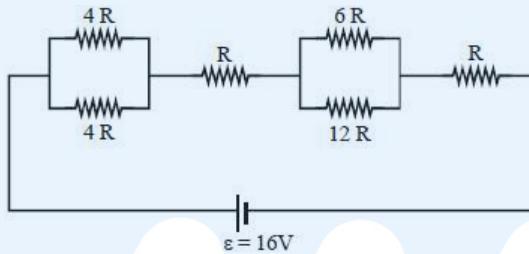
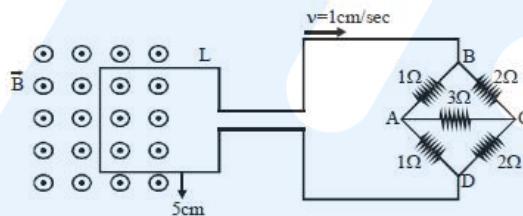


## PART -A (PHYSICS)



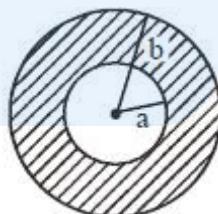
3. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole set up is moving towards right with a constant speed of  $1 \text{ cms}^{-1}$ . At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is  $1.7 \Omega$ , the current in the loop at that instant will be close to :

(A)  $8 \Omega$       (B)  $6 \Omega$   
(C)  $16 \Omega$       (D)  $1 \Omega$

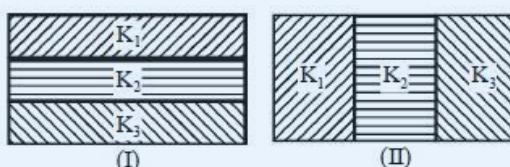


- (A)  $115 \mu\text{A}$       (B)  $170 \mu\text{A}$   
 (C)  $60 \mu\text{A}$       (D)  $150 \mu\text{A}$

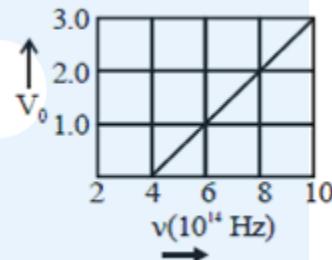
4. A circular disc of radius  $b$  has a hole of radius  $a$  at its centre (see figure). If the mass per unit area of the disc varies as  $\left(\frac{\sigma_0}{r}\right)$ , then the radius of gyration of the disc about its axis passing through the centre is :



- (A)  $\frac{a+b}{3}$       (B)  $\sqrt{\frac{a^2 + b^2 + ab}{3}}$   
(C)  $\frac{a+b}{2}$       (D)  $\sqrt{\frac{a^2 + b^2 + ab}{2}}$



(A)  $\frac{E_1}{E_2} = \frac{K_1 K_2 K_3}{(K_1 + K_2 + K_3)(K_2 K_3 + K_3 K_1 + K_1 K_2)}$  (B)  $\frac{E_1}{E_2} = \frac{9 K_1 K_2 K_3}{(K_1 + K_2 + K_3)(K_2 K_3 + K_3 K_1 + K_1 K_2)}$   
 (C)  $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2 K_3 + K_3 K_1 + K_1 K_2)}{9 K_1 K_2 K_3}$  (D)  $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2 K_3 + K_3 K_1 + K_1 K_2)}{K_1 K_2 K_3}$



11. An excited  $\text{He}^+$  ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number  $n$ , corresponding to its initial excited state is (for photon of wavelength  $\lambda$ , energy  $E = \frac{1240 \text{ eV}}{\lambda(\text{in nm})}$ )

(A)  $n = 4$       (B)  $n = 6$   
 (C)  $n = 5$       (D)  $n = 7$

12. When  $M_1$  gram of ice at  $-10^\circ\text{C}$  (specific heat =  $0.5 \text{ cal g}^{-1}\text{C}^{-1}$ ) is added to  $M_2$  gram of water at  $50^\circ\text{C}$ , finally no ice is left and the water is at  $0^\circ\text{C}$ . The value of latent heat of ice, in  $\text{cal g}^{-1}$  is:

(A)  $\frac{50M_2}{M_1} - 5$       (B)  $\frac{5M_2}{M_1} - 5$   
 (C)  $\frac{50M_2}{M_1}$       (D)  $\frac{5M_1}{M_2} - 50$

13. A shell is fired from a fixed artillery gun with an initial speed  $u$  such that it hits the target on the ground at a distance  $R$  from it. If  $t_1$  and  $t_2$  are the values of the time taken by it to hit the target in two possible ways, the product  $t_1 t_2$  is:

(A)  $2R / g$       (B)  $R / 4g$   
 (C)  $R / g$       (D)  $R/2g$

14. The transfer characteristic curve of a transistor, having input and output resistance  $100 \Omega$  and  $100 \text{ k}\Omega$  respectively, is shown in the figure. The Voltage and Power gain, are respectively :

$I_b (\mu\text{A})$	$I_c (\text{mA})$
100	5
200	10
300	15
400	20

(A)  $5 \times 10^4, 2.5 \times 10^6$       (B)  $5 \times 10^4, 5 \times 10^6$   
 (C)  $5 \times 10^4, 5 \times 10^5$       (D)  $2.5 \times 10^4, 2.5 \times 10^6$

15. Shown in the figure is a shell made of a conductor. It has inner radius  $a$  and outer radius  $b$ , and carries charge  $Q$ . At its centre is a dipole  $\vec{P}$  as shown. In this case :

(A) Surface charge density on the inner surface of the shell is zero everywhere.  
 (B) Electric field outside the shell is the same as that of a point charge at the centre of the shell.  
 (C) Surface charge density on the inner surface is uniform and equal to  $\frac{(Q/2)}{4\pi a^2}$   
 (D) Surface charge density on the outer surface depends on  $|\vec{P}|$

16. Which of the following combinations has the dimension of electrical resistance ( $\epsilon_0$  is the permittivity of vacuum and  $\mu_0$  is the permeability of vacuum)?

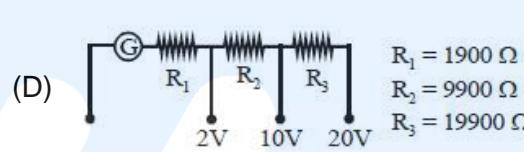
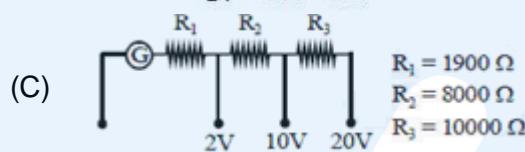
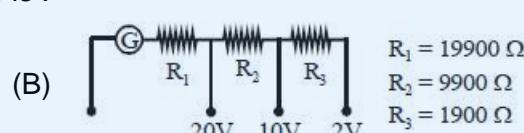
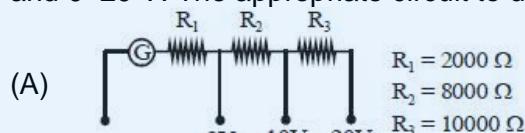
(A)  $\sqrt{\frac{\epsilon_0}{\mu_0}}$

(B)  $\frac{\mu_0}{\epsilon_0}$

(C)  $\frac{\epsilon_0}{\mu_0}$

(D)  $\sqrt{\frac{\mu_0}{\epsilon_0}}$

17. A galvanometer of resistance  $100 \Omega$  has 50 divisions on its scale and has sensitivity of  $20 \mu\text{A} / \text{division}$ . It is to be converted to a voltmeter with three ranges, of  $0-2 \text{ V}$ ,  $0-10 \text{ V}$  and  $0-20 \text{ V}$ . The appropriate circuit to do so is :



18. In a double slit experiment, when a thin film of thickness  $t$  having refractive index  $\mu$  is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of  $t$  is ( $\lambda$  is the wavelength of the light used) :

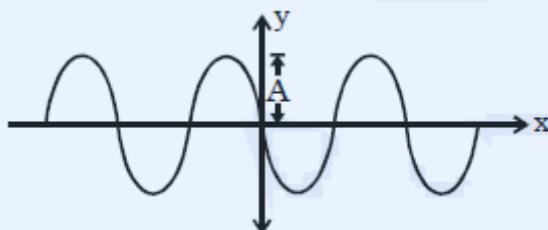
(A)  $\frac{\lambda}{2(\mu - 1)}$

(B)  $\frac{\lambda}{(\mu - 1)}$

(C)  $\frac{\lambda}{(2\mu - 1)}$

(D)  $\frac{2\lambda}{(\mu - 1)}$

19. A progressive wave travelling along the positive  $x$ -direction is represented by  $y(x, t) = A \sin(kx - \omega t + \phi)$ . Its snapshot at  $t = 0$  is given in the figure:



