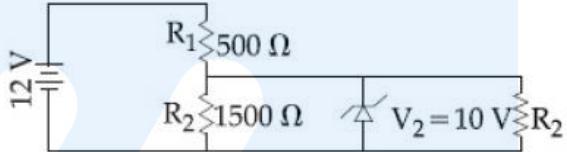


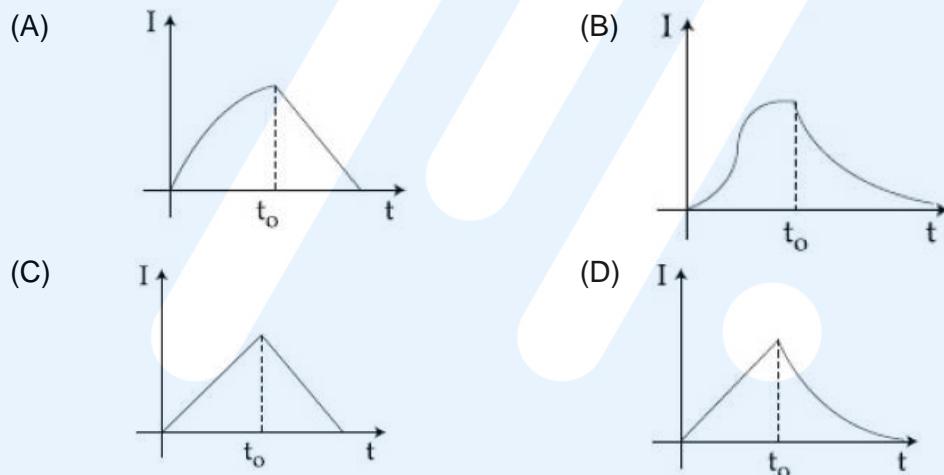
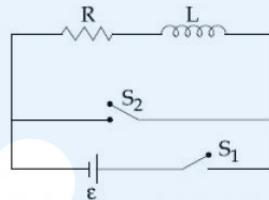
PART -A (PHYSICS)

1. A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1\text{s}$ is R . Neglecting air resistance and taking acceleration due to gravity $g = 10 \text{ ms}^{-2}$, the radius of R is:
 (A) 10.3 m (B) 2.8 m
 (C) 2.5 m (D) 5.1 m
2. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 \AA . The radius of the atom in the excited state, in terms of Bohr radius a_0 , will be:
 $(hc = 12500 \text{ eV} \cdot \text{\AA})$
 (A) $25a_0$ (B) $9a_0$
 (C) $16a_0$ (D) $4a_0$
3. In the given circuit the current through Zener Diode is close to:
 (A) 0.0 mA (B) 6.7 mA
 (C) 4.0 mA (D) 6.0 mA
4. There are two long co – axial solenoids of same length l . The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The ratio of mutual inductance to the self – inductance of the inner – coil is:
 (A) $\frac{n_1}{n_2}$ (B) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$
 (C) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$ (D) $\frac{n_2}{n_1}$
5. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A \sin \frac{\pi t}{90}$. The ratio of kinetic to potential energy of the particle at $t = 210\text{s}$ will be
 (A) $1/9$ (B) 1
 (C) 2 (D) 3
6. A satellite is revolving in a circular orbit at a height h from the earth surface, such that $h < < R$ where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is:
 (A) $\sqrt{2gR}$ (B) \sqrt{gR}
 (C) $\sqrt{\frac{gR}{2}}$ (D) $\sqrt{gR}(\sqrt{2} - 1)$

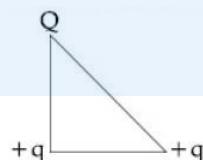


7. The force of interaction between two atoms is given by $F = \alpha\beta \exp\left(-\frac{x^2}{\alpha k T}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is:
- (A) $M^0 L^2 T^{-4}$ (B) $M^2 L T^{-4}$
 (C) $M L T^{-2}$ (D) $M^2 L^2 T^{-2}$
8. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $1/8^{\text{th}}$ of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to:
- (A) 0.74 (B) 0.85
 (C) 0.94 (D) 0.80
9. In the circuit shown,

The switch S_1 is close at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by:

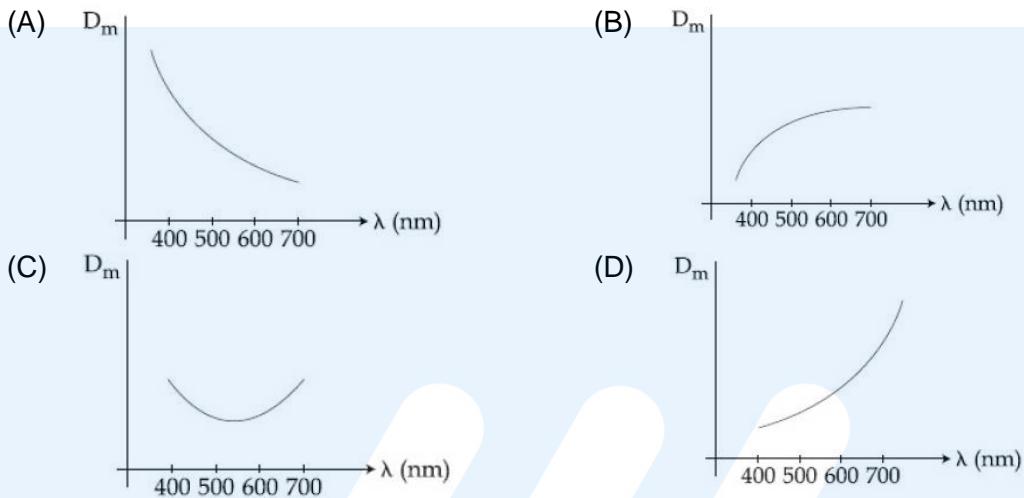
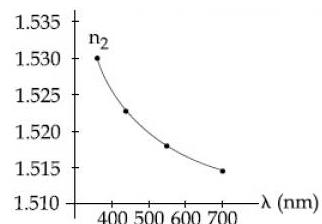


10. Three charges Q , $+q$ and $+q$ are placed at the vertices of a right – angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is:

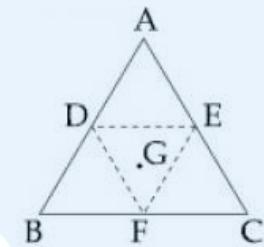


- (A) $+q$ (B) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$
 (C) $\frac{-q}{1+\sqrt{2}}$ (D) $-2q$

11. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following, graph is the correct one, if D_m is the angle of minimum deviation?



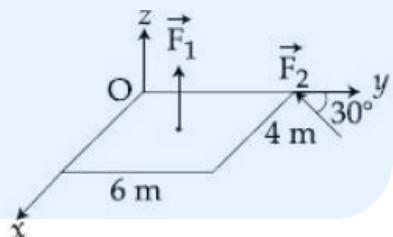
12. The equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0 . If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then:



$$(A) I = \frac{15}{16}I_0 \quad (B) I = \frac{3}{4}I_0$$

$$(C) I = \frac{9}{16}I_0 \quad (D) I = \frac{I_0}{4}$$

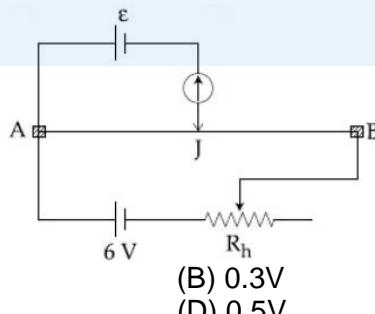
13. A slab is subjected to the two forces \vec{F}_1 and \vec{F}_2 of same magnitude F as shown in the figure. Force \vec{F}_2 is in XY – plane while force F_1 acts along z – axis at the point $(2\hat{i} + 3\hat{j})$. The moment of these forces about point O will be:



$$(A) (3\hat{i} + 2\hat{j} + 3\hat{k})F \quad (B) (3\hat{i} - 2\hat{j} - 3\hat{k})F$$

$$(C) (3\hat{i} + 2\hat{j} - 3\hat{k})F \quad (D) (3\hat{i} + 2\hat{j} + 3\hat{k})F$$

14. Ice at -20°C is added to 50 g of water at 40°C . When the temperature of the mixture reaches 0°C , it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Specific heat of water = $4.2\text{ J/g}/^{\circ}\text{C}$)
 Heat of fusion of water at $0^{\circ}\text{C} = 334\text{ J/g}$
 (A) 50 g (B) 100 g
 (C) 60 g (D) 40 g
15. A particle is moving along a circular path with a constant speed of 10 ms^{-1} . What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?
 (A) $10\sqrt{3}\text{ m/s}$ (B) 0
 (C) $10\sqrt{2}\text{ m/s}$ (D) 10 m/s
16. An amplitude modulated signal is given by
 $V(t) = 10[1 + 0.6\cos(2.2 \times 10^4 t)\sin(5.5 \times 10^5)t]$. Here t is in seconds. The sideband frequencies (in kHz) are. [Given $\pi = 22/7$]
 (A) 1785 and 1715 (B) 1785 and 1715
 (C) 89.25 and 85.75 (D) 892.5 and 857.5
17. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be:
 (A) $2.26 \times 10^{-3}\text{ m/s}$ away from the lens (B) $0.92 \times 10^{-3}\text{ m/s}$ away from the lens
 (C) $3.22 \times 10^{-3}\text{ m/s}$ towards the lens (D) $1.16 \times 10^{-3}\text{ m/s}$ towards the lens
18. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is:
 (A) 15 RT (B) 12 RT
 (C) 4 RT (D) 20 RT
19. Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be.
 (A) 60 W (B) 240 W
 (C) 120 W (D) 30 W
20. The resistance of the meter bridge AB in given figure is 4Ω . With a cell of emf $\epsilon = 0.05\text{ V}$ and rheostat resistance $R_h = 2\Omega$ the null point is obtained at some point J. When the cell is replaced by another one of emf $\epsilon = \epsilon_2$ the same null point J is found for $R_h = 6\Omega$. The emf ϵ_2 is:



- (A) 0.4V (B) 0.3V
 (C) 0.6V (D) 0.5V

21. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss . The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively. Given by:

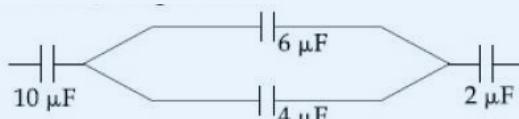
(A) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$

(B) (\sqrt{n}, \sqrt{n})

(C) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

(D) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

22. In the figure shown below, the charge on the left plate of the μF capacitor is $-30\mu\text{C}$. The charge on the right place of the $6\mu\text{F}$ capacitor is:



(A) $-12\mu\text{C}$
(C) $-18\mu\text{C}$

(B) $+12\mu\text{C}$
(D) $+18\mu\text{C}$

23. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The rational between temperature and volume for the process is $TV^x = \text{constant}$, then x is:

(A) $3/5$
(C) $2/3$

(B) $2/5$
(D) $5/3$

24. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% loses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be:

(A) $\frac{1}{4}\rho v^2$
(C) $\frac{1}{2}\rho v^2$

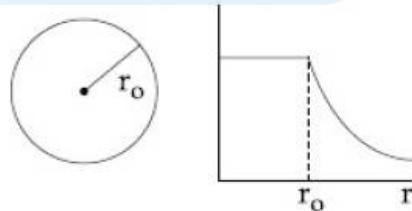
(B) $\frac{3}{4}\rho v^2$
(D) ρv^2

25. If the deBroglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency $6 \times 10^{14} \text{ Hz}$, then the speed of electron is equal to: (Speed of light = $3 \times 10^8 \text{ m/s}$; Planck's constant = $6.63 \times 10^{-34} \text{ J.s}$; Mass of electron = $9.1 \times 10^{-31} \text{ kg}$)

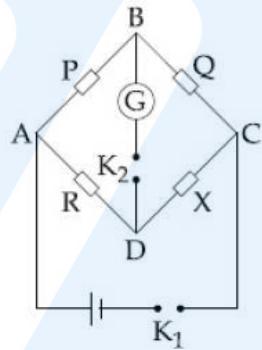
(A) $1.1 \times 10^6 \text{ m/s}$
(C) $1.8 \times 10^6 \text{ m/s}$

(B) $1.7 \times 10^6 \text{ m/s}$
(D) $1.45 \times 10^6 \text{ m/s}$

26. The give graph shown variation (with distance r from centre) of:
- (A) Electric field of a uniformly charged sphere
(B) Potential of a uniformly charged spherical shell
(C) Potential of a uniformly charged sphere
(D) Electric field of a uniformly charged spherical shell

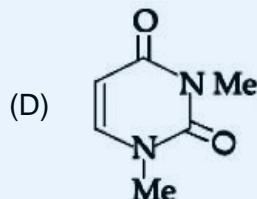
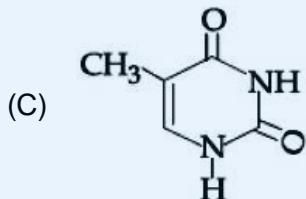
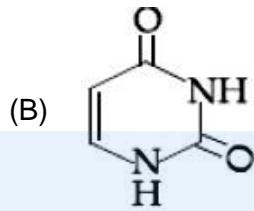
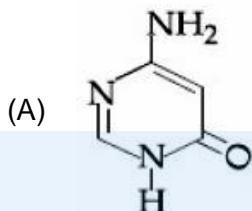


27. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450t - 9x)$ where distance and time are measured in SI units. The tension in the string is:
(A) 10 N (B) 7.5 N
(C) 12.5 N (D) 5 N
28. In an experiment, electrons are accelerated, from rest, by applying, a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied. [Charge of the electron = 1.6×10^{-19} C Mass of the electron = 9.1×10^{-31} kg]
(A) 7.5×10^{-3} m (B) 7.5×10^{-2} m
(C) 7.5 m (D) 7.5×10^{-4} m
29. A body of mass 1 kg falls freely from a height of 100 m, on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x . Given that $g = 10 \text{ ms}^{-2}$, the value of x will be close to:
(A) 40 cm (B) 4 cm
(C) 80 cm (D) 8 cm
30. In a Wheatstone bridge (see figure) Resistance P and Q are approximately equal. When $R = 400 \Omega$, the bridge is balanced. On interchanging P and Q, the value of R, for balance is 405Ω . The value of X is close to:
(A) 401.5 ohm
(B) 404.5 ohm
(C) 403.5 ohm
(D) 402.5 ohm



PART -B (CHEMISTRY)

31. Among the following compounds, which one is found in RNA?



32. The correct match between items I and II is:

Item – I

(Mixture)

- (a) H_2O : Sugar
- (b) H_2O : Aniline
- (c) H_2O : Toluene

(A) a - d, b - r, c - p

(C) a - r, b - p, c - s

Item – II

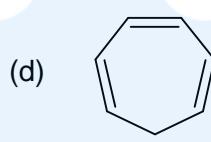
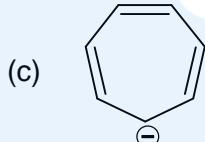
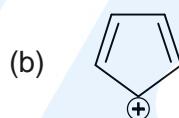
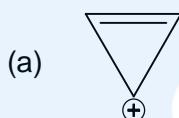
(Separation method)

- p. Sublimation
- q. Recrystallization
- r. Stem distillation
- s. Differential extraction

(B) a - q, b - r, c - s

(D) a - q, b - r, c - p

33. Which compounds out of the following is/are not aromatic?



- (A) b, c and d
- (C) b

- (B) c and d
- (D) a and c

34. A solid having density of $9 \times 10^3 \text{ kg m}^{-3}$ forms face centred cubic crystals of edge length $200\sqrt{2} \text{ pm}$. What is the molar mass of the solid?

[Avogadro constant $\approx 6 \times 10^{23} \text{ mol}^{-1}$, $\pi \approx 3$]

- (A) $0.0432 \text{ kg mol}^{-1}$
- (C) $0.0305 \text{ kg mol}^{-1}$

- (B) $0.0216 \text{ kg mol}^{-1}$
- (D) $0.4320 \text{ kg mol}^{-1}$

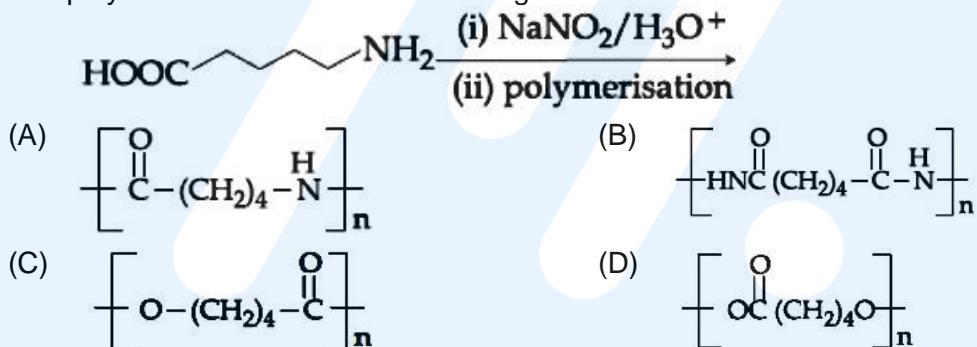
35. For the cell $Zn(s) \mid Zn^{2+}(aq) \parallel M^x+(aq) \mid M(s)$, different half cells and their standard electrode potentials are given below:

$M^{x+}(aq)/M(s)$	$Au^{3+}(aq)/Au(s)$	$Ag^+(aq)/Ag(s)$	$Fe^{3+}(aq)/Fe^{2+}(aq)$	$Fe^{2+}(aq)/Fe(s)$
$E^\circ_{M^{x+}/M}(V)$	1.40	0.80	0.77	-0.44

If $E^\circ_{Zn^{2+}/Zn} = -0.76$ V, which cathode will give a maximum value of E°_{cell} per electron transferred?

