

## PART A - MATHEMATICS

1. If S is the set of distinct values of 'b' for which the following system of linear equations

$$x + y + z = 1$$

$$x + ay + z = 1$$

$$ax + by + z = 0$$

has no solution, then S is:

- (1) an empty set (2) an infinite set
- (3) a finite set containing two or more elements (4) a singleton

*Sol.* (4)

$$\text{Here, } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0 \Rightarrow a = 1$$

For  $a = 1$ , the equations become

$$x + y + z = 1$$

$$x + y + z = 1$$

$$x + by + z = 0$$

These equations give no solution for  $b = 1$

$\Rightarrow S$  is singleton set

- \*2. The following statement  $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$  is:

- (1) a tautology (2) equivalent to  $\sim p \rightarrow q$
- (3) equivalent to  $p \rightarrow \sim q$  (4) a fallacy

*Sol.* (1)

$$\begin{aligned} (p \rightarrow q) \rightarrow ((\sim p \rightarrow q) \rightarrow q) \\ = (p \rightarrow q) \rightarrow ((p \vee q) \rightarrow q) \\ = (p \rightarrow q) \rightarrow ((\sim p \wedge \sim q) \vee q) \\ = (p \rightarrow q) \rightarrow ((\sim p \vee q) \wedge (\sim q \vee q)) \\ = (p \rightarrow q) \rightarrow (\sim p \vee q) \\ = (p \rightarrow q) \rightarrow (p \rightarrow q) \\ = T \end{aligned}$$

- \*3. If  $5(\tan^2 x - \cos^2 x) = 2 \cos 2x + 9$ , then the value of  $\cos 4x$  is:

$$(1) -\frac{3}{5}$$

$$(2) \frac{1}{3}$$

$$(3) \frac{2}{9}$$

$$(4) -\frac{7}{9}$$

*Sol.* (4)

$$5(\sec^2 x - 1 - \cos^2 x) = 2(2\cos^2 x - 1) + 9$$

$$5\left(\frac{1}{t} - 1 - t\right) = 2(2t - 1) + 9$$

$$5(1 - t - t^2) = 4t^2 + 7t$$

$$\Rightarrow 9t^2 + 12t - 5 = 0$$

$$t = \frac{1}{3}, -\frac{5}{3}$$

$$\Rightarrow \cos^2 x = \frac{1}{3}$$

$$\Rightarrow \cos 2x = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$\Rightarrow \cos 4x = -\frac{7}{9}$$

4. For three events A, B and C,  $P(\text{Exactly one of A or B occurs}) = P(\text{Exactly one of B or C occurs}) = P(\text{Exactly one of C or A occurs}) = \frac{1}{4}$  and  $P(\text{All the three events occur simultaneously}) = \frac{1}{16}$ . Then the probability that at least one of the events occurs, is:

(1)  $\frac{7}{32}$   
(3)  $\frac{7}{64}$

(2)  $\frac{7}{16}$   
(4)  $\frac{3}{16}$

**Sol.** (2)

$$P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$$

$$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$$

$$P(A) + P(C) - 2P(A \cap C) = \frac{1}{4}$$

$$\Rightarrow \sum P(A) - \sum P(A \cap B) = \frac{3}{8}$$

$$\Rightarrow P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$$

$$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}.$$

- \*5. Let  $\omega$  be a complex number such that  $2\omega + 1 = z$  where  $z = \sqrt{-3}$ . If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k, \text{ then } k \text{ is equal to:}$$

(1)  $-z$   
(3)  $-1$   
(2)  $z$   
(4)  $1$

**Sol.** (1)

Determinant simplifies to  $3k = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$

$$= 3 \begin{vmatrix} 1 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$$

$$= -3z$$

$$\Rightarrow k = -z$$

- \*6. Let  $k$  be an integer such that the triangle with vertices  $(k, -3k)$ ,  $(5, k)$  and  $(-k, 2)$  has area 28 sq. units. Then the orthocentre of this triangle is at the point:

(1)  $\left(2, -\frac{1}{2}\right)$

(2)  $\left(1, \frac{3}{4}\right)$

(3)  $\left(1, -\frac{3}{4}\right)$

(4)  $\left(2, \frac{1}{2}\right)$

*Sol.* (4)

$$\Delta = \frac{1}{2} \begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = 28$$

$$\Rightarrow k = 2 \text{ (since } k \in \mathbb{I})$$

$$\Rightarrow \text{Orthocentre is } \left(2, \frac{1}{2}\right)$$

- \*7. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is:

(1) 12.5  
(3) 25

(2) 10  
(4) 30

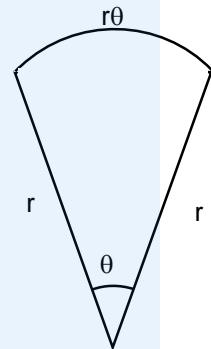
*Sol.* (3)

$$\text{Length of wire} = r(\theta + 2) = 20 \text{ m}$$

$$\text{Area } A = \frac{\theta}{2}r^2$$

$$\Rightarrow A(r) = 10r - r^2$$

$$\Rightarrow \text{Area is maximum if } r = 5. \\ \text{Maximum area } A = 25 \text{ sq. m}$$



8. The area (in sq. units) of the region  $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$  is:

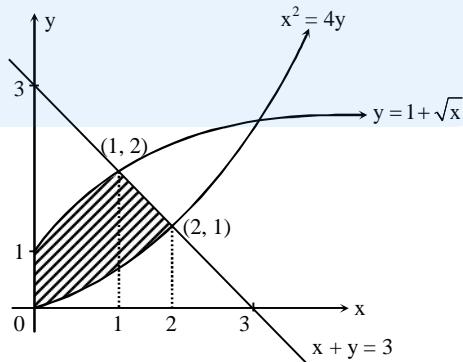
(1)  $\frac{59}{12}$   
(3)  $\frac{7}{3}$

(2)  $\frac{3}{2}$   
(4)  $\frac{5}{2}$

*Sol.* (4)

Required area

$$\begin{aligned} &= \int_0^1 (1 + \sqrt{x}) dx + \frac{1}{2}(3 \times 1) - \int_0^2 \frac{x^2}{4} dx \\ &= \frac{5}{2} \text{ sq. units} \end{aligned}$$



*Sol.* (2)

$$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda$$

Let midpoint of PQ be M which lies on the plane

$$\Rightarrow M(x, y, z) = (1 + \lambda, 4\lambda - 2, 5\lambda + 3)$$

$$2(1 + \lambda) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$$

$$\Rightarrow -6\lambda + 6 = 0 \Rightarrow \lambda = 1$$

$$\Rightarrow M(2, 2, 8), P(1, -2, 3)$$

$$PM = \sqrt{1+16+25} = \sqrt{42}$$

$$\text{PO} = 2\sqrt{42}.$$

10. If for  $x \in \left(0, \frac{1}{4}\right)$ , the derivative of  $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$  is  $\sqrt{x} \cdot g(x)$ , then  $g(x)$  equals:

$$(1) \quad \frac{9}{1+9x^3}$$

$$(3) \quad \frac{3x}{1-9x^3}$$

$$(2) \quad \frac{3x\sqrt{x}}{1-9x^3}$$

$$(4) \quad \frac{3}{1+9x^3}$$

*Sol.* (1)

Here,  $y = 2 \tan^{-1} 3x^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{9x^{\frac{1}{2}}}{1+9x^3} = \sqrt{x}g(x)$$

$$\Rightarrow g(x) = \frac{9}{1+9x^3}$$

11. If  $(2 + \sin x) \frac{dy}{dx} + (y + 1)\cos x = 0$  and  $y(0) = 1$ , then  $y\left(\frac{\pi}{2}\right)$  is equal to:

(1)  $\frac{1}{3}$

$$(2) -\frac{2}{3}$$

$$(3) - \frac{1}{3}$$

$$(4) \quad \frac{4}{3}$$

*Sol.* (1)

$$(2 + \sin x)dy + \cos x(y + 1)dx = 0$$

$$(y + 1)(2 + \sin x) = C$$

$$\Rightarrow (1+1)(2+0) = C = 4$$

$$(y+1) \cdot (2 + \sin x) = 4$$

Put  $x = \frac{\pi}{2}$

$$y = \frac{1}{3}$$

- \*12. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that  $AP = 2AB$ . If  $\angle BPC = \beta$ , then  $\tan \beta$  is equal to:

(1)  $\frac{6}{7}$

(2)  $\frac{1}{4}$

(3)  $\frac{2}{9}$

(4)  $\frac{4}{9}$

**Sol.** (3)

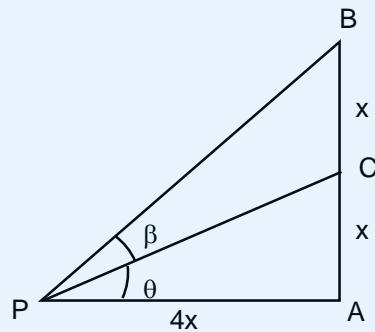
$$\tan(\theta + \beta) = \frac{1}{2}$$

$$\text{and } \tan \theta = \frac{1}{4}$$

$$\Rightarrow \frac{\frac{1}{4} + \tan \beta}{1 - \frac{1}{4} \tan \beta} = \frac{1}{2}$$

$$\Rightarrow 9 \tan \beta = 2$$

$$\Rightarrow \tan \beta = \frac{2}{9}$$



13. If  $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$ , then  $\text{adj}(3A^2 + 12A)$  is equal to:

(1)  $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(2)  $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(3)  $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(4)  $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

**Sol.** (2)

$$\begin{aligned} A^2 &= \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} \end{aligned}$$

$$(3A^2 + 12A) = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{Adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}.$$

- \*14. For any three positive real numbers a, b and c,  $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$ . Then:

(1) b, c and a are in G.P.

(2) b, c and a are in A.P.

(3) a, b and c are in A.P.

(4) a, b and c are in G.P.