For this wave, the phase  $\phi$  is :

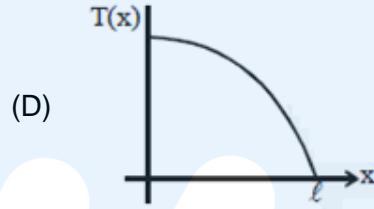
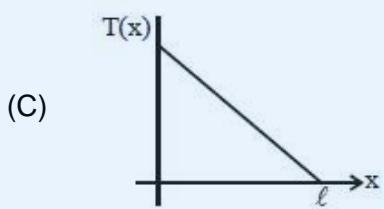
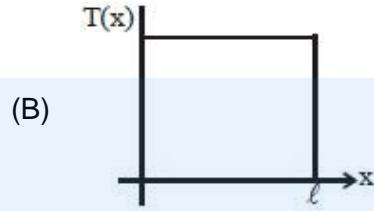
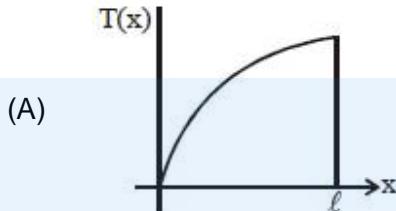
(A)  $\pi$

(B)  $\frac{\pi}{2}$

(C)  $-\frac{\pi}{2}$

(D)  $0$

20. A uniform rod of length  $\ell$  is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is  $T(x)$  at a distance  $x$  from the axis, then which of the following graphs depicts it most closely?



21. A point dipole  $\vec{p} = -p_0 \hat{x}$  is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance  $d$  are, respectively : (Take  $V = 0$  at infinity)

(A)  $\frac{|\vec{p}|}{4\pi\epsilon_0 d^2}, \frac{-\vec{p}}{4\pi\epsilon_0 d^3}$

(B)  $0, \frac{\vec{p}}{4\pi\epsilon_0 d^3}$

(C)  $\frac{|\vec{p}|}{4\pi\epsilon_0 d^2}, \frac{\vec{p}}{4\pi\epsilon_0 d^3}$

(D)  $0, \frac{-\vec{p}}{4\pi\epsilon_0 d^3}$

22. An electromagnetic wave is represented by the electric field  $\vec{E} = E_0 \hat{n} \sin[\omega t + (6y - 8z)]$ . Taking unit vectors in x, y and z directions to be  $\hat{i}, \hat{j}, \hat{k}$ , the direction of propagation  $\hat{s}$ , is :

(A)  $\hat{S} = \left( \frac{-3\hat{j} + 4\hat{k}}{5} \right)$

(B)  $\hat{S} = \left( \frac{4\hat{j} - 3\hat{k}}{5} \right)$

(C)  $\hat{S} = \left( \frac{-4\hat{k} + 3\hat{j}}{5} \right)$

(D)  $\hat{S} = \left( \frac{-3\hat{i} - 4\hat{j}}{5} \right)$

23. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency  $v$ . The value of  $v$  is close to : (Speed of sound in water =  $1500 \text{ ms}^{-1}$ )

(A) 499 Hz  
(C) 504 Hz

(B) 502 Hz  
(D) 507 Hz

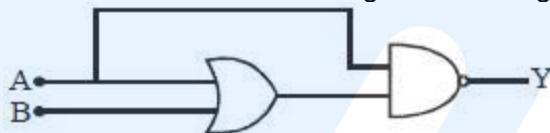
24. The trajectory of a projectile near the surface of the earth is given as  $y = 2x - 9x^2$ . If it were launched at an angle  $\theta_0$  with speed  $v_0$  then ( $g = 10 \text{ ms}^{-2}$ ) :

- (A)  $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$  and  $v_0 = \frac{5}{3} \text{ ms}^{-1}$       (B)  $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$  and  $v_0 = \frac{3}{5} \text{ ms}^{-1}$   
 (C)  $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$  and  $v_0 = \frac{3}{5} \text{ ms}^{-1}$       (D)  $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$  and  $v_0 = \frac{5}{3} \text{ ms}^{-1}$

25. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of  $40\pi \text{ rad s}^{-1}$  about its axis, perpendicular to its plane. If the magnetic field at its centre is  $3.8 \times 10^{-9} \text{ T}$ , then the charge carried by the ring is close to ( $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ ).

- (A)  $2 \times 10^{-6} \text{ C}$       (B)  $7 \times 10^{-6} \text{ C}$   
 (C)  $4 \times 10^{-5} \text{ C}$       (D)  $3 \times 10^{-5} \text{ C}$

26. The truth table for the circuit given in the fig is :



(A)	A	B	Y
	0	0	1
	0	1	0
	1	0	0
	1	1	0

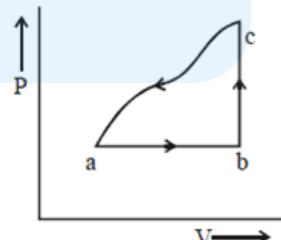
(B)	A	B	Y
	0	0	0
	0	1	0
	1	0	1
	1	1	1

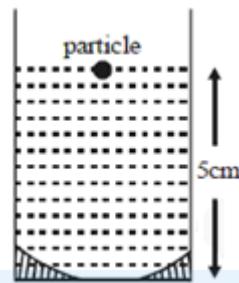
(C)	A	B	Y
	0	0	1
	0	1	1
	1	0	1
	1	1	1

(D)	A	B	Y
	0	0	1
	0	1	1
	1	0	0
	1	1	0

27. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is  $-180 \text{ J}$ . The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is :

- (A) 120 J      (B) 100 J  
 (C) 140 J      (D) 130 J

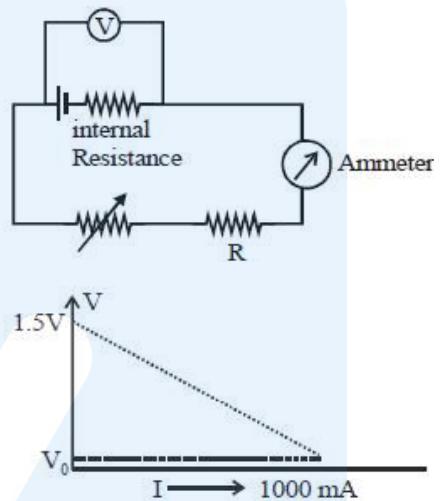




29. To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained:

If  $V_0$  is almost zero, identify the correct statement:

- (A) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA.
  - (B) The emf of the battery is 1.5 V and the value of R is 1.5  $\Omega$
  - (C) The emf of the battery is 1.5 V and its internal resistance is 1.5  $\Omega$
  - (D) The value of the resistance R is 1.5  $\Omega$



30. A person of mass  $M$  is, sitting on a swing of length  $L$  and swinging with an angular amplitude  $\theta_0$ . If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance  $l$  ( $l < L$ ), is close to :

- (A)  $Mg\ell(1 + \theta_0^2)$       (B)  $Mg\ell(1 - \theta_0^2)$   
 (C)  $Mg\ell$       (D)  $Mg\ell\left(1 + \frac{\theta_0^2}{2}\right)$

