University of Sydney
STAT2011 Statistical Models
sydney.edu.au/science/maths/stat2011

Semester 1 2012 Lecturer: Michael Stewart

Tutorial Week 3

1. Suppose two dice (one which is red and one which is green, let us say) each have 6 sides numbered 1,2,...,6 in the usual way. Suppose they are to be rolled together.
   (a) How many possible outcomes are there?
   (b) If each possible outcomes is equally likely, write down the probability distribution of the sum Y of the two numbers showing on each of the dice.
   (c) Evaluate $E(Y)$.

2. Suppose that an integer-valued random variable Z taking only a finite number of different possible values is *symmetric about some integer* μ, so that for each $x = 0, 1, 2, \ldots, m$ (for some m),

$$P(Z = \mu - x) = P(Z = \mu + x)\,.$$

   Prove that $E(Z) = \mu$ (*hint: write* $P(Z = \mu \pm x) = q_x$).

3. Repeat question 1, except suppose that the dice each have n sides. In particular, write down
   (a) how many outcomes there are;
   (b) the list of possible values of the sum;

   By showing that the sum has a *symmetric distribution only* (i.e. not necessarily evaluating the expectation explicitly), state what the expectation of the sum here is.

4. Suppose an urn contains $w \geq 1$ white and $b \geq 1$ black balls and $n \geq 1$ draws are made according to the Polya sampling scheme (after each draw, the colour of the ball drawn is noted and it is replaced into the urn along with another ball of the same colour before the next draw is made). Let X be a random variable describing the number of white balls in the n draws. Then for each $x = 0, 1, \ldots, n$,

$$P(X = x) = \frac{\binom{w-1+x}{x}\binom{b-1+n-x}{n-x}}{\binom{w+b-1+n}{n}}\,.$$

   The expected value of $E(X)$ is given by the sum $\sum_{x=0}^{n} xP(X = x)$ (*note: the final part of Lecture 9 may prove helpful in answering this question.)*
   (a) Explain why this is also equal to the same sum but with the range of summation restricted to $x = 1, \ldots, n$.
   (b) By writing it as a product of fractions, show that we may rewrite $\binom{w-1+x}{x}$ as

$$\frac{w}{x}\binom{w-1+x}{x-1}$$

   (c) Show that we may similarly rewrite $\binom{w+b-1+n}{n}$ as

$$\frac{w+b}{n}\binom{w+b-1+n}{n-1}$$

   (d) Use the previous parts to rewrite $E(X)$ as

$$\frac{nw}{w+b}\sum_{x=1}^{n}\frac{\binom{w-1+x}{x-1}\binom{b-1+n-x}{n-x}}{\binom{w+b-1+n}{n-1}}\,.$$

   (e) By writing $x = y + 1$ and $n = m + 1$ inside the sum, evaluate it and hence the expectation.