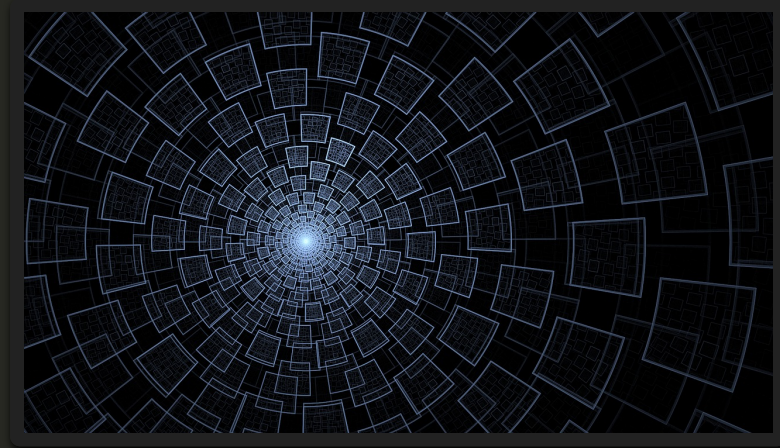


# *A Little Logic*

*arguments, reasoning and proof*



George Matthews, Pennsylvania College of Technology

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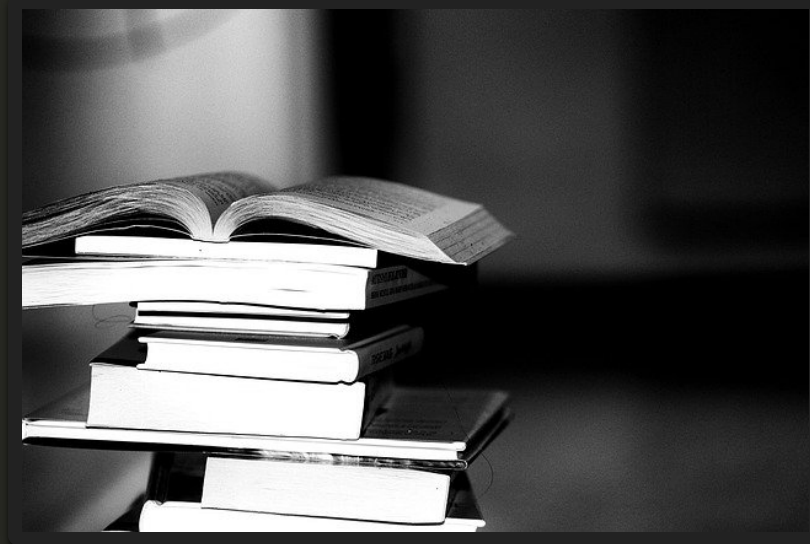
# *What do I know?*

Knowledge is

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- How can we know anything at all? This ancient philosophical question has troubled many people.
- First, note that knowledge is a state of mind, a type of *belief*. But there's more to knowledge than belief.

# *What do I know?*

Knowledge is  
justified,  
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- To count as knowledge our beliefs have to be *true*.
- While it may be hard to figure out just what the facts are, simply believing something strongly is not enough to make it true.

# *What do I know?*

Knowledge is

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- Finally we need *justification* for anything to really count as knowledge since knowledge is more than just lucky guessing.
- How we can justify our claims is one of the concerns of logic.

# *Reasoning & logic*

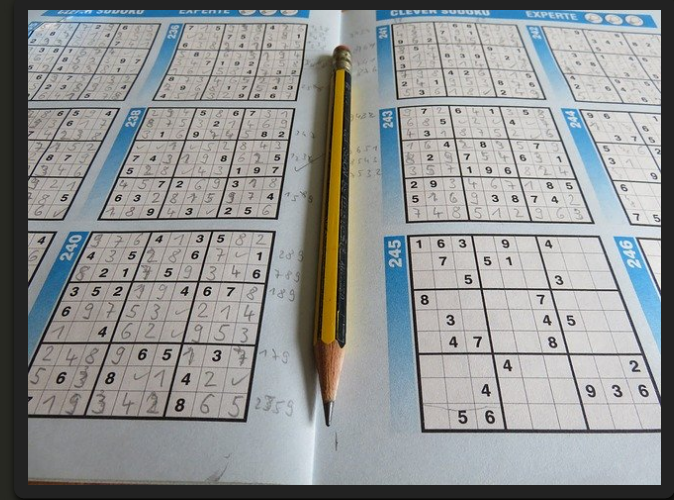
## deduction

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- It shows what else can we figure out based on what we already know.
- Deductive reasoning demonstrates the necessary consequences of given information.