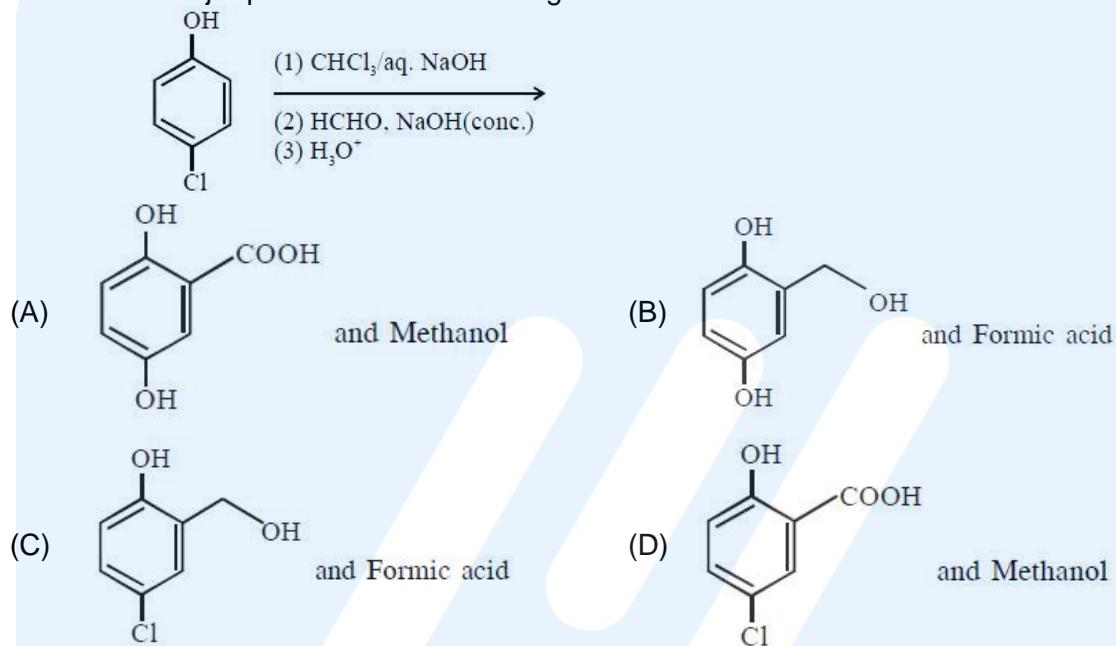
## PART -B (CHEMISTRY)

31. The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality (in mol kg<sup>-1</sup>) of the aqueous solution is

(A)  $13.88 \times 10^{-2}$   
 (C) 13.88

(B)  $13.88 \times 10^{-1}$   
(D)  $13.88 \times 10^{-3}$

32. The major products of the following reaction are:



33. The correct statement among the following is

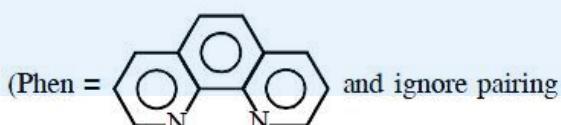
(A)  $(\text{SiH}_3)_3\text{N}$  is planar and less basic than  $(\text{CH}_3)_3\text{N}$ .

(B)  $(\text{SiH}_3)_3\text{N}$  is planar and more basic than  $(\text{CH}_3)_3\text{N}$ .

(C)  $(\text{SiH}_3)_3\text{N}$  is pyramidal and less basic than  $(\text{CH}_3)_3\text{N}$ .

(P)  $(\text{SiH}_3)_2\text{N}$  is pyramidal and more basic than  $(\text{CH}_3)_2\text{N}$ .

34. The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is



energy)

(A)  $[\text{Ni}(\text{phen})_3]^{2+}$

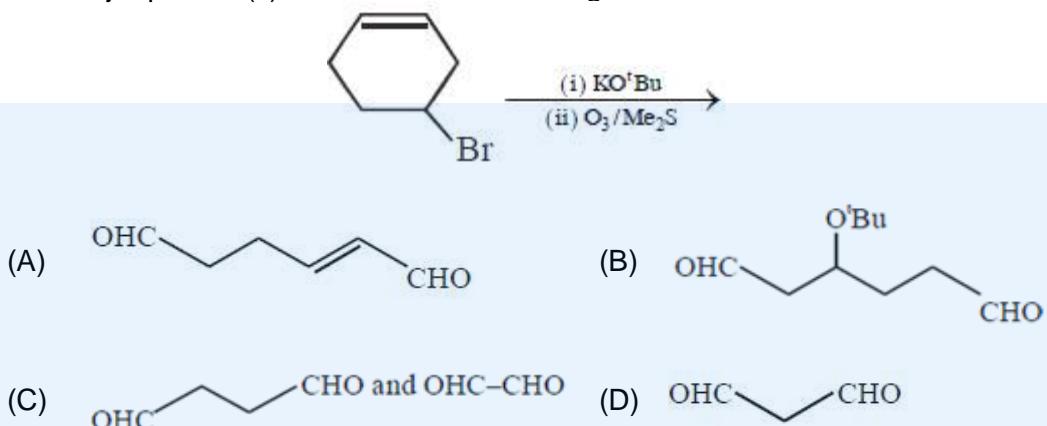
(B)  $[\text{Zn}(\text{phen})_3]^{2+}$

(C)  $[\text{Co}(\text{phen})_3]^{2+}$

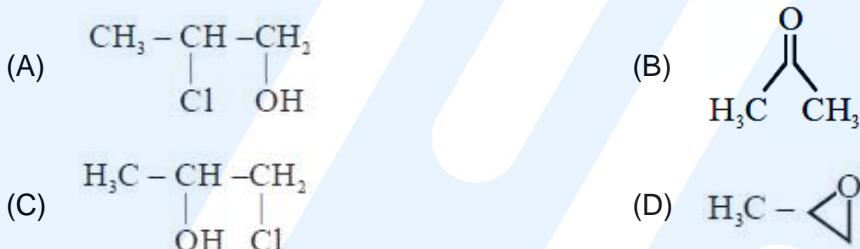
(D)  $[\text{Fe}(\text{phen})_3]^{2+}$

35. The correct set of species responsible for the photochemical smog is :  
 (A) NO, NO<sub>2</sub>, O<sub>3</sub> and hydrocarbons      (B) CO<sub>2</sub>, NO<sub>2</sub> SO<sub>2</sub> and hydrocarbons  
 (C) N<sub>2</sub>, NO<sub>2</sub> and hydrocarbons      (D) N<sub>2</sub>, O<sub>2</sub>, O<sub>3</sub> and hydrocarbons

36. The major product(s) obtained in the following reaction is/ are:



37. The major product of the following addition reaction is:



38. The correct sequence of thermal stability of the following carbonates is :

- (A) BaCO<sub>3</sub> < SrCO<sub>3</sub> < CaCO<sub>3</sub> < MgCO<sub>3</sub>      (B) BaCO<sub>3</sub> < CaCO<sub>3</sub> < SrCO<sub>3</sub> < MgCO<sub>3</sub>  
 (C) MgCO<sub>3</sub> < CaCO<sub>3</sub> < SrCO<sub>3</sub> < BaCO<sub>3</sub>      (D) MgCO<sub>3</sub> < SrCO<sub>3</sub> < CaCO<sub>3</sub> < BaCO<sub>3</sub>

39. The group number, number of valence electrons and valency of an element with atomic number 15, respectively, are:

- (A) 15, 5 and 3      (B) 15, 6 and 2  
 (C) 16, 5 and 2      (D) 16, 6 and 3

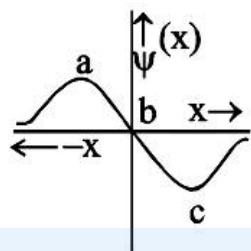
40. Peptization is a :

- (A) process of converting soluble particles to form colloidal solution  
 (B) process of converting precipitate into colloidal solution  
 (C) process of converting a colloidal solution into precipitate  
 (D) process of bringing colloidal molecule into solution

41. The metal that gives hydrogen gas upon treatment with both acid as well as base is:

- (A) iron      (B) magnesium  
 (C) zinc      (D) mercury

42. The electrons are more likely to be found:



- (A) in the region a and b  
 (B) in the region a and c  
 (C) only in the region a  
 (D) only in the region c

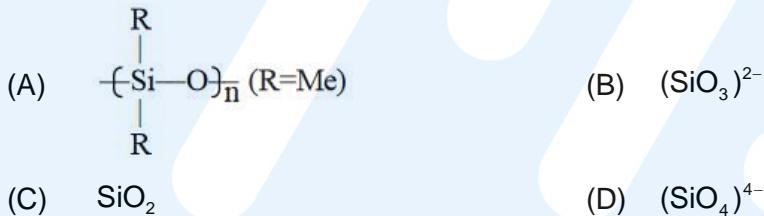
43. An example of a disproportionation reaction is :

- (A)  $2\text{KMnO}_4 \longrightarrow \text{K}_2\text{MnO}_4 + \text{MnO}_2 + \text{O}_2$   
 (B)  $2\text{NaBr} + \text{Cl}_2 \longrightarrow 2\text{NaCl} + \text{Br}_2$   
 (C)  $2\text{CuBr} \longrightarrow \text{CuBr}_2 + \text{Cu}$   
 (D)  $2\text{MnO}_4^- + 10\text{I}^- + 16\text{H}^+ \longrightarrow 2\text{Mn}^{2+} + 5\text{I}_2 + 8\text{H}_2\text{O}$

