Learn Game Theory

Special class

Sonia Motwani • June 19, 2021

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Game Theory

Beginner Level Special Class with Sonia Motwani

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- Special Interest in Finance Companies
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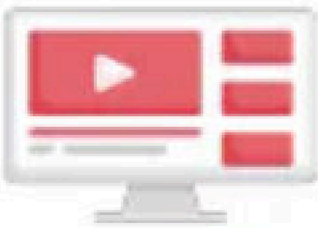

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Did you know...?

That you can predict the winner of a game even before playing the game.😮

Yes, you can do that using Game Theory. If you are new to competitive programming, Game Theory might be something new to you. However, Game Theory programming problems are common in most hackathons.

Types of Game Theory

The different types of games are formed on the basis of number of players involved in a game, symmetry of the game, and cooperation among players.

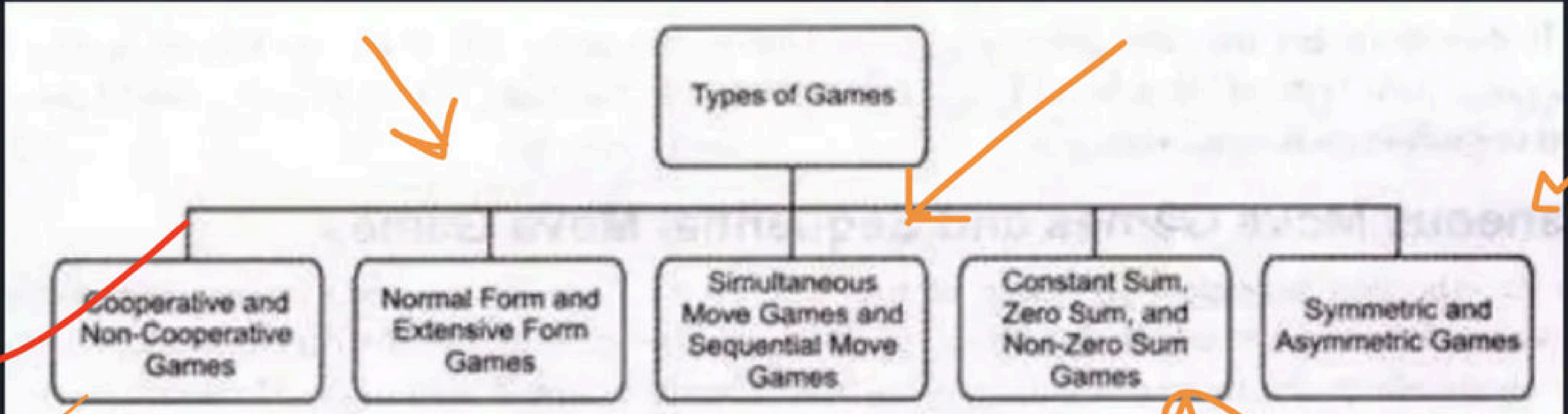
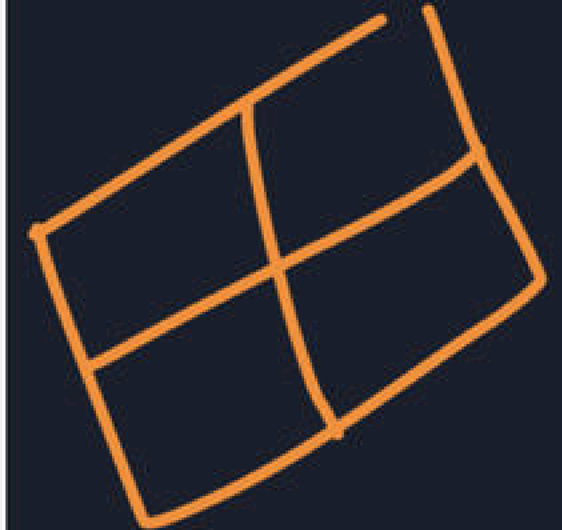


Figure-1: Types of Games

Cooperative and Non-Cooperative Games:

Cooperative games are the one in which players are convinced to adopt a particular strategy through negotiations and agreements between players. - True

However, non-cooperative games refer to the games in which the players decide on their own strategy to maximize their profit. → False



Example - Beauty product organizations. Suppose organizations have high ad-expenditure that they want to reduce. However, they are not sure whether other organizations would follow them or not. This creates a situation of dilemma among organizations. But if the government restricts the advertisement on televisions, this would help in reducing the ad-expenditure.

