Show that the set H of all points in $\mathscr{R}^2$ of the form (3s, 2+5s) is not a vector space by showing that it is not closed under scalar multiplication. (Find a specific vector **u** in H and a scalar *c* such that *c***u** is not in H.)

Determine if the set H of all matrixes of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ is a subspace of M_{2x2}.

Let W be the set of all vectors below. *a*, *b*, *c* represent arbitrary real numbers. In each problem below, either find a set S of vectors that spans W or give an example to show that W is not a vector space.

$$\begin{bmatrix} a - b \\ b - c \\ c - a \\ b \end{bmatrix}$$

$$\begin{bmatrix} -a + 1 \\ a - 6b \\ 2b + a \end{bmatrix}$$

Let H be the set of all vectors of the form $\begin{bmatrix} s \\ 3s \\ 2s \end{bmatrix}$
Find a vector v in $\mathscr{R}^3$ such that H = Span { **v** }.
Why does this show that H is a subspace of $\mathscr{R}^3$?

Let H be the set of all vectors of the form $\begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix}$
Show that H is a subspace of $\mathscr{R}^3$.

Let W be the set of all vectors of the form $\begin{bmatrix} 5b + 2c \\ b \\ c \end{bmatrix}$ where b and c are arbitrary.
Find vectors **u** and **v** such that W = Span { **u**, **v** }.
Why does this show that W is a subspace of $\mathscr{R}^3$?

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$ $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

1. Is **w** in { $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ }? How many vectors are in { $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ }?

2. How many vectors are in Span { $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ }?

3. Is **w** in the subspace spanned by { $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ }? Why?

Using $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ from above; let $\mathbf{w} = \begin{bmatrix} 8 \\ 4 \\ 7 \end{bmatrix}$
Is **w** in the subspace spanned by { $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ }? Why?

Determine if **y** is in the subspace of $\mathscr{R}$ 4 spanned by the columns of A, where

$$\mathbf{y} = \begin{bmatrix} 6 \\ 7 \\ 1 \\ -4 \end{bmatrix} \quad \& \quad A = \begin{bmatrix} 5 & -5 & -9 \\ 8 & 8 & -6 \\ -5 & -9 & 3 \\ 3 & -2 & -7 \end{bmatrix}$$

Consider the following system of homogenous equations:

$$\begin{aligned} +x_1 \quad -3x_2 \quad -2x_3 &= 0 \\ -5x_1 \quad +9x_2 \quad +x_3 &= 0 \end{aligned}$$

Write this system in matrix form.

Determine if **u** belongs to the null space of A. $\mathbf{u} = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$

Let *H* be the set of all vectors in $\mathscr{R}^4$ whose coordinates *a*, *b*, *c*, *d* satisfy the equations

- a - 2b + 5c = d, and
- c - a = b

Show that *H* is a subspace of $\mathscr{R}^4$.

Find a spanning set for the null space of the matrix A. $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

Find a matrix A such that W = Col A. $W = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \text{ in } \mathscr{R}^n \right\}$

Let $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$

If the column space of A is a subspace of $\mathscr{R}^k$, what is k?
If the null space of A is a subspace of $\mathscr{R}^k$, what is k?

Using A from above, let **u** and **v** $\longmapsto$ $\quad \mathbf{u} = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$

Determine if **u** is in Nul A. Could **u** be in Col A?
Determine if **v** is in Col A. Could **v** be in Nul A?

Using A from above, find a nonzero vector in Col A and a nonzero vector in Nul A.

Consider the following two systems of equations:

$$\begin{aligned} +5x_1 + 1x_2 - 1x_3 &= 0 \\ -9x_1 + 2x_2 + 5x_3 &= 1 \\ +4x_1 + 1x_2 - 6x_3 &= 9 \end{aligned} \quad \& \quad \begin{aligned} +5x_1 + 1x_2 - 3x_3 &= 0 \\ -1x_1 + 2x_2 + 5x_3 &= 5 \\ +4x_1 + 1x_2 - 6x_3 &= 45 \end{aligned}$$

It can be shown that the first system has a solution. Use this fact to explain why the second system must also have a solution (without making row operations).

Determine whether **w** is in the column space of A, the null space of A, or both, where

$$w = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -3 \end{bmatrix} \quad \& \quad A = \begin{bmatrix} 7 & 6 & -4 & 1 \\ -5 & -1 & 0 & -2 \\ 9 & -11 & 7 & -3 \\ 19 & -9 & 7 & 1 \end{bmatrix}$$

Given an **m** x **n** matrix A, an element in Col A has the form A**x** for some **x** in $\mathscr{R}^n$. Let A**x** and A**w** represent any two vectors in Col A.

1. Explain why the **zero vector** is in Col A.

2. Show that the vector A**x** + A**w** is in Col A.

3. Given a scalar *c*, show that *c*(Ax) is in Col A.

Find an explicit description of Nul A by listing vectors that span the null space.

$$A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & 0 & 4 & 0 \\ 0 & 0 & 1 & -9 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Determine if **w** is in Nul A.

$$w = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix} \qquad A = \begin{bmatrix} 5 & 21 & 19 \\ 13 & 23 & 2 \\ 8 & 14 & 1 \end{bmatrix}$$

Find A such that the given set is Col A.

$$\left\{ \begin{bmatrix} 2s + 3t \\ r + s - 2t \\ 4r + s \\ 3r - s - t \end{bmatrix} : r, s, t \text{ real} \right\} \qquad \left\{ \begin{bmatrix} b - c \\ 2b + c + d \\ 5c - 4d \\ d \end{bmatrix} : b, c, d, \text{ real} \right\}$$

Let $A = \begin{bmatrix} 7 & -3 & 5 \\ -4 & 1 & -5 \\ -5 & 2 & -4 \end{bmatrix}$ $\quad \mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ $\quad \mathbf{w} = \begin{bmatrix} 7 \\ 6 \\ -3 \end{bmatrix}$

Suppose you know that the equations $A\mathbf{x} = \mathbf{v}$ and $A\mathbf{x} = \mathbf{w}$ are both consistent.
What can you say about the equation $A\mathbf{x} = \mathbf{v} + \mathbf{w}$?

Let $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a - 3b - c = 0 \right\}$

Show that W is a subspace of $\mathscr{R}^3$ in two different ways.