44. 5 moles of  $\text{AB}_2$  weigh  $125 \times 10^{-3}$  kg and 10 moles of  $\text{A}_2\text{B}_2$  weigh  $300 \times 10^{-3}$  kg. The molar mass of A( $M_A$ ) and molar mass of B( $M_B$ ) in  $\text{kg mol}^{-1}$  are :

- (A)  $M_A = 50 \times 10^{-3}$  and  $M_B = 25 \times 10^{-3}$   
 (B)  $M_A = 10 \times 10^{-3}$  and  $M_B = 5 \times 10^{-3}$   
 (C)  $M_A = 5 \times 10^{-3}$  and  $M_B = 10 \times 10^{-3}$   
 (D)  $M_A = 25 \times 10^{-3}$  and  $M_B = 50 \times 10^{-3}$

45. The basic structural unit of feldspar, zeolites, mica and asbestos is :



46. Enthalpy of sublimation of iodine is 24 cal  $\text{g}^{-1}$  at 200°C. If specific heat of  $\text{I}_2(\text{s})$  and  $\text{I}_2(\text{vap})$  are 0.055 and 0.031 cal  $\text{g}^{-1}\text{K}^{-1}$  respectively, then enthalpy of sublimation of iodine at 250°C in cal  $\text{g}^{-1}$  is :

- (A) 2.85  
 (B) 11.4  
 (C) 5.7  
 (D) 22.8

47. Which of the following is a thermosetting polymer?

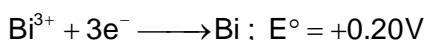
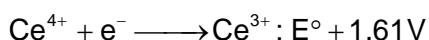
- (A) PVC  
 (B) Bakelite  
 (C) Buna-N  
 (D) Nylon 6

48. An element has a face-centred cubic (fcc) structure with a cell edge of  $a$ . The distance between the centres of two nearest tetrahedral voids in the lattice is:

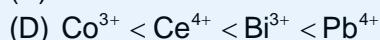
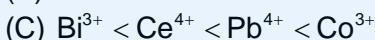
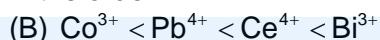
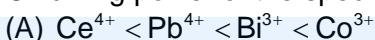
- (A)  $\frac{a}{2}$   
 (B)  $\sqrt{2}a$   
 (C)  $\frac{3}{2}a$   
 (D)  $a$



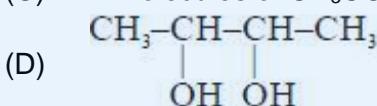
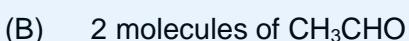
56. Given :



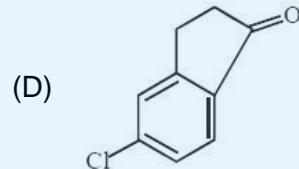
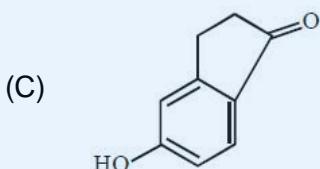
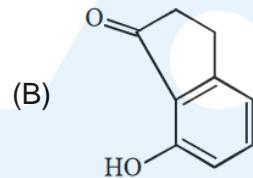
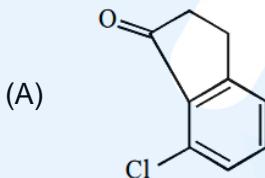
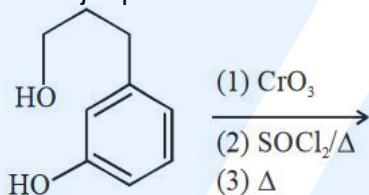
Oxidizing power of the species will increase in the order



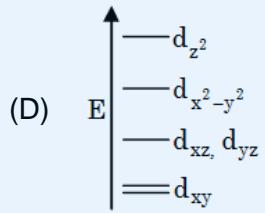
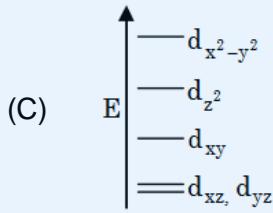
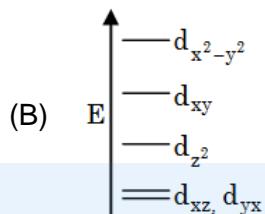
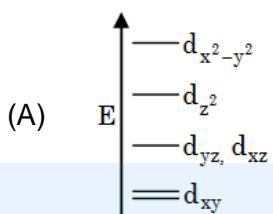
57. But-2-ene on reaction with alkaline  $\text{KMnO}_4$  at elevated temperature followed by acidification will give :



58. The major product of the following reaction is :



59. Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale).



60. An ideal gas is allowed to expand from 1 L to 10 L against a constant external pressure of 1 bar. The work done in kJ is:

(A) - 9.0  
(C) - 2.0

(B) - 0.9  
(D) + 10.0

## PART-C (MATHEMATICS)

68. The equation  $|z - i| = |z - 1|$ ,  $i = \sqrt{-1}$ , represents :

  - (A) a circle of radius  $\frac{1}{2}$
  - (B) the line through the origin with slope 1
  - (C) a circle of radius 1
  - (D) the line through the origin with slope  $-1$

69. If  $m$  is the minimum value of  $k$  for which the function  $f(x) = x\sqrt{kx - x^2}$  is increasing in the interval  $[0, 3]$  and  $M$  is the maximum value of  $f$  in  $[0, 3]$  when  $k = m$ , then the ordered pair  $(m, M)$  is equal to :

  - (A)  $(5, 3\sqrt{6})$
  - (B)  $(4, 3\sqrt{2})$
  - (C)  $(3, 3\sqrt{3})$
  - (D)  $(4, 3\sqrt{3})$

70. If the data  $x_1, x_2, \dots, x_{10}$  is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

  - (A)  $2\sqrt{2}$
  - (B) 2
  - (C) 4
  - (D)  $\sqrt{2}$

71. If the area (in sq. units) of the region  $\{(x, y) : y^2 \leq 4x, x + y \leq 1, x \geq 0, y \geq 0\}$  is  $a\sqrt{2} + b$ , then  $a - b$  is equal to :

  - (A)  $\frac{10}{3}$
  - (B) 6
  - (C)  $\frac{8}{3}$
  - (D)  $-\frac{2}{3}$

72. Consider the differential equation,  $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$ . If value of  $y$  is 1 when  $x = 1$ , then the value of  $x$  for which  $y = 2$ , is :

  - (A)  $\frac{3}{2} - \sqrt{e}$
  - (B)  $\frac{1}{2} + \frac{1}{\sqrt{e}}$
  - (C)  $\frac{3}{2} - \frac{1}{\sqrt{e}}$
  - (D)  $\frac{5}{2} + \frac{1}{\sqrt{e}}$

73. If  $\alpha$  and  $\beta$  are the roots of the equation  $375x^2 - 25x - 2 = 0$ , then  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \alpha^r + \lim_{n \rightarrow \infty} \sum_{r=1}^n \beta^r$  is equal to :

  - (A)  $\frac{1}{12}$
  - (B)  $\frac{29}{358}$
  - (C)  $\frac{7}{116}$
  - (D)  $\frac{21}{346}$



81. For  $x \in \left(0, \frac{3}{2}\right)$ , let  $f(x) = \sqrt{x}$ ,  $g(x) = \tan x$  and  $h(x) = \frac{1-x^2}{1+x^2}$ . If  $\phi(x) = ((h \circ f) \circ g)(x)$ , then  $\phi\left(\frac{\pi}{3}\right)$  is equal to :
- (A)  $\tan \frac{11\pi}{12}$       (B)  $\tan \frac{\pi}{12}$   
 (C)  $\tan \frac{5\pi}{12}$       (D)  $\tan \frac{7\pi}{12}$
82. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is  $90^\circ$ , then the length (in cm) of their common chord is :
- (A)  $\frac{13}{2}$       (B)  $\frac{120}{13}$   
 (C)  $\frac{13}{5}$       (D)  $\frac{60}{13}$
83. If  $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \csc x} dx = m(\pi + n)$ , then  $m \cdot n$  is equal to
- (A) 1      (B)  $\frac{1}{2}$   
 (C)  $-\frac{1}{2}$       (D) -1
84. Let  $S_n$  denote the sum of the first  $n$  terms of an A.P.. If  $S_4 = 16$  and  $S_6 = -48$ , then  $S_{10}$  is equal to :
- (A) -410      (B) -260  
 (C) -320      (D) -380
85. If  $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$  is the inverse of a  $3 \times 3$  matrix  $A$ , then the sum of all values of  $\alpha$  for which  $\det(A) + 1 = 0$ , is :
- (A) 0      (B) -1  
 (C) 1      (D) 2
86. If the line  $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{1}$  intersects the plane  $2x + 3y - z + 13 = 0$  at a point P and the plane  $3x + y + 4z = 16$  at a point Q, then PQ is equal to :
- (A)  $2\sqrt{14}$       (B) 14  
 (C)  $2\sqrt{7}$       (D)  $\sqrt{14}$
87. If the volume of parallelepiped formed by the vectors  $\hat{i} + \lambda \hat{j} + \hat{k}$ ,  $\hat{j} + \lambda \hat{k}$  and  $\lambda \hat{i} + \hat{k}$  is minimum, then  $\lambda$  is equal to :
- (A)  $\sqrt{3}$       (B)  $\frac{1}{\sqrt{3}}$   
 (C)  $-\frac{1}{\sqrt{3}}$       (D)  $-\sqrt{3}$

