CLASSICAL PROBABILITY

We shall use the term "classical probability" to describe a scenario where we have a

~~a~~ sample space with

- a <u>finite number</u> of
- <u>EQUALLY LIKELY</u> outcomes.

There used to be some debate over what "equally likely" means, but we assume this is not an issue.

"fair/balanced"

<u>Example</u>: Flip a coin 3 times in sequence. We can represent the sample space as

$\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

There are $8 = 2^3$ outcomes.

We use the ~~symb~~ symbol S to ~~de~~ denote a generic sample space (as in text) and s to denote a generic outcome in S (i.e. $s \in S$)

(so each $s \in A$ is also in S)

Any ~~set~~ subset $A \subset S$ (or $A \subseteq S$)

is called an EVENT

Note: The empty set ϕ (i.e. with no elements/outcomes) is always considered an event in any sample space S.

Then the probability of any event A is simply given by

$$P(A) = \frac{\text{no elt.s/outcomes in } A}{\text{no. elts/outcomes in } S}$$

(Note! S must be non-empty!)

Note then that $P(\phi) = 0$ always.

3-coin-flip example: Let A = "Head on 1st Flip".

$A = \{HHH, HHT, HTH, HTT\}$ has 4 outcomes, so

$p(A) = \frac{4}{8} = \frac{1}{2}$. (makes sense...).

Let B = "all ~~fl~~ flips the same" $= \{HHH, TTT\}$

so $p(B) = \frac{2}{8} = \frac{1}{4}$.

Let C = "exactly 1 Head" $= \{HTT, THT, TTH\}$, so $P(C) = \frac{3}{8}$.

We can consider other examples with the same basic structure, e.g.:

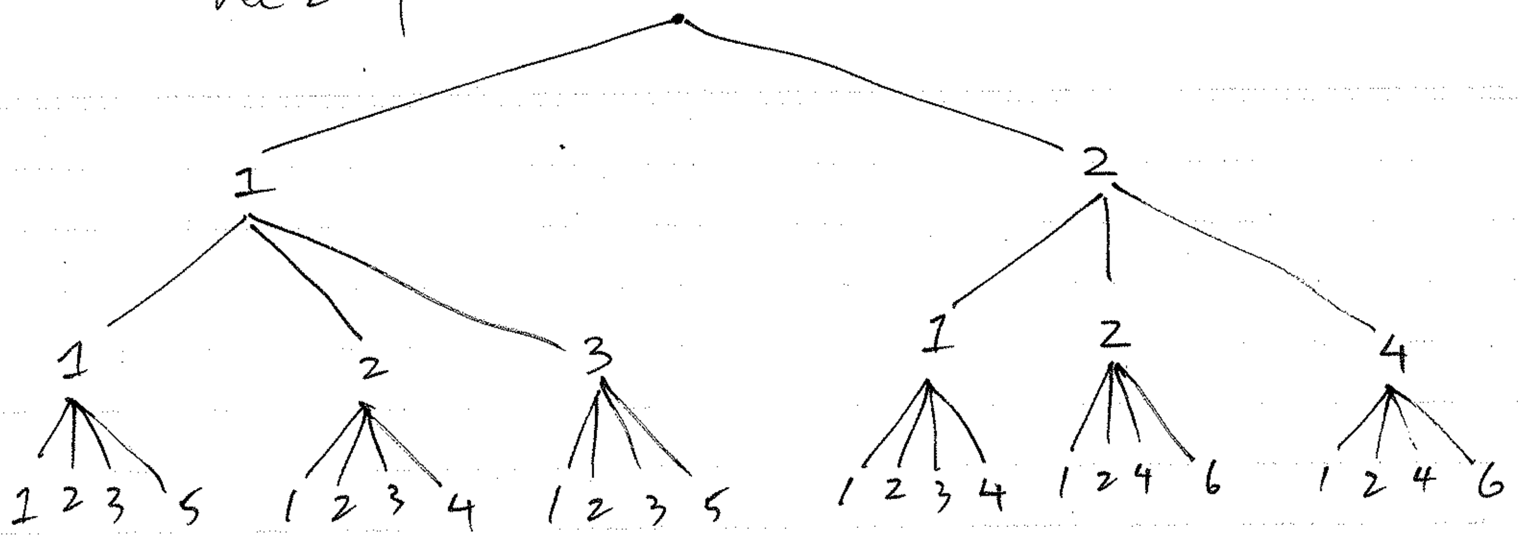
having 3 children "H" $\Leftrightarrow$ Girl
"T" $\Leftrightarrow$ Boy

OR

"An urn contains 1 White ball and 1 Black ball: a ball is drawn "at random", its colour is noted, then replaced; this is repeated twice (3 draws all together)." We can associate e.g. "H" $\Leftrightarrow$ White, "T" $\Leftrightarrow$ Black.

Here is a more complicated example:

Suppose we have 6 balls, numbered 1, 2, ..., 6. Odd Nos are white, Even Nos are black. Balls 1 and 2 are placed in the Urn. 1 ball is drawn out; its colour is noted, it is replaced into the urn, along with the ~~next numbered~~ lowest-numbered remaining ball of the same colour. This process is repeated for 2 more draws



So we have 24 different possible outcomes

(we could have found this total more easily by more general methods we shall review shortly)

111 B	121	131 B	A
112	122 C	132	A
113 B	123	133 B	A
115 B	124 C	135 B	A

211	221 C	241 C
212 C	222 B	242 B
213	224 B	244 B
214 C	226 B	246 B

Consider: A = "Odd on 1st Flop." has 12 outcomes,

So $P(A) = \frac{12}{24} = \frac{1}{2}$

B = "all same colour" = "all odd OR all even" has 12

outcomes, so $P(B) = \frac{12}{24} = \frac{1}{2}$

this is a higher probability than $P(B)$ in the previous example, which makes sense: (its more likely to get the same colour from draw to draw)

$C =$ "exactly 1 Wh" (or "exactly 1 odd") has 6 outcomes so $P(C) = 6/24 = \frac{1}{4}$ (less than the previous example, for the SAME REASON)

RELATIONS between EVENTS

Recall definitions of: (if $A \subseteq S$ and $B \subseteq S$)

UNION: $A \cup B =$ set of all outcomes in at least 1 of A and B.

INTERSECTION: $A \cap B =$ set of all outcomes in both A and B

In the previous examples, $B =$ ~~"all odd"~~ = "all White" $\cup$ "all Black"

If 2 events A and B (both in the same S) have no outcomes in common, we write $A \cap B = \phi$ and we say "A and B are mutually exclusive".

<u>Complement</u>: For any event A in a sample space S, we write its complement as

$$A^c = \{ \text{all } \cancel{\text{elts}} \text{ elts of } S \text{ NOT in } A \}$$

<u>Note</u>: $S^c = \phi$, $\phi^c = S$.