STAT2011 · Tutorial 1 Solutions

1 (a) Since the no. outcomes in S is ~~positive~~ finite $(<\infty)$, (≥ 1)

while the no. outcomes in any A is non-negative (≥ 0)

$$\frac{\text{no. outcomes in } A}{\text{no. outcomes in } S} \geq 0.$$

(b) $P(S) = \dfrac{\text{no outcomes in } S}{} = 1$

(c) no outcomes in $A \cup B$ = (no outcomes A) + (no outcomes in B)

if no overlap

(d) no outcomes A $\leq$ no outcomes B $\leq$ no. outcomes S

(e) (no outcomes in A) + (no outcomes not in A) = no outcomes in S

(f) ~~$A \cup B$~~ $A = (A \cap B) \cup (A \cap B^c)$

$B = (A \cap B) \cup (A^c \cap B)$

using (c)

$A \cup B = (A \cap B) \cup (A \cap B^c) \cup (A^c \cap B)$

no outcomes $A \cup B$ = (no. in ~~$A \cap B$~~ A only) + (no in ~~$A \cap B^c$~~ B only) + (no in both)

$P(A) = P(A \cap B) + P(A \cap B^c)$

$P(B) = P(A \cap B) + P(A^c \cap B)$

$P(A \cup B) = P(A \cap B) + P(A \cap B^c) + P(A^c \cap B)$

$= P(A) + P(B) - P(A \cap B)$ by careful inspection

2. consider (d). The relation $A \subset B$ means we can write $B = A \cup (B \cap A^c)$ and these 2 events are mut. excl. So

then $P(B) = P(A) + P(B \cap A^c)$ by A3.

However $P(B \cap A^c) \geq 0$ by A1 So

$$P(B) \geq P(A).$$

Similar steps replacing A, B with B, S show $P(B) \leq P(S) = 1$ by A2.

Consider (e): $S = A \cup A^c$ where A, A^c mut. excl.

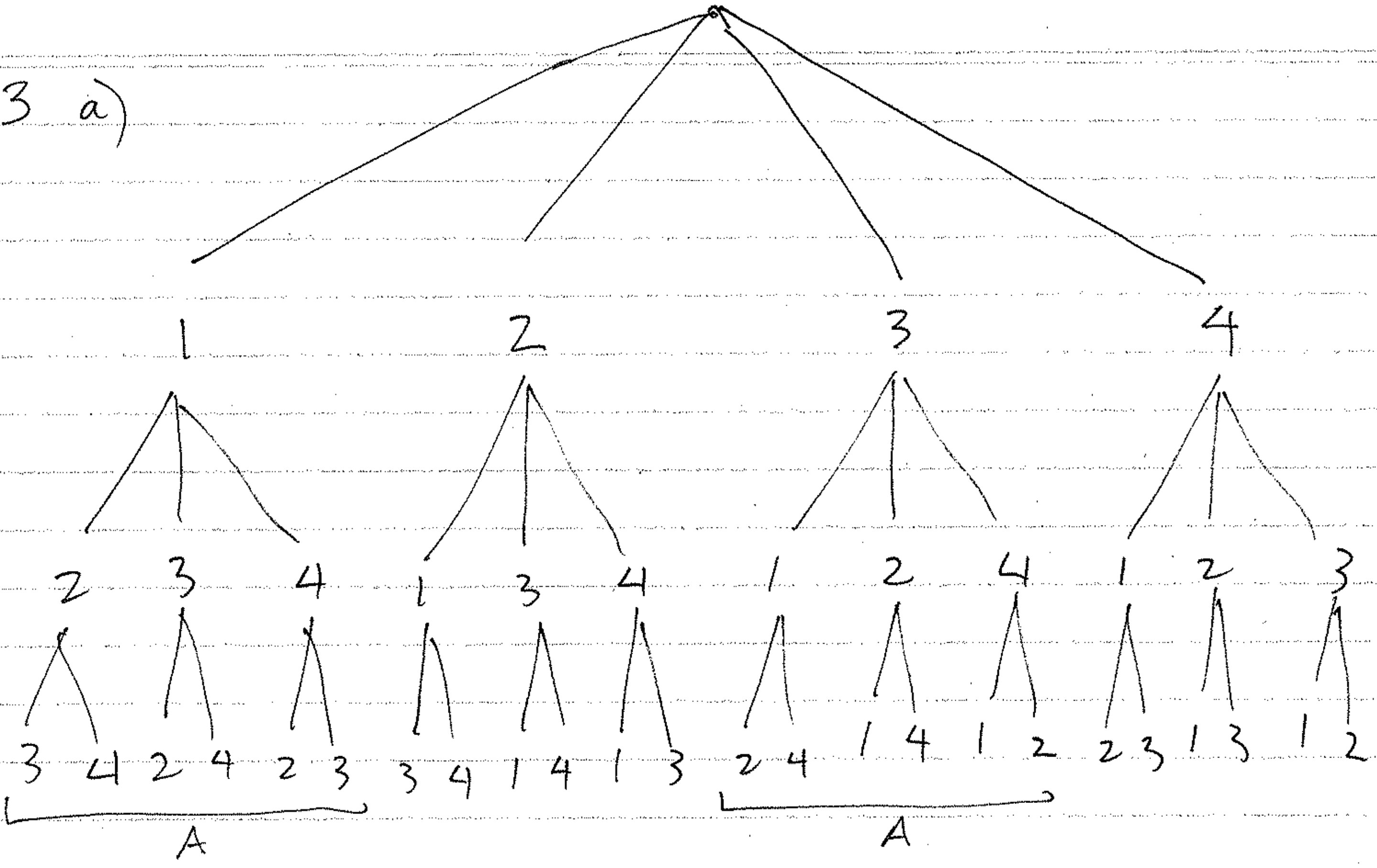
So $P(S) = P(A) + P(A^c)$ by A3

But $P(S) = 1$ by A2, So

$$P(A) + P(A^c) = 1 \Rightarrow P(A^c) = 1 - P(A)$$

(f) is shown using an appropriate method already in Q 1.