- (A) Ag^+/Ag (B) Fe^{3+}/Fe^{2+}
 (C) Au^{3+}/Au (D) Fe^{2+}/Fe
36. The correct match between item I and item II is:
 Item – I Item – II
 a. Norethindrone p. Anti – biotic
 b. Ofloxacin q. Anti – fertility
 c. Equanil r. Hypertension
 s. Analgesics
 (A) a – q, b – r, c – s (B) a – q, b – p, c – r
 (C) a – r, b – p, c – s (D) a – r, b – p, c – r

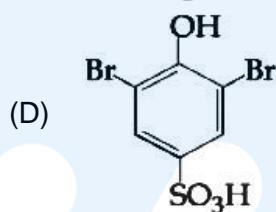
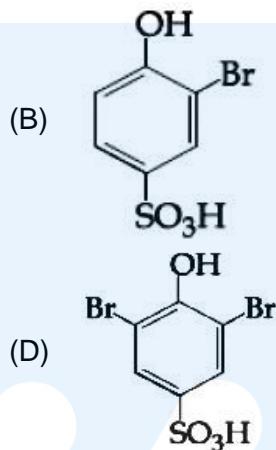
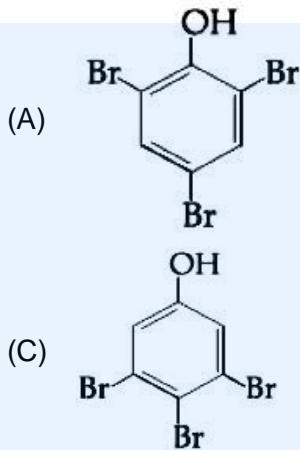
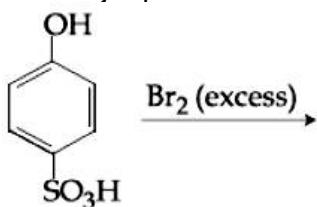
37. The polymer obtained from the following reactions is:



38. NaH is an example of:
 (A) electron rich hydride (B) metallic hydride
 (C) saline hydride (D) molecular hydride
39. The element that usually does NOT show variable oxidation states is:
 (A) Cu (B) Ti
 (C) Sc (D) V
40. An example of solid sol is:
 (A) Paint (B) Gem stones
 (C) Butter (D) Hair cream
41. The concentration of dissolved oxygen (DO) in cold water can go upto:
 (A) 14 ppm (B) 8 ppm
 (C) 10 ppm (D) 16 ppm

42. The correct statements among (a) to (d) regarding H₂ as a fuel are:
- (a) it produces less pollutants than petrol
 - (b) A cylinder of compressed dihydrogen weighs ~30 times more than a petrol tank producing the same amount of energy.
 - (c) Dihydrogen is stored in tanks of metal alloys like NaNi₅
 - (d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively.
- (A) (b) and (d) only (B) (a) and (d) only
 (C) (b), (c) and (d) only (D) (a), (b) and (c) only
43. The major product of the following reaction is:
-
- (A)
- (B)
- (C)
- (D)
44. The freezing point of diluted milk sample is found to be – 0.2°C, while it should have been 0.5°C for pure milk. How much water been added to pure milk to make the diluted sample?
- (A) 1 cup of water to 2 cups of pure milk (B) 3 cups of water to 2 cups of pure milk
 (C) 1 cup of water to 3 cups of pure milk (D) 2 cups of water to 3 cups of pure milk
45. If a reaction follows the Arrhenius equation, the plot lnk v 1/(RT) gives straight line with a gradient (-y) unit. The energy required to activate the reactant is:
- (A) y/R unit (B) y unit
 (C) yR unit (D) -y unit
46. A 10 g effervescent tablet containing sodium bicarbonate and oxalic acid releases 0.25 ml of CO₂ at T = 298.15 K and p = 1 bar. If molar volume of CO₂ is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet? [Molar mass of NaHCO₃ = 84 g mol⁻¹]
- (A) 0.84 (B) 33.6
 (C) 16.8 (D) 8.4
47. The amphoteric hydroxide is:
- (A) be (OH)₂ (B) Ca(OH)₂
 (C) Mg(OH)₂ (D) Sr(OH)₂
48. Peroxyacetyl nitrate (PAN), an eye irritant is produced by:
- (A) classical smog (B) acid rain
 (C) organic waste (D) photochemical smog

49. The major product of the following reactions is:

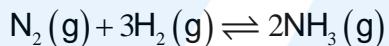


50. An organic compound is estimated through Dumas method and was found to evolve 6 mole of CO_2 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is:

(A) $\text{C}_{12}\text{H}_8\text{N}$
(C) $\text{C}_6\text{H}_8\text{N}_2$

(B) $\text{C}_{12}\text{H}_8\text{N}_2$
(D) $\text{C}_6\text{H}_8\text{N}$

51. Consider the reaction



The equilibrium constant of the above reaction is K_3 . If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $P_{\text{NH}_3} \ll P_{\text{total}}$ at equilibrium)

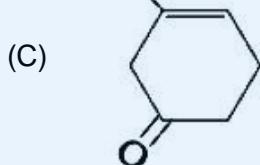
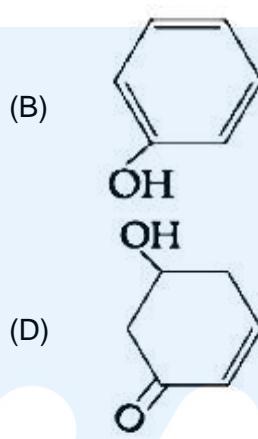
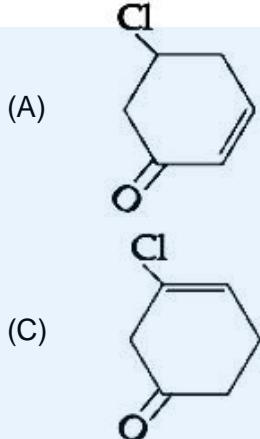
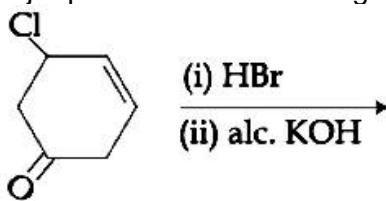
(A) $\frac{3^{3/2} K_p^{1/2} P^2}{16}$

(C) $\frac{K_p^{1/2} P^2}{4}$

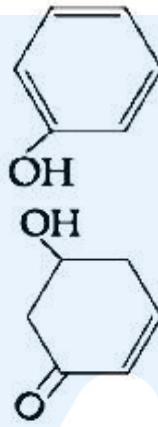
(B) $\frac{K_p^{1/2} P^2}{16}$

(D) $\frac{3^{3/2} K_p^{1/2} P^2}{4}$

52. The major product of the following reaction is:



(D)



53. Two blocks of the same metal having same mass and at temperature T_1 and T_2 respectively, are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS , for this process is:

(A) $C_p \ln \left[\frac{(T_1 + T_2)^2}{4T_1 T_2} \right]$

(B) $2C_p \ln \left[\frac{(T_1 + T_2)^{\frac{1}{2}}}{T_1 T_2} \right]$

(C) $2C_p \ln \left[\frac{(T_1 + T_2)}{4T_1 T_2} \right]$

(D) $2C_p \ln \left[\frac{(T_1 + T_2)}{2T_1 T_2} \right]$

54. Match the metal column I with the coordination compounds/enzymes column II

Column I

Metals

a. Co

b. Zn

c. Rh

d. Mg

(A) a – iii, b – iv, c – i, d - ii

(C) a – ii, b – i, c – iv, d – iii

Column II

Coordination compounds/enzymes

i. Wilkinson catalyst

ii. Chlorophyll

iii. Vitamin B_{12}

iv. Carbonic anhydrase

(B) a – i, b – ii, c – iii, d – iv

(D) a – iv, b – iii, c – i, d – ii

55. For the chemical reaction $X \rightleftharpoons{} Y$, the standard reaction Gibbs energy depends on temperature T (in K) as

$$\Delta_r G^0 \left(\text{in } \text{kJ mol}^{-1} \right) = 120 - \frac{3}{8} T$$

The major component of the reaction mixture at T is:

(A) Y if $T = 300 \text{ K}$

(B) Y if $T = 280 \text{ K}$

(C) X if $T = 350 \text{ K}$

(D) X if $T = 315 \text{ K}$

56. Match the ores (column A) with the metals (column B)

Column A

Ores

I. Siderite

II. Kaolinite

III. Malachite

IV. Calamine

(A) I – a, II – b, III – c, IV – d

(C) I – c, II – d, III – a, IV – b

Column B

Metals

a. Zinc

b. Copper

c. Iron

d. Aluminium

(B) I – c, II – d, III – b, IV – a

(D) I – b, II – c, III – d, IV – a

57. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H – atom is suitable for this purpose?

$$\left[R_H = 1 \times 10^5 \text{ cm}^{-1}, h = 6.6 \times 10^{-34} \text{ Js}, c = 3 \times 10^8 \text{ ms}^{-1} \right]$$

(A) Paschen, $\infty \rightarrow 3$ (C) Balmer, $\infty \rightarrow 2$ (B) Paschen, $5 \rightarrow 3$ (D) Lyman, $\infty \rightarrow 1$

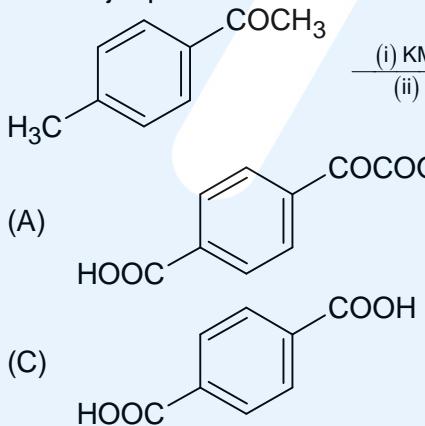
58. The chloride that CANNOT get hydrolysed is

(A) PbCl_4 (C) SnCl_4 (B) CCl_4 (D) SiCl_4

59. The correct order of the atomic radii of C, Cs, Al and S is:

(A) $\text{C} < \text{S} < \text{Al} < \text{Cs}$ (C) $\text{S} < \text{C} < \text{Al}, \text{Cs}$ (B) $\text{S} < \text{C} < \text{Cs} < \text{Al}$ (D) $\text{C} < \text{S} < \text{Cs} < \text{Al}$

60. The major product of the following reaction is



PART-C (MATHEMATICS)

