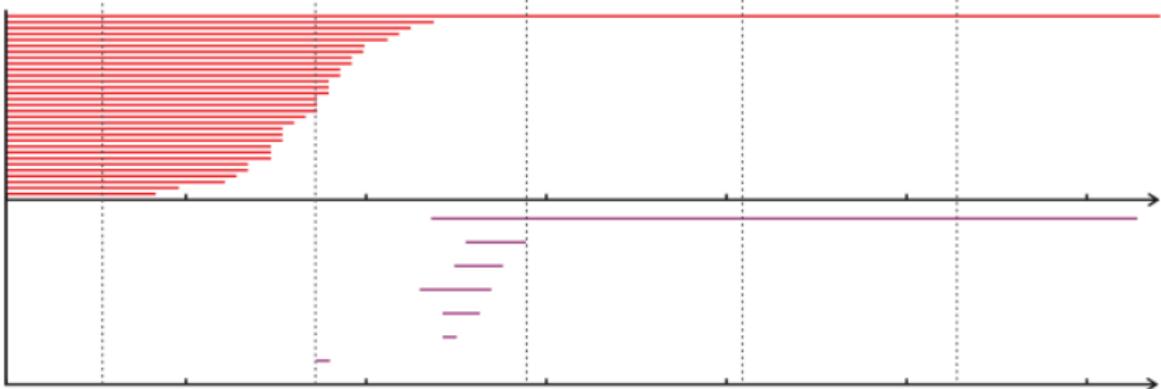
Chain Group and Boundary Homomorphism



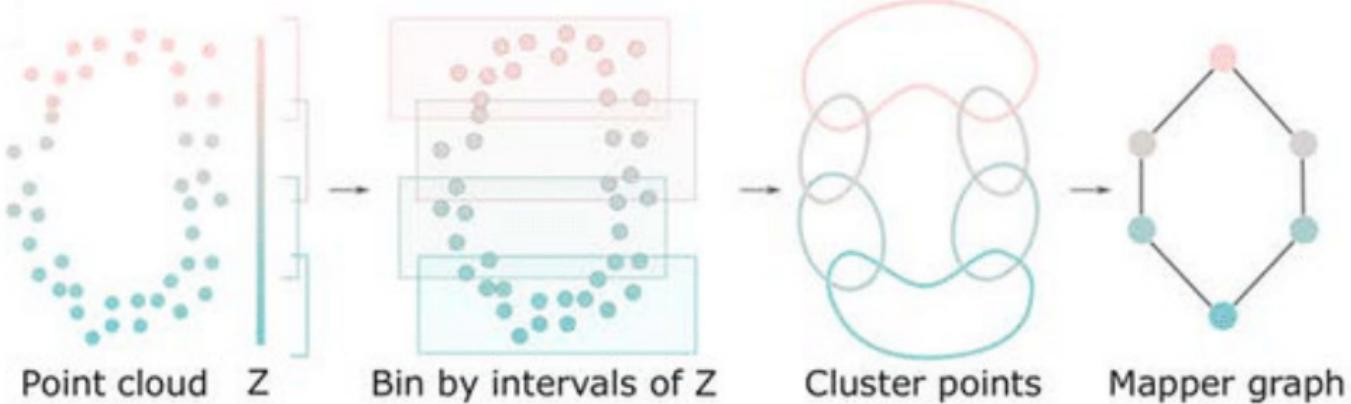
Simplicial
Complex



From Data to Simplicial Complex

 H_0 H_1 

