POKER HANDS

We have a standard 52-card deck, 4 suits (H, D, C, S) and within each suit, 13 ranks (A, 2, 3, ..., 9, T, J, Q, K : note: the ace (A) can function as highest and lowest). A hand consists of 5 cards. Certain types of hand are of interest (NO WILD CARDS) [UNORDERED]

$2{,}598{,}960 = \binom{52}{5}$

ROYAL FLUSH : T, J, Q, K, A, all of the same suit.

(4 of these)

STRAIGHT FLUSH : 5 cards ~~in~~ of same suit in sequential rank. For each suit, we have (A, 2, 3, 4, 5), (2, 3, 4, 5, 6), ..., (9, T, J, Q, K) (NOT including Royal Flush)

9 per suit $\Rightarrow$ (36 of these)

FOUR of a KIND : 4 cards all of the same rank, plus any other card. Construct such a hand by

1) Pick the common rank of the 4-of-a-kind (13 choices)
2) Pick the remaining card : 48 ways. $13 \times 48 = 624$

Note: we could also use

1) Pick the singleton : 52 ways

2) Pick any other rank as the 4-of-a-kind : 12 ways.

note: $52 \times 12 = 13 \times 4 \times 12 = 13 \times 48 = 624$

FULL HOUSE : 3 of one rank, 2 of another (different) rank

1) Pick rank for the 3: 13 ways.

2) Then pick a rank for the 2: 12 ways

3) Choose the 3 : $\binom{4}{3} = 4$ ways

4) Choose the 2: $\binom{4}{2} = \frac{4.3}{2.1} = 6$ ways

Total: $13 \times 12 \times 4 \times 6 = 3,744$

FLUSH: 5 cards of the same suit [not incl Royal Flush OR Straight Flush]

It is easier though to count all (including the fancy ones), and then subtract 40.

1) Pick a suit (4 choices)

2) Pick 5 of that suit : $\binom{13}{5} = \frac{13 \times \cancel{12} \times 11 \times \cancel{10} \times 9}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 13 \times 11 \times 9 = 1287$

Total is $4 \times 1287 = 5148$, then $5148 - 40 = 5108$ plain flushes.

STRAIGHT: 5 cards in sequential rank (not incl. Royal Flush, Str. Flush).

1) Pick the lowest rank (A,2,3,4,5), ..., (T,J,Q,K,A) : 10 ways

2) Pick a suit for each rank

4^5 { a) 4 ways to pick the lowest rank
b) 4 .. .. .. 2nd-lowest ..
⋮
e) 4 .. .. .. highest rank }

So (including fancy ones), total = $10 \times 4^5 = 10{,}240$ ways

Thus excluding fancy ones: $10{,}240 - 40 = 10{,}200$ "plain" straights

OR WE COULD

1) Pick the lowest CARD : any A, 2, .., T : 40 of these

2) Pick the 2ND-lowest card (4 choices, once lowest is determined)

3) .. .. 3RD lowest .. (4 choices)

4) .. .. 2ND highest (4 choices)

5) .. .. highest (4 choices)

Total $40 \times 4 \times 4 \times 4 \times 4 = 10 \times 4^5$ as before

3 of a kind: 3 of 1 rank, 2 cards of other different ranks (i.e. no full house or 4 of a kind)

1) Pick a rank for the 3 .. 13 ways
2) Pick ranks for the 2 singletons: $\binom{12}{2} = 66$ ways
3) Pick suits for the triple: $\binom{4}{3} = 4$ ways
4) Pick suits for 2 singletons: $\binom{4}{1} \times \binom{4}{1} = 16$

Total: $13 \times 66 \times 4 \times 16 = 54{,}912$

Two Pairs: 2 of 1 rank, 2 of another diff. rank plus 1 of a third, different rank.

1) Pick 2 ranks for 2 pairs: $\binom{13}{2} =$
2) Pick rank for the singleton: $\binom{11}{1} = 11$ ways
3) Fill the pairs $\binom{4}{2} \times \binom{4}{2} = 36$ ways
4) Fill the singleton: $\binom{4}{1}$ ways

Total: $\binom{13}{2} \times 11 \times 36 \times 4 = 123{,}552$