61. If the system of linear equations

$$2x + 2y + 3z = a$$

$$3x - y + 5z = b$$

$$x - 3y + 2z = c$$

Where a, b, c are non zero real numbers, has more than one solution, then:

- | | |
|--|--|
| (A) $b - c + a = 0$
(C) $a + b + c = 0$ | (B) $b - c - a = 0$
(D) $b + c - a = 0$ |
|--|--|

62. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \dots$. Then for all $x \in \mathbb{R}$, the value of $f_4(x) - f_6(x)$ is equal to:

- | | |
|---|---|
| (A) $\frac{1}{12}$
(C) $\frac{-1}{12}$ | (B) $\frac{1}{4}$
(D) $\frac{5}{12}$ |
|---|---|

63. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of the square which is nearest to the origin is:

- | | |
|--------------------------|----------------------------|
| (A) 6
(C) $\sqrt{41}$ | (B) $\sqrt{137}$
(D) 13 |
|--------------------------|----------------------------|

64. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

- | | |
|---|---|
| (A) $(p \vee r) \rightarrow (p \wedge r)$
(C) $p \wedge r$ | (B) $(p \wedge r) \rightarrow (p \vee r)$
(D) $p \vee r$ |
|---|---|

65. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is:

- | | |
|------------------------|------------------------|
| (A) $5/4$
(C) $7/8$ | (B) $9/8$
(D) $3/4$ |
|------------------------|------------------------|

66. The value of the integral $\int_{-2}^2 \left[\frac{\sin^2 x}{\pi} + \frac{1}{2} \right] dx$ (where $[x]$ denotes the greatest integer less than or equal to x) is:

- | | |
|----------------|----------------------------------|
| (A) 0
(C) 4 | (B) $\sin 4$
(D) $4 - \sin 4$ |
|----------------|----------------------------------|

67. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals:
- (A) $\frac{2}{3}$ (B) 2
 (C) $\frac{\sqrt{5}}{2}$ (D) $\sqrt{2}$
68. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points?
 (A) (2, 2, 0) (B) (-2, 2, 2)
 (C) (0, -2, 2) (D) (2, 0, -2)
69. Two integers are selected at random from the set {1, 2, ..., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is:
 (A) $\frac{7}{10}$ (B) $\frac{1}{2}$
 (C) $\frac{2}{5}$ (D) $\frac{3}{5}$
70. Two circles with equal radii intersecting at the points (0, 1) and (0, -1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is:
 (A) 1 (B) 2
 (C) $2\sqrt{2}$ (D) $\sqrt{2}$
71. The value of r for which ${}^{20}C_r {}^{20}C_0 + {}^{20}C_{r-1} {}^{20}C_1 + {}^{20}C_{r-2} {}^{20}C_2 + \dots + {}^{20}C_0 {}^{20}C_r$ is maximum is:
 (A) 15 (B) 20
 (C) 11 (D) 10
72. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve:
 (A) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$ (B) $\frac{x^2}{4} + \frac{y^2}{2} = 1$
 (C) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (D) $\frac{x^2}{2} + \frac{y^2}{4} = 1$
73. Let $f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ x^2 - 1, & 0 \leq x \leq 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$, Then, in the interval (-2, 2), g is:
 (A) differentiable at all points (B) not continuous
 (C) not differentiable at two points (D) not differentiable at one point

74. If $x \log_e (\log_e x) - x^2 + y^2 = 4$ ($y > 0$), then $\frac{dy}{dx}$ at $x = e$ is equal to:

(A) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(B) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(C) $\frac{(1+2e)}{\sqrt{4+e^2}}$

(D) $\frac{e}{\sqrt{4+e^2}}$

75. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x) \left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $(A(x))^m$ equals:

(A) $\frac{-1}{27x^9}$

(B) $\frac{-1}{3x^3}$

(C) $\frac{1}{27x^6}$

(D) $\frac{1}{9x^4}$

76. Let $[x]$ denote the greatest integer less than or equal to x . Then:

$$\lim_{x \rightarrow 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x])))^2}{x^2}$$

(A) does not exist

(B) equals π

(C) equal $\pi + 1$

(D) equals 0

77. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in \mathbb{R} : x^2 + 30 \leq 11x\}$ is

(A) -122

(B) -222

(C) 122

(D) 222

78. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in \mathbb{R}$. Then the range of f is:

(A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) $\mathbb{R} - [-1, 1]$

(C) $\mathbb{R} - \left[-\frac{1}{2}, \frac{1}{2}\right]$

(D) $(-1, 1) - \{0\}$

79. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is:

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{2}{9}$

(D) $\frac{4}{9}$

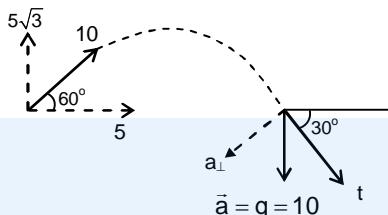
80. The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is:
- (A) $\frac{\sqrt{5}}{2}$ (B) $2\sqrt{5}$
 (C) $\frac{\sqrt{5}}{4}$ (D) $4\sqrt{5}$
81. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y . If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is:
- (A) $\frac{3}{2}y$ (B) $\frac{c}{\sqrt{3}}$
 (C) $\frac{c}{3}$ (D) $\frac{y}{\sqrt{3}}$
82. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is:
- (A) $x + y + 1 = 0$ (B) $x - 2y + 4 = 0$
 (C) $x + 2y + 4 = 0$ (D) $4x + 2y + 1 = 0$
83. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x}\right)^8$ equals 5670 is:
- (A) 0 (B) 6
 (C) 4 (D) 8
84. Let a_1, a_2, \dots, a_{10} be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals:
- (A) 5^4 (B) $4(5^2)$
 (C) 5^3 (D) $2(5^2)$
85. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, $|P|$ then $|p|$ is:
- (A) $\frac{1}{\sqrt{5}}$ (B) $\frac{1}{\sqrt{3}}$
 (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{6}}$

86. If $y(x)$ is the solution of the differential equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$, $x > 0$ where $y(1) = \frac{1}{2}e^{-2}$, then:
- (A) $y(\log_e 2) = \log_e 4$ (B) $y(\log_e 2) = \frac{\log_e 2}{4}$
 (C) $y(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$ (D) $y(x)$ is decreasing in $(0, 1)$
87. The direction ratios of normal to the plane through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$
- (A) $2, -1, 1$ (B) $2, \sqrt{2}, -\sqrt{2}$
 (C) $\sqrt{2}, 1, -1$ (D) $2\sqrt{3}, 1, -1$
88. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is:
- (A) $-10\hat{i} - 5\hat{j}$ (B) $-14\hat{i} - 5\hat{j}$
 (C) $-14\hat{i} + 5\hat{j}$ (D) $-10\hat{i} + 5\hat{j}$
89. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is:
- (A) -81 (B) 100
 (C) 144 (D) -300
90. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x+iy}{27}$ ($i = \sqrt{-1}$), where x and y are real numbers, then $y - x$ equals:
- (A) 91 (B) -85
 (C) 85 (D) -91

HINTS AND SOLUTIONS

PART A – PHYSICS

1.

at $t = 1$

$$u_x = 5, u_y = 5\sqrt{3}$$

$$v_y = 5\sqrt{3} - 10 ; v_x = 5$$

$$\tan \theta = -(2 - \sqrt{3}) \Rightarrow \theta = -30^\circ$$

$$R = \frac{v^2}{a_\perp} = \frac{10^2}{(10 \cos 30^\circ)}$$

$$= \frac{10}{\sqrt{3}} \times 2 = \frac{20}{\sqrt{3}} \text{ m}$$

$$\frac{5^2 + (10 - 5\sqrt{3})^2}{10 \cos \theta} = \frac{200 - 100\sqrt{3}}{10 \times 0.965} = 2.8 \text{ m}$$

2.

Energy supplied

$$E = \frac{12400}{900} = 12.65 \text{ eV}$$

$$\therefore E_n - E_1 = 12.65$$

$$\Rightarrow (13.6) \left(1 - \frac{1}{n^2} \right) = 12.65$$

$$\Rightarrow n^2 \approx 14.3$$

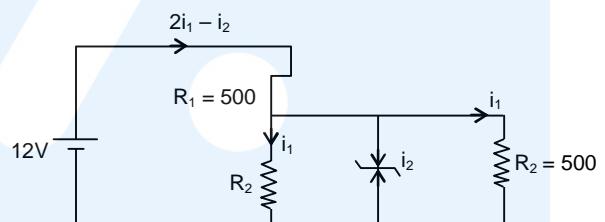
$$\Rightarrow n \approx 4$$

$$r \propto n^2$$

3.

$$12 - 500(2i_1 + i_2) - 10 = 0$$

$$\Rightarrow 2i_1 + i_2 = \frac{2}{500} = \frac{1}{250}$$

< i_1 when zenor has break down.So, $i_2 = 0$.