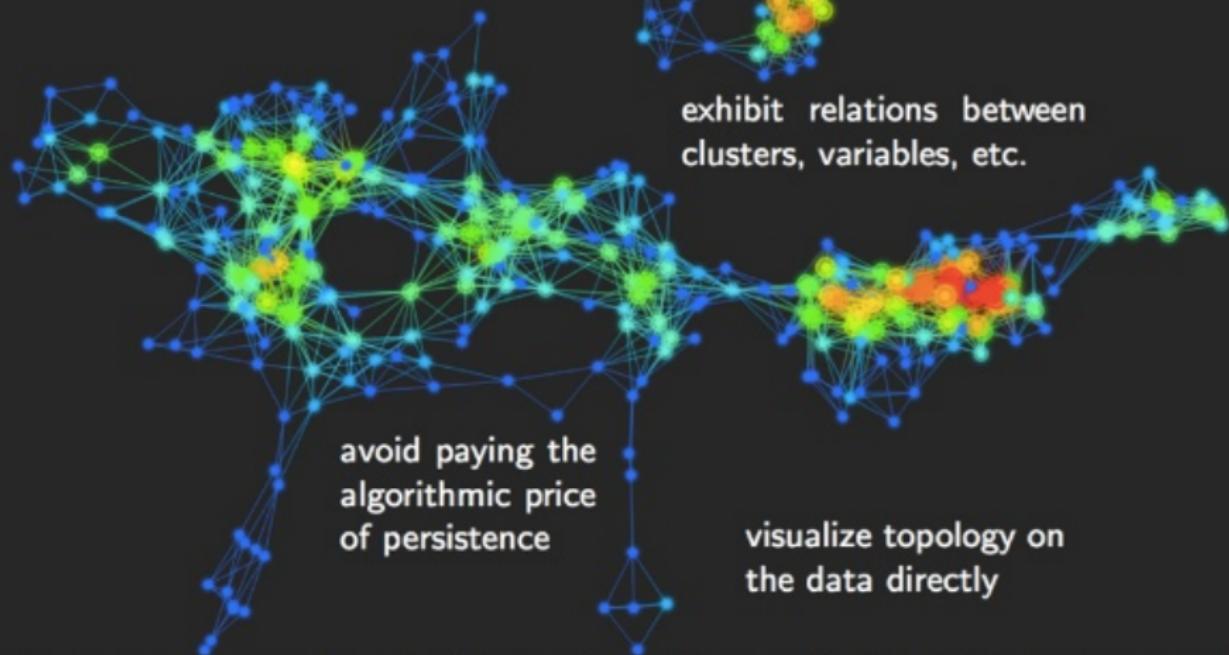
Mapper



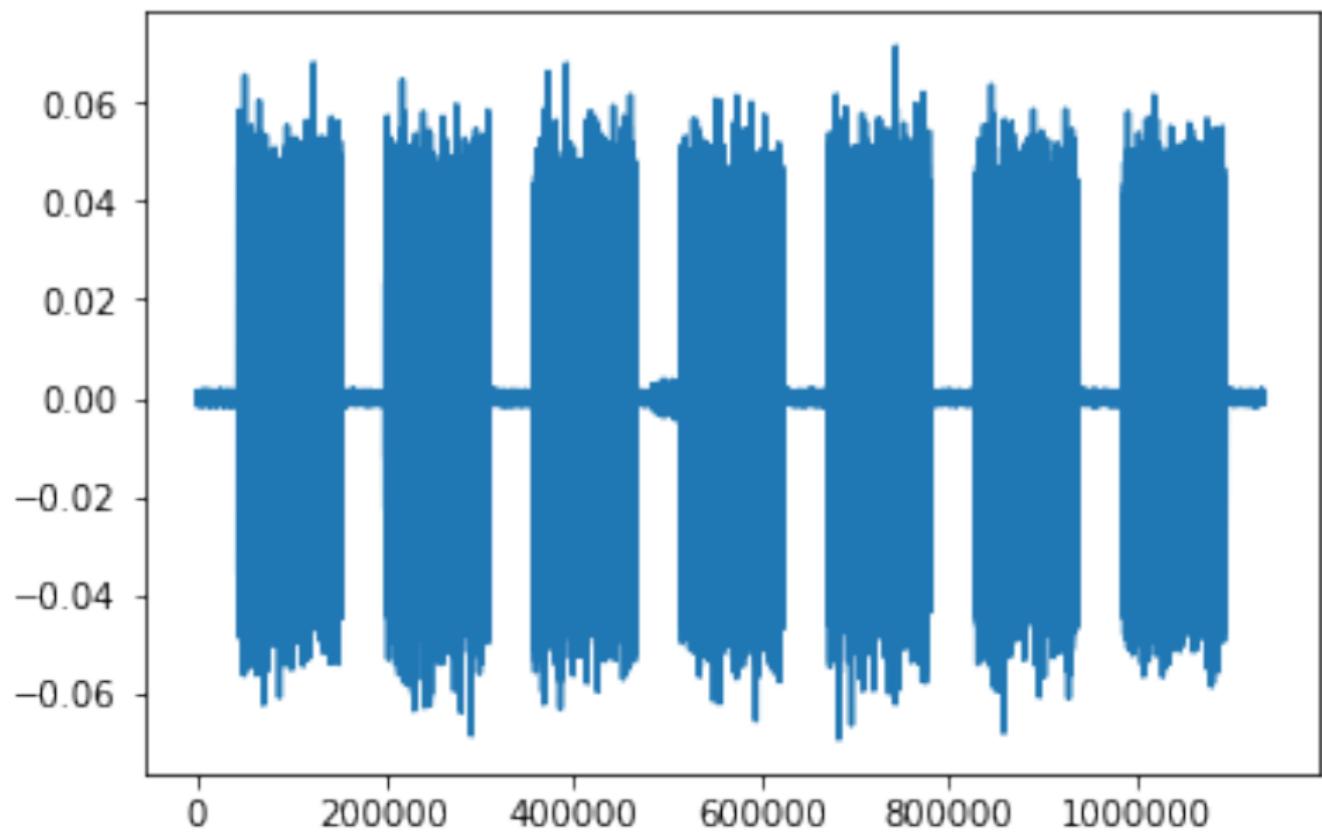
Motivations

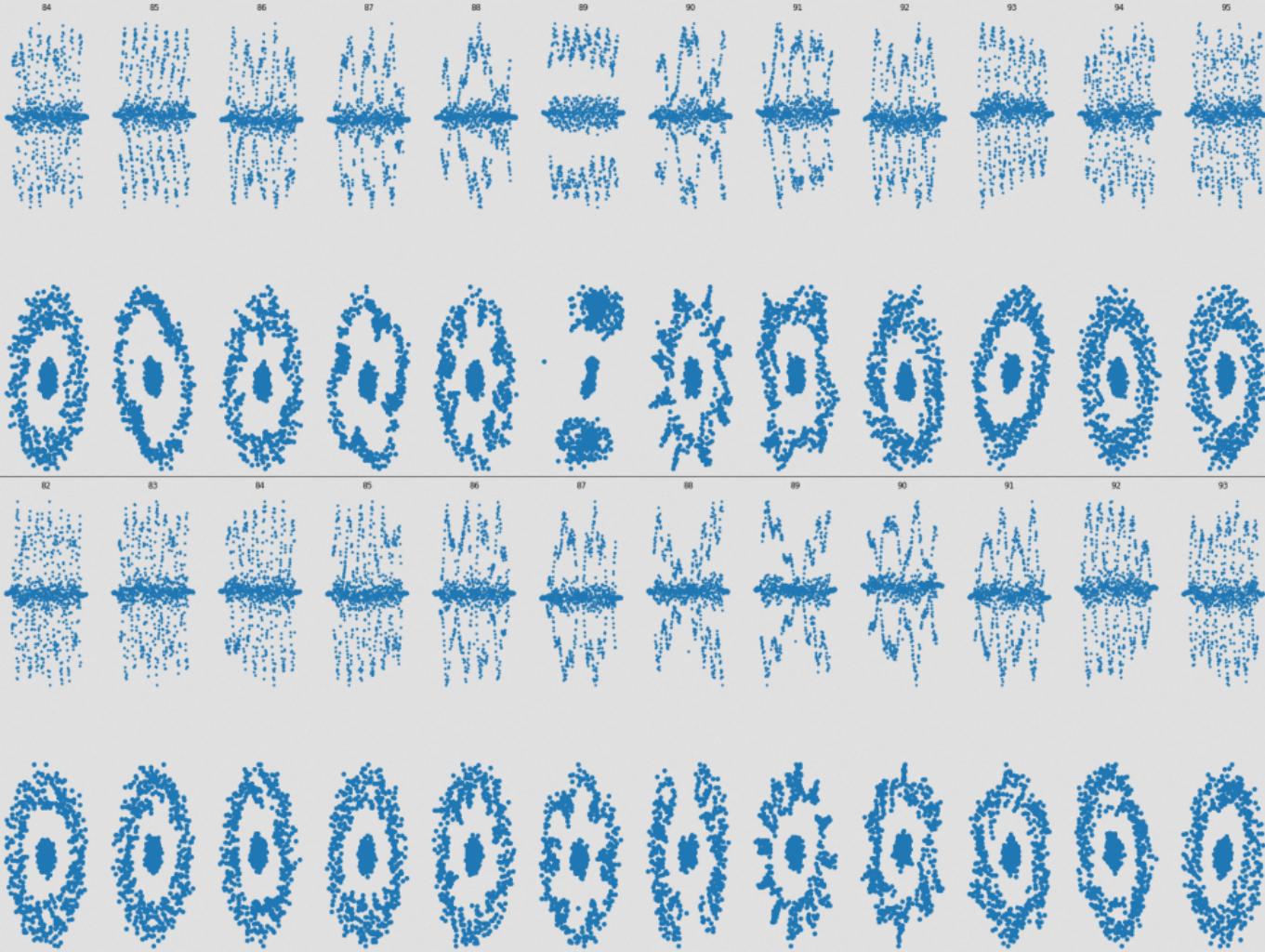
get a higher-level understanding of the structure of data

