Semester 1 2012 — Lecturer: Michael Stewart

Assignment 1

Submission instructions: Please

- note there are **2** questions;
- write out your solutions by hand, in pen on lined A4 paper, on only one side of each page;
- make the first page a signed assignment cover sheet (see `maths.usyd.edu.au/u/UG/asscover.pdf`);
- staple your pages together inside a manilla folder (**no paper clips please**);
- write your name and SID along with a very large first letter of your family name clearly on the front of your folder;
- place your assignment in the white collection box marked "STAT2011 / STAT2012" opposite the lifts on level 8 of the Carslaw building by **2pm Friday 20 April**.

1. In the game of Yahtzee, five ordinary 6-sided dice are rolled. For the purposes of this question it is helpful to imagine that they are either of different colours or are rolled *in sequence*.

   As with poker, different types of "rolls" are arranged in a hierarchy, with different types worth different amounts of points. The table below lists some of these:

Type of roll	Description
Yahtzee	All dice showing the same number
Large Straight	Five numbers in sequence (either $\{1,2,3,4,5\}$ or $\{2,3,4,5,6\}$)
Small Straight	Four numbers in sequence (either $\{1,2,3,4,(\text{not } 5)\}$, $\{2,3,4,5,(\text{not 1 or } 6)\}$) or $\{3,4,5,6,(\text{not } 2)\}$)
Full House	Three of a kind and a pair e.g. $\{2,2,4,4,4\}$
Four-Of-A-Kind	Four dice the same number, plus another different number e.g. $\{3,3,3,3,2\}$.
Three-Of-A-Kind	Three dice the same number, plus two other different numbers e.g. $\{5,5,5,1,3\}$ (note: not a four-of-a-kind, a full house or a Yahtzee)

   Answer the questions below. Note that **simply writing down a single integer is not sufficient**; at the very least you should write each answer first as a product (of course powers of integers, factorials and other combinatorial coefficients are special cases of products) and/or write a short sentence explaining where your answer comes from.

   (a) How many possible outcomes are there?
   (b) How many of these outcomes correspond to a Yahtzee?
   (c) How many outcomes give the numbers $\{1,2,3,4,5\}$?
   (d) How many outcomes give a Large Straight?
   (e) How many outcomes give the numbers $\{1,2,3,4,6\}$?
   (f) How many outcomes give the numbers $\{1,2,3,4,1\}$?
   (g) How many outcomes give a set of numbers of the form $\{1,2,3,4,*\}$ where $*$ is anything *except* 5?
   (h) How many outcomes give a Small Straight?
   (i) How many outcomes give the numbers $\{2,2,2,4,4\}$?
   (j) How many outcomes give a Full House?
   (k) How many outcomes give the numbers $\{3,3,3,3,2\}$?
   (l) How many outcomes give a Four-Of-A-Kind (but not a Yahtzee) with four 3's?
   (m) How many outcomes give a Four-Of-A-Kind (but not a Yahtzee)?
   (n) How many outcomes give the numbers $\{5,5,5,1,3\}$?
   (o) How many outcomes give a Three-Of-A-Kind (but not a Four-of-a-kind, a Yahtzee or a Full House) with three 5's?
   (p) How many outcomes give a Three-Of-A-Kind (but not a Four-of-a-kind, a Yahtzee or a Full House)?

2. Suppose that an urn initially contains w white balls and b black balls and that **3** draws are to be made according to the Pólya sampling scheme: after each ball is drawn its colour is noted and it is replaced into the urn *along with another ball of the same colour.* All possible sequences of balls are equally likely.

   (a) Supposing further that all balls involved are uniquely numbered, how many possible sequences are there?

   (b) Introduce 3 random variables: X, Y and Z which count the number of white balls drawn in the first, second and third draws respectively. There are 8 different sets of values (x, y, z) that the three random variables can take. Write down

   $$P(X = x, Y = y, Z = z)$$

   for all eight possible set of values (x, y, z).

   (c) By adding together four of the probabilities in (b), determine $P(X = 1)$ and hence the marginal distribution, expected value and variance of X. Repeat for Y and Z.

   (d) By adding together two of the probabilities in (b), determine $P(X = 1, Y = 1)$ and hence $E(XY)$, the covariance and (using (c)) the correlation between X and Y. Repeat for the pairs (X, Z) and (Y, Z).