Normal Form and Extensive Form Games

Normal form games refer to the description of game in the form of matrix. In other words, when the payoff and strategies of a game are represented in a tabular form, it is termed as normal form games. Normal form games help in identifying the dominated strategies. In normal form games, the matrix demonstrates the strategies adopted by the different players of the game and their possible outcomes.

Extensive form games are the one in which the description of game is done in the form of a decision tree. Extensive form games help in the representation of events that can occur by chance. These games consist of a tree-like structure in which the names of players are represented on different nodes.

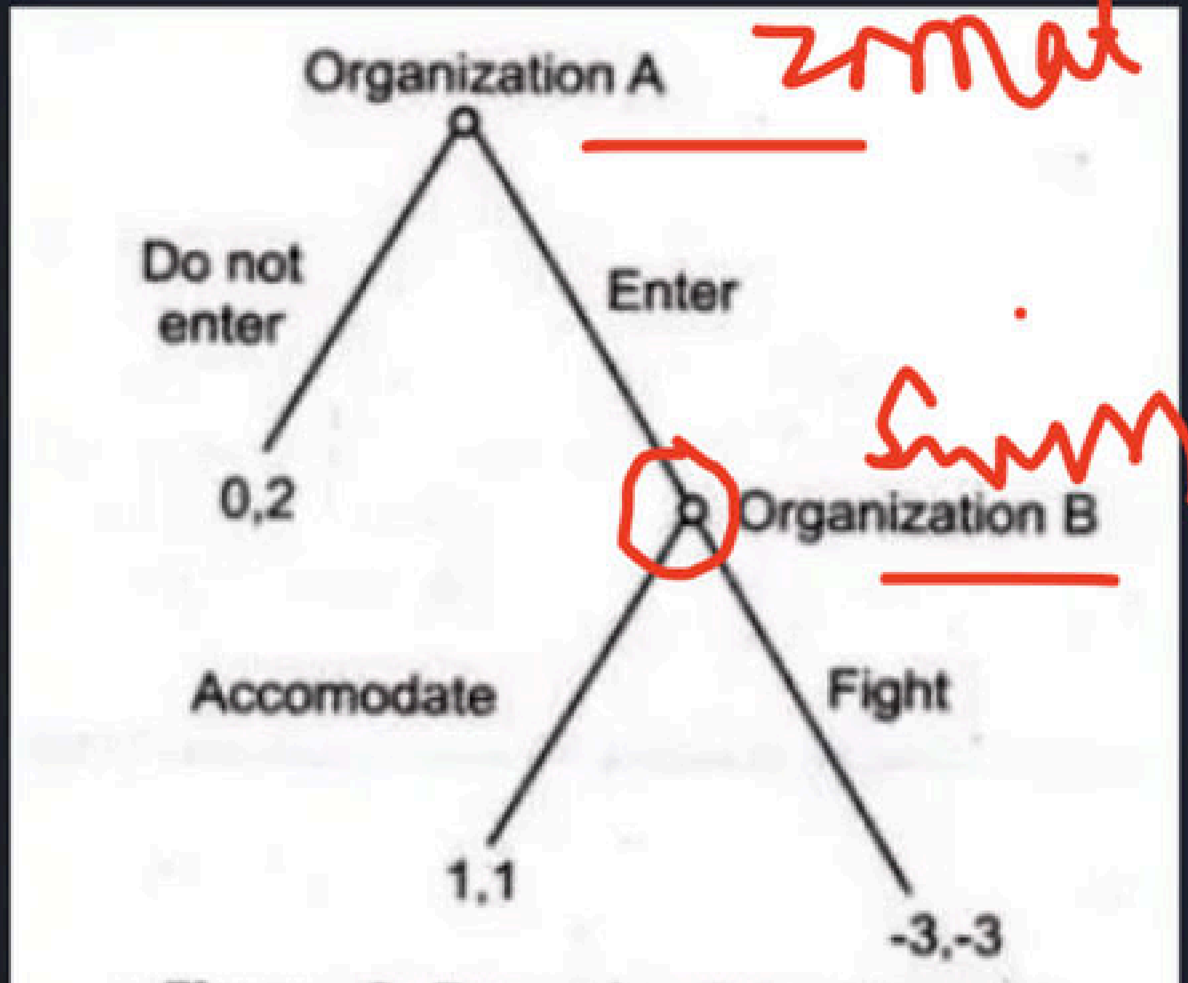


Figure-2: Extensive Form Games

Simultaneous Move Games and Sequential Move Games:

Simultaneous games are the one in which the move of two players (the strategy adopted by two players) is simultaneous. In simultaneous move, players do not have knowledge about the move of other players. On the contrary, sequential games are the one in which players are aware about the moves of players who have already adopted a strategy.

However, in sequential games, the players do not have a deep knowledge about the strategies of other players. For example, a player has knowledge that the other player would not use a single strategy, but he/she is not sure about the number of strategies the other player may use. Simultaneous games are represented in normal form while sequential games are represented in extensive form.

