# PART –A (PHYSICS)

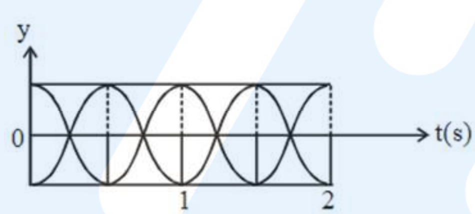
1. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_o\left[1-\frac{1}{2}\left(\frac{V_o}{V}\right)^2\right]$. Here $P_o$ and $V_o$ are constants. Calculate the change in the temperature of the gas if its volume change from $V_o$ to $2V_o$

(A) $\frac{1}{4}\frac{P_oV_o}{R}$
(B) $\frac{1}{2}\frac{P_oV_o}{R}$
(C) $\frac{5}{4}\frac{P_oV_o}{R}$
(D) $\frac{3}{4}\frac{P_oV_o}{R}$

2. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let $I_1$ be the moment of inertia of the disc about its axis and $I_2$ be the moment of inertia of the new sphere about its axis. The ratio of $I_1/I_2$ is given by:

(A) 285
(B) 185
(C) 65
(D) 140

3. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz,

(A)
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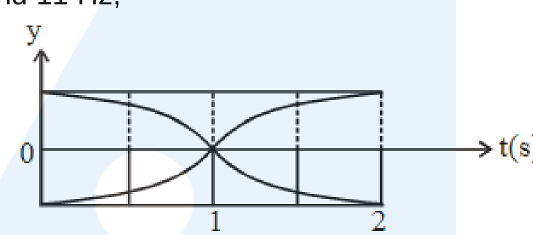


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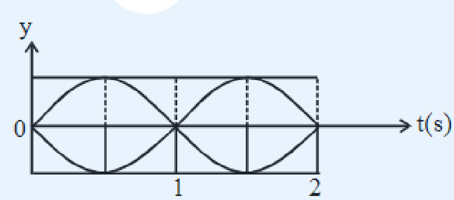


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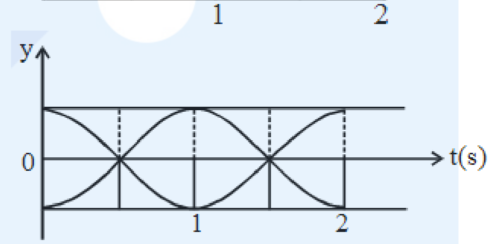



4. In an experiment, brass and steel wires of length 1 m each with areas of cross section 1 $mm^2$ are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress requires to produced a new elongation of 0.2 mm is
[Given, the Young's Modulus for steel and brass are respectively $120 \times 10^9$ $N/m^2$ and $60 \times 10^9$ $N/m^2$]

(A) $1.8 \times 10^6$ $N/m^2$
(B) $0.2 \times 10^6$ $N/m^2$
(C) $1.2 \times 10^6$ $N/m^2$
(D) $4.0 \times 10^6$ $N/m^2$

5. When heat Q is supplied to a diatomic gas of rigid molecules at constant volume its temperature increases by $\Delta T$. The heat required to produce the same change in temperature, at constant pressure is

(A) $\frac{3}{2}Q$ (B) $\frac{5}{3}Q$
(C) $\frac{7}{5}Q$ (D) $\frac{2}{3}Q$

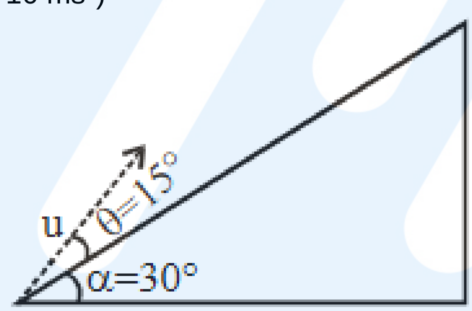
6. A bullet of mass 20 g has an initial speed of 1 $ms^{-1}$ just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistances of $2.5 \times 10^{-2}$ N, the speed of the bullet after emerging from the other side of the wall is close to

(A) 0.7 $ms^{-1}$ (B) 0.3 $ms^{-1}$
(C) 0.1 $ms^{-1}$ (D) 0.4 $ms^{-1}$

7. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

(A) 1.00 mm (B) 1.16 mm
(C) 0.90 mm (D) 1.36 mm

8. A plane is inclined at an angle $\alpha = 30^o$ with respect to the horizontal. A particle is projected with a speed u = 2 $ms^{-1}$, from the base of the plant, making an angle $\theta = 15^o$ with respect to the plane as shown in the figure. The distance from the base at which the particle hits the plane is close to
(Take g = 10 $ms^2$)




(A) 18 cm (B) 14 cm
(C) 26 cm (D) 20 cm

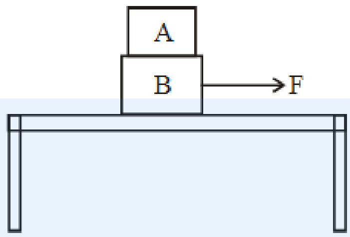
9. The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is: [Take $\mu_o = 4\pi \times 10^{-7}$ $NA^{-2}$]

(A) 9$\mu$T (B) 1$\mu$T
(C) 3$\mu$T (D) 18$\mu$T

10. Two radioactive substances A and B have decay constants $5\lambda$ and $\lambda$ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be

(A) $1/\lambda$ (B) $1/4\lambda$
(C) $2/\lambda$ (D) $1/2\lambda$

11. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontal, so that the block A does not slide over the block B is:
[Take $g = 10\ m/s^2$]

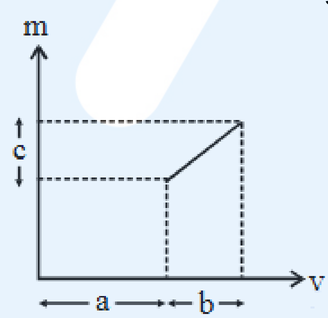



(A) 8 N (B) 16 N
(C) 12 N (D) 40 N

12. The formula $X = 5YZ^2$ X and Z have dimensions of capacitance and magnetic field respectively. What are the dimensions of Y in SI units?
(A) $[M^{-2}\ L^{0}\ T^{-4}\ A^{-2}]$ (B) $[M^{-3}\ L^{-2}\ T^{8}\ A^{-1}]$
(C) $[M^{-2}\ L^{-2}\ T^{6}\ A^{3}]$ (D) $[M^{-1}\ L^{-2}\ T^{4}\ A^{2}]$

13. In $Li^{++}$, electron in first Bohr orbit is excited to a level by a radiation of wavelength $\lambda$. When the ion gets deexcited to the ground state in all possible ways(including intermediate emission) a total of six spectral lines are observed. What is the value of $\lambda$?
(Given: $h = 6.63 \times 10^{34}$ js; $e = 3 \times 10^{8}\ ms^{-1}$)
(A) 10.8 nm (B) 11.4 nm
(C) 9.4 nm (D) 12.3 nm

14. The graph shows how the magnification m produced by a thin lens varies with image distance $v$. What is the focal length of the lens used?




(A) $\frac{b^2}{ac}$ (B) $\frac{a}{c}$
(C) $\frac{b^2c}{a}$ (D) $\frac{b}{c}$

15. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the plant acts on the spaceship. What will be the number of complete revolutions made by the spaceship in 24 hours around the plane?
[Given: Mass of plane = $8 \times 10^{22}$ kg, Radius of planet = $2 \times 10^{6}$ m, Gravitational constant G = $6.67 \times 10–11\ Mn^2/kg^2$]
(A) 9 (B) 11
(C) 13 (D) 17

16. Light is incident normally on a completely absorbing surface with an energy flux of 25 W $cm^{-2}$. If the surface has an area of 25 $cm^2$, the maximum transferred to the surface in 40 min time duration will be

(A) $6.3 \times 10^{-4}$ Ns
(B) $3.5 \times 10^{-6}$ Ns
(C) $5.0 \times 10^{-3}$ Ns
(D) $1.4 \times 10^{-6}$ Ns

17. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$ Its angular momentum with respect to the origin at time t = 2 is .

(A) $-48\hat{k}$
(B) $48(\hat{i} + \hat{j})$
(C) $36\hat{k}$
(D) $-34(\hat{k} - \hat{i})$

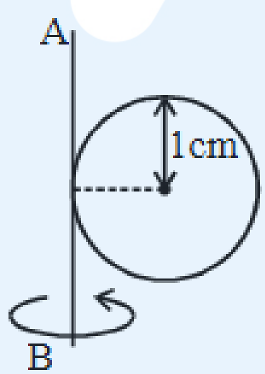
18. Water from a tap emerges vertically downwards with an initial speed of 1.0 $ms^{-1}$. The cross-sectional area of the tap is $10^{-4} m^2$. Assume that the pressure is constant throughout the stream of water and that flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be:
(take g = 110 $ms^{-2}$)

(A) $5 \times 10^{-4}$ $m^2$
(B) $5 \times 10^{-5}$ $m^2$
(C) $1 \times 10^{-5}$ $m^2$
(D) $2 \times 10^{-5}$ $m^2$

19. Space between two concentric conducting spheres of radii a and b (b >a) is filled with a medium of resistivity $\rho$. The resistance between the two spheres will be

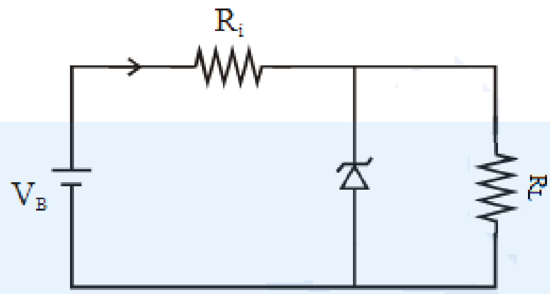
(A) $\frac{\rho}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$
(B) $\frac{\rho}{2\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$
(C) $\frac{\rho}{4\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$
(D) $\frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$

20. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second is 5 s is close to



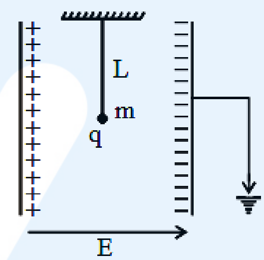

(A) $2.0 \times 10^{-5}$ Nm
(B) $4.0 \times 10^{-6}$ Nm
(C) $1.6 \times 10^{-5}$ Nm
(D) $7.9 \times 10^{-6}$ Nm

21. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is $R_L$= 4kΩ. The series resistance of the circuit is $R_i$ = 1kΩ. If the battery voltage $V_8$ varies from 8 V to 16 V, what are the minimum and maximum values of the current through Zener diode?