88. If A is a symmetric matrix and B is a skew-symmetrix matrix such that  $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ , then AB is equal to :
- (A)  $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$       (B)  $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$   
(C)  $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$       (D)  $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$
89. If the truth value of the statement  $p \rightarrow (\sim q \vee r)$  is false (F), then the truth values of the statement p, q, r are respectively :
- (A) T, T, F      (B) F, T, T  
(C) T, F, T      (D) T, F, F
90. For  $x \in \mathbb{R}$ , let  $[x]$  denote the greatest integer  $\leq x$ , then the sum of the series  $\left[ -\frac{1}{3} \right] + \left[ -\frac{1}{3} - \frac{1}{100} \right] + \left[ -\frac{1}{3} - \frac{2}{100} \right] + \dots + \left[ -\frac{1}{3} - \frac{99}{100} \right]$
- (A) -135      (B) -153  
(C) -133      (D) -131

# HINTS AND SOLUTIONS

## PART A – PHYSICS

1. Numerical aperture of the microscope is given as

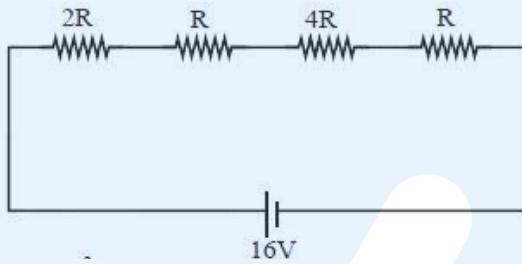
$$NA = \frac{0.61\lambda}{d}$$

Where  $d$  = minimum separation between two points to be seen as distinct

$$d = \frac{0.61\lambda}{NA} = \frac{(0.61) \times (5000 \times 10^{-10})}{1.25} = 2.4 \times 10^{-7} \text{ m}$$

$$= 0.24 \mu\text{m}$$

- 2.



$$P = \frac{16^2}{8R} = 4$$

$$\therefore R = 8\Omega$$

3. Since it is a balanced wheatstone bridge, its equivalent resistance =  $\frac{4}{3}\Omega$

$$\varepsilon = Blv = 5 \times 10^{-4} \text{ V}$$

So total resistance

$$R = \frac{4}{3} + 1.7 \approx 3\Omega$$

$$\therefore i = \frac{\varepsilon}{R} \approx 166 \mu\text{A} \approx 170 \mu\text{A}$$

4.  $dl = (dm)r^2$

$$= (\sigma dA)r^2$$

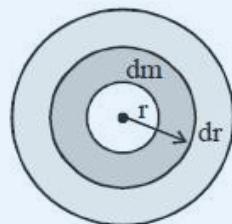
$$= \left( \frac{\sigma_0}{r} 2\pi dr \right) r^2 = (\sigma_0 2\pi r^2 dr)$$

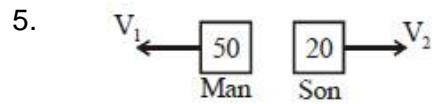
$$I = \int dl = \int_a^b \sigma_0 2\pi r^2 dr$$

$$= \sigma_0 2\pi \left( \frac{b^3 - a^3}{3} \right)$$

$$m = \int dm = \int \sigma dA$$

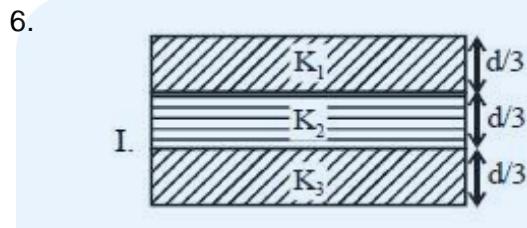
$$= \sigma_0 2\pi \int_a^b dr$$





$$\Rightarrow 0 = 50V_1 - 20V_2 \text{ and } V_1 + V_2 = 0.7$$

$$\Rightarrow V_1 = 0.2$$



$$C_1 = \frac{3\epsilon_0 A K_1}{d}$$

$$C_2 = \frac{3\epsilon_0 A K_2}{d}$$

$$C_3 = \frac{3\epsilon_0 A K_3}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow C_{eq} = \frac{3\epsilon_0 A K_1 K_2 K_3}{d(K_1 K_2 + K_2 K_3 + K_3 K_1)} \quad \dots(i)$$



$$C_1 = \frac{\epsilon_0 K_1 A}{3d}$$

$$C_2 = \frac{\epsilon_0 K_2 A}{3d}$$

$$C_3 = \frac{\epsilon_0 K_3 A}{3d}$$

$$C'_{eq} = C_1 + C_2 + C_3$$

$$= \frac{\epsilon_0 A}{3d} (K_1 + K_2 + K_3) \quad \dots(ii)$$

Now,

$$\frac{E_1}{E_2} = \frac{\frac{1}{2} C_{eq} \cdot V^2}{\frac{1}{2} C'_{eq} V^2} = \frac{9 K_1 K_2 K_3}{(K_1 + K_2 + K_3)(K_1 K_2 + K_2 K_3 + K_3 K_1)}$$

7.  $f_{mix} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} = \frac{2 \times 3 + 3 \times 5}{5} = \frac{21}{5}$

$$C_v = \frac{fR}{5} = \frac{21}{5} \times \frac{R}{2} = 17.4 \text{ J/mol K}$$

8.  $h\nu = \phi + eV_0$

$$V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

$V_0$  is zero for  $\nu = 4 \times 10^{14} \text{ Hz}$

$$0 = \frac{hv}{e} - \frac{\phi}{e}$$

$$\Rightarrow \phi = hv$$

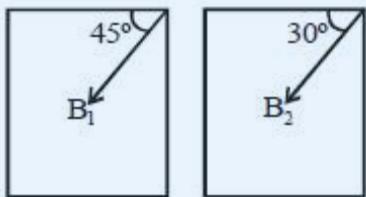
$$= \frac{6.63 \times 10^{-34} \times 4 \times 10^{14}}{1.6 \times 10^{-19}} = 1.66 \text{ eV}$$

9.  $Mg = \left( \frac{Ay}{\ell} \right) \Delta \ell$

$$Mg = (Ay)\alpha \Delta T = 2\pi$$

It is closest to 9.

10.



$$f_1 = \frac{1}{2\pi} \sqrt{\frac{\mu B_1 \cos 45^\circ}{I}}$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{\mu B_2 \cos 30^\circ}{I}}$$

$$\frac{f_1}{f_2} = \frac{B_1 \cos 45^\circ}{B_2 \cos 30^\circ}$$

$$\therefore \frac{B_1}{B_2} \times 0.7$$

11.

$$\frac{1}{\lambda} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) z^2$$

$$\frac{1}{1085} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) 2^2$$

$$\therefore m = 2$$

$$\therefore n = 5$$

12. Heat lost = Heat gain

$$\Rightarrow M_2 \times 1 \times 50 = M_1 \times 0.5 \times 10 + M_1 L_f$$

$$\Rightarrow L_f = \frac{50M_2 - 5M_1}{M_1}$$

$$= \frac{50M_2}{M_1} - 5$$

13. Range will be same for time  $t_1$  and  $t_2$ , so angles of projection will be '0' & ' $90^\circ - \theta$ '



$$t_1 = \frac{2u \sin \theta}{g}, t_2 = \frac{2u \sin(90^\circ - \theta)}{g} \text{ and } R = \frac{u^2 \sin 2\theta}{g}$$

$$t_1 t_2 = \frac{4u^2 \sin \theta \cos \theta}{g^2} = \frac{2}{g} \left[ \frac{2u^2 \sin \theta \cos \theta}{g} \right]$$

$$= \frac{2R}{g}$$

$$14. \quad V_{\text{gain}} = \left( \frac{\Delta I_C}{\Delta I_B} \right) \frac{R_{\text{out}}}{R_{\text{in}}} = \left( \frac{5 \times 10^{-3}}{100 \times 10^{-6}} \right) \times 10^3$$

$$= \frac{1}{20} \times 10^8 = 5 \times 10^4$$

$$P_{\text{gain}} = \left( \frac{\Delta I_C}{\Delta I_B} \right) (V_{\text{gain}})$$

$$= \left( \frac{5 \times 10^{-3}}{100 \times 10^{-6}} \right) (5 \times 10^4)$$

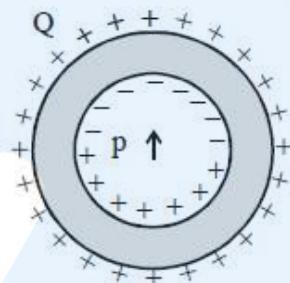
$$= 2.5 \times 10^6$$

15. Total charge of dipole = 0, so charge induced on outside surface = 0.

But due to non uniform electric field of dipole, the charge induced on inner surface is non zero and non uniform.