4.

$$\frac{M}{L} = \frac{\mu_0 n n_1 n_2 n_1^2 \ell}{\mu_0 \pi n_1^2 r_1^2 \ell}$$

$$= \frac{n_2}{n_1}$$

5.

$$K = \frac{1}{2}mv^2 ; U = \frac{1}{2}kx^2 = \frac{1}{2}m^2x^2$$

$$\therefore \frac{k}{U} = \frac{v^2}{\omega^2 x^2} = \left(\frac{\cos(\omega t)}{\sin(\omega t)} \right)^2$$

$$= \cot^2 \left(\frac{\pi}{90} \times 210 \right)$$

$$\begin{aligned}
 &= \cot^2\left(2\pi + \frac{\pi}{3}\right) \\
 &= \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}.
 \end{aligned}$$

6. $\Delta v = v_f - v_i$

$$\begin{aligned}
 &= \sqrt{\frac{2gM_e}{R_e}} - \sqrt{\frac{gM_e}{R_e}} \\
 &= (\sqrt{2} - 1) \sqrt{gR_e}
 \end{aligned}$$

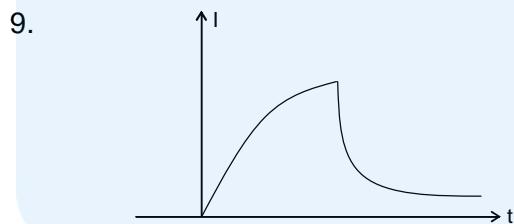
7. Power of e should be dimensionless.

So, $[\lambda] = (\alpha T k)$
 $\Rightarrow L^2 = [\alpha] (ML^2 T^{-2})$
 $\Rightarrow (\alpha) = (M^{-1} T^2)$
 $\Rightarrow E = \frac{1}{2} kT$
 $\Rightarrow (ML^2 T^{-2}) ; (E) = [kT]$
 $\Rightarrow (\alpha\beta) = (F)$
 $\Rightarrow (M^{-1} T^2) (\beta) = (MLT^{-2})$

8. $\Delta x = \frac{\lambda}{8}$

Phase $|\Delta P| = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$

$$\begin{aligned}
 \therefore I_{\text{res}} &= I + I + 2I \cos\left(\frac{\pi}{4}\right) \\
 &= 2I \left(1 + \frac{1}{\sqrt{2}}\right) = 2I \times 1.7 \\
 \therefore \frac{I_{\text{res}}}{I_{\text{max}}} &= \frac{2I \times 1.7}{4I} = 0.85
 \end{aligned}$$



Above is the correct graph for growth and decay of current.

10. $U_{\text{Total}} = \frac{kQq}{a} + \frac{kq^2}{a} + \frac{kQq}{a\sqrt{2}} = 0$

$$\Rightarrow Q = \frac{-q}{\left(1 + \frac{1}{\sqrt{2}}\right)}$$

11. Prism formula

$$D_m = S_m = (n - 1) A \text{ (for thin prism)}$$

So, answer is 1.

12. $I \propto m\ell^2$ (let σ = mass present area)

$$\therefore I_1 \propto \ell^4 \quad \dots(1)$$

$$\text{and } I_2 \propto \left(\frac{\ell}{2}\right)^4 \quad \dots(2)$$

$$\text{So, } I_2 = \frac{I}{16}$$

$$\text{Moment of inertia of remaining sheet} = I - \frac{I}{16}$$

$$= \frac{15I}{16}$$

13. $\vec{\tau}_0 = \vec{\tau}_1 + \vec{\tau}_2$

$$\begin{aligned} &= (6\hat{i}) \times \left(-\frac{F}{2}\hat{i} + \frac{F\sqrt{3}}{2}\hat{j}\right) + (2\hat{i} + 3\hat{j}) \times (F\hat{k}) \\ &= -3F\hat{k}(-2F\hat{j} + 3F\hat{i}) \\ &= F(3\hat{i} - 2\hat{j} - 3\hat{k}) \end{aligned}$$

14. $m \times 0.5 \times 20 + (m - 20) \times 80$

$$= 50 \times 1 \times 40$$

$$\Rightarrow 90m - 1600 = 2000$$

$$\Rightarrow 90m = 3600$$

$$\Rightarrow m = 40 \text{ gm}$$

15. $\Delta \vec{v} = 2v \sin\left(\frac{\theta}{2}\right)$

$$= 2 \times 10 \times \sin(30^\circ)$$

$$= 10 \text{ m/s}$$

16. $f_{so} = f_c \pm f_m$

$$= \frac{\omega_c \pm \omega_m}{2\pi}$$

$$= \frac{(5.5 \pm 0.22) \times 10^5}{2 \times \frac{22}{7}}$$

$$= 89.25, 85.75$$

17. $\frac{1}{V} - \frac{1}{-20} = \frac{1}{30}$

$$\Rightarrow \frac{1}{V} = \frac{1}{0.30} - \frac{1}{20} = +\frac{200 - 3}{60} = \frac{197}{60}$$

$$\Rightarrow -\frac{dV}{dt V^2} + \frac{du}{dt u^2} = 0$$

$$\Rightarrow \frac{dV}{dt} = \frac{V^2}{u^2} \frac{du}{dt} = \left(\frac{3}{197} \right)^2 \times (-5) = -0.00116 \text{ m/s}$$

18. $U_{\text{total}} = U_{O_2} + U_{Ar}$

$$= \frac{3 \times 5 \times RT}{2} + \frac{5 \times 3 \times RT}{2}$$

$$= 15 RT$$

19. Assuming constant voltage supply

$$P = 60 = \frac{V^2}{R_1 + R_2} \quad (1)$$

$$\text{And } P' = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{2V^2}{R_1} = 4P = 4 \times 60$$

$$= 240 \text{ W}$$

20. $0.5 = \frac{6}{(2 + \lambda L)} \lambda x \quad \dots(1)$

$$E_2 = \frac{6}{(6 + \lambda L)} \lambda x \quad \dots(2)$$

So dividing equation (1) and (2)

$$\frac{E_2}{0.5} = \frac{2+4}{6+4} = \frac{3}{5}$$

$$\Rightarrow E_2 = 0.3 \text{ volt.}$$

21. $\frac{E_i}{B_i} = C \quad \dots(1)$

$$\frac{E_f}{B_f} = \frac{C}{n} \quad \dots(2)$$

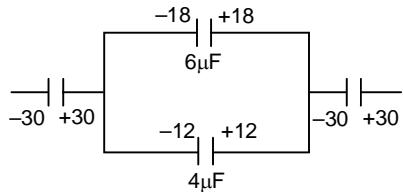
$$\Rightarrow \frac{E_i B_f}{E_f B_i} = \frac{1}{n}$$

$$\Rightarrow \frac{E_i}{E_f} = \frac{1}{n} \frac{B_i}{B_f}$$

$$\left(\because n = \frac{1}{\sqrt{\mu_0 \epsilon_r}} \right)$$

$$\frac{1}{\sqrt{n}} : \sqrt{n}$$

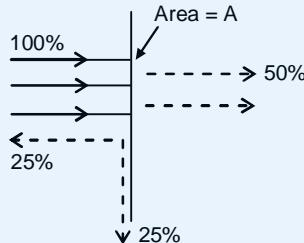
22. $4V + 6V = 30$
 $\Rightarrow V = 3$



23. Equation of adiabatic process
 $TV^{2/f} = \text{constant}$

$$\therefore \frac{2}{f} = \frac{2}{5} = x$$

24. $F = \frac{1}{4} \times \rho A v^2 + \frac{1}{4} 2 \rho a v^2$
 $\Rightarrow \frac{F}{A} = \frac{3}{4} \rho A v^2 = \rho$



25. $\lambda_e = \lambda_{\text{photon}} \times 10^{-3}$
 $\Rightarrow \frac{h}{mv} = \frac{c}{v} \times 10^{-3} \text{ s}$
 $\Rightarrow v = \frac{hv}{mc \times 10^{-3}}$
 $= \frac{6.62 \times 10^{-34} \times 6 \times 10^{14}}{9.1 \times 10^{-31} \times 3 \times 10^8 \times 10^{-3}}$
 $= 1.45 \times 10^6$

26. The potential inside a uniformly charged shell is constant, while it decrease hyperbolically outside.

27. $y = 0.03 \sin \left[450 \left(t - \frac{9x}{450} \right) \right]$

$$\text{So, } v = \frac{450}{9} = 50 \text{ m/s}$$

$$\text{Also, } v = \sqrt{\frac{T}{\lambda}}$$

$$\Rightarrow 50 = \sqrt{\frac{T}{5 \times 10^{-3}}}$$

$$\Rightarrow T = 2500 \times 5 \times 10^{-3} = 12.5 \text{ N}$$

28. $k_e = \frac{p^2}{2M_e} = 500 \text{ e} \quad \dots(i)$

$$\& R = \frac{p}{eB} = \frac{1000 m_e e}{eB} = \frac{10 \cdot 10 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}} \\ = 100 \times 7.541 \times 10^{-6}$$

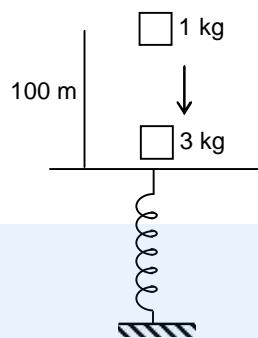
29. $v = \sqrt{2 \times 10 \times 100}$
 $= 20\sqrt{5}$

$$\text{COLM} \rightarrow v_p = \frac{1 \times 20\sqrt{5}}{4}$$

$$= 5\sqrt{5}$$

COTME \rightarrow

$$\begin{aligned} & \frac{1}{2} \times 4 \times (5\sqrt{5})^2 + \frac{1}{2} 1.25 \times 10^6 \left(\frac{30}{k} \right)^2 \\ &= -4 \times 10 \times \left(n - \frac{30}{k} \right) + \frac{1}{2} kn^2 \\ \Rightarrow & 250 + \frac{900}{2 \times 1.25 \times 10^6} = -40x + \frac{1200}{k} + \frac{1}{2} kx^2 \\ \Rightarrow & x \approx 2\text{cm} \end{aligned}$$