(A) 0.5 mA; 0.6 mA (B) 1 mA; 8.5 mA
(C) 1.5 mA; 8.5 mA (D) 0.5 mA; 8.5 mA

22. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. the time period of the pendulum is given by




(A) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$ (B) $2\pi\sqrt{\dfrac{L}{\left(g+\dfrac{qE}{m}\right)}}$

(C) $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{qE}{m}\right)}}$ (D) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2-\dfrac{q^2E^2}{m^2}}}}$

23. In free space, a particle A of charge 1 μC is held fixed at a point P. Another particle B of the same charge and mass 4 μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is

$$\left[\text{Take } \frac{1}{4\pi\varepsilon_o} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}\right]$$

(A) $1.5 \times 10^2$ m/s (B) $2.0 \times 10^3$ m/s
(C) 1.0 m/s (D) $3.0 \times 10^4$ m/s

24. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is
[Given Planck's constant $h = 6.6 \times 10^{-34}$ Js, speed of light $c = 3.0 \times 10^8$ m/s]
(A) $1 \times 10^{16}$ (B) $1.5 \times 10^{16}$
(C) $2 \times 10^{16}$ (D) $5 \times 10^{15}$

25. A coil of self inductance 10 mH and resistance 0.1 $\Omega$ is connected through a switch to a battery of internal resistance 0.9 $\Omega$. After the switch is closed the time taken for the current to attain 80% of the saturation values is: [take ln 5 = 1.6]
(A) 0.016 s (B) 0.324 s
(C) 0.002 s (D) 0.103 s

26. A submarine experiences a pressure of $5.05 \times 10^6$ Pa at a depth of $d_1$ in a sea. When it goes further to a depth of $d_2$, it experiences a pressure of $8.08 \times 10^6$ Pa. Then $d_2 - d_1$ is approximately (density of water = $10^3$ kg/m$^3$ and acceleration due to gravity = 10 ms$^{-2}$)
(A) 400 m (B) 500 m
(C) 600 m (D) 300 m

27. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water?
[Take density of water = $10^3$ kg/m$^3$]
(A) 46.3 kg (B) 65.4 kg
(C) 30.1 kg (D) 87.5 kg

28. In a Young's double slit experiment the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to central fringe on the screen will be
(A) $\left(\sqrt{3}+1\right)^4 : 16$ (B) 25 : 9
(C) 9 : 1 (D) 4:1

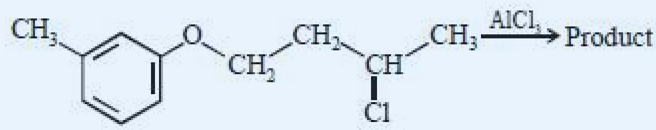
29. A source of sound S is moving with the velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the sound as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him?
(Take velocity of sound in air is 350 m/s)
(A) 1143 Hz (B) 857 Hz
(C) 750 Hz (D) 807 Hz

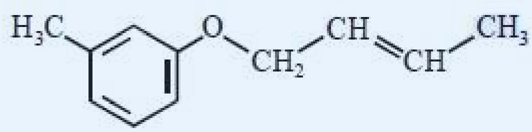
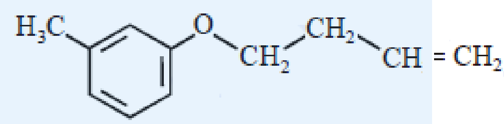
30. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be
(A) $\frac{m}{\pi}$ (B) $\frac{3m}{\pi}$
(C) $\frac{4m}{\pi}$ (D) $\frac{2m}{\pi}$

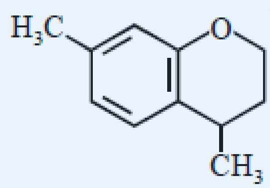
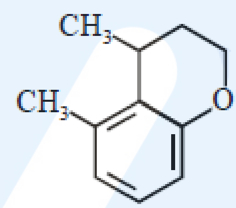
# PART –B (CHEMISTRY)

31. The difference between $\Delta H$ and $\Delta U$ ($\Delta H$ - $\Delta U$), when the combustion of one mole of heptane(l) is carried out a temperature T is equal to
(A) -4 RT (B) -3 RT
(C) 3 RT (D) 4 RT

32. The major product obtained in the given reaction is



(A)  (B) 

(C)  (D) 

33. The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are:
(A) Lyman and Paschen (B) Brackett and Pfund
(C) Paschen and Pfund (D) Balmer and Brackett

34. The correct order of the first ionization enthalpies is
(A) Mn < Ti < Zn < Ni (B) Zn < Ni < Mn < Ti
(C) Ti < Mn < Zn < Ni (D) Ti < Mn < Ni < Zn

35. The correct statements among (a) to (b) are:
(a) saline hydrides produce $H_2$ gas when reacted with $H_2O$.
(b) reaction of $LiAH_4$ with $BF_3$ leads to $B_2H_6$.
(c) $PH_3$ and $CH_4$ are electron - rich and electron-precise hydrides, respectively.
(d) HF and $CH_4$ are called as molecular hydrides.
(A) (c) and (d) only (B) (a), (b) and (c) only
(C) (a), (b), (c) and (d) (D) (a), (c) and (d) only

36. Air pollution that occurs in sunlight is:
(A) oxidising smog (B) acid rain
(C) reducing smog (D) fog

37. For the re action of $H_2$ w i t h $I_2$, the rate constant is $2.5 \times 10^{-4}$ $dm^3$ $mol^{-1}$ $s^{-1}$ at 327°C and 1.0 $dm^3$ $mol^{-1}$ $s^{-1}$ at 527°C. The activation energy for the reaction, in kJ $mol^{-1}$ is:
(R = 8.314 J $K^{-1}$ $mol^{-1}$)
(A) 72 (B) 166
(C) 150 (D) 59

38. In chromatography, which of the following statements is INCORRECT for $R_f$?
(A) $R_f$ value depends on the type of chromatography.
(B) The value of $R_f$ can not be more than one.
(C) Higher $R_f$ value means higher adsorption.
(D) $R_f$ value is dependent on the mobile phase.

39. A hydrated solid X on heating initially gives a monohydrated compound Y. Y upon heating above 373K leads to an anhydrous white powder Z. X and Z, respectively, are:
(A) Washing soda and soda ash.
(B) Washing soda and dead burnt plaster.
(C) Baking soda and dead burnt plaster.
(D) Baking soda and soda ash.

40. The INCORRECT statement is:
(A) the spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar.
(B) the spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83 BM.
(C) the gemstone, ruby, has $Cr^{3+}$ ions occupying the octahedral sites of beryl.
(D) the color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light.

41. Which of these factors does not govern the stability of a conformation in acyclic compounds?
(A) Torsional strain
(B) Angle strain
(C) Steric interactions
(D) Electrostatic forces of interaction

42. The correct statement is:
(A) zincite is a carbonate ore
(B) aniline is a froth stabilizer
(C) zone refining process is used for the refining of titanium
(D) sodium cyanide cannot be used in the metallurgy of silver

43. For the reaction,
$$2SO_2(g) + O_2(g) \longrightarrow 2SO_3(g)$$
$\Delta_H = -57.2$ kJ mol$^{-1}$ and
$K_C = 1.7 \times 10^{16}$
Which of the following statement is INCORRECT?
(A) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.
(B) The equilibrium will shift in forward direction as the pressure increase.
(C) The equilibrium constant decreases as the temperature increases.
(D) The addition of inert gas at constant volume will not affect the equilibrium constant.