So, for any observer outside the shell, the resultant electric field is due to Q uniformly distributed on outer surface only and it is equal to.

$$E = \frac{KQ}{r^2}$$



$$16. \quad [\epsilon_0] = M^{-1} L^{-3} T^4 A^2$$

$$[\mu_0] = M L T^{-2} A^{-2}$$

$$[R] = M L^2 T^{-3} A^{-2}$$

$$[R] = \left[ \sqrt{\frac{\mu_0}{\epsilon_0}} \right]$$

$$17. \quad 20 \times 50 \times 10^{-6} = 10^{-3} \text{ Amp.}$$

$$V_1 = \frac{2}{10^{-3}} = 100 + R_1$$

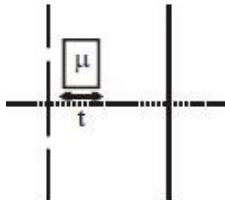
$$1900 = R_1$$

$$V_2 = \frac{10}{10^{-3}} = (2000 + R_2)$$

$$R_2 = 8000$$

$$V_3 = \frac{20}{10^{-3}} = 10 \times 10^3 + R_3 = 10 \times 10^3 R_3$$

- 18.

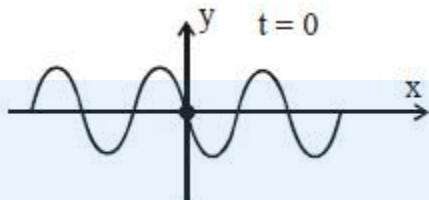


$$\Delta x = (\mu - 1)t = 1\lambda$$

for one maximum shift

$$t = \frac{\lambda}{\mu - 1}$$

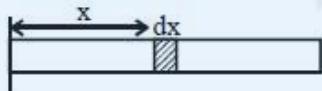
19.



$$y = A \sin(kx - wt + \phi)$$

at  $x = 0, t = 0$  and slope is negative  
 $\Rightarrow \phi = \pi$

20.



$$T = \int_{x=x}^{x=\ell} dm \omega^2 x = \int_{x=x}^{x=\ell} \frac{m}{l} dx \omega^2 x T$$

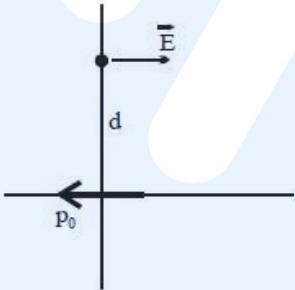
$$= \frac{m \omega^2}{2\ell} (\ell^2 - x^2)$$

$$T = \frac{m \omega^2}{2\ell} (\ell^2 - x^2)$$

21.  $V = 0$ 

$$\vec{E} = -\frac{\vec{K}\vec{P}}{r^3}$$

$$= -\frac{\vec{p}}{4\pi\epsilon_0 d^3}$$



$$\vec{E} = E_0 \hat{n} \sin(\omega t + (6y - 8z)) = E_0 \hat{n} \sin(\omega t + \vec{k} \cdot \vec{r})$$

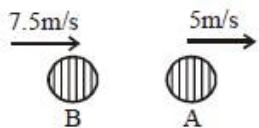
$$\text{where } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \text{ and } \vec{k} \cdot \vec{r} = 6y - 8z$$

$$\Rightarrow \vec{k} = 6\hat{j} - 8\hat{k}$$

direction of propagation  $\hat{s} = -\hat{k}$

$$= \left( \frac{-3\hat{j} + 4\hat{k}}{5} \right)$$

23.



$f_0 = 500 \text{ Hz}$  frequency received by B again =  
 $\leftarrow 1500$

$$(B) \quad 7.5 \text{ m/s} \rightarrow \quad (A) \& \Rightarrow \quad \rightarrow 5 \text{ m/sec}$$

$$f_2 = \left( \frac{1500 + 7.5}{1500 + 5} \right) \times \left( \frac{1500 - 5}{1500 - 7.5} \right) f_0 = 502 \text{ Hz}$$

24. Equation of trajectory is given as

$$y = 2x - 9x^2 \quad \dots (\text{A})$$

Comparing with equation:

$$y = x \tan \theta - \frac{g}{2u^2 \cos^2 \theta} \cdot x^2 \quad \dots (\text{B})$$

We get,  $\tan \theta = 2$ 

$$\therefore \cos \theta = \frac{1}{\sqrt{5}}$$

$$\text{Also, } \frac{g}{2u^2 \cos^2 \theta} = 9$$

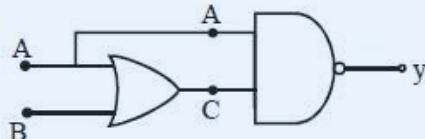
$$\Rightarrow \frac{10}{2 \times 9 \times \left( \frac{1}{\sqrt{5}} \right)^2} = u^2 \quad ; \quad u^2 = \frac{25}{9}$$

$$\Rightarrow u = \frac{5}{3} \text{ m/s}$$

$$25. \quad B = \frac{\mu_0 i}{2R} = \frac{\mu_0 q \omega}{2R 2\pi}$$

$$\Rightarrow q = 3 \times 10^{-5} \text{ C}$$

26.



$$C = A + B$$

$$\text{and } y = \overline{A.C}$$

A	B	$C = (A + B)$	$A.C$	$y = \overline{A.C}$
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0

27.

	$\Delta E$	$\Delta W$	$\Delta Q$
ab			250
bc		0	60
ca	-180		

	$\Delta E$	$\Delta W$	$\Delta Q$
ab	120	130	250
bc	60	0	60
ca	-180		

28. Light incident from particle P will be reflected at mirror.

$$u = -5 \text{ cm}, f = m - \frac{R}{2} = -20 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} ; \quad v_1 = +\frac{20}{3} \text{ cm}$$

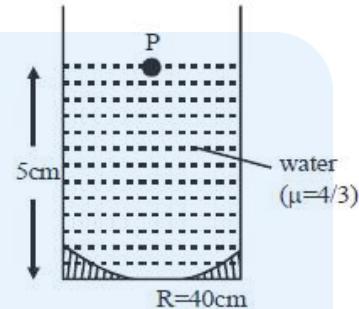
This image will act as object for light getting refracted at water surface.

$$\text{So, object distance } d = 5 + \frac{20}{3} = \frac{35}{3} \text{ cm}$$

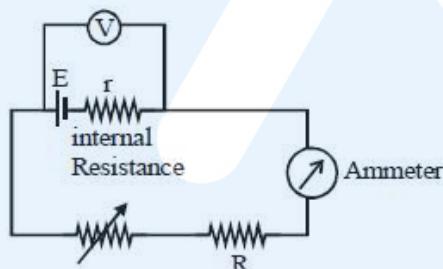
Below water surface.