30. $\frac{P}{Q} = \frac{400}{S} \quad \dots(1)$

and $\frac{Q}{P} = \frac{405}{5} \quad \dots(2)$

Solving $S^2 = 400 \times 405$

$\Rightarrow S = 402.5 \Omega$

PART B – CHEMISTRY

31. Fact based

32. Fact based

33. b \rightarrow antiaromatic
d \rightarrow Non aromatic (no cyclic conjugation)

34. Density $= \frac{Z \times M}{N_{AV} \times a^3}$

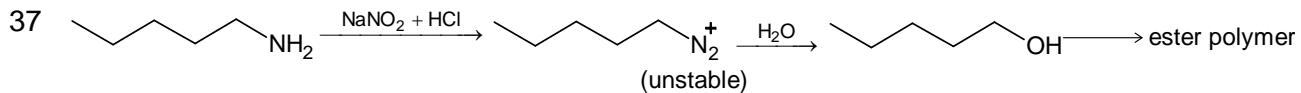
Here $Z = 4$, $a = 200\sqrt{2}$

$N_{AV} = 6.02 \times 10^{23}$, $d = 9 \times 10^3 \frac{\text{Kg}}{\text{m}^3}$

35. $E_{\text{Cell}}^{\circ} = E_{\text{Cathode}}^{\circ} - E_{\text{Zn}^{2+}/\text{Zn}}^{\circ}$

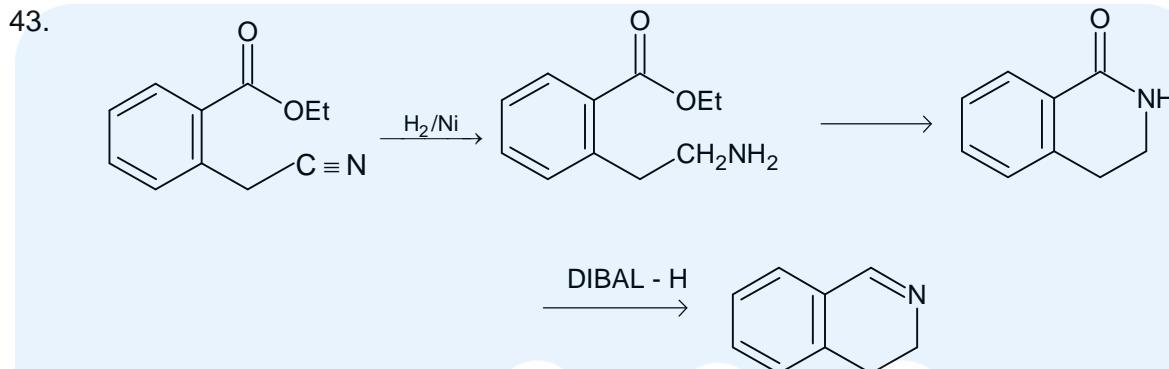
Since $\text{Au}^{3+} \rightarrow \text{Au}$ has maximum value.

36. Fact based.



38. NaH is an example of saline hydride.

39. Sc^{3+} has noble gas configuration hence only +3 exists.
40. An example of solid sol is gem stones.
41. In cold water, dissolved oxygen can reach a concentration upto 10 ppm
42. Fact based.



44. $0.5 \alpha \frac{1}{2}, 0.2 \alpha \frac{1}{x}$
Here $\frac{0.5}{0.2} = \frac{x}{2}$; $x = 5$, hence 3 cup

45. $\ln K = -\frac{E_a}{RT} + \ln A$
 $\therefore \text{Slope} = -E_a = -y$

46. Let $\text{NaHCO}_3 = x \text{ gm}$
Then, $\text{H}_2\text{C}_2\text{O}_4 = (10 - x) \text{ gm}$

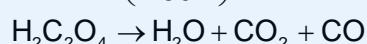
$$\therefore n_{\text{NaHCO}_3} = \frac{x}{84}$$



$$\therefore n_{\text{CO}_2} = \frac{x}{168}$$

$$\text{Total CO}_2 = \frac{x}{168} + \frac{10-x}{90} = \frac{0.25}{25}$$

$$n_{\text{H}_2\text{C}_2\text{O}_4} = \left(\frac{10-x}{90} \right)$$



$$\therefore n_{\text{CO}_2} = \left(\frac{10-x}{90} \right)$$

On solving 'x'

$$\% = \frac{x}{10} \times 100 = 10x$$

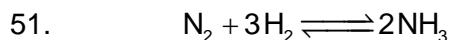
47. Be – O – H
Both bond has same dissociation energy.
48. Fact based.

49. $-\text{SO}_3\text{H}$ will be replaced by Br this is called ipso effect.

50. $\text{CO}_2 = 6$ mole, $\text{N}_1 = 1$ mole

$$\text{C}_{\text{atom}} = 6, \text{N}_{\text{atom}} = 2$$

Hence $\text{C}_6\text{H}_8\text{N}_2$



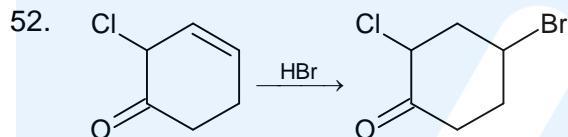
$$\text{equ}^m \quad x \quad 3x \quad P_1$$

$$P_T = 4x \quad K_p = \frac{P_1^2}{x \times 27 \times 3}$$

$$x = \left(\frac{P}{4} \right)$$

$$P_1 = \sqrt{27x^4 K_p}$$

$$\sqrt{27} (K_p)^{1/2} \left(\frac{P_T}{4} \right)^2 = \frac{3^{3/2} K_p^{1/2} P^2}{16}$$



53. $\Delta S_{\text{Total}} = C_p \ln \frac{(T_1 + T_2)}{2T_1} + C_p \ln \frac{(T_1 + T_2)}{2T_2}$

54. Co \rightarrow Vitamin B₁₂

Zn \rightarrow Carbonic anhydrase

Rh \rightarrow Wilkinson catalyst

Mg \rightarrow Chlorophyll

55. $\Delta G^\circ = \left(120 - \frac{3}{8} T \right) = 0$

Then $T = 320 \text{ K}$

Hence $T > 320 \text{ K}$ Y formed

$T < 320 \text{ K}$ X formed

56. Siderite \rightarrow Iron

Kaolinite \rightarrow Aluminium

Malachite \rightarrow Copper

Calamine \rightarrow Zinc

57. [i] $900 \text{ nm} = 9000 \text{ \AA}^\circ$

It is in far infra red region hence paschen.

58. Central atom has no vacant orbital.

59. On moving down size increases.

60. It is the case of side chain oxidation.

PART C – MATHEMATICS

61. $2x + 2y + 3z = a \quad (1)$

$3x - y + 5z = b \quad (2)$

$x - 3y + 2z = c \quad (3)$

$$(2x + 2y + 3z) + (x - 3y + 2z) - (3x - y + 5z) = 0$$

$$\Rightarrow a + c - b = 0$$

62. $F_4(x) = \frac{\sin^4 x + \cos^4 x}{4} = \frac{1 - 2\sin^2 x \cdot \cos^2 x}{4} = \frac{1}{4} - \frac{1}{2}\sin^2 x \cdot \cos^2 x$

$$F_6(x) = \frac{\sin^6 x + \cos^6 x}{6} = \frac{1 - 3\sin^2 x \cdot \cos^2 x (\sin^2 x + \cos^2 x)}{6}$$

$$= \frac{1}{6} - \frac{1}{2}\sin^2 x \cdot \cos^2 x$$

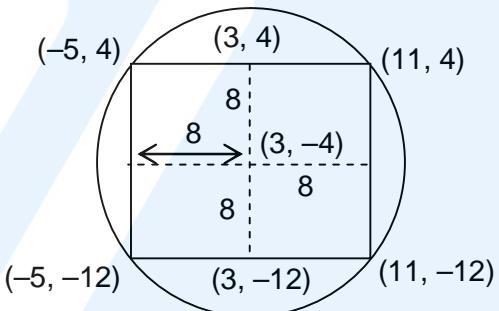
$$F_4(x) - F_6(x) = \frac{1}{4} - \frac{1}{6} = \frac{6-4}{24} = \frac{2}{24} = \frac{1}{12}$$

63. Centre $(3, -4)$

$$\text{Radius} = \sqrt{9 + 16 + 103} = \sqrt{128} = 8\sqrt{2}$$

$\therefore (-5, 4)$ will be nearer to the $(0, 0)$

$$\therefore \text{Ans. } \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}$$



64. As $(p \wedge q) \leftrightarrow r$ is true

\therefore If $p \wedge q$ is true and r is true. Or $p \wedge q$ is F and r is F.

As q is false

$p \wedge q$ can not be true

This case is not possible

\therefore There will be two cases for p, q, r .

T, F, F, or F, F, F respectively

Now check the options.