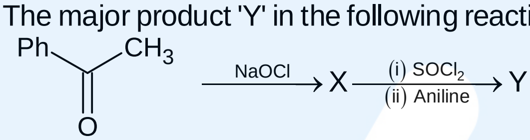
44. The increasing order of nucleophilicity of the following nucleophiles is :
(a) $CH_3CO_2^-$
(b) $H_2O$
(c) $CH_3SO_3^-$
(d) $\overset{-}{O}H$
(A) (b) < (c) < (a) < (d)
(B) (a) < (d) < (c) < (b)
(C) (d) < (a) < (c) < (b)
(D) (b) < (c) < (d) < (a)

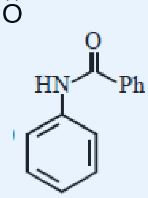
45. The correct match between Item-I and Item-II is:

| | Item-I | | Item-II |
|---|---|---|---|
| (a) | High density polythene | (I) | Peroxide catalyst |
| (b) | Polyacrylonitrile | (II) | Condensation at high temperature & pressure |
| (c) | Novolac | (III) | Ziegler-Natta Catalyst |
| (d) | Nylon 6 | (IV) | Acid or base catalyst |

(A) (a) → (III), (b)→(I), (c)→(II), (d)→(IV)
(B) (a)→(IV), (b)→(II), (c)→(I), (d)→(III)
(C) (a)→(II), (b)→(IV), (c)→(I), (d)→(III)
(D) (a)→(III), (b)→(I), (c)→(IV), (d)→(II)

46. The major product 'Y' in the following reaction is:-




(A)
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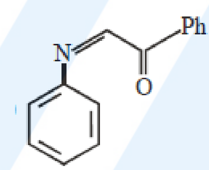


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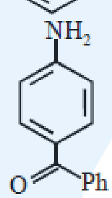


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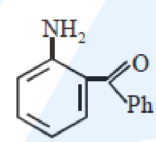
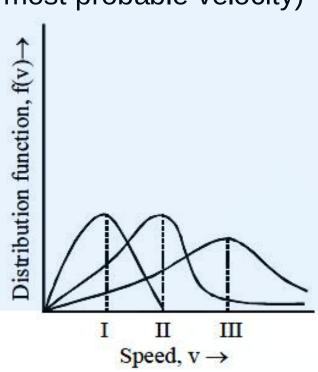


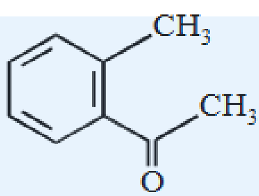

47. Points I, II and III in the following plot respectively correspond to ($V_{mp}$ : most probable velocity)

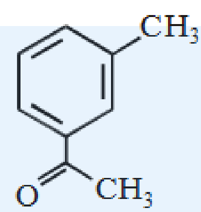


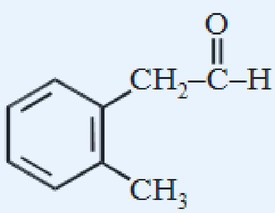

(A) $V_{mp}$ of $N_2$ (300K); $V_{mp}$ of $H_2$(300K); $V_{mp}$ of $O_2$(400K)
(B) $V_{mp}$ of $H_2$ (300K); $V_{mp}$ of $N_2$(300K); $V_{mp}$ of $O_2$(400K)
(C) $V_{mp}$ of $O_2$ (400K); $V_{mp}$ of $N_2$(300K); $V_{mp}$ of $H_2$(300K)
(D) $V_{mp}$ of $N_2$ (300K); $V_{mp}$ of $O_2$(400K); $V_{mp}$ of $H_2$(300K)

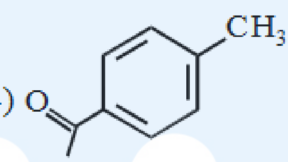
48. The highest possible oxidation states of uranium and plutonium, respectively, are
(A) 6 and 4
(B) 7 and 6
(C) 4 and 6
(D) 6 and 7

49. Compound A ($C_9H_{10}O$) shows positive iodoform test. Oxidation of A with $KMnO_4$/KOH gives acid B($C_8H_6O_4$). Anhydride of B is used for the preparation of phenolphthalein. Compound A is:-

(1) 

(2) 

(3) 

(4) 

50. The noble gas that does NOT occur in the atmosphere is:
(A) He
(B) Ra
(C) Ne
(D) Kr

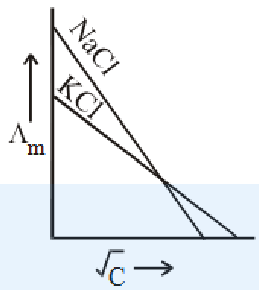
51. The pH of a 0.02 M $NH_4Cl$ solution will be
[given $K_b$ ($NH_4OH$) = $10^{-5}$ and log 2 = 0.301]
(A) 2.65
(B) 5.35
(C) 4.35
(D) 4.65

52. The crystal field stabilization energy (CFSE) of $[Fe(H_2O)_6]Cl_2$ and $K_2[NiCl_4]$, respectively, are :-
(A) $-0.4\Delta_o$ and $-0.8\Delta_t$
(B) $-0.4\Delta_o$ and $-1.2\Delta_t$
(C) $-2.4\Delta_o$ and $-1.2\Delta_t$
(D) $-0.6\Delta_o$ and $-0.8\Delta_t$

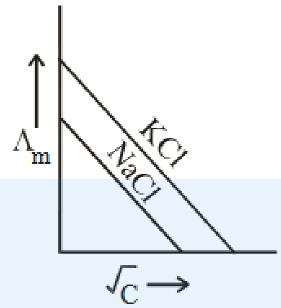
53. The correct option among the following is:
(A) Colloidal particles in lyophobic sols can be precipitated by electrophoresis.
(B) Brownian motion in colloidal solution is faster the viscosity of the solution is very high.
(C) Colloidal medicines are more effective because they have small surface area.
(D) Addition of alum to water makes it unfit for drinking.

54. Which one of the following graphs between molar conductivity ($\Lambda_m$) versus $\sqrt{C}$ is correct?

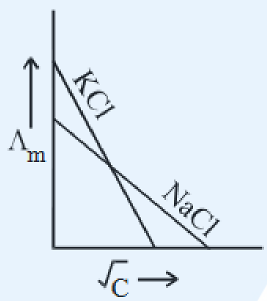
(A)



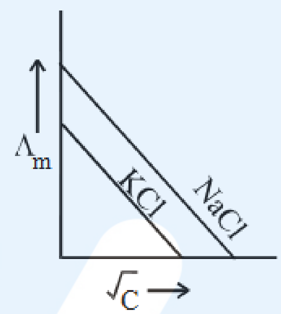

(B)




(C)




(D)




55. 1 g of non-volatile non-electrolyte solute is dissolved in 100g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points, $\frac{\Delta T_b(A)}{\Delta T_b(B)}$ is :

(A) 5 : 1
(B) 10 : 1
(C) 1 : 5
(D) 1 : 0.2

56. Which of the following is NOT a correct method of the preparation of benzylamine from cyanobenzene?

(A) (i) $HCl/H_2O$ (ii) $NaBH_4$
(B) (i) $LiAlH_4$ (ii) $H_3O^+$
(C) (i) $SnCl_2$ + HCl(gas) (ii) $NaBH_4$
(D) $H_2/Ni$

57. The number of pentagons in $C_{60}$ and trigons (triangles) in white phosphorus, respectively, are:

(A) 12 and 3
(B) 20 and 4
(C) 12 and 4
(D) 20 and 3

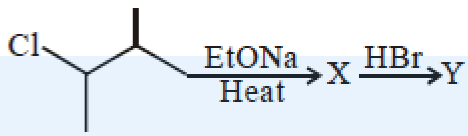
58. The minimum amount of $O_2$(g) consumed per gram of reactant is for the reaction :
(Given atomic mass : Fe = 56, O = 16, Mg = 24, P = 31, C = 12, H = 1)

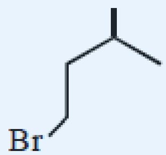
(A) $C_3H_8(g) + 5O_2(g) \rightarrow 3\,CO_2(g) + 4\,H_2O(l)$
(B) $P_4(s) + 5O_2(g) \rightarrow P_4O_{10}(s)$
(C) $4Fe(s) + 3O_2(g) \rightarrow 2\,Fe_2O_3(s)$
(D) $2\,Mg(s) + O_2(g) \rightarrow 2\,MgO(s)$

59. Number of stereo centers present in linear and cyclic structures of glucose are respectively

(A) 4 & 5
(B) 5 & 5
(C) 4 & 4
(D) 5 & 4

60. The major product 'Y' in the following reaction is:




(A)
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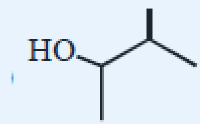
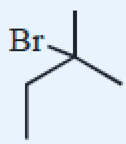
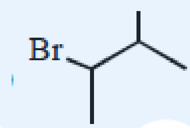



(C)



(D)


# PART–C (MATHEMATICS)

61. If $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=3$, then a + b is equal to

(A) 5 (B) 1
(C) -4 (D) -7

62. The sum of the real roots of the equation

$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x=2 \end{vmatrix}=0$ is equal to

(A) -4 (B) 0
(C) 6 (D) 1

63. Lines are drawn parallel to the line 4x – 3y + 2 = 0 at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines?

(A) $\left(-\frac{1}{4},\frac{2}{3}\right)$ (B) $\left(\frac{1}{4},\frac{1}{3}\right)$
(C) $\left(\frac{1}{4},-\frac{1}{3}\right)$ (D) $\left(-\frac{1}{4},-\frac{2}{3}\right)$

64. If the tangent to the curve $y=\frac{x}{x^2-3}, x\in R, \left(x\neq\pm\sqrt{3}\right)$ at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line 2x + 6y – 11 = 0 then

(A) $|2\alpha + 6\beta| = 11$ (B) $|2\alpha + 6\beta| = 19$
(C) $|6\alpha + 2\beta| = 19$ (D) $|6\alpha + 2\beta| = 9$

65. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, -4) and parallel to the vector $6\hat{i}+3\hat{j}-4\hat{k}$ is