After refraction, final image is at

$$\begin{aligned} d' &= d \left( \frac{\mu_2}{\mu_1} \right) = \left( \frac{35}{3} \right) \left( \frac{1}{4/3} \right) \\ &= \frac{35}{4} = 8.75 \text{ cm} \\ &\approx 8.8 \text{ cm} \end{aligned}$$



29.



$$V = E - Ir$$

$$\text{When } V = V_0 = 0 \Rightarrow 0 = E - Ir$$

$$\therefore E = r$$

$$\text{When } I = 0, V = E = 1.5 \text{ V}$$

$$\therefore r = 1.5 \Omega$$

30. Angular momentum conservation

$$MV_0 L = MV_1(L - \ell)$$

$$V_1 = V_0 \left( \frac{L}{L - \ell} \right)$$

$$w_g + w_p = \Delta KE$$

$$-mg\ell + w_p = \frac{1}{2}m(V_1^2 - V_0^2)$$

$$w_p = mg\ell + \frac{1}{2}mV_0^2 \left( \left( \frac{L}{L-\ell} \right)^2 - 1 \right)$$

$$= mg\ell + \frac{1}{2}mV_0^2 \left( \left( 1 - \frac{L}{L-\ell} \right)^{-2} - 1 \right)$$

Now,  $\ell \ll L$

By, Binomial approximation

$$= mg\ell + \frac{1}{2}mV_0^2 \left( \left( 1 + \frac{L}{L-\ell} \right)^{-2} - 1 \right)$$

$$= mg\ell + \frac{1}{2}mV_0^2 \left( \frac{2\ell}{L} \right)$$

$$W_P = mg\ell + mV_0^2 \frac{\ell}{L}$$

$$\text{Here, } V_0 = \text{maximum velocity} = \omega \times A = \left( \sqrt{\frac{g}{L}} \right) (\theta_0 L)$$

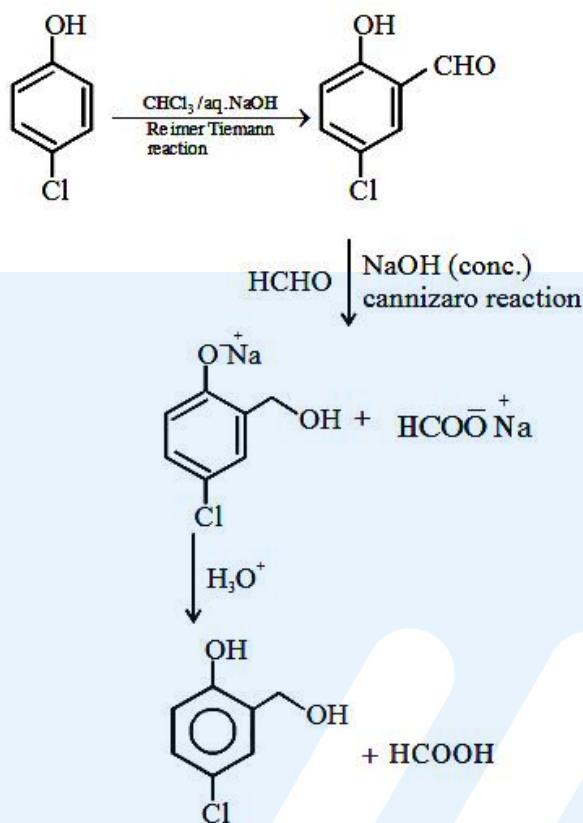
$$\text{So, } w_p = mg\ell + m \left( \theta_0 \sqrt{gL} \right)^2 \frac{\ell}{L}$$

$$= mg\ell \left( 1 + \theta_0^2 \right)$$

## PART B – CHEMISTRY

31.  $X_{\text{solvent}} = 0.8 = 8/10$   
 $N_{\text{Total}} = 10, n_{\text{solutent}} = 8, n_{\text{solute}} = 2$   
 Wt of solvent =  $8 \times 18$   
 $\text{Molality} = \frac{2 \times 1000}{8 \times 18}$

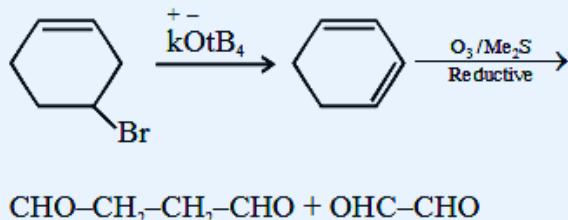
32.

33.  $p\pi-d\pi$  bonding in  $\text{N}(\text{SiH}_3)_3$ .34. When  $\text{Fe}^{2+}$  oxidizes to  $\text{Fe}^{3+}$   
3d

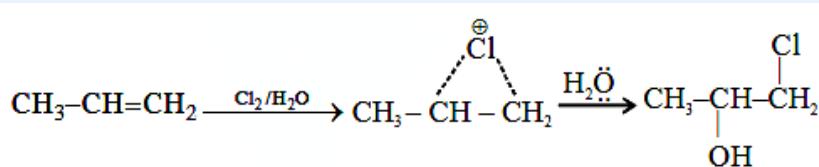
X	1	1	1	1	1
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35. Smog photochemical is  $\text{NO}$ ,  $\text{NO}_2$ ,  $\text{O}_3$ .

36.



37.



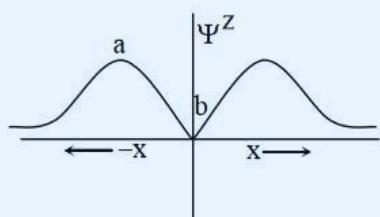
38. Smaller the size of cation, more polarisability high thermal decomposition of carbonates.

39.  $15 \rightarrow 1s^2, 2s^2, 2p^6, 3s^2, 3p^3 \}$  Period- III, Group-V

40. Peptization is process to form colloidal solution by adding peptizing agent.



42.



At a & c, probability of finding electron is maximum



44. Mol. wt is of 1 mol

$$AB_2 \quad A + 2B = 25$$

$$A_2B_2 \quad 2A + 2B = 30$$

45. Basic unit is silicate  $(SiO_4)^{4-}$

46.  $\Delta C_p = \frac{\Delta H_2 - \Delta H_1}{T_2 - T_1}$

47. Thermosetting are cross-linked polymer.

48. Two nearest tetrahedral voids are along one body-diagonal.

49. Canary yellow ppt comes in test of  $PO_4^{3-}$  ion.

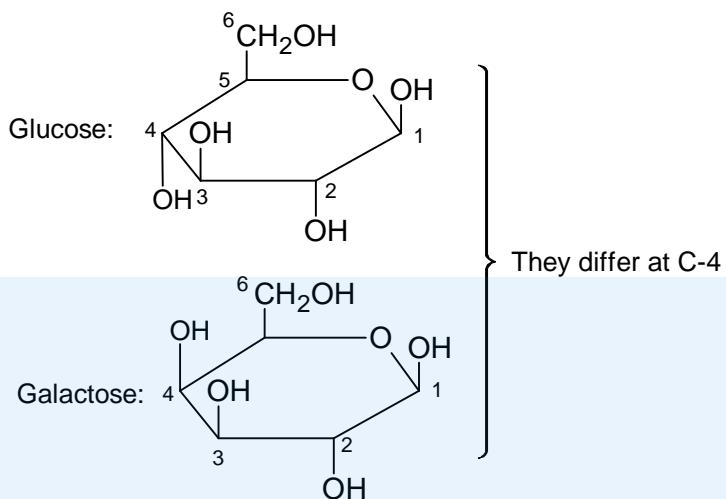
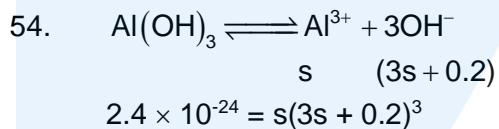
50. Concentration(on dressing), i.e. washing of ore.

51.  $-\frac{1}{x} \left( \frac{dA}{dt} \right) = \frac{1}{y} \left( \frac{dB}{dt} \right)$

$$\log_{10} \left[ -\frac{d(A)}{dt} \right] = \left[ \log_{10} \left( \frac{dB}{dt} \right) \right] + \log \frac{x}{y} \quad ] \log \frac{x}{y} = \log 0.3$$

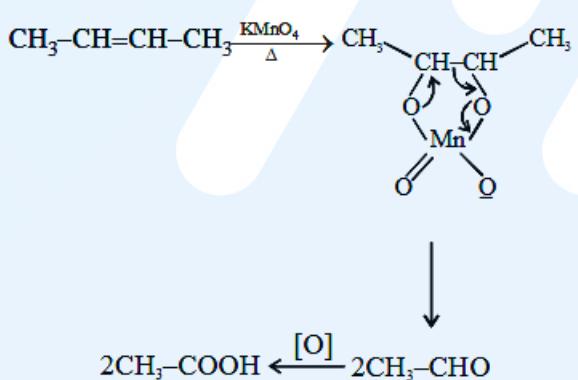
$$\left( \frac{x}{y} = 2 \right)$$

52.