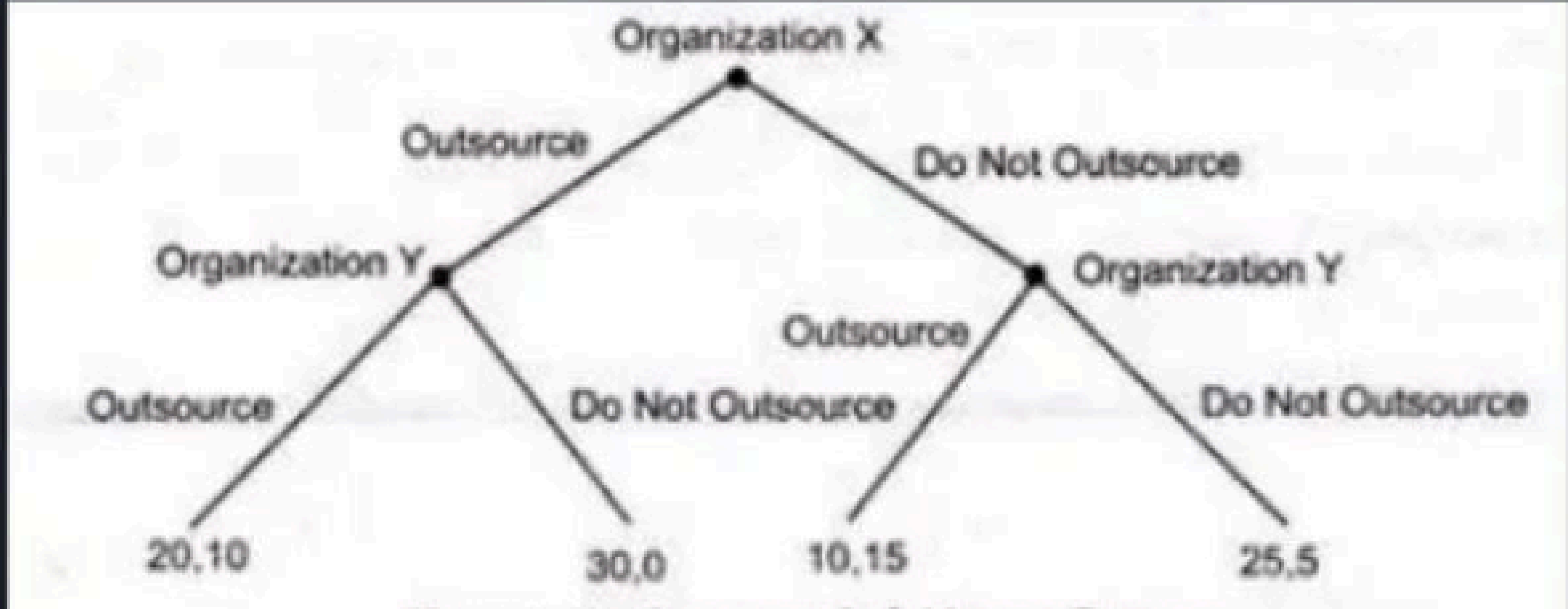


Figure-3: Sequential Move Game

Constant Sum, Zero Sum, and Non-Zero Sum Games:

Constant sum game is the one in which the sum of outcome of all the players remains constant even if the outcomes are different. Zero sum game is a type of constant sum game in which the sum of outcomes of all players is zero. In zero sum game, the strategies of different players cannot affect the available resources.

Moreover, in zero sum game, the gain of one player is always equal to the loss of the other player. On the other hand, non-zero sum game are the games in which sum of the outcomes of all the players is not zero.

