Semester 1 2012 Lecturer: Michael Stewart

Tutorial Week 1

1. A "classical probability scenario" is where the sample space S consists of a finite positive number of *equally likely outcomes*. Then for any event $A \subset S$,

$$P(A) = \frac{\text{no. outcomes in } A}{\text{no. outcomes in } S}.$$

Use this definition directly to prove that the following properties always hold for such a scenario (remember the empty set $\emptyset$ and the whole sample space S are both always considered events):

(a) for any $A \subset S$, $P(A) \geq 0$;

(b) $P(S) = 1$;

(c) for any A and B with $A \cap B = \emptyset$, $P(A \cup B) = P(A) + P(B)$;

(d) if $A \subset B \subset S$ then $P(A) \leq P(B) \leq 1$;

(e) $P(A^c) = 1 - P(A)$;

(f) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

2. [*A little more challenging*] Show that (d), (e) and (f) above can be verified using (a), (b), (c) above and properties of events only (i.e. not needing to know anything more about $P(\cdot)$ other than it obeys (a), (b) and (c)).

3. Suppose we have 4 balls numbered 1,2,3,4, the odd numbered ones coloured white, the evens black and they are all placed in an urn. Consider drawing out 3 balls in order *without replacement*. Define the events

- $A =$ "first ball is white",
- $B =$ "all balls the same colour" and
- $C =$ "exactly 1 ball is white".

Determine their probabilities in two different ways:

(a) Use a tree diagram/exhaustive listing approach to firstly list all outcomes in the sample space and then count directly how many of these occur in each event.

(b) Use a "multiplication principle" approach to determine the number of outcomes in

   i. the sample space and

   ii. each of A, B and C

   which does not require exhaustively listing all possible outcomes.

4. Suppose we have an urn containing 10 white balls and 10 black balls. Five draws are made: a ball is drawn out, its colour noted and then replaced, along with another ball of the same colour. Suppose each possible sequence of balls is equally likely. What is the probability that

(a) the first ball is white;

(b) all balls are the same colour;

(c) exactly 1 ball is white?

5. Repeat the previous question if the sampling is instead done *without replacement*.