**Sol.** (2)

$$(15a - 3b)^2 + (15a - 5c)^2 + (3b - 5c)^2 = 0$$

$$\text{Let, } 15a = 3b = 5c = 45\lambda$$

$$\Rightarrow a = 3\lambda; b = 15\lambda; c = 9\lambda$$

$$\Rightarrow 2c = a + b$$

$$b, c, a \text{ are in A.P.}$$

15. The distance of the point  $(1, 3, -7)$  from the plane passing through the point  $(1, -1, -1)$ , having normal perpendicular to both the lines  $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$  and  $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$ , is:

(1)  $\frac{20}{\sqrt{74}}$

(2)  $\frac{10}{\sqrt{83}}$

(3)  $\frac{5}{\sqrt{83}}$

(4)  $\frac{10}{\sqrt{74}}$

**Sol.** (2)

Equation of plane is  $\begin{vmatrix} x-1 & y+1 & z+1 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 0$

$5x + 7y + 3z + 5 = 0$

Distance from  $(1, 3, -7)$  is  $\frac{|5+21-21+5|}{\sqrt{83}} = \frac{10}{\sqrt{83}}$

16. Let  $I_n = \int \tan^n x \, dx$ , ( $n > 1$ ). If  $I_4 + I_6 = a \tan^5 x + bx^5 + C$ , where  $C$  is a constant of integration, then the ordered pair  $(a, b)$  is equal to:

(1)  $\left(-\frac{1}{5}, 1\right)$

(2)  $\left(\frac{1}{5}, 0\right)$

(3)  $\left(\frac{1}{5}, -1\right)$

(4)  $\left(-\frac{1}{5}, 0\right)$

**Sol.** (2)

$$I_n = \int \tan^n x \, dx, n > 1$$

$$= \int \tan^{n-2} x (\sec^2 x - 1) \, dx$$

$$= \frac{\tan^{n-1} x}{n-1} - I_{n-2} + C$$

$$\Rightarrow I_n + I_{n-2} = \frac{\tan^{n-1} x}{n-1} + C$$

$$\Rightarrow I_6 + I_4 = \frac{\tan^5 x}{5} + C$$

$$\text{Given, } I_4 + I_6 = a \tan^5 x + bx^3 + C$$

$$\Rightarrow a = \frac{1}{5}, b = 0$$

- \*17. The eccentricity of an ellipse whose centre is at the origin is  $\frac{1}{2}$ . If one of its directrices is  $x = -4$ , then the

equation of the normal to it at  $\left(1, \frac{3}{2}\right)$  is:

(1)  $2y - x = 2$

(2)  $4x - 2y = 1$

(3)  $4x + 2y = 7$

(4)  $x + 2y = 4$

**Sol.** (2)

$$\text{Eccentricity, } e = \frac{1}{2}$$

Let  $2a$  be the length of major axis and  $2b$  be the length of minor axis

$$\Rightarrow \frac{a}{e} = 4$$

$$\Rightarrow a = 2$$

$$\text{Also, } b = \sqrt{3}, \text{ as } e = \frac{1}{2}$$

$$\Rightarrow \text{Equation of ellipse is } \frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\Rightarrow \text{Equation of normal at } \left(1, \frac{3}{2}\right) \text{ is } 4x - 2y = 1$$

- \*18. A hyperbola passes through the point  $P(\sqrt{2}, \sqrt{3})$  and has foci at  $(\pm 2, 0)$ . Then the tangent to this hyperbola at  $P$  also passes through the point:

$$(1) (3\sqrt{2}, 2\sqrt{3})$$

$$(2) (2\sqrt{2}, 3\sqrt{3})$$

$$(3) (\sqrt{3}, \sqrt{2})$$

$$(4) (-\sqrt{2}, -\sqrt{3})$$

**Sol.** (2)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{4-a^2} = 1$$

$$\Rightarrow a^2 = 8, 1, (a^2 \neq 8)$$

$$\Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1.$$

$$\text{Hence equation of tangent at } P(\sqrt{2}, \sqrt{3}) \text{ is } \frac{x\sqrt{2}}{1} - \frac{y\sqrt{3}}{3} = 1$$

$$\Rightarrow \sqrt{6}x - y = \sqrt{3}$$

19. The function  $f : R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$  defined as  $f(x) = \frac{x}{1+x^2}$ , is:

(1) invertible.

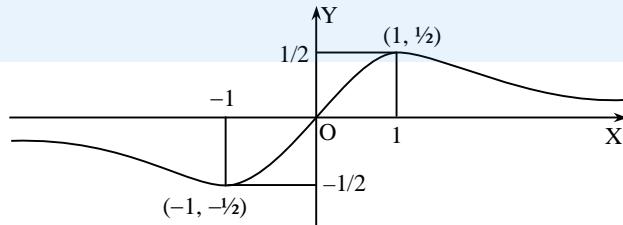
(2) injective but not surjective.

(3) surjective but not injective.

(4) neither injective nor surjective.

**Sol.** (3)

For,  $f(x) = \frac{x}{1+x^2}$  the curve has graph as shown



Which is onto but not one-one for,  $f : R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$

20.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$  equals:

(1)  $\frac{1}{24}$

(2)  $\frac{1}{16}$

(3)  $\frac{1}{8}$

(4)  $\frac{1}{4}$

*Sol.* (2)

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x - \cot x}{8\left(x - \frac{\pi}{2}\right)^3}$$

$$\text{Put } x - \frac{\pi}{2} = t; x = t + \frac{\pi}{2}$$

$$\lim_{t \rightarrow 0} \frac{1}{8} \cdot \frac{-\sin t + \tan t}{t^3}$$

$$\lim_{t \rightarrow 0} \frac{1}{8} \cdot \frac{\sin t(1 - \cos t)}{t \cdot \cos t \cdot t^2}$$

$$= \frac{1}{8} \cdot 1 \cdot \frac{1}{2} = \frac{1}{16}$$

21. Let  $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$  and  $\vec{b} = \hat{i} + \hat{j}$ . Let  $\vec{c}$  be a vector such that  $|\vec{c} - \vec{a}| = 3$ ,  $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$  and the angle between  $\vec{c}$  and  $\vec{a} \times \vec{b}$  be  $30^\circ$ . Then  $\vec{a} \cdot \vec{c}$  is equal to:

(1)  $\frac{25}{8}$

(2) 2

(3) 5

(4)  $\frac{1}{8}$

*Sol.* (2)

$$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c} = 9$$

$$\Rightarrow |\vec{c}|^2 - 2\vec{a} \cdot \vec{c} = 0$$

$$\text{and } |\vec{a} \times \vec{b}| \cdot |\vec{c}| \cdot \sin 30^\circ = 3$$

$$\Rightarrow 3 \times |\vec{c}| \times \frac{1}{2} = 3$$

$$\Rightarrow |\vec{c}| = 2$$

$$\therefore \vec{a} \cdot \vec{c} = 2$$

22. The normal to the curve  $y(x - 2)(x - 3) = x + 6$  at the point where the curve intersects the y-axis passes through the point:

(1)  $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(2)  $\left(\frac{1}{2}, \frac{1}{2}\right)$

(3)  $\left(\frac{1}{2}, -\frac{1}{3}\right)$

(4)  $\left(\frac{1}{2}, \frac{1}{3}\right)$

**Sol.** (2)

$$\frac{dy}{dx} \text{ at } x = 0 \text{ is } 1$$

$\Rightarrow$  Slope of normal at (0, 1) is -1

$\Rightarrow$  Equation of normal is  $x + y = 1$

23. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is:

(1)  $\frac{6}{55}$

(2)  $\frac{12}{55}$

(3)  $\frac{14}{45}$

(4)  $\frac{7}{55}$

**Sol.** (1)

Consider two sequences : 0, 4, 8 and 2, 6, 10

Take both numbers from either of these sequences.

$$\text{Hence, probability} = \frac{{}^3C_2 + {}^3C_2}{{}^{11}C_2} = \frac{6}{55}.$$

- \*24. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is:

(1) 485  
(3) 469

(2) 468  
(4) 484

**Sol.** (1)

X : 4L, 3M; Y : 3L, 4M

Possible combinations

	(1)	(2)	(3)	(4)
X	3L	2L, 1M	1L, 2M	3M
Y	3M	1L, 2M	2L, 1M	3L

$$\therefore \text{Number of ways} = {}^4C_3 \cdot {}^4C_3 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^3C_3 \cdot {}^3C_3 \\ = 485$$

- \*25. The value of  $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \dots + ({}^{21}C_{10} - {}^{10}C_{10})$  is:

(1)  $2^{21} - 2^{11}$   
(3)  $2^{20} - 2^9$

(2)  $2^{21} - 2^{10}$   
(4)  $2^{20} - 2^{10}$

**Sol.** (4)

$$\text{Let } S = ({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + \dots + ({}^{21}C_{10} - {}^{10}C_{10})$$

$$\Rightarrow S = ({}^{21}C_0 + {}^{21}C_1 + \dots + {}^{21}C_{10}) - ({}^{10}C_0 + {}^{10}C_1 + \dots + {}^{10}C_{10})$$

$$\Rightarrow S = 2^{20} - 2^{10}.$$

26. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is:

(1)  $\frac{12}{5}$

(2) 6

(3) 4

(4)  $\frac{6}{25}$

**Sol.**

$$p = \frac{15}{25}, q = \frac{10}{25}, n = 10$$

$$\sigma^2 = npq = 10 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{5}.$$

27. Let  $a, b, c \in \mathbb{R}$ . If  $f(x) = ax^2 + bx + c$  is such that  $a + b + c = 3$  and  $f(x + y) = f(x) + f(y) + xy, \forall x, y \in \mathbb{R}$ ,

then  $\sum_{n=1}^{10} f(n)$  is equal to:

- (1) 330  
(3) 190

- (2) 165  
(4) 255

**Sol.****(1)**

Partially differentiating, we get

$$f'(x) - x = \text{constant} = \lambda$$

$$\Rightarrow f(x) = \frac{x^2}{2} + \lambda x + k$$

$$f(0) = 0 \Rightarrow k = 0$$

$$\frac{1}{2} + \lambda = 3 \Rightarrow \lambda = \frac{5}{2}$$

$$\sum_{n=1}^{10} f(n) = a \sum_{n=1}^{10} n^2 + b \sum_{n=1}^{10} n$$

$$= \frac{1}{2} \frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \times \frac{n(n+1)}{2}$$

$$= \frac{1}{2} \cdot \frac{10 \times 11 \times 21}{6} + \frac{5}{2} \times \frac{10 \times 11}{2} = 330.$$

- \*28. The radius of a circle, having minimum area, which touches the curve  $y = 4 - x^2$  and the lines,  $y = |x|$  is:

- (1)  $2(\sqrt{2} + 1)$   
(3)  $4(\sqrt{2} - 1)$

- (2)  $2(\sqrt{2} - 1)$   
(4)  $4(\sqrt{2} + 1)$

**Sol.****(3)**

Let  $P\left(\frac{t}{2}, 4 - \frac{t^2}{4}\right)$  be point where circle touches the parabola  $y = 4 - x^2$

$\Rightarrow$  Normal at  $P$ :  $ty - x + \frac{t^3}{4} - \frac{7t}{2} = 0$  to the parabola

passes through centre ( $C$ ) of the circle  $(0, \beta)$ .

$$\Rightarrow t^3 - 14t + 4\beta t = 0 \quad \dots\dots (1)$$

$$\text{Also, radius } r = \frac{|\beta|}{\sqrt{2}}$$

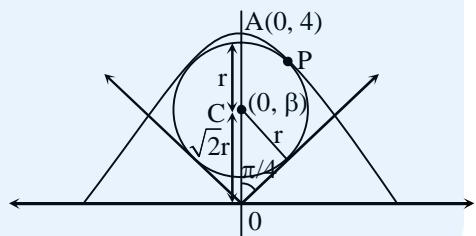
$$\Rightarrow t^4 + 4t^2 + 8\beta t^2 - 32t^2 - 128\beta + 256 + 16\beta^2 = 16r^2$$

$$\Rightarrow t^4 + (8\beta - 28)t^2 - 128\beta + 256 + 8\beta^2 = 0 \quad \dots\dots (2)$$

From equation (1) and (2), we get

Either  $\beta = 8 \pm 4\sqrt{2}$  for  $t = 0$

$$\text{or } \beta = \frac{-\sqrt{2} \pm \sqrt{17}}{\sqrt{2}} \text{ for } t^2 = 14 - 4\beta$$



$$\text{As, } r = \frac{|\beta|}{\sqrt{2}} \Rightarrow r = 4\sqrt{2} \pm 4, \frac{\sqrt{17} - \sqrt{2}}{2}$$

$$\Rightarrow \text{Minimum possible radius, } r = \frac{\sqrt{17} - \sqrt{2}}{2}$$

[But of the given options  $r = 4(\sqrt{2} - 1)$  is minimum]

29. If, for a positive integer  $n$ , the quadratic equation,  $x(x+1) + (x+1)(x+2) + \dots + (x+n-1)(x+n) = 10n$  has two consecutive integral solutions, then  $n$  is equal to:  
 (1) 12   (2) 9  
 (3) 10   (4) 11

**Sol.**

(4)

$$x^2 + nx + \frac{n^2 - 31}{3} = 0$$

Let  $I$  and  $I+1$  be the roots of the equation  
 $2I+1 = -n$    ... (1)

$$I(I+1) = \frac{n^2 - 31}{3} \quad \dots (2)$$

Eliminating  $I$  from (1) and (2), we get

$$\frac{n^2 - 1}{4} = \frac{n^2 - 31}{3}$$

$$\Rightarrow n^2 = 121$$

$$\Rightarrow n = 11.$$

30. The integral  $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$  is equal to:  
 (1) -2   (2) 2  
 (3) 4   (4) -1

**Sol.**

(2)

$$\begin{aligned} I &= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\csc^2 x - \csc x \cdot \cot x) dx \\ &= 2 \end{aligned}$$

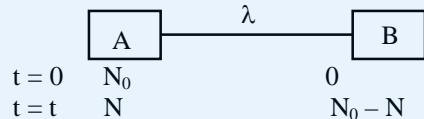
## PART B - PHYSICS

**ALL THE GRAPHS/DIAGRAMS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE.**

31. A radioactive nucleus A with a half life T, decays into a nucleus B. At  $t = 0$ , there is no nucleus B. At sometime  $t$ , the ratio of the number of B to that of A is 0.3. Then,  $t$  is given by :
- (1)  $t = \frac{T}{\log(1.3)}$       (2)  $t = \frac{T \log 2}{2 \log 1.3}$   
 (3)  $t = T \frac{\log 1.3}{\log 2}$       (4)  $t = T \log(1.3)$

**Sol.** (3)

$$\begin{aligned} \frac{N_0 - N}{N} &= 0.3 \\ \Rightarrow N &= \frac{N_0}{1.3} \\ N &= N_0 e^{-\lambda t} \\ \Rightarrow \frac{1}{1.3} &= e^{-\lambda t} \\ \Rightarrow t &= \frac{\ell \ln(1.3)}{\lambda} = T \frac{\ell \ln(1.3)}{\ell \ln(2)} \\ \therefore \lambda &= \frac{\ell \ln 2}{T}. \end{aligned}$$



32. The following observations were taken for determining surface tension T of water by capillary method:  
 diameter of capillary,  $D = 1.25 \times 10^{-2}$  m  
 rise of water,  $h = 1.45 \times 10^{-2}$  m

Using  $g = 9.80 \text{ m/s}^2$  and the simplified relation  $T = \frac{r h g}{2} \times 10^3 \text{ N/m}$ , the possible error in surface tension is

closest to :

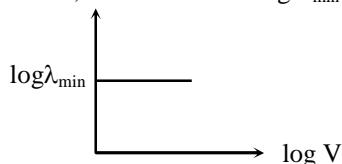
- (1) 10 %      (2) 0.15 %  
 (3) 1.5 %      (4) 2.4 %

**Sol.** (3)

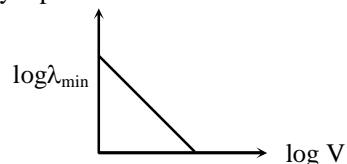
$$\begin{aligned} T &= \frac{r h g}{2} \times 10^3 \text{ N/m} \\ \frac{\Delta T}{T} &= \left| \frac{\Delta r}{r} \right| + \left| \frac{\Delta h}{h} \right| = \frac{0.01}{1.25} + \frac{0.01}{1.45} \\ \% \text{ error} &= \frac{\Delta T}{T} \times 100 = \frac{1}{1.25} + \frac{1}{1.45} = 0.8 + 0.69 \approx 1.5\% \end{aligned}$$

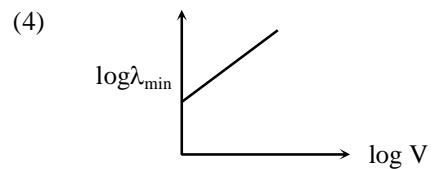
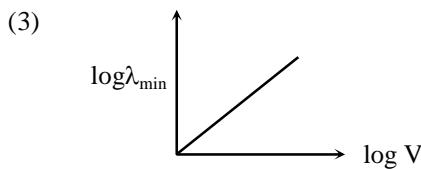
33. An electron beam is accelerated by a potential difference  $V$  to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If  $\lambda_{\min}$  is the smallest possible wavelength of X-ray in the spectrum, the variation of  $\log \lambda_{\min}$  with  $\log V$  is correctly represented in:

(1)



(2)



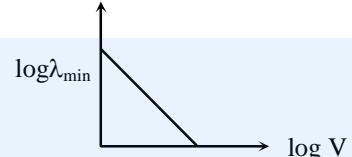
**Sol.**

(2)

$$\frac{hc}{\lambda_{\min}} = eV$$

$$\log \frac{hc}{e} - \log \lambda_{\min} = \log V$$

$$\Rightarrow \log \lambda_{\min} = k - \log V$$



- \*34. The moment of inertia of a uniform cylinder of length  $\ell$  and radius  $R$  about its perpendicular bisector is  $I$ . What is the ratio  $\ell/R$  such that the moment of inertia is minimum?

(1)  $\frac{3}{\sqrt{2}}$

(3)  $\frac{\sqrt{3}}{2}$

(2)  $\sqrt{\frac{3}{2}}$

(4) 1

**Sol.**

(2)

$$I = \frac{m}{12} [3R^2 + \ell^2]$$

$$\left( R^2 = \frac{m}{\pi \rho \ell} \right)$$

$$= \frac{m}{12} \left[ \frac{3m}{\pi \rho} \ell^{-1} + \ell^2 \right]$$

$$\frac{dI}{d\ell} = \frac{m}{12} \left[ -\frac{3m}{\pi \rho \ell^2} + 2\ell \right]$$

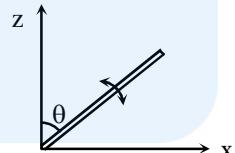
For minima

$$0 = \frac{-3m}{\pi \rho \ell^2} + 2\ell$$

$$\Rightarrow \frac{3\pi R^2 \ell \rho}{\pi \rho \ell^2} = 2\ell$$

$$\Rightarrow \frac{\ell}{R} = \sqrt{\frac{3}{2}}$$

- \*35. A slender uniform rod of mass  $M$  and length  $\ell$  is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle  $\theta$  with the vertical is:



(1)  $\frac{2g}{3\ell} \cos \theta$

(2)  $\frac{3g}{2\ell} \sin \theta$

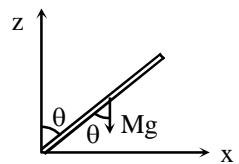
(3)  $\frac{2g}{3\ell} \sin \theta$

(4)  $\frac{3g}{2\ell} \cos \theta$

**Sol.**

(2)

$$\begin{aligned}\tau &= I\alpha \\ \Rightarrow Mg \frac{\ell}{2} \sin \theta &= \frac{M\ell^2}{3} \alpha \\ \Rightarrow \alpha &= \frac{3g}{2\ell} \sin \theta\end{aligned}$$