A non-zero sum game can be transformed to zero sum game by adding one dummy player. The losses of dummy player are overridden by the net earnings of players. Examples of zero sum games are chess and gambling. In these games, the gain of one player results in the loss of the other player. However, cooperative games are the example of non-zero games. This is because in cooperative games, either every player wins or loses.

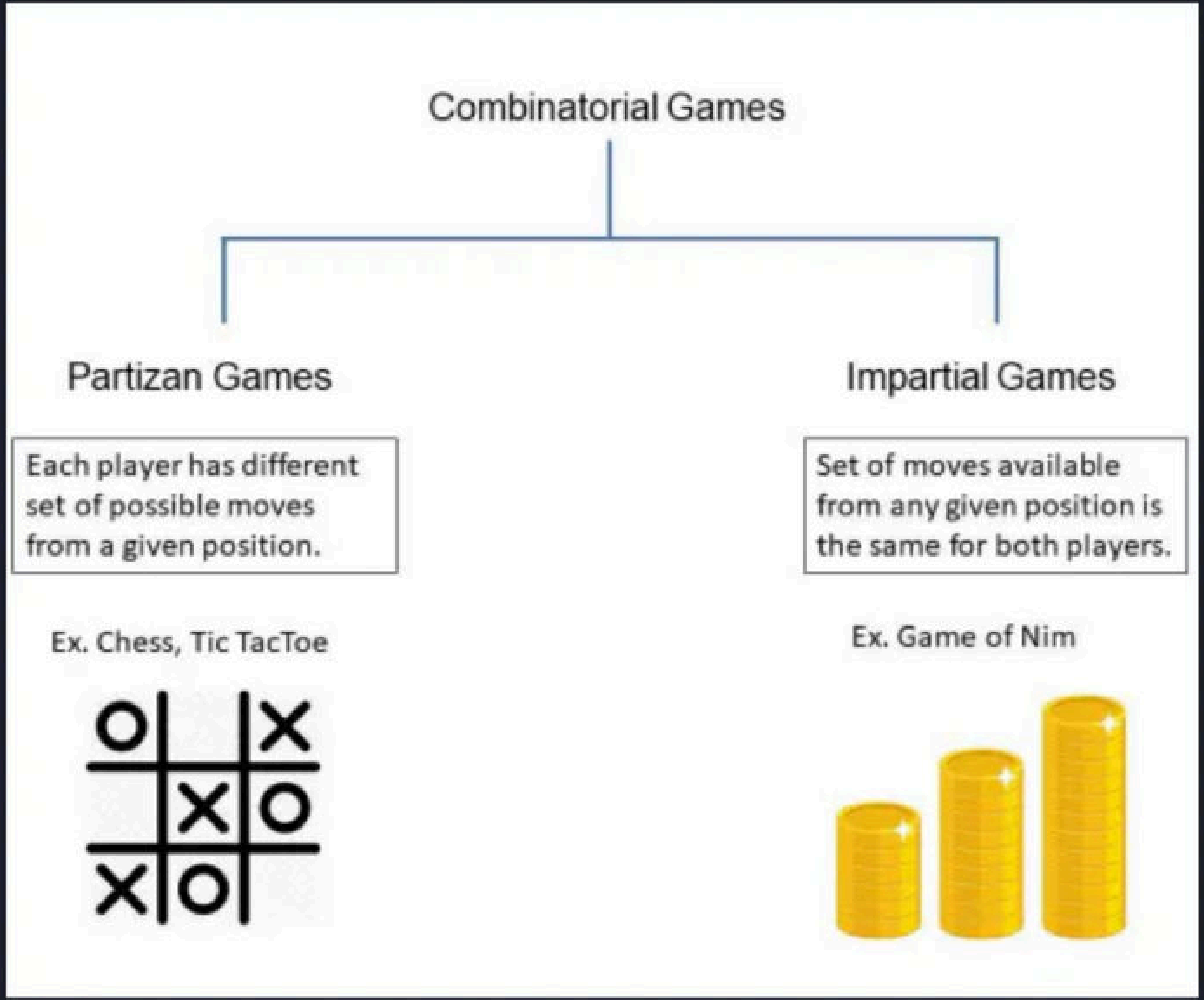
Symmetric and Asymmetric Games:

In symmetric games, strategies adopted by all players are same. Symmetry can exist in short-term games only because in long-term games the number of options with a player increases. The decisions in a symmetric game depend on the strategies used, not on the players of the game. Even in case of interchanging players, the decisions remain the same in symmetric games. Example of symmetric games is prisoner's dilemma.

On the other hand, asymmetric games are the one in which strategies adopted by players are different. In asymmetric games, the strategy that provides benefit to one player may not be equally beneficial for the other player. However, decision making in asymmetric games depends on the different types of strategies and decision of players. Example of asymmetric game is entry of new organization in a market because different organizations adopt different strategies to enter in the same market.

Using Game Theory, you can predict winners but only in **combinatorial games**. Combinatorial Games, are games that will be played by two persons and these games will not have randomization (Ex. Coin Toss). Also, these games will be always finished in either win or lose state.

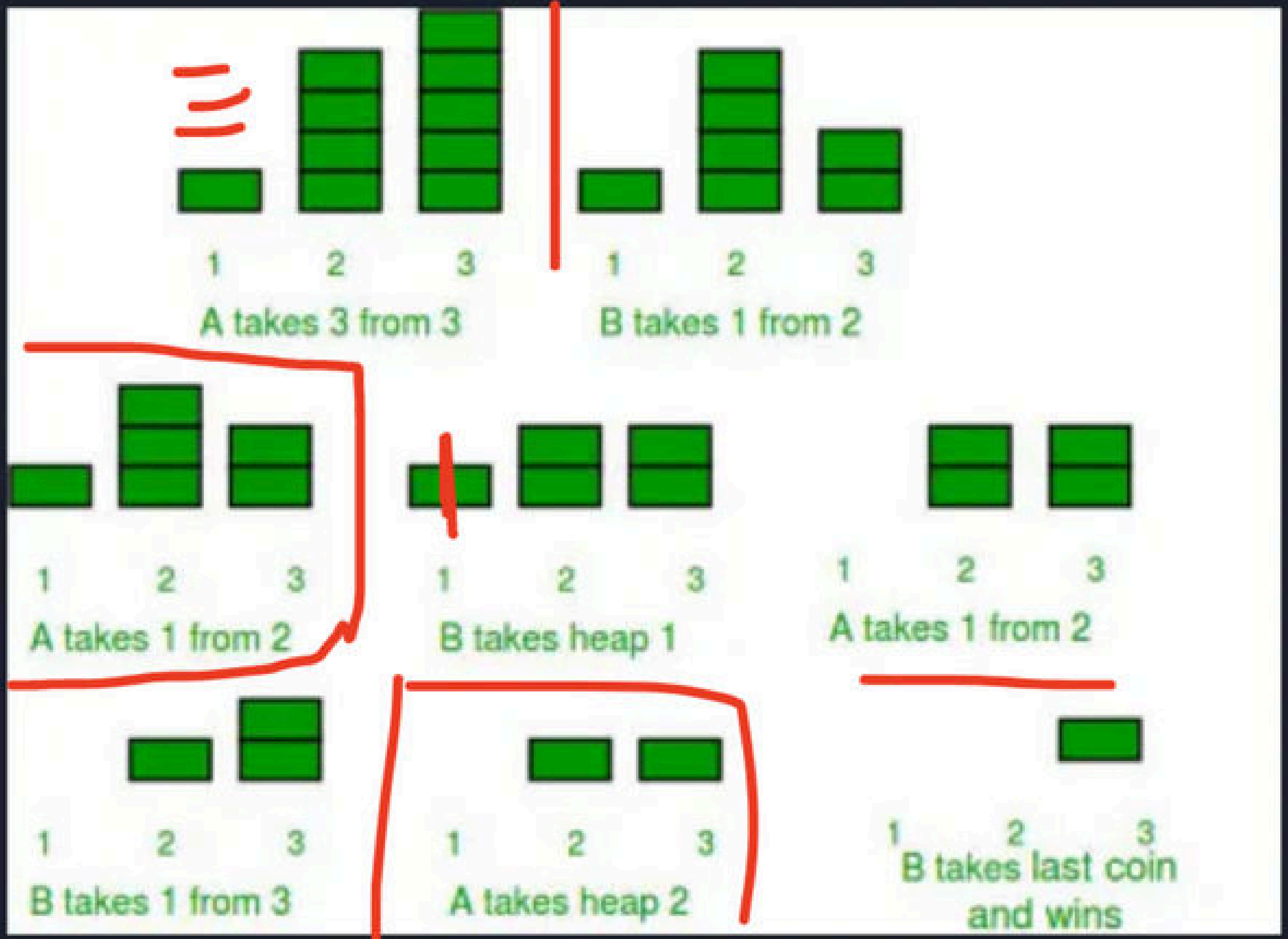
For example, games like Chess, Tic Tac Toe, and Game of Nim come under the category of combinatorial games.



The game of nim

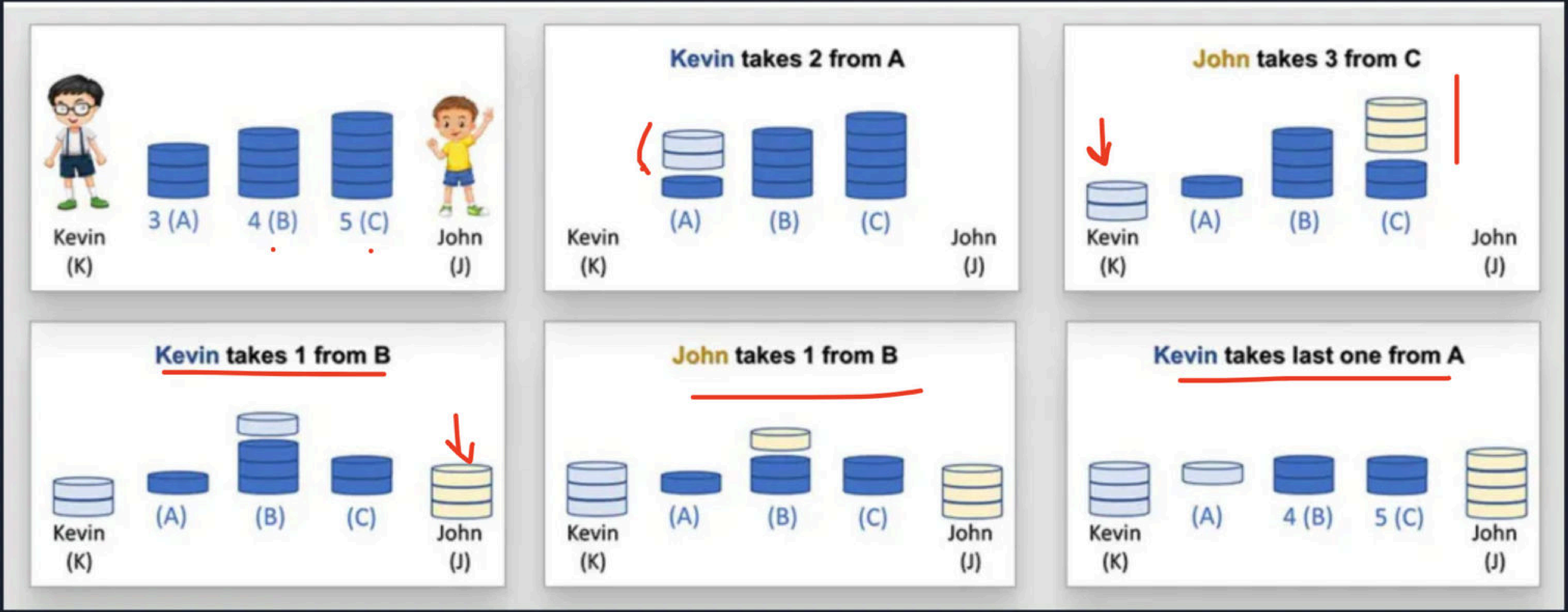
The ‘Game of Nim’ is simple. There are ‘n’ piles of coins, (in between two players) each player can take at least one coin from any piles. The player who makes the last move will win. Both players will play optimally. (playing to win)