(A) 7 (B) $4\sqrt{3}$
(C) $2\sqrt{13}$ (D) 6

66. If the line ax + y = c, touches both the curves $x^2 + y^2 = 1$ and $y^2-4\sqrt{2}x$, then |c| is equal to

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
(C) $\frac{1}{2}$ (D) 2

67. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If $\alpha$ is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then

(A) $a\alpha^2 + b\alpha - a = 2\alpha^2$ (B) $a\alpha^2 - b\alpha - a = 0$
(C) $a\alpha^2 - b\alpha - a = 1$ (D) $a\alpha^2 + b\alpha + a = 0$

68. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is

(A) $(5, 0)$
(B) $\left(\frac{5}{3}, 0\right)$
(C) $(-5, 0)$
(D) $\left(-\frac{5}{3}, 0\right)$

69. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, where $-1 \le x \le 1$, $-2 \le y \le 2$, $x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy \cos\alpha + y^2$ is equal to

(A) $4\sin^2\alpha - 2x^2y^2$
(B) $4\cos^2\alpha + 2x^2y^2$
(C) $2\sin^2\alpha$
(D) $4\sin^2\alpha$

70. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant is

(A) $x = \sqrt{1+2y}, y \ge 0$
(B) $y = \sqrt{1+4x}, x \ge 0$
(C) $x = \sqrt{1+4y}, y \ge 0$
(D) $y = \sqrt{1+2x}, x \ge 0$

71. Let $a_1, a_2, a_3, \ldots\ldots$ be and A.P with $a_6 = 2$. Then the common difference of this A.P., which maximizes the product $a_1a_4a_5$ is

(A) $\frac{3}{2}$
(B) $\frac{8}{5}$
(C) $\frac{2}{3}$
(D) $\frac{6}{5}$

72. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$ is

(A) 38
(B) 58
(C) 23
(D) 35

73. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is

(A) 210
(B) 180
(C) 170
(D) 190

74. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 $cm^3$/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of ice decreases is

(A) $\frac{1}{36\pi}$
(B) $\frac{5}{6\pi}$
(C) $\frac{1}{9\pi}$
(D) $\frac{1}{18\pi}$

75. If both the means and the standard deviation of 50 observations $x_1$, $x_2$, ………, $x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2$, $(x_2 - 4)^2$, …., $(x_{50} - 4)^2$ is
(A) 400 (B) 380
(C) 525 (D) 480

76. The number of real roots of the equation
$5 + |2^x - 1| = 2^x(2^x - 2)$ is
(A) 4 (B) 3
(C) 2 (D) 1

77. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area(in sq. units) of the triangle PQR is
(A) $\frac{34}{15}$ (B) $\frac{68}{15}$
(C) $\frac{14}{3}$ (D) $\frac{16}{3}$

78. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane x + y + z = 3 such that the foot of the perpendicular Q also lies on the plane x – y + z = 3. Then the co-ordinates of Q are
(A) (2, 0, 1) (B) (-1, 0, 4)
(C) (1, 0, 2) (D) (4, 0, -1)

79. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to
(A) $3^{5/6} - 3^{2/3}$ (B) $3^{5/3} - 3^{1/3}$
(C) $3^{7/6} - 3^{5/6}$ (D) $3^{4/3} - 3^{1/3}$

80. The angles A, B and C of a triangle ABC are in A.P and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is
(A) $2\sqrt{3}$ (B) $\frac{4}{\sqrt{3}}$
(C) $4\sqrt{3}$ (D) $\frac{2}{\sqrt{3}}$

81. Let a, b and c be in G.P with common ratio r, where a ≠ 0 and $0 < r \le \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4th term of this A.P is
(A) $\frac{2}{3}a$ (B) $\frac{7}{3}a$
(C) 5a (D) a

82. Let y = y(x) be the solution of the differential equation
$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y(0) = 1. Then
(A) $y'\left(\frac{\pi}{4}\right) + y'\left(\frac{-\pi}{4}\right) = -\sqrt{2}$ (B) $y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - 2$
(C) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$ (D) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

83. If the plane 2x – y + 2z + 3 = 0 has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes 4x – 2y + 4z + $\lambda$ = 0 and 2x – y + 2z + $\mu$ = 0, respectively, then the maximum value of $\lambda + \mu$ us equal to

(A) 15 (B) 13
(C) 5 (D) 9

84. The area(in sq. units) of the region bounded by the curves y = $2^x$ and y = |x +1| in the first quadrant is

(A) $\frac{3}{2}$ (B) $\log_e 2+\frac{3}{2}$
(C) $\frac{3}{2}-\frac{1}{\log_e 2}$ (D) $\frac{1}{2}$

85. Let $\lambda$ be a real number for which the system of linear equations
x + y + z = 6
4x + $\lambda$y - $\lambda$z = $\lambda$ - 2
3x + 2y – 4z = -5
Has indefinitely many solutions. Then $\lambda$ is a root of the quadratic equation

(A) $\lambda^2 - \lambda - 6 = 0$ (B) $\lambda^2 - 3\lambda - 4 = 0$
(C) $\lambda^2 + 3\lambda - 4 = 0$ (D) $\lambda^2 + \lambda - 6 = 0$

86. The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+......+\frac{1^3+2^3+3^3+.....+15^3}{1+2+3+.....+15}-\frac{1}{2}(1+2+3+......+15)$ is equal to

(A) 620 (B) 1860
(C) 1240 (D) 660

87. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is

(A) 8 (B) 6
(C) 7 (D) 5

88. The negation of the Boolean expression ~ s$\vee$(~r$\wedge$s) is equivalent to

(A) s$\vee$r (B) ~s $\wedge$~r
(C) r (D) s $\wedge$ r

89. If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2}+c$, where c is a constant of integration, then g(-1) is equal to

(A) -1 (B) 1
(C) $-\frac{5}{2}$ (D) $-\frac{1}{2}$

90. If z and w are two complex numbers such that |zw| = 1 and arg(z) – arg(w) = $\frac{\pi}{2}$, then

(A) $\bar{z}w = i$ (B) $z\bar{w}=\frac{-1+i}{\sqrt{2}}$
(C) $z\bar{w}=\frac{1-i}{\sqrt{2}}$ (D) $\bar{z}w=-i$

# HINTS AND SOLUTIONS

## PART A – PHYSICS

1. n = 1 mole

$$P = P_o\left\{1-\frac{1}{2}\left(\frac{V_o}{V}\right)^2\right\} \quad ; \quad PV = nRT = RT$$

$$P = \frac{RT}{V}$$

$$\frac{RT}{V} = P_o\left\{1-\frac{V_o^2}{2V^2}\right\}$$

$$T = \frac{P_oV}{R}\left\{1-\frac{V^2}{2V^2}\right\} = \frac{P_o}{R}\left\{V-\frac{V_o^2}{2V^2}\right\}$$

$$\Delta T = \frac{P_o}{R}\left\{(2V_o - V_o) - \frac{V_o^2}{2}\left(\frac{1}{2V_o}-\frac{1}{V_o}\right)\right\}$$

$$= \frac{P_o}{R}\left\{V_o - \frac{V_o^2}{2}\right\}$$

$$\Delta T = \frac{P_o}{R}\left\{(2V_o - V_o) - \frac{V_o^2}{2}\left(\frac{1}{2V_o}-\frac{1}{V_o}\right)\right\}$$

$$= \frac{P_o}{R}\left\{V_o - \frac{V_o^2(1-2)}{2\times 2V_o}\right\}$$

$$= \frac{P_o}{R}\left\{V_o - \frac{V_o}{4}\right\} = \frac{3}{4}\frac{P_oV_o}{R}$$

2. $$I_1 = \frac{\left(\frac{7M}{8}\right)(ZR)^2}{2} = \frac{7M\times 4R^2}{2\times 8} = \frac{7MR^2}{4}$$

$$I_2 = \frac{2}{5}\frac{M}{8}\left(\frac{R}{2}\right)^2 = \frac{2M}{5\times 8}\frac{R^2}{4} = \frac{MR^2}{80}$$

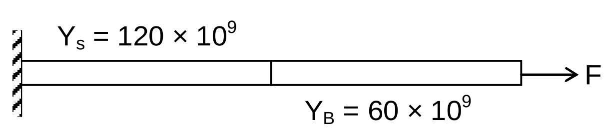
$$\frac{I_1}{I_2} = \frac{7MR^2\times 80}{4MR^2} = 140$$

3. By looking into graph.

4. $\ell$ = 1 M

$A = 10^{-6}\ M^2$

Stress = $\dfrac{F}{A}$ $\qquad Y_s = 120 \times 10^9$




Stress = $\dfrac{\text{Stress}}{Y}$

$$\Delta\ell = \frac{\ell \times F}{AY}$$

$$\Delta\ell_1 + \Delta\ell_2 = \frac{\ell_1 F}{AY_1} + \frac{\ell_2 F}{AY_2} = 0.2 \times 10^{-3}$$

$$\frac{F}{A} = \frac{0.2 \times 10^{-3}}{\frac{\ell}{Y_1} + \frac{\ell}{Y_2}}$$

$$= \frac{0.2 \times 10^{-3}}{\frac{1}{120 \times 10^9} + \frac{1}{60 \times 10^9}} = \frac{0.2 \times 10^{-3} \times 10^9 \times 120}{1+2}$$

$$= \frac{0.2 \times 10^6 \times 120}{3} = 8 \times 10^6$$

5. $Q = C_V \Delta T$

$Q' = C_P \Delta T$

$$Q' = \frac{C_P}{C_V} Q = \left(1 + \frac{2}{5}\right) Q = \frac{7}{5} Q$$

6. $2.5 \times 10^{-2} \times 0.2 = \frac{1}{2} \times 20 \times 10^{-3} \left\{-V^2 + 1^2\right\}$