- \*36.  $C_p$  and  $C_v$  are specific heats at constant pressure and constant volume respectively. It is observed that  $C_p - C_v = a$  for hydrogen gas  
 $C_p - C_v = b$  for nitrogen gas  
The correct relation between  $a$  and  $b$  is

- (1)  $a = 28b$  (2)  $a = \frac{1}{14}b$   
(3)  $a = b$  (4)  $a = 14b$

*Sol.* (4)

For ideal gas

$$C_p - C_v = R/M$$

If  $C_p$  and  $C_v$  are specific heats ( $J/kg \cdot {}^\circ C$ )

$M$  = molar mass of gas

$$\Rightarrow a = R/2$$

$$\Rightarrow b = R/28$$

$$\Rightarrow a = 14b$$

- \*37. A copper ball of mass 100 gm is at a temperature  $T$ . It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be  $75^\circ C$ .  $T$  is given by: (Given : room temperature =  $30^\circ C$ , specific heat of copper =  $0.1 \text{ cal/gm}^\circ C$ )
- (1)  $825^\circ C$  (2)  $800^\circ C$   
(3)  $885^\circ C$  (4)  $1250^\circ C$

*Sol.* (3)

Final temperature of calorimeter and its contents is given,  $T_0 = 75^\circ C$

$$\Rightarrow 100 \times 0.1 \times (75 - T) + 100 \times 0.1 (75 - 30) + 170 \times 1 \times (75 - 30) = 0$$

$$\Rightarrow 75 - T + 45 + 765 = 0$$

$$\Rightarrow T = 885^\circ C$$

38. In amplitude modulation, sinusoidal carrier frequency used is denoted by  $\omega_c$  and the signal frequency is denoted by  $\omega_m$ . The bandwidth ( $\Delta\omega_m$ ) of the signal is such that  $\Delta\omega_m \ll \omega_c$ . Which of the following frequencies is **not** contained in the modulated wave?

- (1)  $\omega_c - \omega_m$  (2)  $\omega_m$   
(3)  $\omega_c$  (4)  $\omega_m + \omega_c$

*Sol.* (2)

Modulated signal can be written as

$$C_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$$

$$\Rightarrow C_m(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2} \cos(\omega_c + \omega_m)t$$

$$\text{where } \mu = \frac{A_m}{A_c}$$

- \*39. The temperature of an open room of volume  $30 \text{ m}^3$  increases from  $17^\circ C$  to  $27^\circ C$  due to the sunshine. The atmospheric pressure in the room remains  $1 \times 10^5 \text{ Pa}$ . If  $n_i$  and  $n_f$  are the number of molecules in the room before and after heating, then  $n_f - n_i$  will be :

- (1)  $-2.5 \times 10^{25}$  (2)  $-1.61 \times 10^{23}$   
(3)  $1.38 \times 10^{23}$  (4)  $2.5 \times 10^{25}$

**Sol.** (1)

$$\text{Using, } n = \left( \frac{PV}{RT} \right)$$

$$\begin{aligned} n_f - n_i &= \frac{PV}{R} \left( \frac{1}{T_f} - \frac{1}{T_i} \right) \text{ moles} \\ &= \frac{1 \times 10^5 \times 30}{8.32} \left( \frac{1}{300} - \frac{1}{290} \right) \times 6.023 \times 10^{23} \text{ molecules} \\ &= -2.5 \times 10^{25} \text{ molecules} \end{aligned}$$

40. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is

- (1) 15.6 mm                                  (2) 1.56 mm  
 (3) 7.8 mm                                    (4) 9.75 mm

**Sol.** (3)

$$y = \frac{m \times 650 \times 10^{-9} \times D}{d} = \frac{n \times 520 \times 10^{-9} \times D}{d}$$

$$\Rightarrow \frac{m}{n} = \frac{4}{5} \Rightarrow \text{minimum values of } m \text{ and } n \text{ will be 4 and 5 respectively.}$$

$$\begin{aligned} y &= \frac{4 \times 650 \times 10^{-9} \times 1.5}{5 \times 10^{-4}} \text{ meter} \\ &= 7.8 \text{ mm} \end{aligned}$$

41. A particle A of mass  $m$  and initial velocity  $v$  collides with a particle B of mass  $\frac{m}{2}$  which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths  $\lambda_A$  to  $\lambda_B$  after the collision is:

$$(1) \frac{\lambda_A}{\lambda_B} = \frac{1}{2}$$

$$(3) \frac{\lambda_A}{\lambda_B} = 2$$

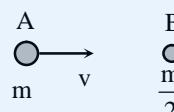
$$(2) \frac{\lambda_A}{\lambda_B} = \frac{1}{3}$$

$$(4) \frac{\lambda_A}{\lambda_B} = \frac{2}{3}$$

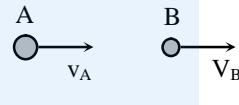
**Sol.** (3)

$$mv = mv_A + \frac{m}{2}v_B \text{ (conservation of linear momentum)}$$

$$\therefore v = v_A + \frac{v_B}{2} = v_B - v_A \text{ (elastic collision)}$$



Before collision



After collision

$$\therefore \frac{v_B}{v_A} = 4$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \frac{m_B v_B}{m_A v_A} = 2$$

42. A magnetic needle of magnetic moment  $6.7 \times 10^{-2} \text{ Am}^2$  and moment of inertia  $7.5 \times 10^{-6} \text{ kg m}^2$  is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is:

- (1) 8.76 s                                    (2) 6.65 s  
 (3) 8.89 s                                    (4) 6.98 s

**Sol.** (2)

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

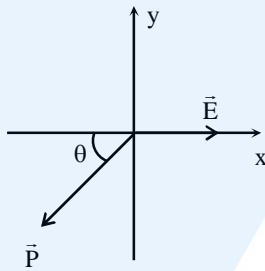
$$= 2\pi \sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665 \text{ sec}$$

So, time of 10 oscillations = 6.65 sec

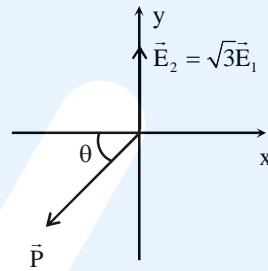
43. An electric dipole has a fixed dipole moment  $\vec{p}$ , which makes angle  $\theta$  with respect to x-axis. When subjected to an electric field  $\vec{E}_1 = E\hat{i}$ , it experiences a torque  $\vec{T}_1 = \tau\hat{k}$ . When subjected to another electric field  $\vec{E}_2 = \sqrt{3}E_1\hat{j}$  it experiences a torque  $\vec{T}_2 = -\vec{T}_1$ . The angle  $\theta$  is:
- (1)  $90^\circ$  (2)  $30^\circ$   
 (3)  $45^\circ$  (4)  $60^\circ$

**Sol.** (4)

From the given information



Case - I

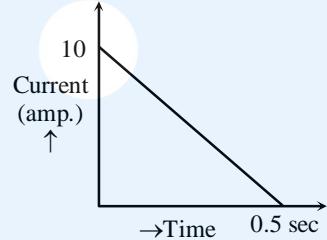


Case - II

$$\therefore PE_1 \sin \theta = \sqrt{3}PE_1 \sin\left(\frac{\pi}{2} + \theta\right)$$

$$\therefore \theta = 60^\circ$$

44. In a coil of resistance  $100 \Omega$ , a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is:
- (1) 275 Wb  
 (2) 200 Wb  
 (3) 225 Wb  
 (4) 250 Wb

**Sol.** (4)

$$\text{Change in flux} = R \int i dt = 250 \text{ Wb}$$

- \*45. A time dependent force  $F = 6t$  acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be:
- (1) 18 J (2) 4.5 J  
 (3) 22 J (4) 9 J

**Sol.** (2)

From impulse momentum theorem

$$\int_0^1 6tdt = mv$$

$$\therefore v = 3 \text{ m/s}$$

So, work done by the force =  $\Delta \text{K.E.} = \frac{1}{2}(1)(3)^2 = 4.5\text{J}$

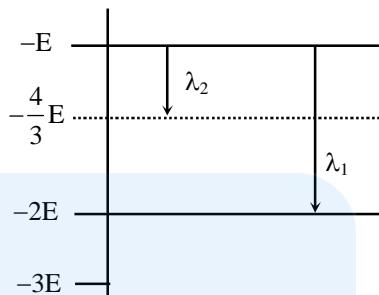
46. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths  $r = \lambda_1/\lambda_2$ , is given by:

(1)  $r = \frac{1}{3}$

(2)  $r = \frac{4}{3}$

(3)  $r = \frac{2}{3}$

(4)  $r = \frac{3}{4}$



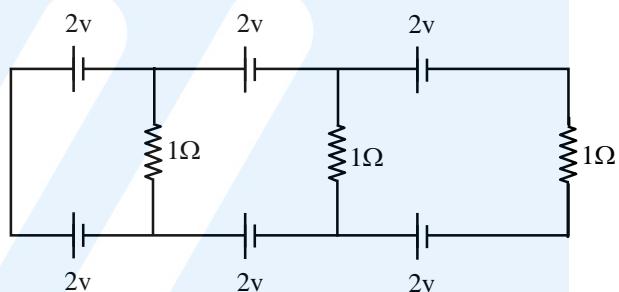
**Sol.** (1)

$$\Delta E \propto \frac{1}{\lambda}$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{1}{3}$$

47. In the given circuit, the current in each resistance is:

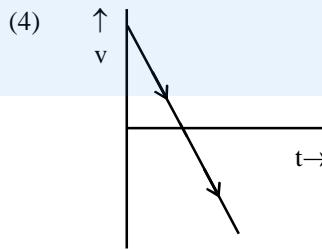
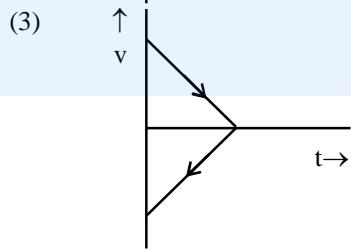
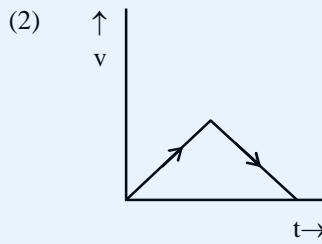
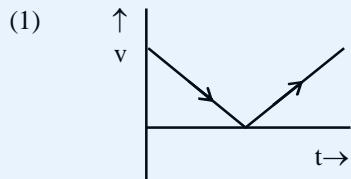
- (1) 0 A  
(2) 1 A  
(3) 0.25 A  
(4) 0.5 A



**Sol.** (1)

Potential difference across each resistor is zero.