Mapper Algorithm

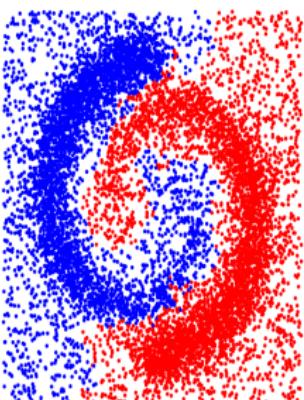
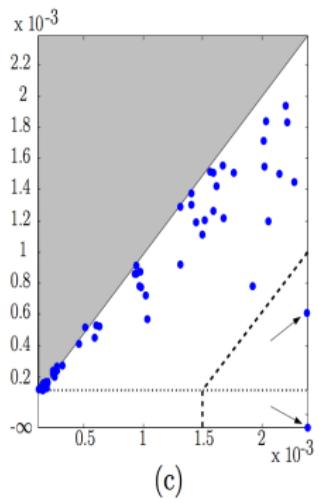
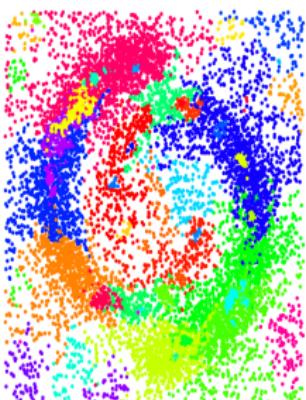
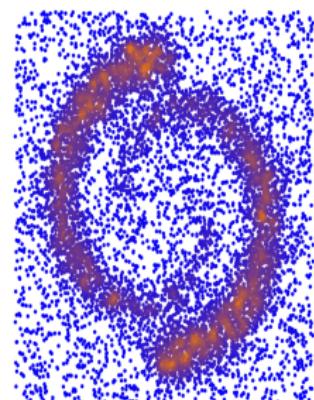
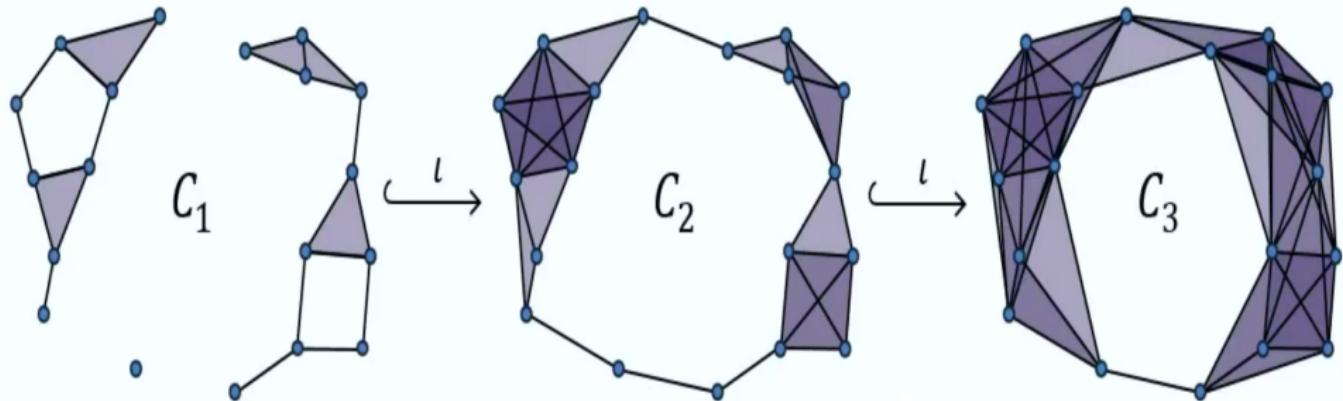


principle: summarize the topological structure of a map $f : X \rightarrow \mathbb{R}$ through a graph





Filtration:

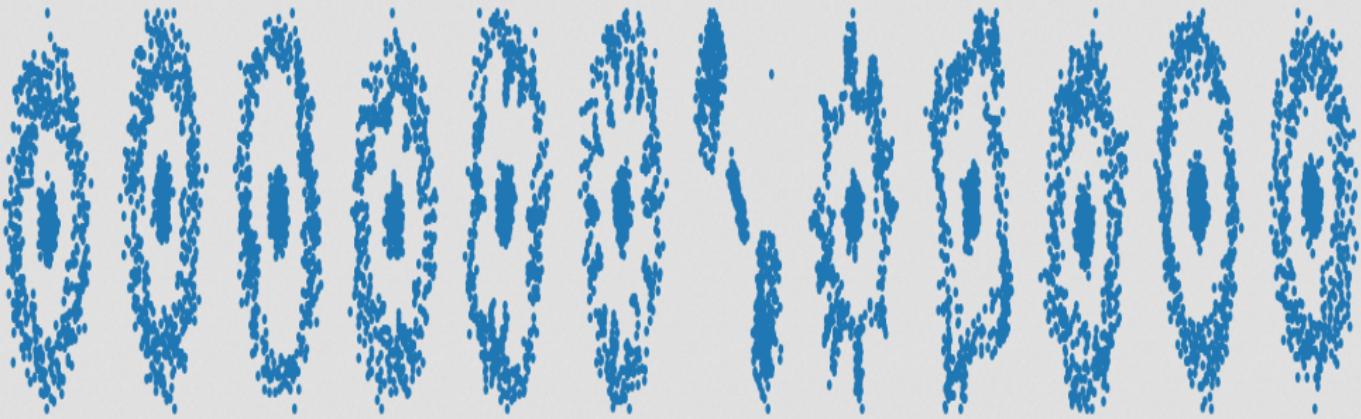
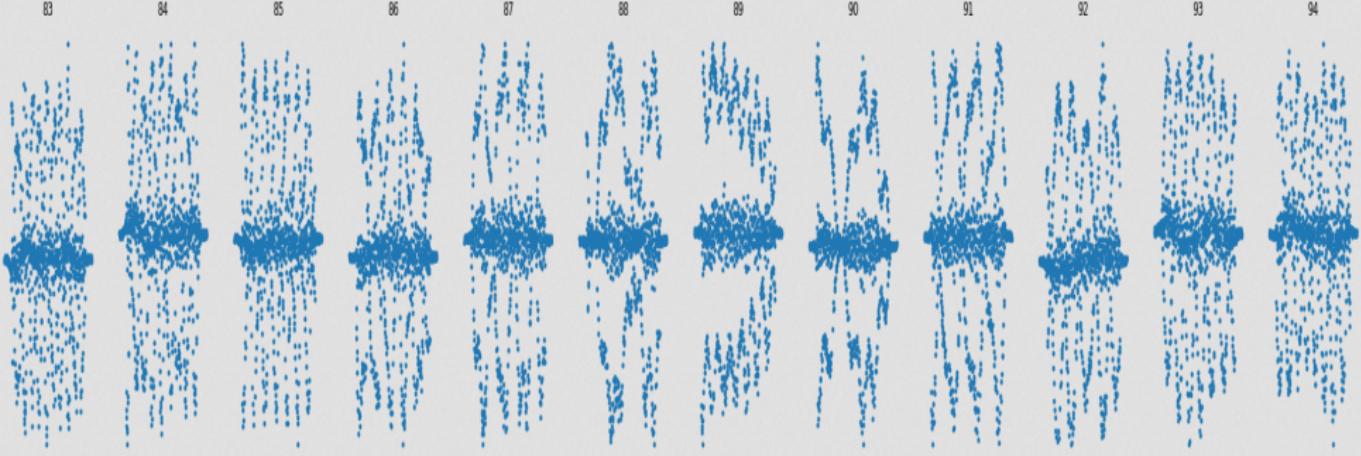