65. $x^2 = 4y \quad \dots\dots(1)$

$x + 2 = 4y \quad \dots\dots(2)$

Solve (1) and (2)

$$x + 2 = x^2$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = 2, -1$$

$$\begin{aligned}
 \text{Area} &= \int_{-1}^2 \left(\frac{x+2}{4} - \frac{x^2}{4} \right) dx \\
 &= \frac{1}{4} \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2 \\
 &= \frac{1}{4} \left[\left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right) \right] \\
 &= \frac{1}{4} \left[\frac{10}{3} - \frac{3-12+2}{6} \right] = \frac{1}{4} \left[\frac{10}{3} + \frac{7}{6} \right] \\
 &= \frac{1}{4} \times \frac{27}{6} = \frac{9}{8}
 \end{aligned}$$

66.

$$\begin{aligned}
 &\int_0^2 \left(\frac{\sin^2 x}{\frac{1}{2} + \left[\frac{x}{\pi} \right]} + \frac{\sin^2 x}{\frac{1}{2} + \left[-\frac{x}{\pi} \right]} \right) dx \\
 &= \int_0^2 \left(\frac{\sin^2 x}{1 + \left[\frac{x}{\pi} \right]} + \frac{\sin^2 x}{\frac{1}{2} - 1 - \left[\frac{x}{\pi} \right]} \right) dx = \int_0^2 dx = 0
 \end{aligned}$$

67. Variance remains same if same number is subtracted from each observation. (subtract 10 from each observation)

$$\therefore \frac{10(-d)^2 + 10(0)^2 + 10(d)^2}{30} - \left(\frac{10(-d) + 10(0) + 10(d)}{30} \right)^2 = \frac{4}{3}$$

$$\frac{20d^2}{30} = \frac{4}{3}$$

$$\Rightarrow d^2 = 2$$

$$(d) = \sqrt{2}$$

68. Equation of required plane is

$$a(x-3) + b(y+2) + c(z-1) = 0$$

[as it contains the point (3, -2, 1)]

$$2a - b + 3c = 0$$

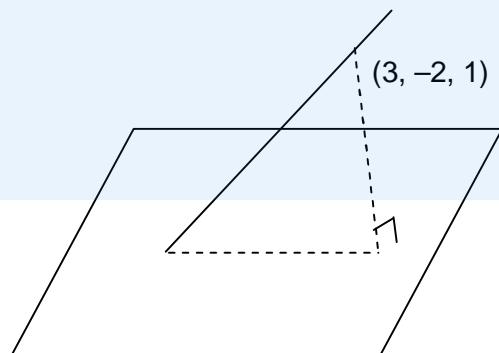
$$2a + 3b - c = 0$$

$$\Rightarrow \frac{a}{-8} = \frac{b}{8} = \frac{c}{8}$$

$$\Rightarrow \frac{a}{+1} = \frac{b}{-1} = \frac{c}{1}$$

∴ equation of plane is
 $x - 3 - y - 2 - z + 1 = 0$

$$\Rightarrow x - y - z = 4$$



69. $P = \frac{\text{both number is even}}{\text{number of ways selecting two numbers such that their sum is even.}}$

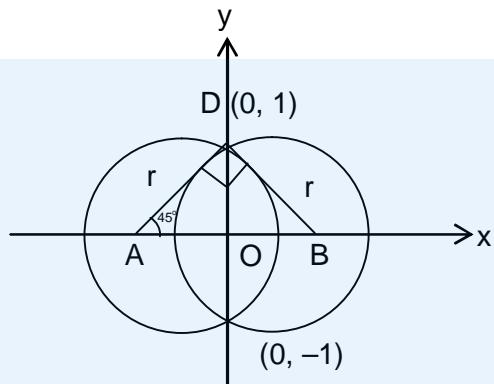
$$= \frac{^5C_2}{^5C_2 + ^6C_2} = \frac{10}{10+15} = \frac{10}{25} = \frac{2}{5}$$

70. The two circles will be orthogonal

$$OD = 1$$

$$\therefore OA = OB = OD = 1$$

$$\Rightarrow AB = 2$$



71. ${}^{20}C_r \cdot {}^{20}C_0 + {}^{20}C_{r-1} \cdot {}^{20}C_1 + \dots + {}^{20}C_0 \cdot {}^{20}C_r =$

Selecting r students from 20 boys and 20 girls $= {}^{40}C_r$

${}^{40}C_r$ will be maximum if $r = 20$.

72. Equation of tangent is

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$$

$$A \text{ is } \left(\frac{a}{\cos \theta}, 0 \right)$$

$$B \text{ is } \left(0, \frac{b}{\sin \theta} \right)$$

Let P(h, k) is mid point

$$2h = \frac{a}{\cos \theta}$$

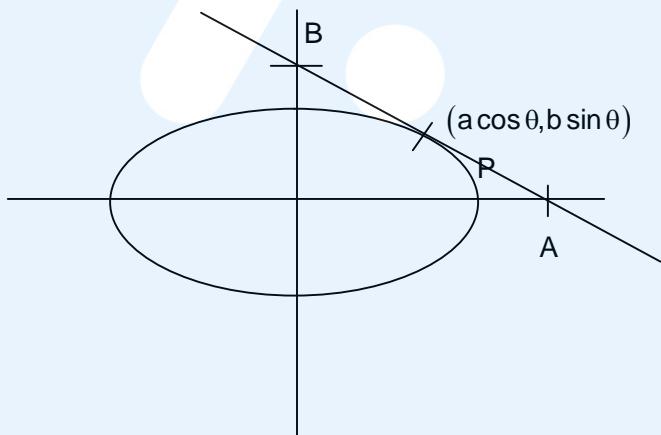
$$2k = \frac{b}{\sin \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\Rightarrow \frac{a^2}{4h^2} + \frac{b^2}{4k^2} = 1$$

$$\Rightarrow \frac{2}{4x^2} + \frac{1}{4y^2} = 1$$

$$\Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$



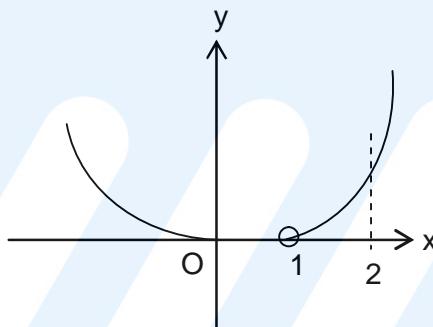
$$73. \quad |f(x)| = \begin{cases} 1 & -2 \leq x < 0 \\ |x^2 - 1| & 0 \leq x \leq 2 \end{cases}$$

$$f(|x|) = \begin{cases} -1 & -2 \leq |x| < 0 \\ x^2 - 1 & 0 \leq |x| \leq 2 \end{cases}$$

$$\Rightarrow f(|x|) = \{x^2 - 1 \mid -2 \leq x \leq 2\}$$

$$g(x) = |f(x)| + f(|x|) : \begin{cases} 1 + x^2 - 1 & -2 \leq x < 0 \\ |x^2 - 1| + x^2 - 1 & 0 \leq x \leq 2 \end{cases}$$

$$g(x) = \begin{cases} x^2 & -2 < x < 0 \\ 0 & 0 \leq x < 1 \\ 2(x^2 - 1) & 1 \leq x < 2 \end{cases}$$



$\therefore g(x)$ is not differentiable at $x = 1$

$$74. \quad x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \log_e(\log_e x) - 2x + 2y \cdot \frac{dy}{dx} = 0$$

Put $x = e$

$$1 - 2e + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2e - 1}{2y} \quad (1)$$

Put $x = e$ in original equation

$$0 - e^2 + y^2 = 4$$

$$y^2 = 4 + e^2$$

Now put in (i)

$$\frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}}$$

$$75. \quad \int \frac{\sqrt{1-x^2}}{x^4} dx = \int \frac{x\sqrt{\frac{1}{x^2}-1}}{x^4} dx = \int \frac{1}{x^3} \sqrt{\frac{1}{x^2}-1} dx$$

$$\text{Let } \frac{1}{x^2} - 1 = t$$

$$-\frac{2}{x^3} dx = dt$$

Integration becomes $-\frac{1}{2} \int \sqrt{t} dt = -\frac{1}{2} \cdot \frac{3}{2} t^{3/2}$

$$= -\frac{1}{3} \left(\frac{1}{x^2} - 1 \right)^{3/2}$$

$$= -\frac{1}{3x^3} (1-x^2)^{3/2}$$

$$= -\frac{1}{3x^3} (\sqrt{1-x^2})^3$$

$$\Rightarrow (A(x))^3 = \left(-\frac{1}{3x^3} \right)^3 = \frac{1}{27x^9}$$

76. $\lim_{x \rightarrow 0} \frac{\tan(\pi \sin^2 x)}{\pi \sin^2 x} \cdot \frac{\pi \sin^2 x}{x^2} + \left(\frac{|x| - \sin(x[x])}{|x|} \right)^2$
 $= 1(\pi) \cdot (1)^2 + \lim_{x \rightarrow 0} \left(1 - \frac{\sin x[x]}{x[x]} \cdot \frac{x[x]}{|x|} \right)^2 \dots \dots \dots \text{(i)}$

$$\lim_{x \rightarrow 0^-} \frac{x[x]}{|x|} = \frac{x}{-x} (-1) = 1$$

$$\lim_{x \rightarrow 0^+} \frac{x[x]}{[x]} = \frac{x(0)}{x} = 0$$

Put in equation (i)

\therefore Limit does not exist.