A



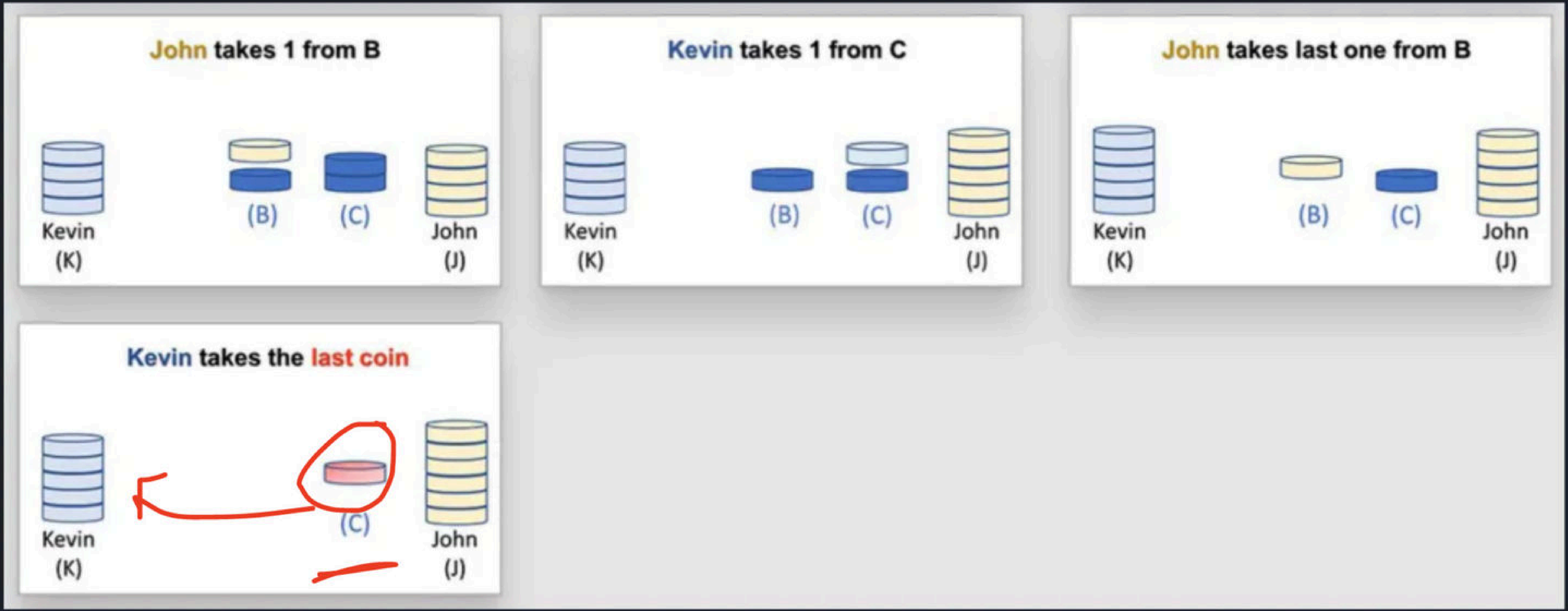
B

There are two players, John and Kevin. They decided to play this 'Game of Nim'. So they built 3 piles of coins, each with coins respectively 3, 4, and 5. One player must take either 1, 2, or 3 coins at each round. If John makes the first move, and both players play optimally. This is one scenario that could occur.



Kevin takes the last coin. Therefore, Kevin wins the game.

Could we have predicted this?



Predicting the winner

g(4)
g(3)
g(2)

Game Theory implies that the winner of this game will surely depend on two factors.

01. Who starts the game.

02. The number of coins in each pile. (not the total coins in all the piles)

This doesn't suggest that the person who is starting the game will win all the time, there can be a state which Kevin will lose at our example. (by changing the number of coins)

Let's make life simpler. Assume that in our example, there is only one pile. This Pile has 5 coins. Two players will take turns and one player can either take 1,2, or 3 coins. Then the winner will be,

If both players play optimally, then the player starting first is guaranteed to win, if the grundy value at the beginning of the game is non-zero. Otherwise, if the grundy value equals to zero, then player starting first will lose definitely.

What is Grundy Value?

We want to calculate the grundy value, at the beginning of our game. Grundy Value is the MEX value of all the possible states you can go to at the beginning. This Grundy Values define each game state. Therefore if total coins are equal to 5,

Grundy(5) = MEX(the possible states that you can go to)

MEX (Minimum Excludant)

MEX or otherwise Minimum Excludant of a set is the smallest non-negative number which is not present in the set.