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Sudoku puzzles are pure deduction, no guesswork is required.

# *Reasoning & logic*

## induction

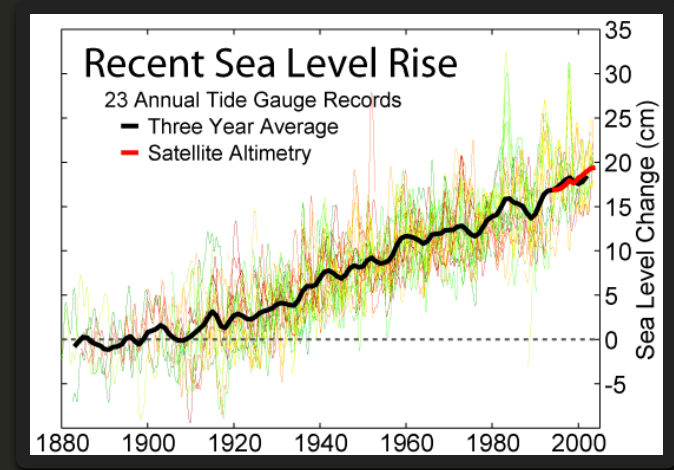
- Inductive logic is the logic of *data analysis*.
- It shows what is likely to be true given the data that we have.
- In spite of inductive reasoning being weaker than proof we rely on this kind of reasoning every day.



# Reasoning & logic

## induction

- Inductive logic is the logic of *data analysis*.
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Probability theory and statistics enable us to make reliable guesses with incomplete information.

# *Reasoning & logic*

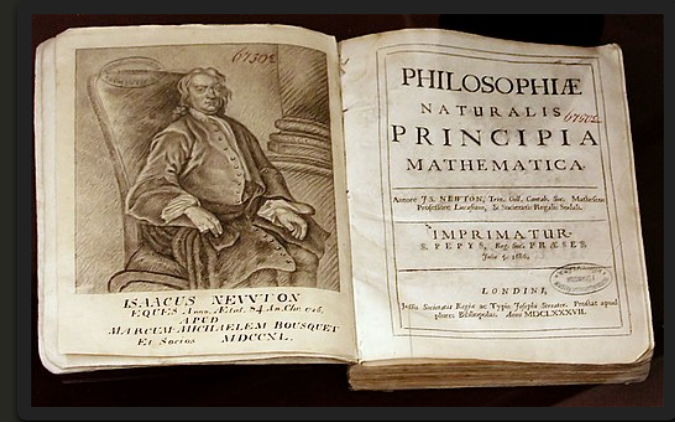
## abduction

- Abductive reasoning is reasoning about the *best explanation* for something.
- It (tries to) show why one way of explaining the facts is better than other ways.
- One explanation is typically better than another when it is simpler and has more predictive power.

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Scientific progress happens when scientists come up with better explanations. Isaac Newton did just that when he showed how all motion in the heavens and on earth followed a few basic laws.

## *Basic concepts*

All humans are mortal.

Socrates is human.

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The premises are the information we start out with and the conclusion is what we claim *logically follows from* that information.

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*note:* I'll write technical terms in all capitals -- it's not shouting, just a reminder that we are using these words in particular ways. See **glossary** for definitions.

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So is this argument VALID?

- To check we *assume* that the premises are **TRUE** (don't worry we'll get rid of that assumption later).
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- If not, then the argument is **VALID**, as this one clearly is.

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- So this argument is **SOUND**!