77. $x^2 - 11x + 30 \leq 0$

$$(x-6)(x-5) \leq 0$$



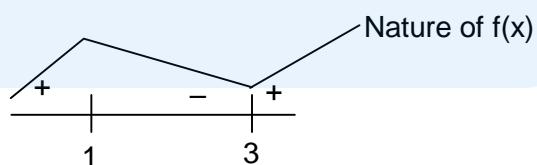
$$5 \leq x \leq 6$$

$$f(x) = 3x^3 - 18x^2 + 27x - 40$$

$$f'(x) = 9x^2 - 36x + 27$$

$$= 9(x^2 - 4x + 3) = 9(x-3)(x-1)$$

$\therefore f(x)$ will be maximum when $x = 6$



$$f(6) = 3(6)^3 - 18(6)^2 + 27(6) - 40$$

$$= 36(18-18) + 162 - 40 = 122$$

78. $y = \frac{x}{x^2 + 1}$

$$yx^2 - x + y = 0$$

$$D \geq 0$$

$$\Rightarrow 1 - 4y^2 \geq 0$$

$$\Rightarrow y^2 \leq \frac{1}{4}$$

$$y \in \left[-\frac{1}{2}, \frac{1}{2} \right]$$

79. $\frac{a}{1-r} = 3$

Cube both sides

$$\frac{a^3}{(1-r)^3} = 27 \quad \dots\dots\dots(1)$$

$$\text{and } \frac{a^3}{1-r^3} = \frac{27}{19} \quad \dots\dots\dots(2)$$

$$(1) / (2) \text{ gives } \frac{1-r^3}{(1-r)^3} = 19$$

$$r = \frac{2}{3}$$

80. Equation of circle $x(x-1) + \left(y - \frac{1}{2}\right)y = 0$

$$x^2 + y^2 - x - \frac{y}{2} = 0$$

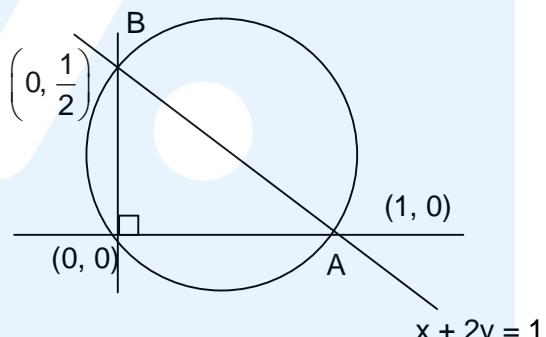
Equation of tangent at $(0, 0)$

$$x \cdot 0 + y \cdot 0 - \frac{x+0}{2} - \frac{y+0}{2 \times 2} = 0$$

$$2x + y = 0 \quad \dots\dots\dots(1)$$

Sum of distance of A and B from Line (i) is

$$\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$$



81. $x^2 - c^2 = y$

$$(a+b)^2 - c^2 = ab$$

$$a^2 + b^2 - c^2 = -ab$$

$$\frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{2}$$

$$\cos c = -\frac{1}{2}$$

$$c = \frac{2\pi}{3}$$

$$\sin C = \frac{\sqrt{3}}{2}$$

$$\frac{c}{\sin c} = 2R \Rightarrow R = \frac{c}{2\sin c} = \frac{c}{\sqrt{3}}$$

82. Equation of tangent to parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$ (1)

Now solve it with $xy = 2$

$$\left(mx + \frac{1}{m}\right)x = 2$$

$$\Rightarrow mx^2 + \frac{1}{m}x - 2 = 0$$

For common tangent $D = 0$

$$\Rightarrow \frac{1}{m^2} + 8m = 0$$

$$m = -\frac{1}{2}$$

$$\text{Put it in equation (i)} \quad y = -\frac{1}{2}x - 2$$

$$\Rightarrow 2x + y + 4 = 0$$

83. 5th term will be the middle term.

$$t_{4+1} = {}^8C_4 \left(\frac{x^3}{3}\right)^4 \left(\frac{3}{x}\right)^4 = 5670$$

$$= {}^8C_4 \cdot x^8 = 5670$$

$$= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2} x^8 = 5670$$

$$= x^8 = \frac{567}{7} = 81$$

$$= x^8 - 81 = 0$$

$$\Rightarrow \text{Real value of } x = \pm\sqrt{3}$$

84. $\frac{a_3}{a_1} = \frac{a_1 r^2}{a_1} = r^2$

$$\Rightarrow r^2 = 25$$

$$\text{Now } \frac{a_9}{a_5} = \frac{a_1 r^8}{a_1 r^4} = r^4 = (25)^2 = 5^4$$

85. A is orthogonal matrix

$$\therefore 4q^2 + r^2 = p^2 + q^2 + r^2 = 1 \quad \dots\dots\dots(1)$$

$$p^2 - q^2 - r^2 = 0 \quad \dots\dots\dots(2)$$

and $2q^2 - r^2 = 0 \quad \dots\dots\dots(3)$

Solving (1), (2) and (3)

$$p^2 = \frac{1}{2}$$

$$|p| = \frac{1}{\sqrt{2}}$$

86. I.F. $= e^{\int \left(\frac{2+1}{x}\right) dx} = e^{2x} \cdot x$

Solution will be $y(xe^{2x}) = \int e^{-2x} \cdot xe^{2x} dx + c$

$$xye^{2x} = \frac{x^2}{2} + c$$

$$1\left(\frac{1}{2}e^{-2}\right) \cdot e^2 = \frac{1}{2} + c \quad (\text{Put } x = 1 \text{ and } y = \frac{1}{2}e^{-2})$$

$$c = 0$$

$$\therefore y = \frac{x}{2}e^{-2x}$$

$$\frac{dy}{dx} = \frac{-2xe^{-2x} + e^{-2x}}{2} = \frac{e^{-2x}}{2}(1-2x)$$

$$\frac{dy}{dx} = 0 \Rightarrow x = \frac{1}{2}$$

$$\frac{dy}{dx} < 0 \text{ if } x \in \left(\frac{1}{2}, 1\right)$$

87. d. r. of AB

$$= (0, 1, 1)$$

$$a \cdot 0 + b \cdot 1 + c \cdot 1 = 0$$

$$b + c = 0$$

$$b + c = 0 \quad \dots\dots\dots(i)$$

$$\cos \frac{\pi}{4} = \frac{a \cdot 0 + b(1) + c(-1)}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{2}}$$

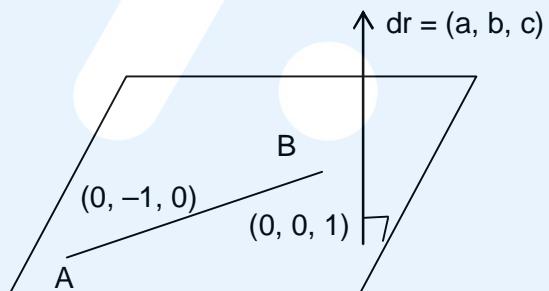
$$\frac{1}{\sqrt{2}} = \frac{b - c}{\sqrt{a^2 + b^2 + c^2}} \cdot \sqrt{2}$$

$$= b^2 + c^2 - 2bc$$

$$\Rightarrow a^2 = -2bc \quad \dots\dots\dots(ii)$$

$$\Rightarrow a^2 = -2b(-b)$$

$$a^2 = 2b^2 \quad \Rightarrow \quad a = \pm \sqrt{2}b$$



$$\frac{a}{\sqrt{2}} = \frac{b}{1}$$

$$\text{From (i)} \quad \frac{a}{\sqrt{2}} = \frac{b}{1} = \frac{c}{-1}$$

$$\text{OR} \quad \frac{a}{-\sqrt{2}} = \frac{b}{1}$$

$$\Rightarrow \frac{a}{-\sqrt{2}} = \frac{b}{1} = \frac{c}{-1} \quad \text{from (i)}$$

$$\frac{a}{\sqrt{2}} = \frac{b}{-1} = \frac{c}{1}$$

dr. will be $(\sqrt{2}, 1, -1)$ or $(\sqrt{2}, -1, 1)$

88. $\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix} = 0$

$$\left| \begin{array}{ccc} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{array} \right|_{R_3 \rightarrow R_3 - 2R_1} = 0$$

$$\left| \begin{array}{ccc} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 0 & 0 & \lambda^2 - 9 \end{array} \right| = 0$$

$$(\lambda^2 - 9)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = 2 \quad \text{OR} \quad \lambda^2 = 5$$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix}$$

$$= \hat{i}(2\lambda^2 - 2 - 16) - \hat{j}(\lambda^2 - 1 - 8)$$

$$= (\lambda^2 - 9)(2\hat{i} - \hat{j}) = -5(2\hat{i} - \hat{j}) \quad (\text{put } \lambda = 2)$$

89. $\alpha + \alpha^3 = -\frac{K}{81} \quad \dots \dots \dots (1)$

$$\alpha^4 = \frac{256}{81}$$

$$\alpha = \pm \frac{4}{3} \quad \dots \dots \dots (2)$$

From (1) and (2)

$$\frac{4}{3} + \frac{64}{27} = \frac{-K}{81}$$

$$K = -300$$

90. $\left(-2 - \frac{i}{3}\right)^3 = \left(\frac{x+iy}{27}\right)$

$$\left(-1\right)^3 \left(2^3 + \frac{i^3}{27} + 3(2)\frac{i^2}{9} + 3(2)^2 \cdot \frac{i}{3}\right) = \frac{x-iy}{27}$$
$$-\left[8 - \frac{i}{27} - \frac{2}{3} + 4i\right] = \frac{x+iy}{27}$$
$$\Rightarrow \frac{x}{27} = -8 + \frac{2}{3}$$

and $\frac{y}{27} = \frac{1}{27} - 4$

$$\frac{y-x}{27} = \frac{1}{27} - 4 + 8 - \frac{2}{3}$$
$$= \frac{1 + 27 \times 4 - 18}{27}$$
$$= \frac{109 - 18}{27}$$
$$= \frac{91}{27}$$
$$\Rightarrow y - x = 91$$