$5 \times 10^{-3} = 10 \times 10^{-3}\ (1 - V^2)$

$1 - V^2 = \frac{1}{2}$ ; $V^2 = \frac{1}{2}$ ; $V = \frac{1}{\sqrt{2}} = 0.7$

7. $$\frac{400}{\frac{\pi}{4} d^2} = 379 \times 10^6$$

$$d^2 = \frac{4 \times 400 \times 10^{-6}}{\pi \times 379} = 0.336 \times 10^{-6} \times 4$$

$$d = 2\sqrt{0.336} \times 10^{-3}\ M \simeq 1.16\ mm$$

8. $$T = \frac{2u \sin\theta}{g \cos\alpha}$$

$$R = u \cos\theta\, T - \frac{1}{2} g \sin\alpha\, T^2$$

$$= \frac{u \cos\theta\, 2u \sin\theta}{g \cos\alpha} - \frac{g \sin\alpha}{2} \frac{4u^2 \sin^2\theta}{g^2 \cos^2\alpha}$$

$$= \frac{u^2 \sin^2\theta}{g \cos\alpha} - \frac{u^2 \sin\alpha}{g \cos^2\alpha} \{1 - \cos 2\theta\}$$

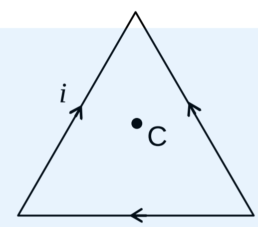
$$= \frac{4 \times \frac{1}{2}}{10 \times \frac{\sqrt{3}}{2}} - \frac{u^2 \sin\alpha}{g \cos^2\alpha} \left\{1 - \frac{\sqrt{3}}{2}\right\}$$

$$= \frac{4}{10\sqrt{3}} - \frac{8}{30}\left\{1 - \frac{\sqrt{3}}{2}\right\}$$

$$= \frac{4}{5\sqrt{3}} - \frac{8}{30} = \frac{8\sqrt{3}-8}{30} = \frac{8(\sqrt{3}-1)}{30} = 20 \text{ cm}$$

9. $i = 10$ A

$\ell = 1$ m




$\mu_o = 4\pi \times 10^{-7} \frac{N}{A^2}$

$$B = \frac{\mu_o i}{\frac{4\pi\sqrt{3}\,\ell}{2}} \times 3$$

$$= \frac{\mu_o i\sqrt{3}}{2\pi\ell} = \frac{4\pi \times 10^{-7} \times 10 \times \sqrt{3}}{2\pi \times 1} = 20\sqrt{3} \times 10^{-7}$$

$= 3\ \mu T$

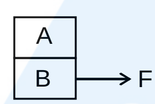
10. $\frac{1}{e^2} = e^{\lambda t - 5\lambda t}$

$$t = \frac{1}{2\lambda}$$

11. $M_A = 1$ kg, $M_B = 3$ kg

$\mu_{AB} = 0.2$

$\mu_B = 0.2$



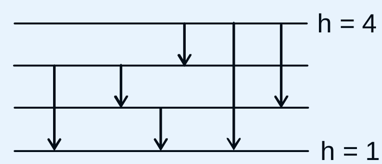

$F_{max} = (M_A + M_B) \times 0.2 \times 10 + (M_A + M_B) \times 0.2 \times$
10

$= 4 \times 2 + 4 \times 2 = 16$

12. $X = 5YZ^2$

$$Y = \frac{X}{5Z^2} = M^{-3} L^{-2} T^8 A^4$$

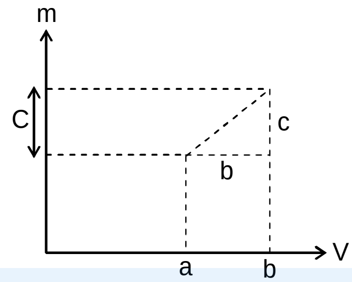
13. $\frac{hc}{\lambda} = 13.6 \text{ eV(g)}\left\{1 - \frac{1}{16}\right\}$




$$\frac{1240 \text{ eV}}{\lambda} = \frac{15}{16} \times 9 \times 13.6 \text{ eV}$$

$$\lambda = \frac{1240 \times 16}{15 \times 9 \times 13.6} = 10.8 \text{ nm}$$

14. $f = \frac{b}{c}$




15. $\frac{mV^2}{r} = \frac{GMm}{r^2}$

$V = \sqrt{\frac{GM}{r}}$

$n = \frac{VT}{2\pi r} = \sqrt{\frac{GM}{r}} \; \frac{T}{2\pi r}$

$= \left(\sqrt{\frac{GM}{r^3}}\right) \times \frac{T}{2\pi} = \sqrt{\frac{6.67\times10^{-11}\times8\times10^{22}}{(202\times10^4)^3}} \times \frac{T}{2\pi}$

$= \frac{24\times3600}{2\times3.14}\sqrt{\frac{6.67\times8\times10^{11}}{(202)^3\times10^{12}}} = \frac{24\times3600}{2\times3.14\times1242.8} = \frac{24\times3600}{78.51} \simeq 11$

16. $I = 25\frac{W}{cm^2} = 25\times10^4 \text{ W / m}^2$

P = 25 × 25 ; W = 625 W

$\frac{hc}{\lambda}\frac{dn}{dt} = P$

$F = \frac{h}{\lambda}\frac{dn}{dt} = \frac{P}{C} = \frac{625}{3\times10^8}$

Momentum = $\frac{625\times40\times60}{3\times10^8}$ = $5 \times 10^{-3}$ Ns

17. $\vec{v} = 2\hat{i} - 6 + \hat{j}$

At t = 2

$\vec{v} = 2\hat{i} - 12\hat{j}$

$\vec{P} = m\vec{v} = 4i - 24\hat{j}$

At t = 2

$\vec{r} = 4\hat{i} - 12\hat{j}$

$\vec{L} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -12 & 0 \\ 4 & -24 & 0 \end{vmatrix}$

$= \{4(-24) + 4\times12\}\,\hat{k}$

$= (-96 \; + \; 48)\,\hat{k}$

$= (-)\,48\,\hat{k}$

18. $10^{-4} \times 1 = \sqrt{(1)^2 + 2 \times 10 \times 0.15} \times A$

$A = \frac{10^{-4}}{2} = 5 \times 10^{-5}$

19. $R = \int_a^b \frac{\rho\, dx}{4\pi x^2}$

$= \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$

20. $m = 5 \times 10^{-3}$ kg, $r = 10^{-2}$ m

$\omega = 25 \times 2\pi$ rad/5

$= 50\ \pi$ rad/sec

$\omega = \frac{\tau}{I} t$

$\tau = \frac{I\omega}{t} = \frac{5mr^2}{4} \times \frac{\omega}{t}$

$= \frac{5 \times 5 \times 10^{-3} \times 10^{-4} \times 50\ \pi}{4 \times 5}$

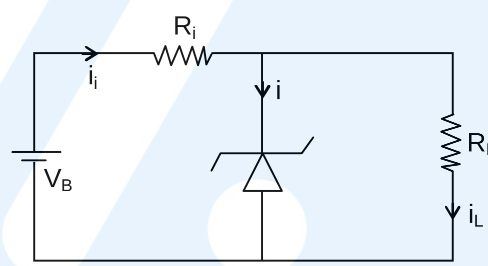
$= \frac{25\pi}{4} \times 10^{-6} = 2 \times 10^{-5}$

21. $V_{breakdwon} = 6V$, $R_L = 4k\Omega$, $R_i = 1\ k\Omega$

$i_L = \frac{6}{4} \times 10^{-3} = 1.5 \times 10^{-3} = 1.5$ mA

$i_i = 2 \times 10^{-3}$

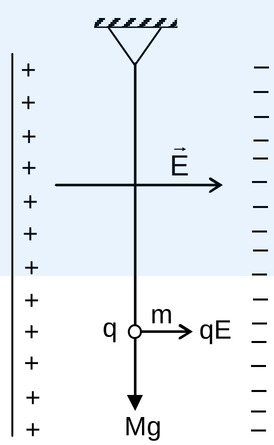
$i = i_1 - i_L = 0.5$ mA – minimum current