- \*48. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?



**Sol.** (4)

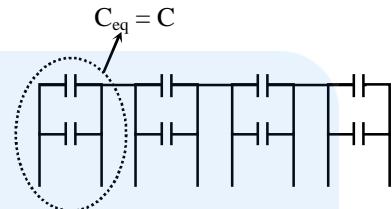
$$v = u - gt$$

49. A capacitance of  $2 \mu\text{F}$  is required in an electrical circuit across a potential difference of  $1.0 \text{ kV}$ . A large number of  $1 \mu\text{F}$  capacitors are available which can withstand a potential difference of not more than  $300 \text{ V}$ . The minimum number of capacitors required to achieve this is:  
 (1) 32                          (2) 2  
 (3) 16                          (4) 24

**Sol.** (1)

$$\frac{C}{4} = 2 \Rightarrow C = 8 \mu\text{F}$$

Which requires eight  $1 \mu\text{F}$  capacitors in parallel.  
 ⇒ Minimum number of capacitors required is 32.



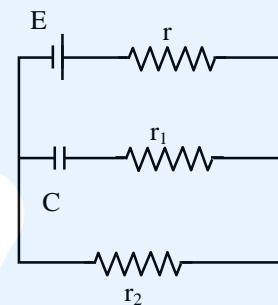
50. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance  $C$  will be:

$$(1) CE \frac{r_1}{(r_1 + r)}$$

$$(3) CE \frac{r_1}{(r_2 + r)}$$

$$(2) CE$$

$$(4) CE \frac{r_2}{(r + r_2)}$$



**Sol.** (4)

$$q = CV \\ = \frac{CEr_2}{r + r_2}$$

51. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be:

$$(1) 180^\circ$$

$$(3) 90^\circ$$

$$(2) 45^\circ$$

$$(4) 135^\circ$$

**Sol.** (1)

In common emitter amplifier circuit the output voltage is out of phase w.r.t. input voltage.

52. Which of the following statements is **false**?

(1) Kirchhoff's second law represents energy conservation.

(2) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

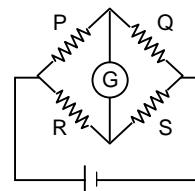
(3) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

(4) A rheostat can be used as a potential divider.

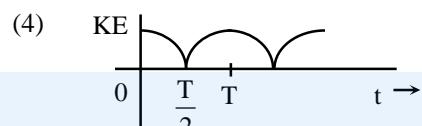
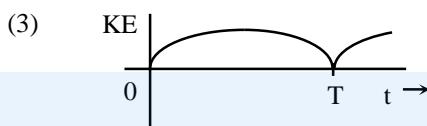
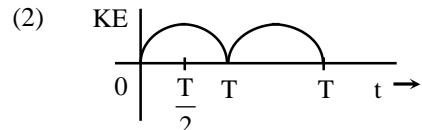
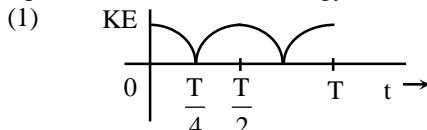
**Sol.** (3)

The balanced condition is given by  $\frac{P}{Q} = \frac{R}{S}$ ; When battery and Galvanometer

are exchanged, it become  $\frac{P}{R} = \frac{Q}{S}$ ; which is same as previous



- \*53. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like:



**Sol.** (1)

For given SHM  $x = A \sin \omega t$

$$v = \frac{dx}{dt} = A\omega \cos \omega t$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t = KE_{\max} \left( \frac{1+\cos 2\omega t}{2} \right)$$

54. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light =  $3 \times 10^8 \text{ ms}^{-1}$ )

(1) 15.3 GHz

(2) 10.1 GHz

(3) 12.1 GHz

(4) 17.3 GHz

**Sol.** (4)

$$f' = \left( \sqrt{\frac{1+\beta}{1-\beta}} \right) f, \text{ where } \beta = \frac{v}{c}$$

So,  $f' = 17.3 \text{ GHz}$

- \*55. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of:

(1)  $\frac{1}{81}$

(2) 9

(3)  $\frac{1}{9}$

(4) 81

**Sol.** (2)

$$\text{Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{\rho \ell A g}{A} = \rho \ell g$$

$$\text{So, } \frac{\text{Stress}_f}{\text{Stress}_i} = 9$$

56. When a current of 5 mA is passed through a galvanometer having a coil of resistance  $15\Omega$ , it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10V is:

(1)  $4.005 \times 10^3 \Omega$

(2)  $1.985 \times 10^3 \Omega$

(3)  $2.045 \times 10^3 \Omega$

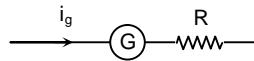
(4)  $2.535 \times 10^3 \Omega$

**Sol.**

$$i_g(R + R_g) = V$$

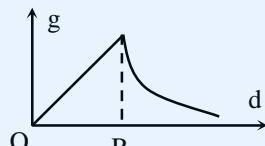
$$R = \frac{V}{i_g} - R_g$$

$$R = \frac{10}{5 \times 10^{-3}} - 15 = 1.985 \times 10^3 \Omega$$

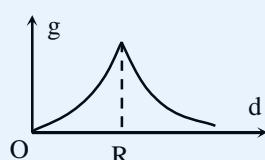


- \*57. The variation of acceleration due to gravity  $g$  with distance  $d$  from centre of the earth is best represented by ( $R$  = Earth's radius):

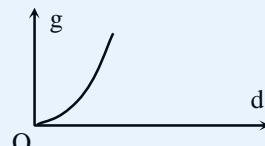
(1)



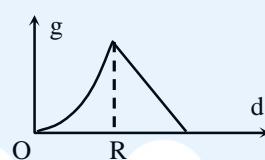
(3)



(2)



(4)

**Sol.**

(1)

The variation of magnitude of acceleration due to gravity is given by

$$g = \left( \frac{GM}{R^3} \right) d, \text{ where } 0 \leq d \leq R$$

$$= \frac{GM}{d^2}, \text{ where } d > R$$

- \*58. An external pressure  $P$  is applied on a cube at  $0^\circ\text{C}$  so that it is equally compressed from all sides.  $K$  is the bulk modulus of the material of the cube and  $\alpha$  is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:

$$(1) 3PK\alpha$$

$$(2) \frac{P}{3\alpha K}$$

$$(3) \frac{P}{\alpha K}$$

$$(4) \frac{3\alpha}{PK}$$

**Sol.**

(2)

$$\text{By applying pressure, } \Delta P = -\frac{B\Delta V}{V}$$

$$\Rightarrow -\frac{\Delta V}{V} = \frac{\Delta P}{B} = \frac{P}{K} \quad (\text{given } B = K)$$

By increasing temperature, fractional increase in volume

$$-\frac{\Delta V}{V} = 3\alpha\Delta\theta$$

$$\frac{P}{K} = 3\alpha\Delta\theta$$

$$\Delta\theta = \frac{P}{3\alpha K}$$

59. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15cm from a converging lens of magnitude of focal length 20cm. A beam of parallel light falls on the diverging lens. The final image formed is:
- real and at a distance of 6 cm from the convergent lens.
  - real and at a distance of 40 cm from convergent lens.
  - virtual and at a distance of 40 cm from convergent lens.
  - real and at a distance of 40 cm from the divergent lens.

**Sol.****(2)**

For diverging lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{\infty} = \frac{1}{-25}$$

$$\Rightarrow v = -25 \text{ cm}$$

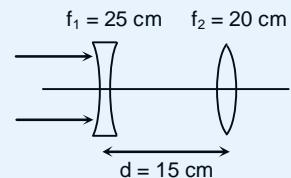
First image is formed at a distance 25cm left to the diverging lens.

For the converging lens.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-40} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$$\Rightarrow v = +40 \text{ cm}$$



- \*60. A body of mass  $m = 10^{-2} \text{ kg}$  is moving in a medium and experiences a frictional force  $F = -kv^2$ . Its initial speed is  $v_0 = 10 \text{ ms}^{-1}$ . If, after 10 s, its energy is  $\frac{1}{8}mv_0^2$ , the value of k will be:
- $10^{-1} \text{ kg m}^{-1}\text{s}^{-1}$
  - $10^{-3} \text{ kg m}^{-1}$
  - $10^{-3} \text{ kg s}^{-1}$
  - $10^{-4} \text{ kg m}^{-1}$

**Sol.****(4)**

$$\frac{1}{2}mv_f^2 = \frac{1}{8}mv_0^2 \Rightarrow v_f = \frac{v_0}{2}$$

$$\text{Now, } \frac{mdv}{dt} = -kv^2$$

$$\Rightarrow m \int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = -k \int_0^{10} dt$$

$$\Rightarrow m \left[ -\frac{1}{v} \right]_{v_0}^{\frac{v_0}{2}} = -k [t]_0^{10}$$

$$\Rightarrow m \left( \frac{2}{v_0} - \frac{1}{\frac{v_0}{2}} \right) = 10k$$

$$\Rightarrow \frac{m}{v_0} = 10k$$

$$\Rightarrow k = \frac{m}{10v_0} = \frac{10^{-2}}{10 \times 10} = 10^{-4} \text{ kg m}^{-1}$$

### PART C - CHEMISTRY

\*61. 1 gram of a carbonate ( $M_2CO_3$ ) on treatment with excess HCl produces 0.01186 mole of  $CO_2$ . The molar mass of  $M_2CO_3$  in g  $mol^{-1}$  is:

- (1) 84.3    (2) 118.6  
 (3) 11.86    (4) 1186

**Sol.** (1)



Moles of  $M_2CO_3$  = Moles of  $CO_2$  produced.