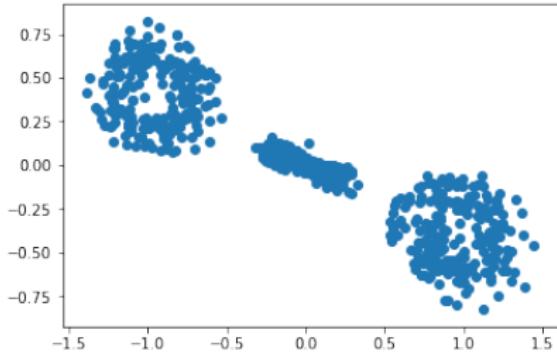
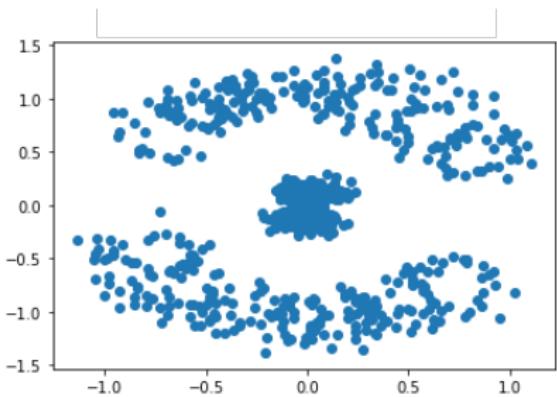
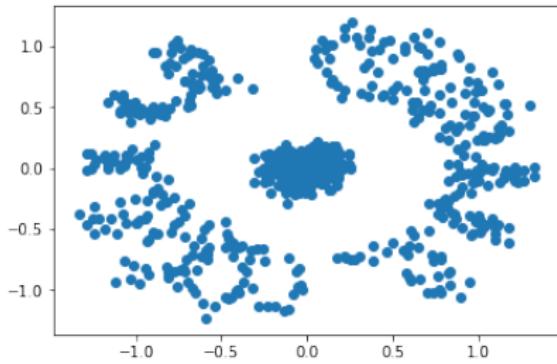
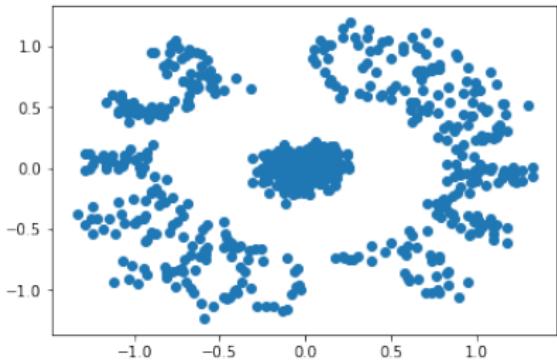
(a)

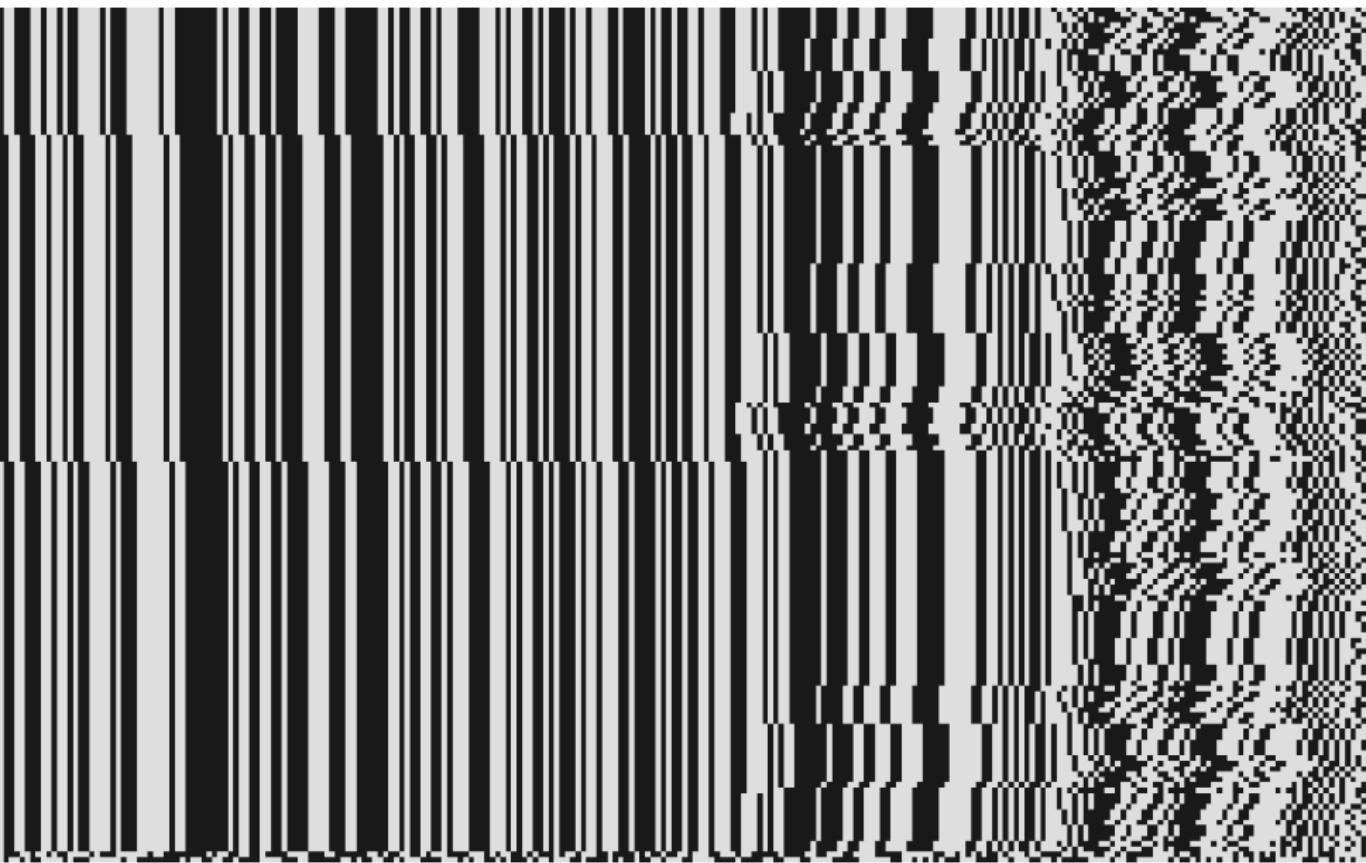
(b)

(c)

