Semester 1 2012 Lecturer: Michael Stewart

Tutorial Week 2

This week we compute the probabilities of various prizes in different "games" run by NSW Lotteries. In later weeks we compare these to the prices for playing and prizes offered. Eventually we will have a good idea of how much revenue this generates for the state!

1. **Powerball** One of the "games" run by NSW Lotteries is Powerball. There are two "machines" each containing 45 balls, numbered 1 to 45.

   - 5 balls (the "winning numbers") are drawn out of the first machine;
   - 1 ball (the "Powerball") is drawn out of the second machine.

   An entry form for a single game consists of two panels ("top" and "bottom"), each with 45 boxes (numbered 1 to 45). To play a single game, a player marks 5 numbers in the top panel and a single number in the bottom panel.

   There are a range of ways a player can "win", corresponding to 7 different "divisions":

Div	Short description	Long description
1	5 winning numbers + Powerball	top panel choices include 5 numbers from 1st machine bottom panel choice matches number from 2nd machine
2	5 winning numbers	top panel choices include all 5 numbers from 1st machine bottom panel choice irrelevant
3	4 winning numbers + Powerball	top panel choices include 4 numbers from 1st machine bottom panel choice matches number from 2nd machine
4	3 winning numbers + Powerball	top panel choices include 3 numbers from 1st machine bottom panel choice matches number from 2nd machine
5	4 winning numbers	top panel choices include 4 numbers from 1st machine bottom panel choice irrelevant
6	2 winning numbers + Powerball	top panel choices include 2 numbers from 1st machine bottom panel choice matches number from 2nd machine
7	3 winning numbers	top panel choices include 3 numbers from 1st machine bottom panel choice irrelevant

   (a) How many different ways can the game turn out? Assuming each is equally likely, what is the probability that a single (given) entry (say top: 1 2 3 4 5, bottom: 1) wins the division 1 prize? You may leave your answer as a function of combinatorial coefficients (you can evaluate them in the computer class).

   (b) In the same setup as (1a), what is the probability of winning the division 2 prize?

   (c) In the same setup as (1a), what is the probability of winning the division 3 prize?

   (d) In the same setup as (1a), what is the probability of winning the division 4 prize?

   (e) In the same setup as (1a), what is the probability of winning the division 5 prize?

   (f) In the same setup as (1a), what is the probability of winning the division 6 prize?

   (g) In the same setup as (1a), what is the probability of winning the division 7 prize?

2. **Lotto** In this game, a single machine contains 45 balls numbered 1 to 45. Eight balls are drawn out, in 2 stages:

   - firstly 6 numbers are drawn out (the so-called "winning numbers");
   - secondly another 2 numbers are drawn out (the so-called "supplementary numbers").

   (a) How many elements in the sample space (i.e. how many ways can the game turn out?)?. The prize schedule for Lotto is as follows:

Div	Description
1	All 6 winning numbers
2	5 winning numbers and 1 or 2 supplementary numbers[1]
3	5 winning numbers
4	4 winning numbers
5	3 winning numbers and 1 or 2 supplementary numbers
6	1 or 2 winning numbers and 2 supplementary numbers

   (b) Assuming each possible outcome is equally likely, compute the probability that a single given entry (say, 1 2 3 4 5 6) qualifies for each of the 6 different prize divisions.

3. **$2 Jackpot Lottery** (*Time permitting*) In this game, 225,000 tickets are sold[2]. Then there are two rounds of draws made. In the first round, 3,215 tickets are drawn out in 9 sequential "batches" (without replacement), tickets within each batch receiving a certain cash prize. For each such "winning ticket", 2 "one-off" consolation prizes are awarded to the tickets immediately numbered above and below the winning number[3]. The main and consolation prizes for each batch are as given below:

Batch	No. tickets	Main Prize	Consolation Prize
1	1	$100,000	$1,000
2	1	$10,000	25 free tickets in a future game
3	1	$5,000	10 free tickets in a future game
4	2	$500	5 free tickets in a future game
5	10	$200	5 free tickets in a future game
6	20	$100	2 free tickets in a future game
7	100	$50	1 free ticket in a future game
8	600	$20	1 free ticket in a future game
9	2,480	$10	1 free ticket in a future game

   Then, a final, *second* draw is made from all 225,000 tickets, the *jackpot number*. If this number matches one of the 3,215 *winning tickets*,[4] then the holder wins the "jackpot". Otherwise, the holder of the jackpot number wins a consolation prize of 10 free tickets in a future game.

   If there is no jackpot winner, a certain amount of money (currently $105,000) is added to the "jackpot pool" for the next game. The minimum jackpot pool amount is $500,000.

   (a) Describe the sample space: how many ways can the whole "game" turn out?

   (b) Assuming each outcome is equally likely, what is the probability that a single ticket wins

      i. a prize;

      ii. a cash prize;

      iii. a cash prize of at least $1,000?

   (c) What is the probability that a single ticket wins the jackpot?

   (d) What is the probability that the jackpot prize is won at all?

[1] For a "standard game" this could only be at most 1 supplementary number, but it is perhaps possible to get 5 winning + 2 supplementary numbers using a "systems" entry.

[2] Note that it is not possible to request certain numbers: an (otherwise unknown) batch of randomly picked or sequential numbers may be bought though.

[3] If ticket 1 is a winning ticket, ticket 2 wins a corresponding "double consolation" prize; as does ticket 224,999 if ticket 225,000 is a winnning ticket

[4] There is some ambiguity on the NSW Lotteries website, suggesting that a winner of a batch 1 consolation (cash) prize may also qualify for the jackpot.