Ex:- if S = {0, 1, 3, 8, 12} then MEX(S) = 2

2 4 5 6 7

G(5)
G(4)
G(1)

What is Grundy Value?

- the state of taking one coin – Grundy(4)
- the state of taking two coins – Grundy(3)
- the state of taking three coins – Grundy(2)

Therefore,

Grundy(5) = MEX({Grundy(4), Grundy(3), Grundy(2)})

Now you can clearly see a recursive function here. No wonder why the combinatorial game theory and programming are the perfect match😀. Let's calculate all the Grundy values for our example.

			Result
$Grundy(0)$	(base-condition)	–	0
$Grundy(1)$	$MEX(\{G(0)\})$	$MEX(0)$	1
$Grundy(2)$	$MEX(\{G(1), G(0)\})$	$MEX(\{1, 0\})$	2
$Grundy(3)$	$MEX(\{G(2), G(1), G(0)\})$	$MEX(2, 1, 0)$	3
$Grundy(4)$	$MEX(\{G(3), G(2), G(1)\})$	$MEX(3, 2, 1)$	0

From the above table, let's calculate Grundy(5),

Grundy(5)=MEX({ Grundy(4), Grundy(3), Grundy(2) })

Grundy(5)=MEX({0,3,2})→ Grundy(5)=1

Grundy(5) equals to a non zero value. Therefore, if both players play optimally, the player starting first will win. Now, that brings up the question, how to solve when there are more than one piles of coins? That's where Sprague — Grundy Theorem comes in.

True False

Chess (Partisan or Impartial)?

In the game of chess, one player can only move the black pieces and the other one can only move the white ones. Thus, there is a restriction on both the players. Their set of moves are different and hence such a game is classified under the category of Partisan Games.

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Software Engineer at Flipkart | Former SDE and Instructor @ InterviewBit | Google Women TechMakers Scholar 2018

Thank you!

Should we do a series of classes on this topic?
Min Max algorithm next! :)