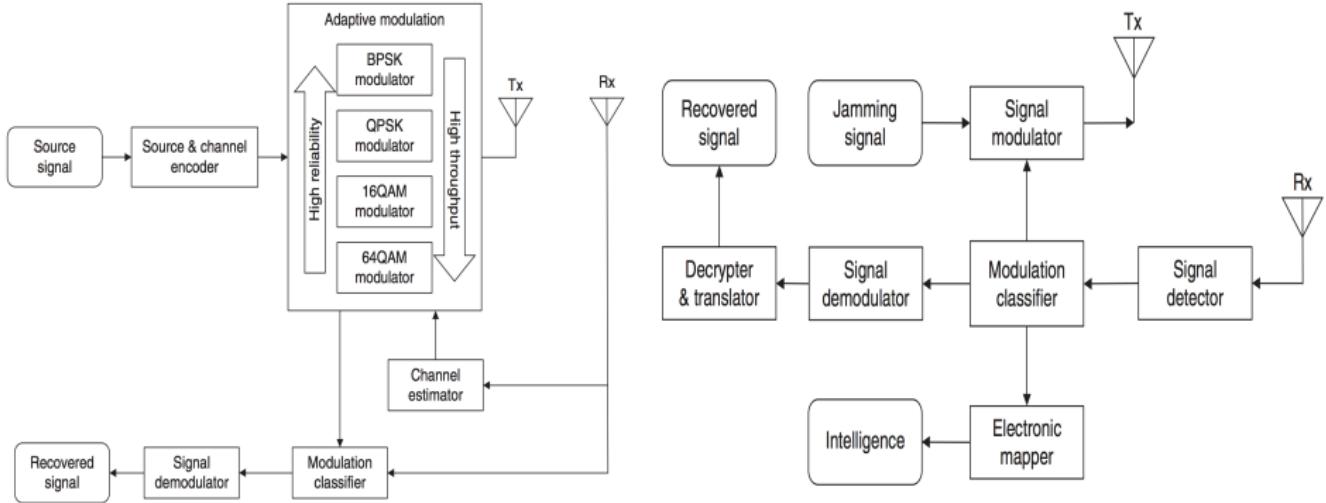
(d)

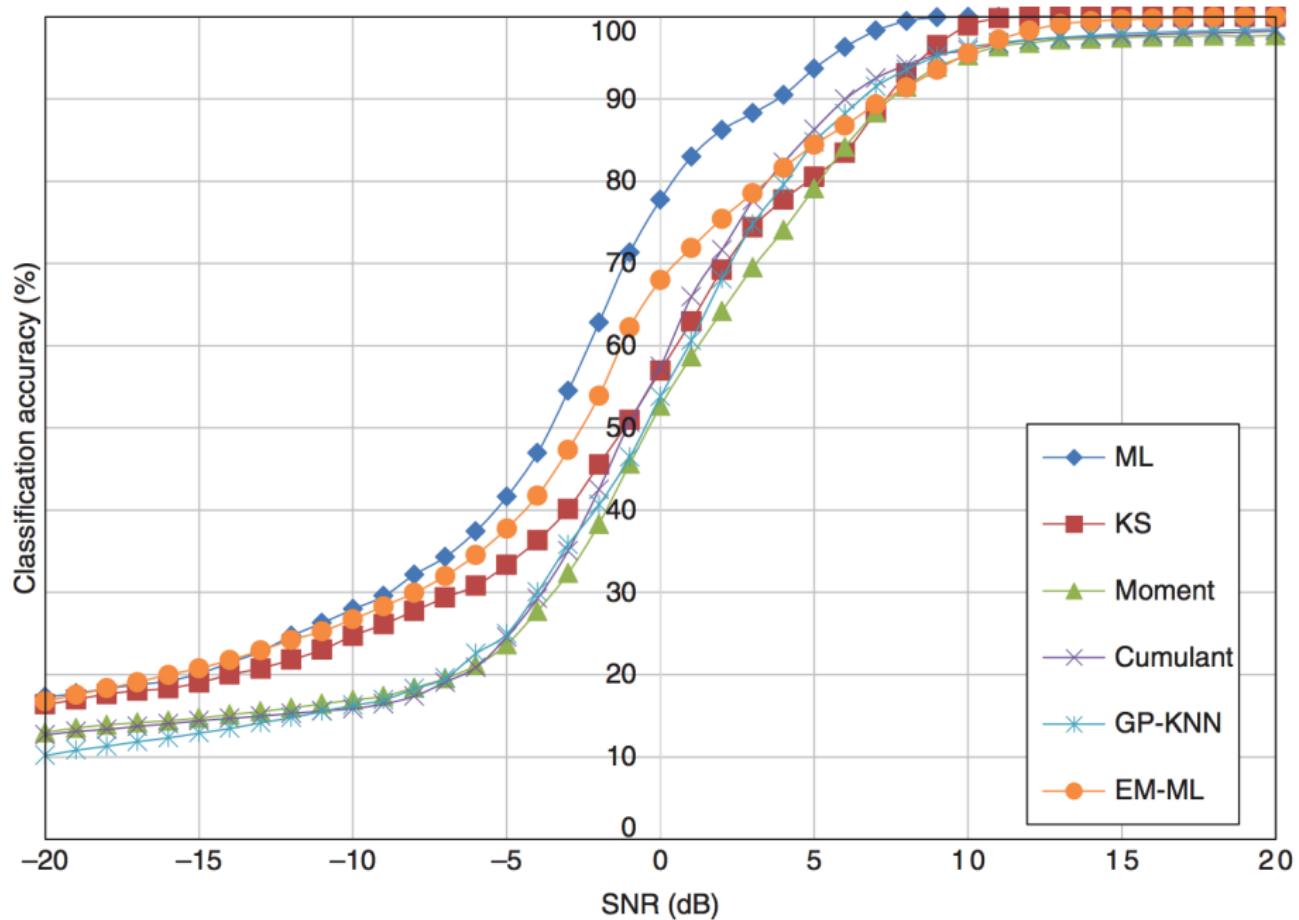






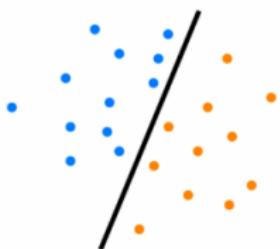
Automatic Modulation Classification from Nandi and Zhu