$i_i = 10 \times 10^{-3} = 10$ mA

$i_{max} = 8.5$ mA

22. $T = 2\pi \sqrt{\frac{L}{\sqrt{g^2 + \frac{q^2E^2}{M^2}}}}$



23. $q_A = 1\ \mu c$ ; $q_B = 1\ \mu c$, $m_B = 4 \times 10^{-9}$ kg, $r_{AB} = 10^{-3}$ m

$$\frac{1}{2}M_B V^2 = k\, q_A q_B \left\{\frac{1}{10^{-3}} - \frac{1}{9\times 10^{-3}}\right\}$$

$$\frac{1}{2}4\times 10^{-9} V^2 = 9\times 10^9 \times 10^{-6} \times 10^{-6} \times \frac{8}{9} \times 10^3$$

$$V^2 = \frac{8}{2}\times 10^9 = 4\times 10^9$$

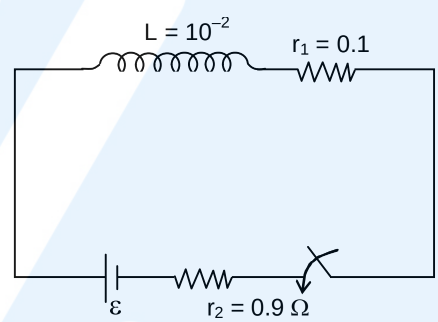
24. $$2\times 10^{-3} = \frac{hc}{\lambda}\frac{dn}{dt}$$

$$\frac{dn}{dt} = \frac{2\times 10^{-3}\lambda}{hc}$$

$$= \frac{2\times 10^{-3}\times 500\times 10^{-9}}{6.6\times 10^{-34}\times 3\times 10^8}$$

$$= \frac{1000}{6.6\times 3}\times 10^{14} = 5 \times 10^{15}$$

25. $L = 10 \times 10^{-3}$ H, $r_1 = 0.1\ \Omega$




$i = \varepsilon\left\{1 - e^{-t/2}\right\}$

$i_{saturation} = \varepsilon$

80% $i_{saturation} = 0.8\ \varepsilon$

$0.8\ \varepsilon = \varepsilon\left\{1 - e^{-t/2}\right\}$

$0.8 = 1 - e^{-t/2}$ ; $e^{-t/2} = 0.2$

$e^{t/L} = 5$

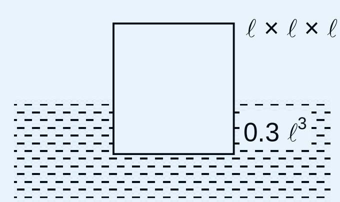
$t = L \ln 5 = 10 \times 10^{-3} \times 1.6 = 16 \times 10^{-3}$

26. $P_1 = 5.05 \times 10^6$ ; $P_2 = 8.08 \times 10^6$

$P_2 - P_1 = \rho g(d_2 - d_1)$

$$d_2 - d_1 = \frac{3.03\times 10^6}{10^3\times 10} = 3.03 \times 10^2 = 303$$

27. $0.3\ \ell^3 \rho_\omega = \ell^3 \rho$




$$\rho = 300\frac{kg}{m^3}$$

$m + \ell^3\rho = \ell^3\rho_\omega$

$M = \ell^3(\rho_w - \rho) = (5)^3\{1000 - 300\} = 700 \times (5)^3$

$= 87.5$ kg

28. $$\frac{I_{Max}}{I_{Min}} = \frac{9}{1}$$

29. $f_a = \dfrac{V}{V - V_s} f_o$ = 1000 Hz

(s) ⟶ V = 50 m/s

$f'_a = \dfrac{V}{V + V_s} f_o$

$\dfrac{f'_a}{f_a} = \dfrac{V - V_s}{V + V_s} = \dfrac{350 - 50}{350 + 50} = \dfrac{300}{400} = \dfrac{3}{4}$

$f'_a = \dfrac{3}{4} \times 1000$ = 750 Hz

30. m = $I\ell^2$ 2πr = $4\ell$

$m' = \dfrac{I4\ell^2}{\pi}$ $r = \dfrac{2\ell}{\pi}$

$\dfrac{m'}{m} = \dfrac{4}{\pi}$ $\pi r^2 = \dfrac{\pi 4\ell^2}{\pi^2} = \dfrac{4\ell^2}{\pi}$

$m' = \dfrac{4}{\pi} m$

## PART B – CHEMISTRY

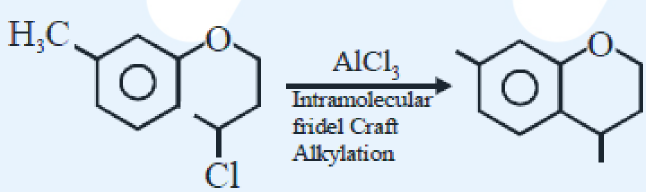
31. $C_7H_{16}(\ell) + 11O_2(g) \longrightarrow 7CO_2(g) + 8H_2O(\ell)$

$\Delta n_g = n_p - n_r = 7 - 11 = -4$

$\because \quad \Delta H = \Delta U + \Delta n_g RT$

$\therefore \quad \Delta H - \Delta U = -4\ RT$

32.




33. $$\frac{\dfrac{1}{\lambda_2} = R_H\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)Z^2}{\dfrac{1}{\lambda_1} = R_H\left(\dfrac{1}{n_1^1} - \dfrac{1}{n_2^1}\right)Z^2}$$

As for shortest wavelength both $n_1$ and $n_2^1$ are $\infty$

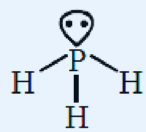
$\therefore \dfrac{\lambda_1}{\lambda_2} = \dfrac{9}{1} = \dfrac{n_1^1}{n_1^2}$

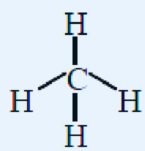
Now if $n_1^1$ = 3 and $n_1$ is 1 it will justify the statement hence Lyman and Paschen is correct.

34. As Zn is fully filled and left to right in group IP increases.

35. (a) $\underset{\text{Ionic hydride}}{\overset{\oplus}{M}\overset{\ominus}{H}} + HOH \longrightarrow MOH + H_2$

(b) $4BF_3 + 3LiAlH_4 \longrightarrow 2B_2H_6 + 3LiF + 3AlF_3$

(c)  $\rightarrow$ phosphorous is electron rich hydride due to presence of lone pair

 $\rightarrow$ It is electron precise hydride.

(d) HF & $CH_4$ are molecular hydride due to they are covalent molecules.

36. Fact based

37. $H_2(g) + I_2(g) \rightarrow 2HI(g)$

Apply Arrhenius equation

$$\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{600} - \frac{1}{800}\right)$$

$$\log\frac{1}{2.5\times10^{-4}} = \frac{E_a}{2.303\times8.31}\left(\frac{200}{600\times800}\right)$$

$\therefore\ E_a \approx 166\text{kJ/mol}$

38. $R_f$ value can't measure the extent of adsorption.

39. $\underset{\substack{\text{washing soda}\\(X)}}{Na_2CO_3.10H_2O(s)} \xrightarrow{\Delta} \underset{(Y)}{Na_2CO_3.H_2O}$

$\downarrow \Delta\ T > 373k$

$\underset{\substack{\text{(soda ash)}\\(Z)}}{Na_2CO_3}$

40. In gemstone, ruby has $Cr^{3+}$ ion occupying the octahedral sites of aluminium oxide ($Al_2O_3$) normally occupied by $Al^{3+}$ion.

41. Angle strain govern stability in cyclic compound.

42. Fact based.

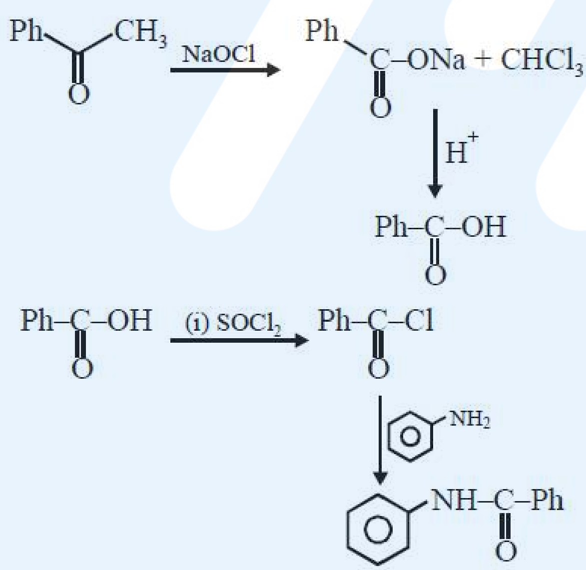
43. In option (B)- $\Delta n_g$ is –ve therefore increase in pressure will bring reaction in forward direction.
In option (C)- as the reaction is exothermic therefore increase in temperature will decrease the equilibrium constant.
In option (D)- Equilibrium constant changes only with temperature. Hence, option (B), (C) and (D) are correct therefore option (1) is incorrect choice.

44.
$$\underbrace{\overset{-}{O}H > CH_3-\underset{\underset{O}{\|}}{C}-O^- > CH_3-\overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{S}}-O^-}_{\text{Charged ion}} > \underbrace{H_2O}_{\text{neutral system}}$$

ione pair donating tendency on oxygen is reduced, nucleophilicity reduced b < c < a < d

45
| | | | |
|---|---|---|---|
| (a) | High density | (III) | Ziegler-Natta Catalyst |
| (b) | Polyacrylonitrile | (I) | Peroxide catalyst |
| (c) | Novolac | (IV) | Acid or base catalyst |
| (d) | Nylon 6 | (II) | Condensation at high temperature & pressure |

46.



47.

$$V_{mp} = \sqrt{\frac{2RT}{M}} \Rightarrow V_{mp} \propto \sqrt{\frac{T}{M}}$$

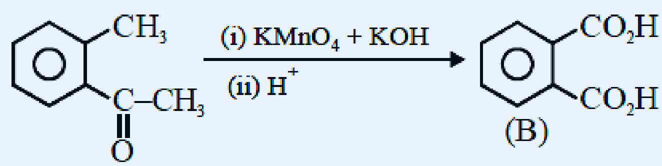
For $N_2$, $O_2$, $H_2$

$$\sqrt{\frac{300}{28}} < \sqrt{\frac{400}{32}} < \sqrt{\frac{300}{2}}$$

$$V_{mp} \text{ of } N_2(300K) < V_{mp} \text{ of } O_2(400K) < V_{mp} \text{ of } H_2(300K)$$

48. The highest oxidation state of U and Pu is 6+ and 7+ respectively.

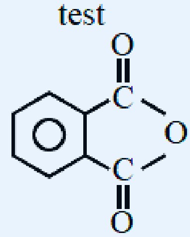
49.




