The Evolution of the Goodwin Economics Model

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1 Introduction

Exogenous economics models are those that assume the economy is stable and operates in equilibrium, such that only external factors can cause a potential crash (Ganti, 2019). However, the real world reflects that a macro-economy can itself destabilize due to internal factors such as wage share, employment rate, and private debt (Minsky, 1992). For this, R. M. Goodwin proposed an endogenous economics model in 1967 that, at its core, mimics Lotka-Volterra prey-predator dynamics (Goodwin, 1982). The extension of Goodwin's model by Keen (1995) focused on modeling Hayman Minsky's "financial instability hypothesis". In essence, Minsky argues that the economy can fall into a crisis given an accumulation of debt by the private sector (Minsky, 1992). This Goodwin-Keen model looks into the impact of three parameters on a simplified macro-economy: employment rate (λ), wage share (ω), and private debt (d) (Grasselli and Lima, 2012; Maheshwari, 2015).

2 Proposal

Through the framework of endogenous economic fluctuation models (Boldrin and Woodford, 1990), the potential long-term equilibrium of the Canadian economy is thoroughly investi-

gated for the purposes of assessing the impact of wage share, employment rate, and private debt on economic stability. For the purposes of this project, economic stability is defined as the long-term equilibrium between labour share and employment (Weitzman, 1983). To do so, we use the Goodwin growth model and later the extension into the Goodwin-Keen model. While Goodwin's original model seeks to explain the dynamics between wage share and employment (Goodwin, 1982), Keen's addendum to incorporate Minsky's thesis aims to estimate the convergence of the economy to a stable or unstable long-term equilibrium by introducing the role of debt in the private sector (Keen, 1995). These predictions will be made using previous data from Statistics Canada. The following equations describe the Goodwin-Keen model, as per Grasselli and Lima (2012). Please refer to the Nomenclature section for a description of used parameters (Grasselli and Lima, 2012; Maheshwari, 2015).

$$\begin{aligned}
\frac{\mathrm{d}\lambda}{\mathrm{d}t} &= \lambda \cdot \left(\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta \right) \\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \omega \cdot (\Phi(\lambda) - \alpha) \\
\frac{\mathrm{d}d}{\mathrm{d}t} &= d \cdot \left(r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta \right) + \kappa(1-\omega-rd) - (1-\omega).
\end{aligned}$$

We note that $\lambda(t)$ and $\omega(t)$ are unknowns; hence, we want to determine their behaviour as functions of time based on a set of initial conditions (λ_0, ω_0). Knowing these, we can assess the economic stability as defined by:

$$\boldsymbol{\varepsilon} = \lim_{t \to \infty} [\lambda(t), \omega(t)].$$

Our research project will focus on studying the long-term equilibrium impact that these real variables can have on the simplified macro-economy. To study the long-term dynamics of the models, we will test each model at various initial conditions and conduct sensitivity analysis to systematically test the effects of parameter values on the equilibrium (e.g. the population growth rate and labour productivity). Finally, we plan to determine regions of convergence towards equilibrium by using phase plane analysis to evaluate the stability of the system.

References

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Nomenclature

λ	Employment rate
ω	Wage share
d	Debt ratio
α	The exponential growth of disembodied productivity growth rate, Technological growth
β	Population growth rate
δ	Constant depreciation rate
κ	Acceleration relation for the total real capital stock
ν	Constant capital-to-output ratio
$\Phi(\lambda)$	Function of the rate of employment, Phillips curve
r	Constant real interest rate