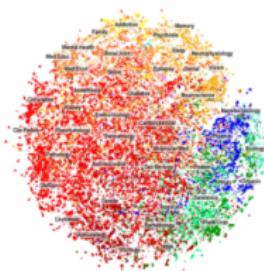


Unsupervised Machine Learning

Supervised



Unsupervised

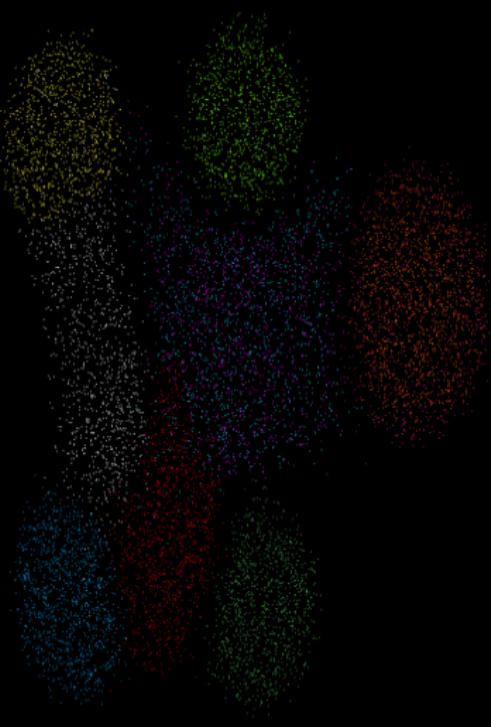


Reinforcement

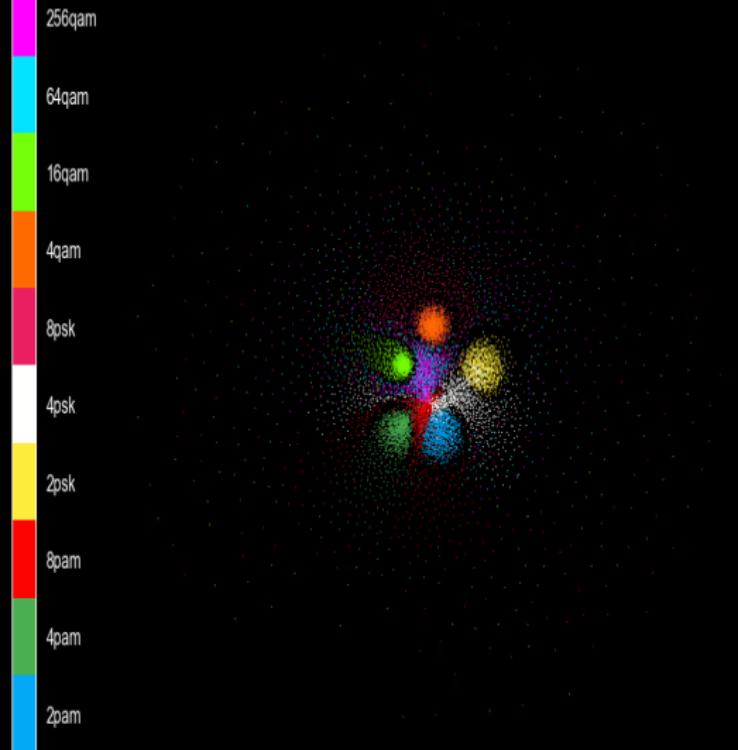


Results with and without Topology

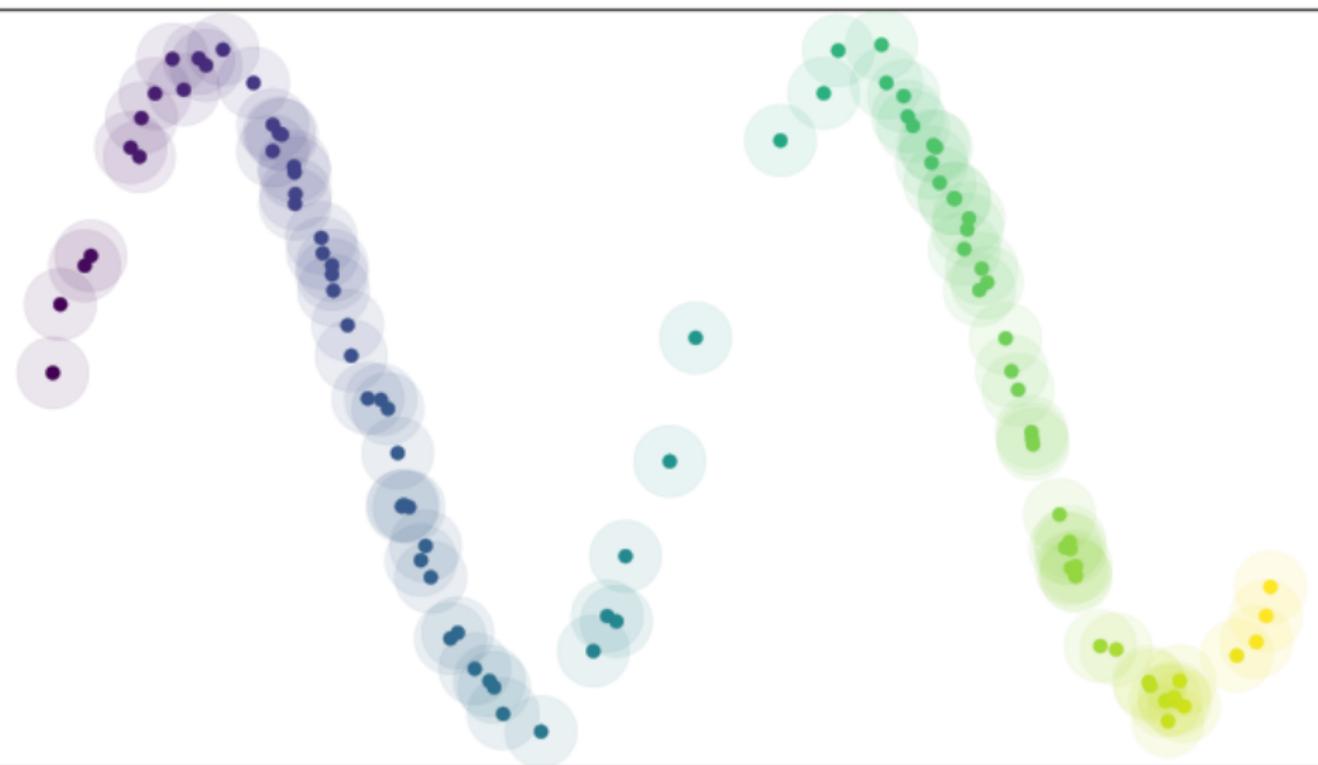
Uniform Manifold Approximation and Projection