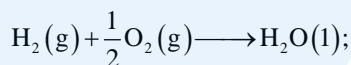
$$\text{moles of } M_2CO_3 = \frac{w}{\text{molar mass}} = 0.01186$$

$\therefore$  Molar mass = 84.3 g  $mol^{-1}$

So, option (1) is correct.

\*62. Given  $C_{(\text{graphite})} + O_2(g) \longrightarrow CO_2(g)$ ;

$$\Delta_r H^\circ = -393.5 \text{ kJ mol}^{-1}$$

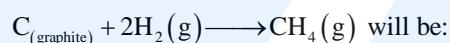


$$\Delta_r H^\circ = -285.8 \text{ kJ mol}^{-1}$$



$$\Delta_r H^\circ = +890.3 \text{ kJ mol}^{-1}$$

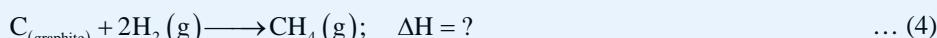
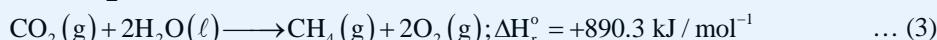
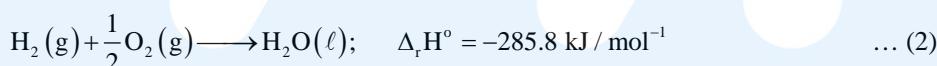
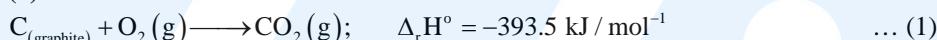
Based on the above thermochemical equations, the value of  $\Delta_r H^\circ$  at 298 K for the reaction



- (1) +144.0 kJ  $mol^{-1}$   
 (3) -144.0 kJ  $mol^{-1}$

- (2) -74.8 kJ  $mol^{-1}$   
 (4) +74.8 kJ  $mol^{-1}$

**Sol.** (2)



[Eq. (1) + Eq. (3)] + [2 × Eq. (2)] = Eq. (4)

$$\therefore [\Delta H_1 + \Delta H_3] + [2 \times \Delta H_2] = \Delta H_4$$

$$[(-393.5) + (890.3)] + [2(-285.8)] = -74.8 \text{ kJ / mol}$$

$$= -74.8 \text{ kJ/mol}^{-1}$$

63. The freezing point of benzene decreases by  $0.45^0\text{C}$  when 0.2 g of acetic acid is added to 20g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be:  
 ( $K_f$  for benzene = 5.12 K kg  $mol^{-1}$ )

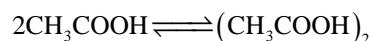
- (1) 80.4 %    (2) 74.6 %  
 (3) 94.6 %    (4) 64.6 %

**Sol.** (3)

$$\Delta T_f = i \times K_f \times m$$

$$\Rightarrow 0.45 = i \times 5.12 \times \frac{0.2 \times 1000}{60 \times 20}$$

$$i = 0.527$$



$$1 - \alpha \quad \frac{\alpha}{2}$$

$$i = 1 - \alpha + \frac{\alpha}{2}$$

$$\alpha = 0.946$$

$\therefore$  % dissociation is 94.6%.

- \*64. The most abundant elements by mass in the body of a healthy human adult are: Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all  $^1\text{H}$  atoms are replaced by  $^2\text{H}$  atoms is:
- (1) 37.5 kg
  - (2) 7.5 kg
  - (3) 10 kg
  - (4) 15 kg

**Sol.** (2)

$$\text{Total hydrogen } (^1\text{H}) = \frac{10}{100} \times 75 = 7.5 \text{ kg}$$

If it is replaced by  $^2\text{H}$  then mass will be doubled so now hydrogen mass = 15 kg  
So, mass of person will be increased by 7.5 kg.

- \*65.  $\Delta U$  is equal to
- |                     |                    |
|---------------------|--------------------|
| (1) Isobaric work   | (2) Adiabatic work |
| (3) Isothermal work | (4) Isochoric work |

**Sol.** (2)

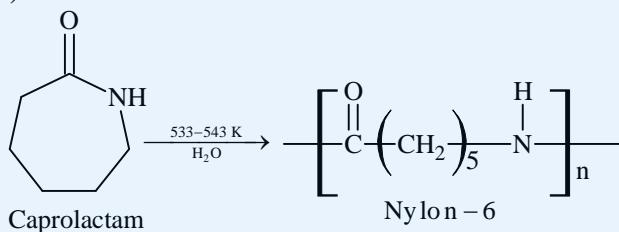
$$\Delta U = q + w$$

$q = 0$  in adiabatic process.

So,  $\Delta U = w$

66. The formation of which of the following polymers involves hydrolysis reaction?
- |              |                |
|--------------|----------------|
| (1) Bakelite | (2) Nylon 6, 6 |
| (3) Terylene | (4) Nylon 6    |

**Sol.** (4)



67. Given

$$E_{\text{Cl}_2/\text{Cl}^-}^0 = 1.36 \text{ V}, E_{\text{Cr}^{3+}/\text{Cr}}^0 = -0.74 \text{ V}$$

$$E_{\text{Cr}_2\text{O}_7^{2-}/\text{Cr}^{3+}}^0 = 1.33 \text{ V}, E_{\text{MnO}_4^-/\text{Mn}^{2+}}^0 = 1.51 \text{ V}.$$

Among the following, the strongest reducing agent is

- |                      |                      |
|----------------------|----------------------|
| (1) $\text{Mn}^{2+}$ | (2) $\text{Cr}^{3+}$ |
| (3) $\text{Cl}^-$    | (4) Cr               |

**Sol.** (4)

Reduction potential of

$$E_{\text{Cr}^{3+}/\text{Cr}}^{\circ} = -0.74 \text{ V}$$

$$\text{So, } E_{\text{Cr}/\text{Cr}^{3+}} = +0.74 \text{ V}$$

$\therefore$  Cr would be strongest reducing agent.

68.

The Tyndall effect is observed only when following conditions are satisfied:

- (a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.
- (b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.
- (c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.
- (d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(1) (b) and (d)

(2) (a) and (c)

(3) (b) and (c)

(4) (a) and (d)

**Sol.**

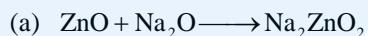
(1)

Tyndall effect is observed only when

- (i) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.
  - (ii) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.
- So, (b) and (d) are correct.

\*69.

In the following reactions, ZnO is respectively acting as a/an:



(1) base and base

(2) acid and acid

(3) acid and base

(4) base and acid

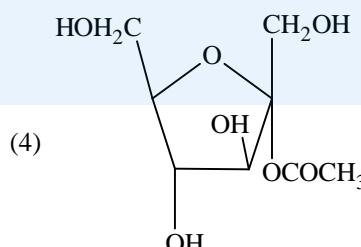
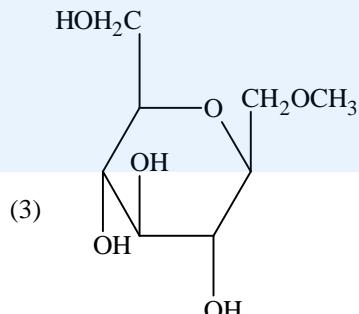
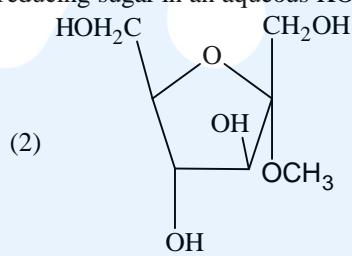
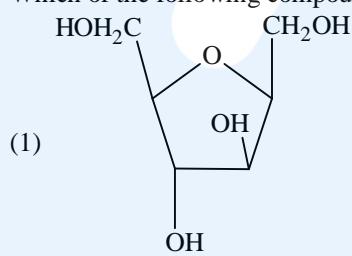
**Sol.**

(3)

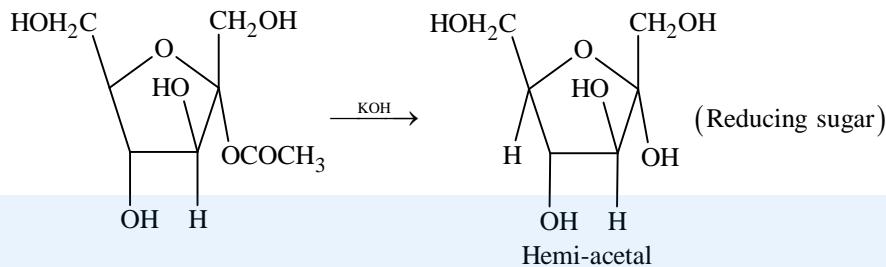
 $\text{ZnO} + \text{Na}_2\text{O} \longrightarrow \text{Na}_2\text{ZnO}_2$ ; ZnO behaving as an acid. $\text{ZnO} + \text{CO}_2 \longrightarrow \text{ZnCO}_3$ ; ZnO behaving as a base.

70.

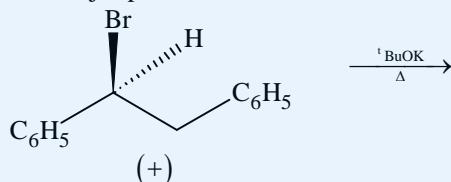
Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?