(B)

$(C_8H_6O_4)$ Phthalic acid

+ve iodoform test



is used for prepareation of phenolphthalein indicator

Phthalic anhydride

50. Fact based.

51. For the salt of strong acid and weak base

$$H^+ = \sqrt{\frac{K_w \times C}{K_b}}$$

$$\left[H^+\right] = \sqrt{\frac{10^{-14} \times 2 \times 10^{-2}}{10^{-5}}}$$

$$-\log\left[H^+\right] = 6 - \frac{1}{2}\log 20$$

$\therefore$ pH = 5.35

52. CFSE = [-0.4$n_{t2g}$ + 0.6 $n_{eg}$] $\Delta_o$

53. In electrophoresis precipitation occurs at the electrode which is oppositely charged therefore (A) is correct.

54. Both NaCl and KCl are strong electrolytes and as $Na^+$(aq.) has less conductance than $K^+$ (aq.) due to more hydration therefore the graph of option (B) is correct.

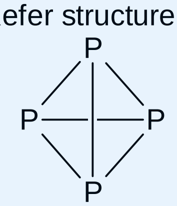
55. $\Delta T_b = K_b \times m$

$$\therefore \quad \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{K_{b(A)}}{K_{b(B)}} \text{ as } m_A = m_B$$

$$\therefore \quad \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{1}{5}$$

56. Benzylamine will not give cyanobenzene with $HCl/H_2O$ & $NaBH_4$.

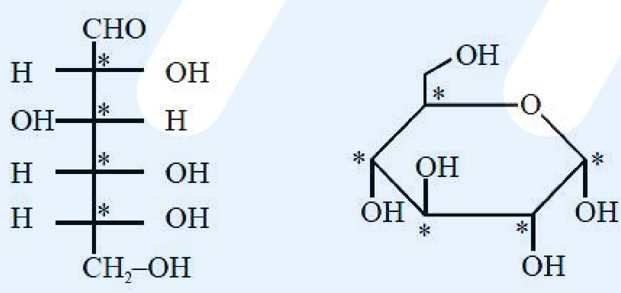
57. Refer structure of $C_{60}$ & $P_4$




58. 4 mol of Fe require $3 \times 32$ gram

$\frac{1}{56}$ mol of Fe require $= \frac{3 \times 32}{4} \times \frac{1}{56} = 0.428$ g

59.

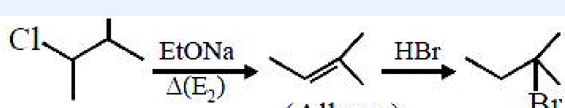



D–Glucose
(Linear structure)

α-D–Glucose
(cyclic structure)

∗ :- Stereocenter

60.




(Alkene)
(Saytzeff prod.)

# PART C – MATHEMATICS

61. $\lim\limits_{x \to 1} \dfrac{x^2 - ax + b}{x - 1} = 5$

$1 - a + b = 0$ ........(i)

$2 - a = 5$ . ........(ii)

$\Rightarrow a + b = -7$

62. By expansion, we get

$-5x^3 + 30x - 30 + 5x = 0$

$\Rightarrow -5x^3 + 35x - 30 = 0$

$\Rightarrow x^3 - 7x + 6 = 0$, All roots area real

So, sum of roots = 0

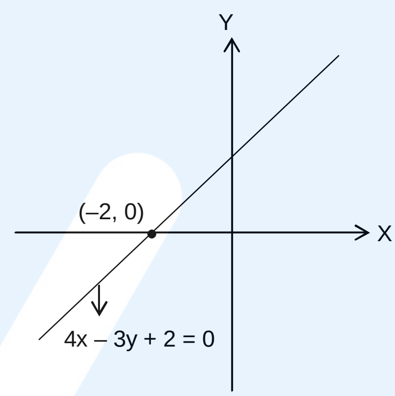
63. Required line is $4x - 3y + \lambda = 0$

$\left|\dfrac{\lambda}{5}\right| = \dfrac{3}{5}$

$\Rightarrow \lambda = \pm 3$

So, required equation of line is $4x - 3y + 3 = 0$ and $4x - 3y - 3 = 0$

(1) $4\left(-\dfrac{1}{4}\right) - 3\left(\dfrac{2}{3}\right) + 3 = 0$




64. $\left.\dfrac{dy}{dx}\right|_{(\alpha,\beta)} = \dfrac{-\alpha^2 - 3}{\left(\alpha^2 - 3\right)^2}$

Given that :

$\dfrac{-\alpha^2 - 3}{\left(\alpha^2 - 3\right)^2} = -\dfrac{1}{3}$

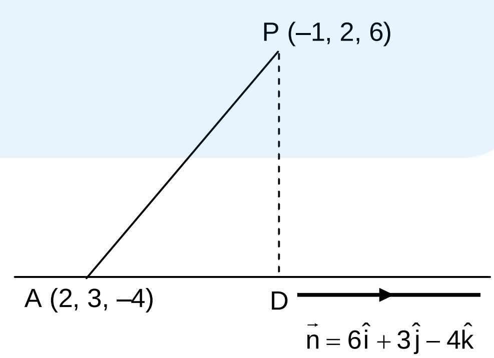
$\Rightarrow \alpha = 0, \pm 3 \qquad (\alpha \neq 0)$

$\Rightarrow \beta = \pm \dfrac{1}{2}. \qquad (\beta \neq 0)$

$|6\alpha + 2\beta| = 19$

65. $AD = \left|\dfrac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \sqrt{61}$

$\Rightarrow PD = \sqrt{AP^2 - AD^2} = \sqrt{110 - 61} = 7$



66. Tangent to $y^2 = 4\sqrt{2}x$ is $y = mx + \frac{\sqrt{2}}{m}$ it is also tangent to $x^2 + y^2 = 1$

$\Rightarrow \left|\frac{\sqrt{2}/m}{\sqrt{1+m^2}}\right| = 1 \Rightarrow m = \pm 1$

$\Rightarrow$ Tangent will be $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$ compare with $y = -ax + C$

$\Rightarrow a = \pm 1$ and $C = \pm\sqrt{2}$

67. $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$

$(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

68. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$a = 3, b = 4$ and $e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

corresponding focus will be (–ae, 0) i.e. $(-5, 0)$.

69. $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$

$\cos\left(\cos^{-1} x - \cos^{-1} \frac{y}{2}\right) = \cos\alpha$

$\Rightarrow x \times \frac{y}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\Rightarrow \left(\cos\alpha - \frac{xy}{2}\right)^2 = (1-x^2)\left(1-\frac{y^2}{4}\right)$

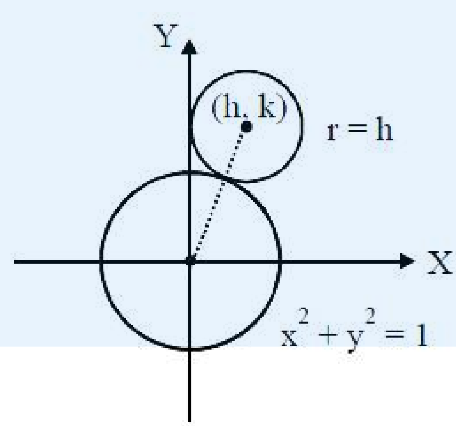
$x^2 + \frac{y^2}{4} - xy\cos\alpha = 1 - \cos^2\alpha = \sin^2\alpha$

70. $\sqrt{h^2 + k^2} = |h| + 1$

$\Rightarrow x^2 + y^2 = x^2 + 1 + 2x$

$\Rightarrow y^2 = 1 + 2x$

$\Rightarrow y = \sqrt{1+2x}; x \geq 0$




71. Let a is first term and d is common difference then, $a + 5d = 2$ (given) ……(1)

$f(d) = (2 - 5d)(2 - 2d)(2 - d)$

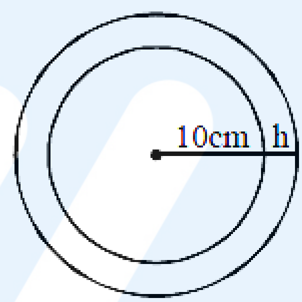
$$f'(x)=0 \Rightarrow d=\frac{2}{3},\frac{8}{5}$$
$$f''(d)<0 \text{ at } d=\frac{8}{5}$$
$$\Rightarrow d=\frac{8}{5}$$

72. $T_{r+1}=\sum_{r=0}^{n} {}^nC_r\, x^{2n-2r}.x^{-3r}$

$2n-5r=1 \Rightarrow 2n=5r+1$ for $r=15, n=38$ smallest value of n is 38.