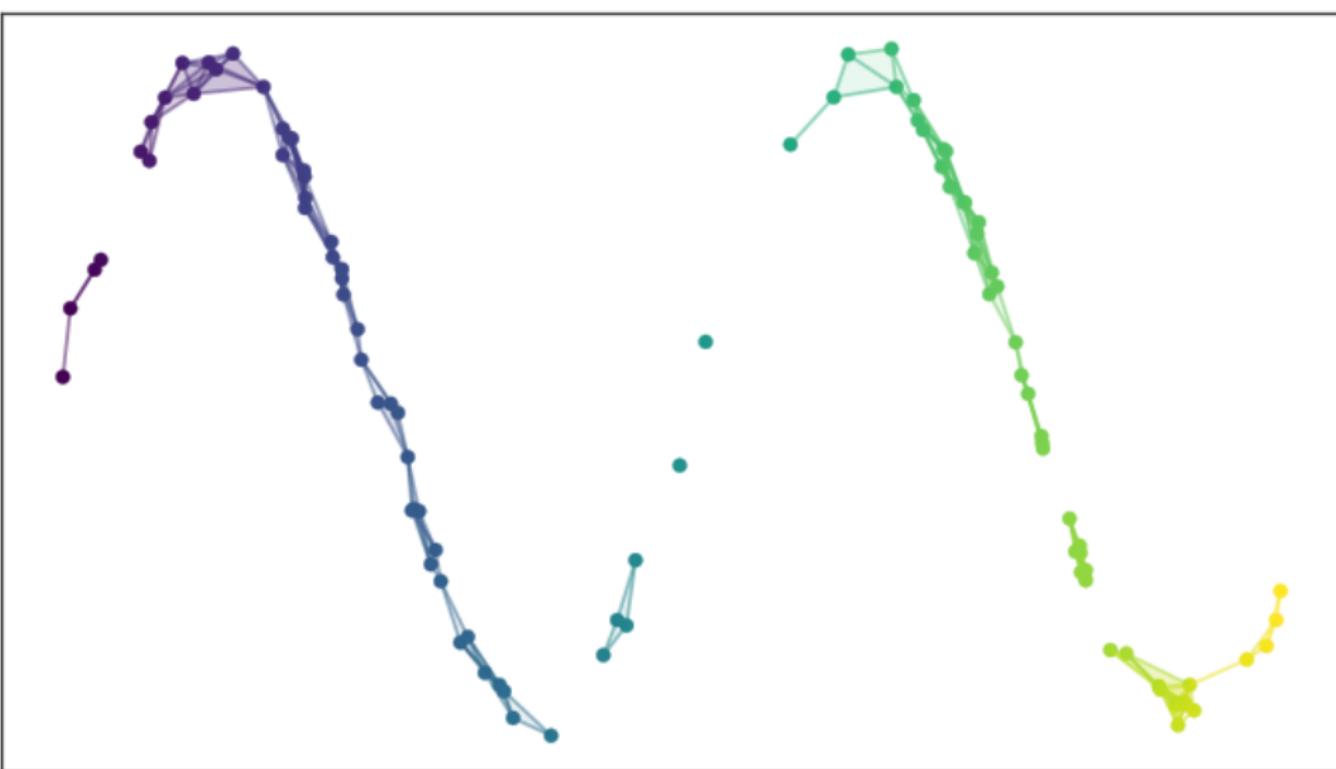


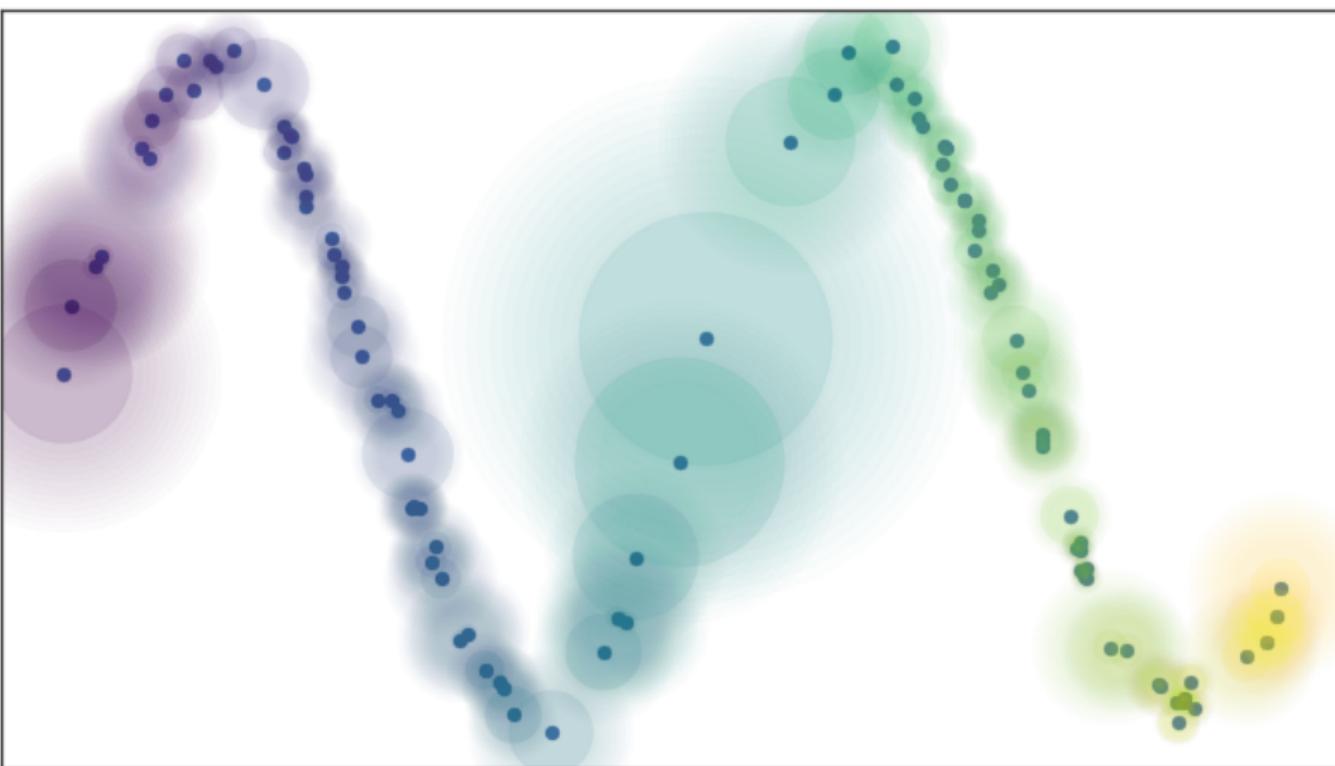
t-distributed Stochastic Neighbor Embedding

