For any (finite) sample space $S$ with $A \subseteq S$, $B \subseteq S$, the following relations ~~ar~~ are apparent:

(1) $A = (A \cap B) \cup (A \cap B^c)$

(2) $B = (A \cap B) \cup (A^c \cap B)$

(3) $(A \cup B) = (A \cap B) \cup (A \cap B^c) \cup (A^c \cap B)$

Notes: if $A \subseteq B$, RHS of ~~(1) becomes~~, then $A \cap B = A$
$A \cap B^c = \phi$

So RHS of (1) becomes $A \cup \phi = A$.

if $A \cap B = \phi$ then $A \cap B^c = A$, $A^c \cap B = B$

and RHS of (3) becomes $\phi \cup A \cup B$.

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"AXIOMATIC" PROPERTIES of CLASSICAL PROB.

The following 3 properties are easily checked in any classical prob scenario:

A1) $P(A) \geq 0$ for all events $A \subseteq S$ (even $\phi$)

A2) $P(S) = 1$

A3) If $A \cap B = \phi$, $P(A \cup B) = P(A) + P(B)$

as are the following

* for $A \subseteq B$, $P(A) \leq P(B)$
* $P(A^c) = 1 - P(A)$
* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(the more mathematically inclined might be able to see that the last 3 can be ~~ob~~ deduced using just (A1), (A2) and (A3) and properties of events (i.e. not relying on the special def'n of $P(\cdot)$ other than it obeys A1, A2, A3)
(see Tutorial).

## The Multiplication Principle

To compute "classical" probabilities we simply need to be able to count how many outcomes make up any event of interest (and in the sample space of course).

However, in only moderately complicated examples this can become rather difficult for certain events.

In our "urn example with (positive) dependence" – also called a Polya Urn Model. – we were able to exhaustively list all outcomes and thus compute any desired probability. We can, however, develop methods for use when exhaustive listing is not feasible.

The process for generating outcomes in that example had a certain special form, namely

- it is made up ~~a~~ of several stages
- a varying no. choices are available at each stage
- the actual choices avail. at each stage depend on choices made at earlier stages BUT
- the number of choices avail. at each stage is the same regardless of the past.

at stage 1: 2 choices

at stage 2: if 1st choice "1", choices are 1, 2, 3

" " " "2", choices are 1, 2, 4

but in EITHER CASE, there are 3 choices.

at stage 3: if 1st 2 choices are (1,1), choices are 1,2,3,5

" " " (1,2), " " " 1,2,3,4

etc

in ANY CASE, 4 choices

In such a process, if (regardless of earlier choices) there are $n_j$ choices at step $j$, $(j=1,2,\dots,k)$ then the total no. choices in $k$ steps is the product

$$n_1 n_2 \cdots n_k$$

Thus in Lec 2's example, $n_1 = 2$, $n_2 = 3$, $n_3 = 4$,

So total no. choices is $2 \times 3 \times 4 = \underline{\underline{24}}$

We can also apply this "multiplication principle" in more creative ways to count ways of "constructing" outcomes in various events

The idea is to write down a systematic, multi-step procedure for constructing outcomes and see if we can apply the mult. principle.

E.g. consider the event C = "Exactly 1 Wh/odd" in the Polya Urn example.

Each such outcome has a single Wh (i.e. 1) and 2 Blacks (either (2,2) OR (2,4)) in some order. So a way of constructing such an outcome is:

1) Choose which of (2,2) or (2,4) it has (2 choices)

2) Pick a position for the 1 (3 choices)

These ~~No. cho~~ completely determine the outcome.

there are thus $2 \times 3 = 6$ different outcomes in C (as we saw using an exhaustive listing).

We have thus determined $P(C) = 6/24 = 1/4$ purely using the mult. principle (and some creative thought). The nice thing is that this method can be easily extended to large-scale problems, while exhaustive listing cannot (See Tutorial 1).