**Sol.** (4)

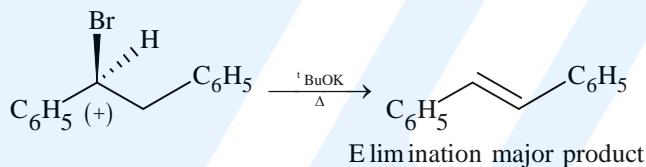


71. The major product obtained in the following reaction is:



- (1)  $\text{C}_6\text{H}_5\text{CH}=\text{CHC}_6\text{H}_5$   
 (2) (+)  $\text{C}_6\text{H}_5\text{CH(O}^t\text{Bu)}\text{CH}_2\text{C}_6\text{H}_5$   
 (3) (-)  $\text{C}_6\text{H}_5\text{CH(O}^t\text{Bu)}\text{CH}_2\text{C}_6\text{H}_5$   
 (4) ( $\pm$ )  $\text{C}_6\text{H}_5\text{CH(O}^t\text{Bu)}\text{CH}_2\text{C}_6\text{H}_5$

**Sol.** (1)



\*72. Which of the following species is **not** paramagnetic?

- (1) CO  
 (2)  $\text{O}_2$   
 (3)  $\text{B}_2$   
 (4) NO

**Sol.** (1)

- (14) CO – diamagnetic  
 (16)  $\text{O}_2$  – paramagnetic  
 (10)  $\text{B}_2$  – paramagnetic  
 (15) NO – paramagnetic

73. On treatment of 100 mL of 0.1 M solution of  $\text{CoCl}_3 \cdot 6\text{H}_2\text{O}$  with excess  $\text{AgNO}_3$ ;  $1.2 \times 10^{22}$  ions are precipitated. The complex is:

- (1)  $[\text{Co}(\text{H}_2\text{O})_3 \text{Cl}_3] \cdot 3\text{H}_2\text{O}$   
 (2)  $[\text{Co}(\text{H}_2\text{O})_6] \text{Cl}_3$   
 (3)  $[\text{Co}(\text{H}_2\text{O})_5 \text{Cl}] \text{Cl}_2 \cdot \text{H}_2\text{O}$   
 (4)  $[\text{Co}(\text{H}_2\text{O})_4 \text{Cl}_2] \text{Cl} \cdot 2\text{H}_2\text{O}$

**Sol.** (4)

Moles of  $\text{CoCl}_3 \cdot 6\text{H}_2\text{O}$   $\rightarrow 100 \text{ mL} \times 0.1 \text{ M} = 10 \times 10^{-3}$  moles

Ions  $\rightarrow 6.023 \times 10^{23} \times 0.01$

$= 6.023 \times 10^{21}$  ions

Precipitated ions  $= 1.2 \times 10^{22}$

$\therefore 1 \text{ Ag}^+$  ion and  $1 \text{ Cl}^-$  ion.

So  $[\text{Co}(\text{H}_2\text{O})_5 \text{Cl}] \text{Cl} \cdot \text{H}_2\text{O}$  is correct.

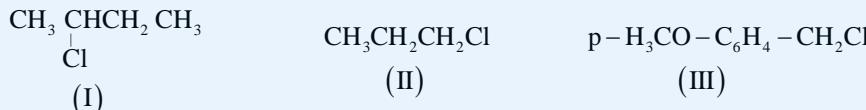
- \*74.  $pK_a$  of a weak acid (HA) and  $pK_b$  of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is:

- (1) 6.9 (2) 7.0  
(3) 1.0 (4) 7.2

**Sol.** (1)

$$\begin{aligned} \text{pH} &= 7 + \frac{pK_a}{2} - \frac{pK_b}{2} \\ &= 7 + \frac{3.2}{2} - \frac{3.4}{2} \\ &= 6.9 \end{aligned}$$

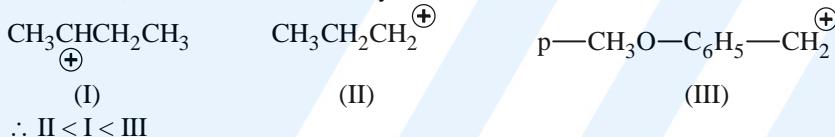
75. The increasing order of the reactivity of the following halides for the  $S_N1$  reaction is:



- (1) (II) < (I) < (III) (2) (I) < (III) < (II)  
(3) (II) < (III) < (I) (4) (III) < (II) < (I)

**Sol.** (1)

Rate of  $S_N1 \propto$  carbocation stability

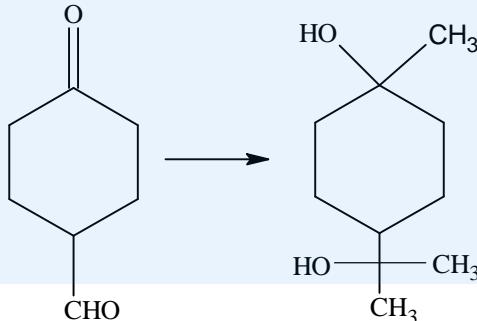


- \*76. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect, is:

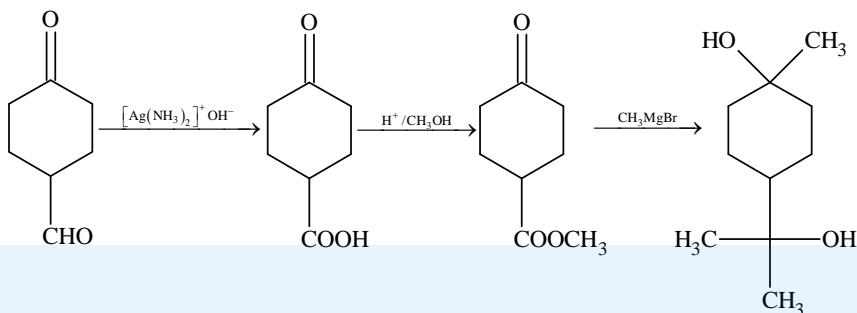
- (1) both form soluble bicarbonates  
(2) both from nitrides  
(3) nitrates of both Li and Mg yield  $\text{NO}_2$  and  $\text{O}_2$  on heating  
(4) both form basic carbonates

**Sol.** (1)

77. The correct sequence of reagents for the following conversion will be:

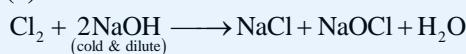


- (1)  $\text{CH}_3\text{MgBr}, \text{H}^+ / \text{CH}_3\text{OH}, [\text{Ag}(\text{NH}_3)_2]^+ \text{OH}^-$   
(2)  $\text{CH}_3\text{MgBr}, [\text{Ag}(\text{NH}_3)_2]^+ \text{OH}^-, \text{H}^+ / \text{CH}_3\text{OH}$   
(3)  $[\text{Ag}(\text{NH}_3)_2]^+ \text{OH}^-, \text{CH}_3\text{MgBr}, \text{H}^+ / \text{CH}_3\text{OH}$   
(4)  $[\text{Ag}(\text{NH}_3)_2]^+ \text{OH}^-, \text{H}^+ / \text{CH}_3\text{OH}, \text{CH}_3\text{MgBr}$

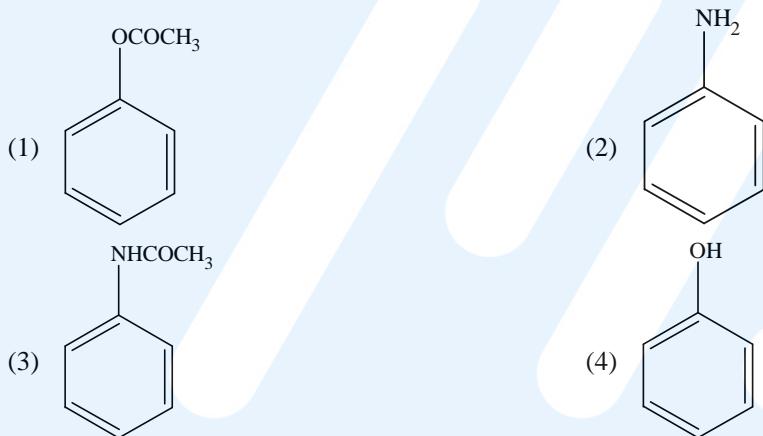
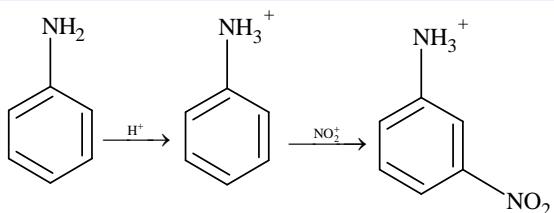
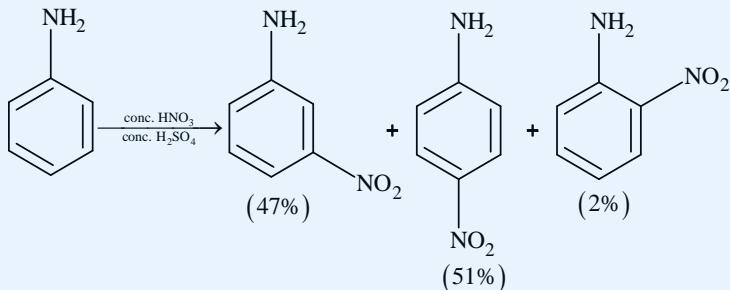
**Sol.** (4)

78. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are:

- (1)  $\text{ClO}_2^-$  and  $\text{ClO}_3^-$       (2)  $\text{Cl}^-$  and  $\text{ClO}^-$   
 (3)  $\text{Cl}^-$  and  $\text{ClO}_2^-$       (4)  $\text{ClO}^-$  and  $\text{ClO}_3^-$

**Sol.** (2)

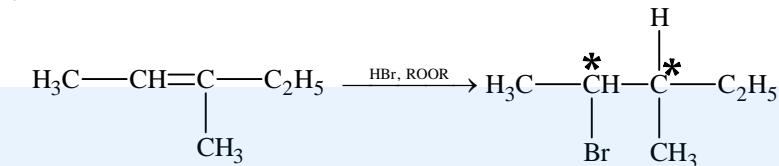
79. Which of the following compounds will form significant amount of *meta* product during mono-nitration reaction?