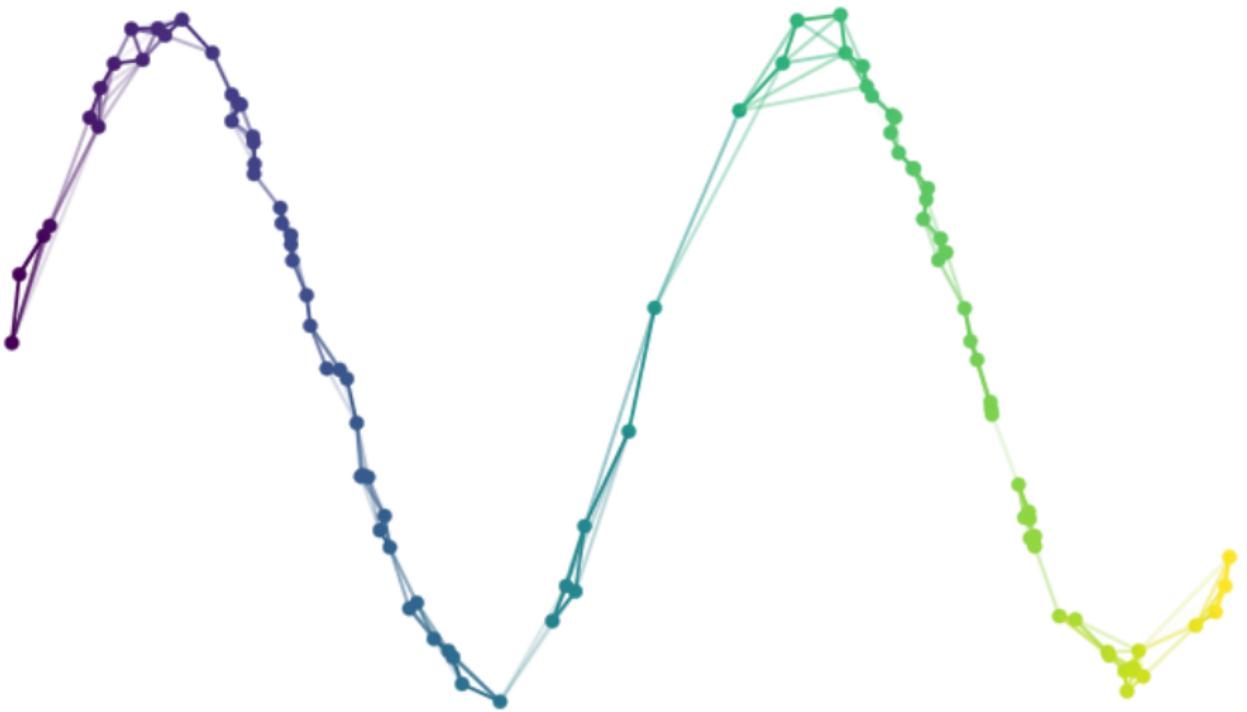


Uniform Manifold Approximation and Projection by McInnes

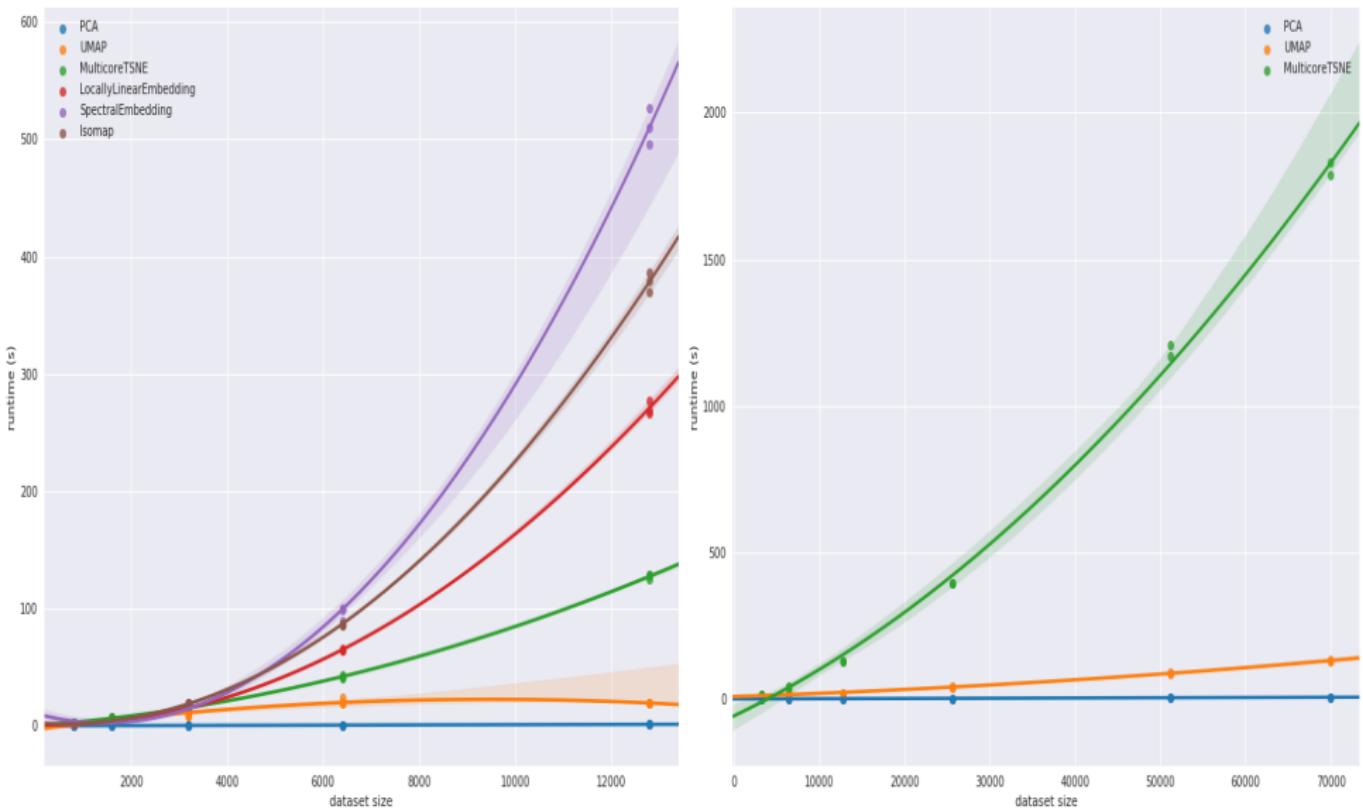






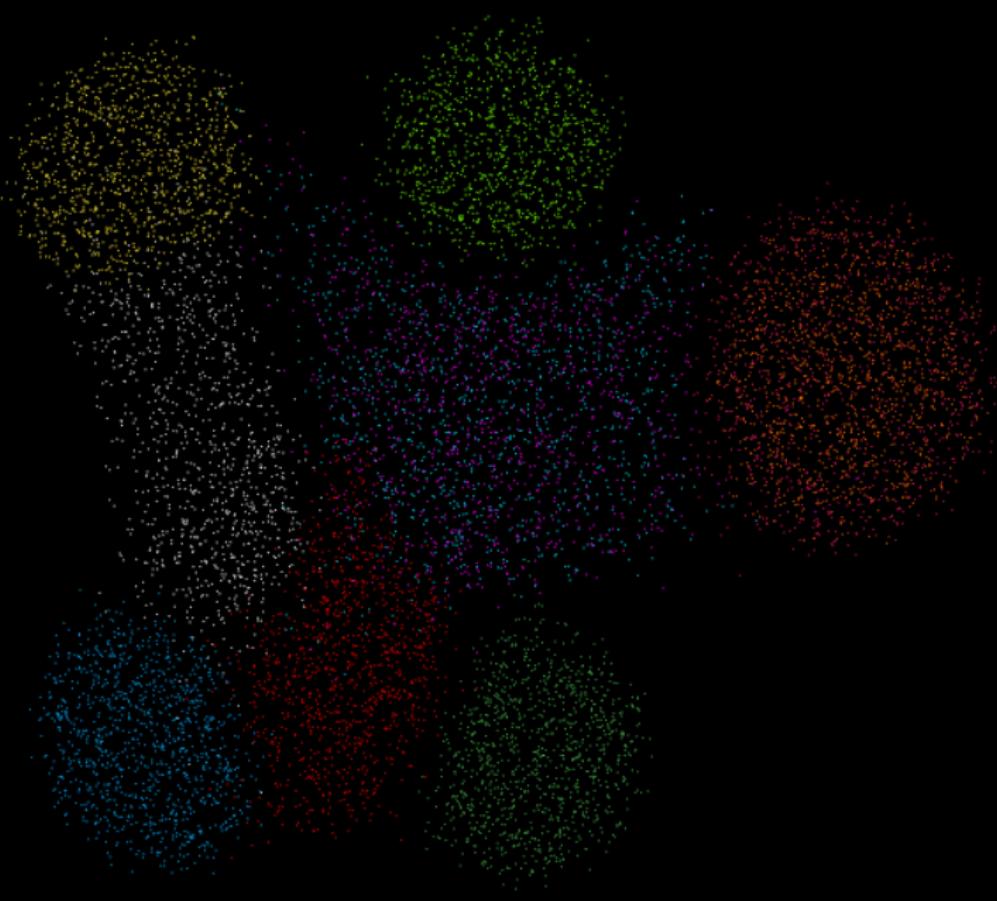


Performance



Results for Modulation Embedding

Uniform Manifold Approximation and Projection



Conclusion



dreamstime



Thank
you!

stevenrafaelturner@gmail.com

[https://
www.shapeways.com/
product/6CJQ9GXW
W/topology-joke](https://www.shapeways.com/product/6CJQ9GXWW/topology-joke)

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