73. Total cases = number of diagonals in 20 sided polygon.
$={}^{20}C_2-20=170$

74. $V=\frac{4}{3}\pi\left((10+h)^3-10^3\right)$




e

$$\frac{dV}{dt}=4\pi(10+h)^2\frac{dh}{dt}$$
$$-50=4\pi(10+5)^2\frac{dh}{dt}$$
$$\Rightarrow \frac{dh}{dt}=-\frac{1}{18\pi}\frac{cm}{min}$$

75. Mean $(\mu)=\frac{\sum x_i}{50}=16$

Standard deviation $(\sigma)=\sqrt{\frac{\sum{}_i^2}{50}-(\mu)^2}=16$

$$\Rightarrow (256)\times 2=\frac{\sum x_i^2}{50}$$
$\Rightarrow$ New mean
$$=\frac{\sum(x_i-4)^2}{50}=\frac{\sum x_i^2+16\times 50-8\sum x_i}{50}$$
$$=(256)\times 2+16-8\times 16=400$$

76. Let $2^x = t$

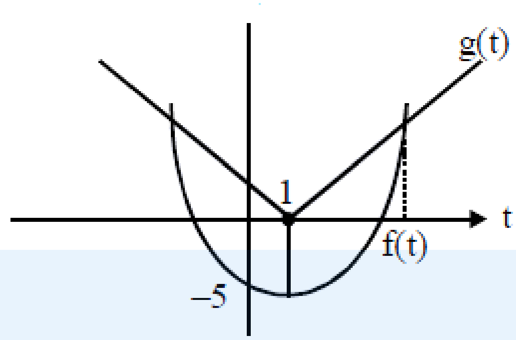
$5 + |t-1| = t^2 - 2t$

$\Rightarrow |t-1| = (t^2 - 2t - 5)$

$g(t) \qquad f(t)$

From the graph

So, number of real root is 1.




77. $3x^2 + 5y^2 = 32$

$$\left.\frac{dy}{dx}\right|_{(2,2)} = -\frac{3}{5}$$

Tangent : $y - 2 = -\frac{3}{5}(x-2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$

Normal : $y - 2 = \frac{5}{3}(x-2) \Rightarrow R\left(\frac{4}{5}, 0\right)$

Area is $= \frac{1}{2}(QR) \times 2 = QR = \frac{68}{15}$

78. Let point P on the line is $(2\lambda+1, -\lambda-1, \lambda)$ foot of perpendicular Q is given by

$$\frac{x-2\lambda-1}{1} = \frac{y+\lambda+1}{1} = \frac{z-\lambda}{1} = \frac{-(2\lambda-3)}{3}$$

$\because$ Q lies on $x+y+z=3$ and $x-y+z=3$

$\Rightarrow x+z=3$ and $y=0$

$y = 0 \Rightarrow \lambda + 1 = \frac{-2\lambda+3}{3} \Rightarrow \lambda = 0$

$\Rightarrow$ Q is (2, 0, 1)

79. $I = \int \frac{1}{\cos^{2/3} x \sin^{1/3} x . \sin x} dx$

$= \int \frac{\tan^{2/3} x}{\tan^2 x} . \sec^2 x . dx$

$= \int \frac{\sec^2 x}{\tan^{4/3} x} . dx \qquad \{\tan x = t, \sec^2 x dx = dt\}$

$= \int \frac{dt}{\tan^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3(t^{-1/3})$

$\Rightarrow 1 = -3 \tan(x)^{-1/3}$

$$\Rightarrow I = \left.\frac{3}{(\tan x)^{1/3}}\right|_{\pi/6}^{\pi/3} = -3\left(\frac{1}{(\sqrt{3})^{1/3}} - (\sqrt{3})^{1/3}\right)$$

$= 3^{7/6} - 3^{5/6}$

80. $\angle B=\frac{\pi}{3}$, by sine Rule

$\sin A=\frac{1}{2}$

$\Rightarrow A=30^{o}, a=2, b=2\sqrt{3}, c=4$

$\Delta=\frac{1}{2}\times 2\sqrt{3}\times 2=2\sqrt{3}$ sq.cm

81. $b=ar$

$c=ar^2$

$3a, 7b$ and $15c$ are in A.P.

$\Rightarrow 14b=3a+15c$

$\Rightarrow 14(ar)=3a+15ar^2$

$\Rightarrow 14r=3+15r^2$

$\Rightarrow 15r^2-14r+3=0 \qquad \Rightarrow (3r-1)(5r-3)=0$

$r=\frac{1}{3}, \frac{3}{5}$

Only acceptable value is $r=\frac{1}{3}$, because $r\in\left(0, \frac{1}{2}\right]$

$\therefore$ c.d $=7b-3a=7ar-3a=\frac{7}{3}a-3a=-\frac{2}{3}a$

$\therefore$ $4^{th}$ term $=15c-\frac{2}{3}a=\frac{15}{9}a-\frac{2}{3}a=a$

82. $\frac{dy}{dx}+y(\tan x)=2x+x^2\tan x$

I.F. $=e^{\pm\int \tan x\, dx}=e^{\ln .\sec x}=\sec x$

$\therefore$ $y.\sec x=\int\left(2x+x^2\tan x\right)\sec x.dx$

$=\int 2x\sec x\, dx+\int x^2(\sec x.\tan x)dx$

$y\sec x=x^2\sec x+\lambda$

$\Rightarrow y=x^2+\lambda\cos x$

$y(0)=0+\lambda=1 \qquad \Rightarrow \lambda=1$

$y=x^2+\cos x$

$y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$

$y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$

$y'(x)=2x-\sin x$

$y'\left(\frac{\pi}{4}\right)=\frac{\pi}{2}-\frac{1}{\sqrt{2}}$

$$y'\left(\frac{-\pi}{4}\right)=\frac{-\pi}{2}+\frac{1}{\sqrt{2}}$$
$$y'\left(\frac{\pi}{4}\right)-y'\left(\frac{-\pi}{4}\right)=\pi-\sqrt{2}$$

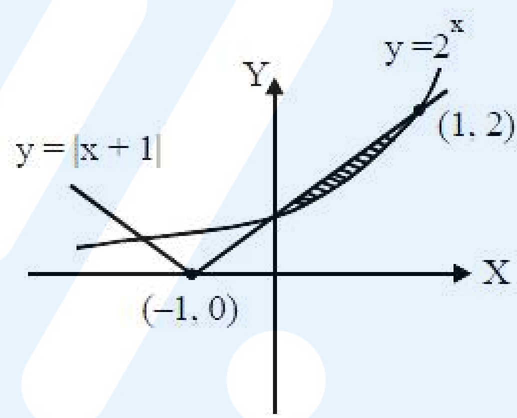
83. $4x-2y+4z+6=0$

$$\frac{|\lambda-6|}{\sqrt{16+4+16}}=\left|\frac{\lambda-6}{6}\right|=\frac{1}{3}$$
$$|\lambda-6|=2$$
$$\lambda=8,4$$
$$\frac{|\mu-3|}{\sqrt{4+4+1}}=\frac{2}{3}$$
$$|\mu-3|=2$$
$$\mu=5,1$$

$\therefore$ Maximum value of $(\mu+\lambda)=13$.

84. Required Area

$$\int_0^1\left((x+1)-2^x\right)dx$$
$$=\left(\frac{x^2}{2}+x-\frac{2^x}{\ln 2}\right)_0^1$$
$$=\left(\frac{1}{2}+1-\frac{2}{\ln 2}\right)-\left(0+0-\frac{1}{\ln 2}\right)$$
$$=\frac{3}{2}-\frac{1}{\ln 2}$$




85. $D=0$

$$\begin{vmatrix}1 & 1 & 1\\ 4 & \lambda & \lambda\\ 3 & 2 & -4\end{vmatrix}=0\Rightarrow\lambda=3$$

86. Sum $=\sum_{n=1}^{15}\frac{1^3+2^3+.....n^3}{1+2+.....+n}-\frac{1}{2}.\frac{15.16}{2}$

$$=\sum_{n=1}^{15}\frac{n(n+1)}{2}-60$$
$$=\sum_{n=1}^{15}\frac{n(n+1)(n+2-(n-1))}{6}-60$$
$$=\frac{15.16.17}{6}-60=620$$

87. $1-\left(\frac{1}{2}\right)^{n} > \frac{99}{100}$

$\Rightarrow \left(\frac{1}{2}\right)^{n} < \frac{1}{100}$

$\Rightarrow n = 7$

88. $\sim(\sim s \vee (\sim \wedge s))$

$s \wedge (r \vee \sim s)$

$(s \wedge r) \vee (s \wedge \sim s)$

$(s \wedge r) \vee (\phi)$

$(s \wedge r)$

89. Let $x^2 = t$ $\quad 2xdx = dt$

$\Rightarrow \frac{1}{2}\int t^2.e^{-t}dt = \frac{1}{2}\left[-t^2.e^{-t} + \int 2t.e^{-t}, dt\right]$

$= \frac{1}{2}\left(-t^2.e^{-t}\right) + \left(-t.e^{-t} + \int 1.e^{-t}.dt\right)$

$= -\frac{t^2e^{-t}}{2} - te^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1\right)e^{-t}$

$= \left(-\frac{x^4}{2} - x^2 - 1\right)e^{-x^2} + C$

for $k = 0$

$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$

90. $|z|.|w| = 1$ $\quad z = re^{i\left(\theta + \frac{\pi}{2}\right)}$ and $w = \frac{1}{r}e^{i\theta}$

$\bar{z}.w = e^{-i\left(\theta + \frac{\pi}{2}\right)}.e^{i\theta} = e^{-i\left(\frac{\pi}{2}\right)} = -i$

$z.\bar{w} = e^{i\left(\theta + \frac{\pi}{2}\right)}.e^{-i\theta} = e^{i\left(\frac{\pi}{2}\right)} = i$