**Sol.** (2)

\*80. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is :

- |          |         |
|----------|---------|
| (1) Zero | (2) Two |
| (3) Four | (4) Six |

**Sol.** (3)



$$\text{Stereoisomers} = 2^n = 2^2 = 4$$

81. Two reactions  $R_1$  and  $R_2$  have identical pre-exponential factors. Activation energy of  $R_1$  exceeds that of  $R_2$  by  $10 \text{ kJ mol}^{-1}$ . If  $k_1$  and  $k_2$  are rate constants for reactions  $R_1$  and  $R_2$  respectively at  $300 \text{ K}$ , then  $\ln(k_2/k_1)$  is equal to :

- $$(R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1})$$
- |        |       |
|--------|-------|
| (1) 12 | (2) 6 |
| (3) 4  | (4) 8 |

**Sol.** (3)

$$k_1 = Ae^{-E_{a1}/RT}$$

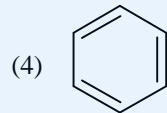
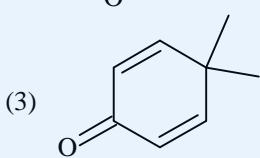
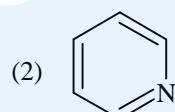
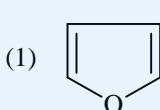
$$k_2 = Ae^{-E_{a2}/RT}$$

$$\frac{k_2}{k_1} = e^{-\frac{(E_{a2}-E_{a1})}{RT}}$$

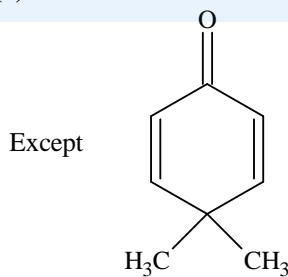
$$\frac{k_2}{k_1} = e^{+\frac{10 \times 10^3}{8.314 \times 300}}$$

$$\ln \frac{k_2}{k_1} = 4$$

\*82. Which of the following molecules is least resonance stabilized?



**Sol.** (3)



\*83. The group having isoelectronic species is:

- (1)  $O^-$ ,  $F^-$ ,  $Na$ ,  $Mg^{2+}$   
 (2)  $O^{2-}$ ,  $F^-$ ,  $Na$ ,  $Mg^{2+}$   
 (3)  $O^-$ ,  $F^-$ ,  $Na^+$ ,  $Mg^{2+}$   
 (4)  $O^{2-}$ ,  $F^-$ ,  $Na^+$ ,  $Mg^{2+}$

Sol. (4)

$O^{2-}$ ,  $F^-$ ,  $Na^+$  and  $Mg^{2+}$ , all have 10 electrons each.

\*84. The radius of the second Bohr orbit for hydrogen atom is:

(Planck's Const.  $h = 6.6262 \times 10^{-34}$  Js; mass of electron =  $9.1091 \times 10^{-31}$  kg; charge of electron  $e = 1.60210 \times 10^{-19}$  C; permittivity of vacuum  $\epsilon_0 = 8.854185 \times 10^{-12}$  kg $^{-1}$  m $^{-3}$  A $^2$ )

- (1)  $4.76 \text{ \AA}^\circ$   
 (2)  $0.529 \text{ \AA}^\circ$   
 (3)  $2.12 \text{ \AA}^\circ$   
 (4)  $1.65 \text{ \AA}^\circ$

Sol. (3)

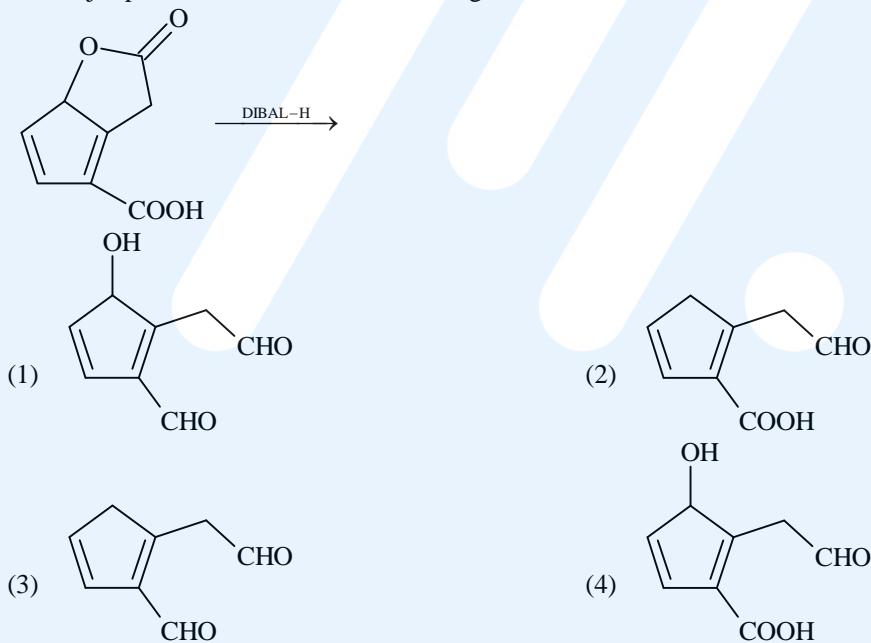
$$r_n = \frac{0.53n^2}{Z} \text{ \AA}^\circ$$

$$n = 2$$

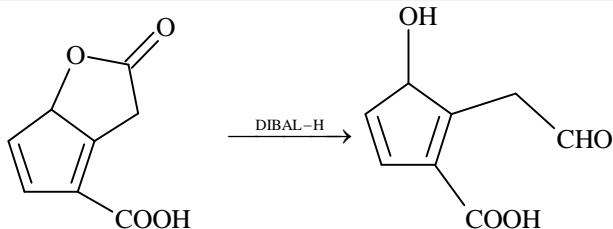
$$Z = 1$$

$$r_2 = 0.53 \times 4 \text{ \AA}^\circ = 2.12 \text{ \AA}^\circ$$

85. The major product obtained in the following reaction is:



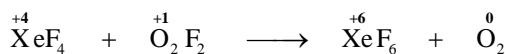
Sol. (4)



\*86. Which of the following reactions is an example of a redox reaction?

- (1)  $\text{XeF}_2 + \text{PF}_5 \longrightarrow [\text{XeF}]^+ \text{PF}_6^-$   
 (2)  $\text{XeF}_6 + \text{H}_2\text{O} \longrightarrow \text{XeOF}_4 + 2\text{HF}$   
 (3)  $\text{XeF}_6 + 2\text{H}_2\text{O} \longrightarrow \text{XeO}_2\text{F}_2 + 4\text{HF}$   
 (4)  $\text{XeF}_4 + \text{O}_2\text{F}_2 \longrightarrow \text{XeF}_6 + \text{O}_2$

Sol. (4)



Xenon oxidises and oxygen gets reduced.

87. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is ‘a’, the closest approach between two atoms in metallic crystal will be:

- (1)  $2\sqrt{2}a$   
 (2)  $\sqrt{2}a$   
 (3)  $\frac{a}{\sqrt{2}}$   
 (4)  $2a$

Sol. (3)

In FCC structure

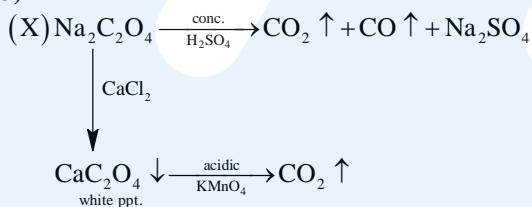
$$4r = \sqrt{2}a$$

$$2r = \frac{a}{\sqrt{2}} = \text{closest approach between two atoms.}$$

88. Sodium salt of an organic acid ‘X’ produces effervescence with conc.  $\text{H}_2\text{SO}_4$ . ‘X’ reacts with the acidified aqueous  $\text{CaCl}_2$  solution to give a white precipitate which decolourises acidic solution of  $\text{KMnO}_4$ . ‘X’ is:

- (1)  $\text{HCOONa}$   
 (2)  $\text{CH}_3\text{COONa}$   
 (3)  $\text{Na}_2\text{C}_2\text{O}_4$   
 (4)  $\text{C}_6\text{H}_5\text{COONa}$

Sol. (3)



\*89. A water sample has ppm level concentration of following anions

$$\text{F}^- = 10; \text{SO}_4^{2-} = 100; \text{NO}_3^- = 50$$

The anion/anions that make/makes the water sample unsuitable for drinking is/are:

- (1) both  $\text{SO}_4^{2-}$  and  $\text{NO}_3^-$   
 (2) only  $\text{F}^-$   
 (3) only  $\text{SO}_4^{2-}$   
 (4) only  $\text{NO}_3^-$

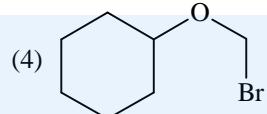
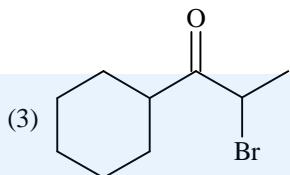
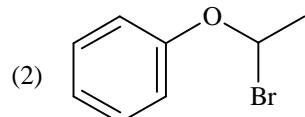
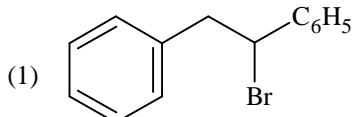
Sol. (2)

Permissible limit for  $\text{SO}_4^{2-} = 500 \text{ ppm}$

Permissible limit for  $\text{NO}_3^- = 50 \text{ ppm}$

Permissible limit for  $\text{F}^- = 1 \text{ ppm}$

90. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolorize the colour of bromine?



Sol. (4)