3 a)



There are 24 possible sequences

A: first ball is white is where first is 1 or 3.

This takes in half of all possible outcomes

$P(A) = 1/2$

B: NO OUTCOMES have ALL ODD or ALL EVEN

(there is only 2 of each colour ...) $P(B) = 0$

C: We can list all outcomes with 1 odd and 2 even:

124, 142, 214, 234, 241, 243, 324, 342,

412, 421, 432, 423. 12 outcomes

$P(C) = 12/24 = 1/2$

b) : • there are 4 choices for first draw
• Regardless of 1st choice, there are 3 choices for 2nd
• " " 1st + 2nd ", " " 2 choices for 3rd

Mult principle: $4 \times 3 \times 2 = \underline{\underline{24 \text{ possible outcomes}}}$.

For $\underline{A}$ we can construct a favourable outcome by

1) choosing either 1 or 3
2) choosing 1 of remaining 3 balls
3) choosing 1 of remaining 2 balls.

Mult principle: $2 \times 3 \times 2 = 12$ ways

So $P(A) = \frac{12}{24}$.

For B : $P(B) = 0$ (no outcomes satisfy cond'n for B)

For C : construct an outcome by

1) choosing 1 or 3 (2 choices)
2) choosing a position for this white ball (3 choices)

_ _ _

3) choosing between (2,4) and (4,2) to fill remaining 2 spots (2 choices)

$2 \times 3 \times 2 = 12$

Note: Consider the colour of ball NOT selected.

this is equally likely W or B.

So $P(2W) = P(1W) = \frac{1}{2}$.

4. Suppose there are 28 numbered balls, odd ones white, even ones black. At the first draw there are 20 choices. Regardless of ~~a~~ first draw, at 2nd draw there are 21 choices, etc etc so there are

$20 \times 21 \times 22 \times 23 \times 24$ outcomes in the sample space.

(a) ~~P(A)~~ Of course, $P(\text{first wh}) = \frac{1}{2}$.

(b) $P(\text{all Wh}) = ?$ Well, to have all Wh, there are - 10 choices for first position
11 choices .. second
⋮
etc

So there are $10 \times 11 \times 12 \times 13 \times 14$ choices for all wh.

$$\text{So } P(\text{all Wh}) = \frac{\cancel{10} \times \cancel{11} \times \cancel{12} \times 13 \times 14}{\cancel{20} \times 21 \times \cancel{22} \times 23 \times \cancel{24}} \simeq 0.0471$$

$$= \frac{13 \times 14}{8 \times 21 \times 23} = \frac{13 \times \cancel{2} \times \cancel{7}}{\cancel{2} \times 4 \times 3 \times \cancel{7} \times 23} = \frac{13}{12 \times 23} = \frac{13}{276}$$

Similarly by symmetry,

$$P(\text{all Black}) = \frac{13}{276} \quad \text{so}$$

$$P(\text{all same colour}) = \frac{26}{276} \approx 0.0942\ldots$$

(c) We need to "construct" an appropriate outcome.
(ie with 1 wh + 4 blacks)

One way is to

1) Pick a position for the single white (5 choices)
2) Pick the white (10 ways)
3) Choose the 4 blacks: $10 \times 11 \times 12 \times 13$ ways

These determine the outcome.

$$\text{so } P(\text{exactly 1 wh}) = \frac{5 \times 10 \times 11 \times 12 \times 13 \times 10}{20 \times 21 \times 22 \times 23 \times 24}$$

~~Hotly~~ $$= \frac{630}{3864} \approx 0.168\ldots$$

5. (a) $P(\cancel{\text{first wh}}) = \frac{1}{2}$

~~(b) P(all wh) = 10×9×...~~

<u>Note:</u> there are $20 \times 19 \times 18 \times 17 \times 16$ outcomes

(imagine each ball is uniquely numbered)

(a) Exactly $\frac{1}{2}$ of these have a wh as the first one

So $P(\text{first Wh}) = \frac{1}{2}$.

(b) There are $10 \times 9 \times 8 \times 7 \times 6$ outcomes where

all are white, similarly for all black.

So $P(\text{all same colour}) = \dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 2}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}$

$= \dfrac{21}{646} \simeq 0.0325$

(c) We can construct an appropriate outcome by

1) Picking one wh (10 ways)
2) Picking 4 Blacks ($10 \times 9 \times 8 \times 7$ ways)
3) Picking position for the white (5 ways)

So $P(\text{exactly 1 wh}) = \dfrac{10 \times 9 \times 8 \times 7 \times 10 \times 5}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16} = \dfrac{175}{1292} \simeq 0.1354\ldots$