## *More examples*

All cats are animals.

My brother is an animal.

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- Does the conclusion **have to be true** as well?
- Clearly not, just ask him. Sound it is INVALID and thus also UNSOUND.
- The premises here refer to three *different* categories which are related in some way, but not in the way the conclusion states.

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All cats are fish.

All fish are made of wood.

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- Yes, so it is **VALID**.
- If cats were part of the larger class "fish" and fish were themselves part of a larger class "things made of wood" then cats would have to be made of wood. (This is a mini PROOF of validity, more on that later.)

## *Proving validity*

Either Fred or Betty killed Mr. Slate at the quarry.

Either Fred or Wilma were home since somebody let Dino in.

But Wilma was working late.

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note: Even though the argument is VALID it is not SOUND, since it is a *fictional* case.

# *Counterexamples*



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Wilma is younger than Fred.

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a counterexample

person	age
Barney	35
Betty	40
Fred	45
Wilma	38

# Glossary

- **Argument:** a series of statements in which the premises are intended to logically support the conclusion.
- **Valid:** an argument is valid when it is impossible for the premises to be true and the conclusion false.
- **Sound:** an argument is sound when it is valid and has true premises.
- **True:** said only of statements, not of arguments, and surprisingly hard to define.
- **False:** the opposite of true and once again said only of individual statements, not arguments.
- **Proof:** a step by step demonstration of the validity of an argument. In a proof we spell out exactly how we can derive the conclusion from nothing but the information contained in the premises.
- **Counterexample:** a possible case in which the premises of an argument are true and the conclusion is false -- used to show that an argument is invalid.

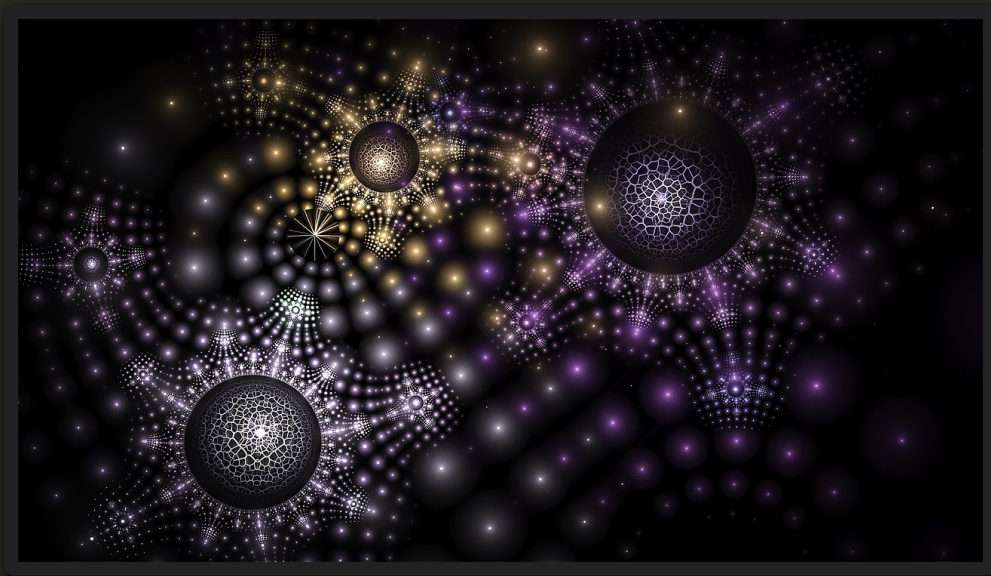
## *Find out more*

**Critical Thinking Web:** A great site with over 100 free tutorials on many aspects of logic and critical thinking. A nice way to hone your logical thinking skills.

**Deductive and Inductive Arguments:** An in depth look at the subject at the Internet Encyclopedia of Philosophy.

**Abduction:** A close look at the logic of scientific explanation. Gets technical, but the introduction is accessible.

**The Irrationality of Politics:** Michael Huemer is a professor of philosophy at the University of Colorado. This TED Talk by him addresses the question of why we are so irrational when it comes to politics.



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