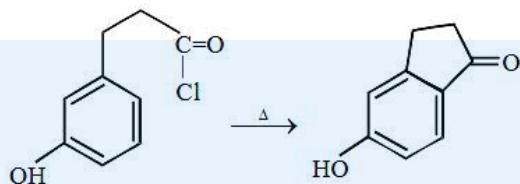
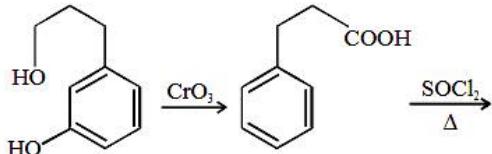
53. Not always double stranded  $\alpha$ -helix.55.  $\text{CH}_3\text{O}-$ ,  $\text{CH}_3-$ , H is electron donating, But  $-\text{NO}_2$  is electron withdrawing group.

56. Lower the standard reduction potential, more the ability to get reduced higher the oxidizing power.

57.



58.



59. Ligand filed exert mass repulsion along x, y axis as compared to Z-axis so  $d_{x^2-y^2}$  and  $d_{xx}$  will have increase in energy y
60.  $W = -P_{\text{ext}}(V_2 - V_1)$

## PART C – MATHEMATICS

61. Tangents  $y^2 = 12x \Rightarrow y = 2x + \frac{3}{m}$

$$\frac{x^2}{1} - \frac{y^2}{8} = 1 \Rightarrow y = mx \pm \sqrt{m^2 - 8}$$

Common tangent gives

$$\therefore \frac{3}{m} = \pm \sqrt{m^2 - 8}$$

$$m^4 - 8m^2 - 9 = 0$$

$$m = \pm 3$$

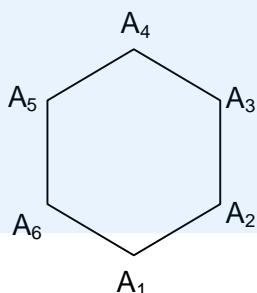
$$\therefore y = 3x + 1 \quad P\left(-\frac{1}{3}, 0\right)$$

$$y = -3x - 1$$

P divides SS' in 5 : 4.

62. Only two equilateral triangle are possible  $A_1A_3A_5$  and  $A_2A_5A_6$

$$\frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$$



$$\begin{aligned}
 63. \quad & \lim_{x \rightarrow 2} \frac{\int_2^x 4t^3 dt}{x-2} \\
 &= \lim_{x \rightarrow 2} \frac{4 \cdot f^3(x) \cdot f'(x)}{1} \\
 &= 4f^3(2)f'(2) = 18
 \end{aligned}$$

$$\begin{aligned}
 64. \quad & \int \frac{2x^3 - 1}{x^4 + x} dx \\
 &= \int \frac{2x - \frac{1}{x^2}}{x^2 + \frac{1}{x}} dx \\
 &= x^2 + \frac{1}{x} = t \\
 &\left(2x - \frac{1}{x^2}\right) dx = dt \\
 &\int \frac{dt}{t} = \ln(t) + C \\
 &= \ln\left(x^2 + \frac{1}{x}\right) + C
 \end{aligned}$$

$$\begin{aligned}
 65. \quad & (1+x)(1-x)^{10}(1+x+x^2)^9 \\
 & (1-x^2)(1-x^3)^9 \\
 & {}^9C_6 = 84
 \end{aligned}$$

$$\begin{aligned}
 66. \quad & \text{Since } {}^{21}C_0 + \dots + {}^{21}C_{10} + {}^{21}C_{11} + \dots + {}^{21}C_{21} = 2^{21} \\
 & \Rightarrow \text{but we have to select 10 objects and } {}^{21}C_0 + \dots + {}^{21}C_{10} = {}^{21}C_{11} + \dots + {}^{21}C_{21} \\
 & \quad \left( {}^{21}C_0 + \dots + {}^{21}C_{10} \right) = 2^{20}
 \end{aligned}$$

$$\begin{aligned}
 67. \quad & np = 8 \\
 & npq = 4 \\
 & q = \frac{1}{2} \Rightarrow p = \frac{1}{2} \\
 & n = 16 \\
 & p(x=r) = {}^{16}C_r \left(\frac{1}{2}\right)^{16} \\
 & p(x \leq 2) = \frac{{}^{16}C_0 + {}^{16}C_1 + {}^{16}C_2}{2^{16}} \\
 & = \frac{137}{2^{16}}
 \end{aligned}$$

68.  $|z - i| = |z - 1|$   
gives  $y = x$

69.  $f(x) = x\sqrt{kx - x^2}$   
 $f'(x) = \frac{3kx - 4x^2}{2\sqrt{kx - x^2}}$

For increasing  $f(x) \geq 0$

$$kx - x^2 \geq 0$$

$$x^2 - kx \leq 0$$

$$x(x - k) \leq 0 \text{ so } x \in [0, k]$$

$$\text{+ve } \boxed{x \geq 3}$$

$$3kx - 4x^2 \geq 0$$

$$4x^2 - 3kx \leq 0$$

$$4x\left(x - \frac{3k}{4}\right) \leq 0$$

$$3 - \frac{3k}{4} \leq 0$$

$$k \geq 4$$

minimum value of  $k$  is  $m = 4$

$$f(x) = x\sqrt{kx - x^2}$$

$$= 3\sqrt{4 \times 3 - 3^2} = 3\sqrt{3}, M = 3\sqrt{3}$$

70.  $x_1 + \dots + x_4 = 44$

$$x_5 + \dots + x_{10} = 96$$

$$\bar{x} = 14, \sum x_i = 140$$

$$\text{Variance} = \frac{\sum x_i^2}{n} - \bar{x}^2 = 4$$

Standard deviation = 2

71.  $\{(x, y) : y^2 \leq 4x, x + y \leq 1, x \geq 0, y \geq 0\}$

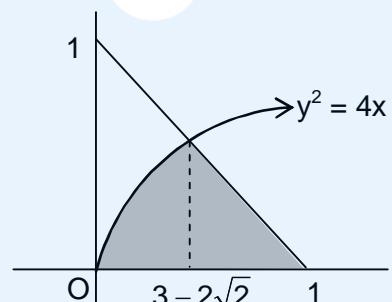
$$A \int_0^{3-2\sqrt{2}} 2\sqrt{x} dx + \frac{1}{2}(1 - (3 - 2\sqrt{2}))(1 - (3 - 2\sqrt{2}))$$

$$= \frac{2[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2} + \frac{1}{2}(2\sqrt{2} - 2)(2\sqrt{2} - 2)$$

$$= \frac{8\sqrt{2}}{3} + \left(-\frac{10}{3}\right)$$

$$a = \frac{8}{3}, b = -\frac{10}{3}$$

$$a - b = 6$$



72.  $y^2 dx + x dy = \frac{dy}{y}$

$$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$$

$$\text{IF} = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$$

$$e^{\frac{1}{y}} \cdot x = \int e^{\frac{1}{y}} \cdot \frac{1}{y^3} dy + C$$

$$xe^{\frac{1}{y}} = e^{\frac{1}{y}} + \frac{e^{-\frac{1}{y}}}{y} + C$$

$$C = -\frac{1}{e}$$

$$x = \frac{3}{2} - \frac{1}{\sqrt{e}} \text{ when } y = 2$$

73.  $375x^2 - 25x - 2 = 0$

$$\alpha + \beta = \frac{25}{375}, \alpha\beta = \frac{-2}{375}$$

$$\Rightarrow (\alpha + \alpha^2 + \dots \text{upto infinite terms}) + (\beta + \beta^2 + \dots \text{upto infinite terms})$$

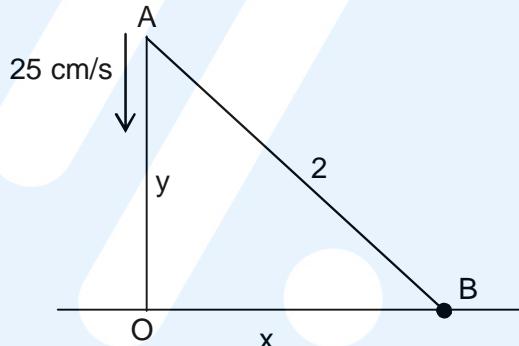
$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{1}{12}$$

74.  $x^2 + y^2 = 4 \left( \frac{dy}{dt} = -25 \right)$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$$

$$\sqrt{3} \frac{dx}{dt} - 1(25) = 0$$

$$\frac{dx}{dt} = \frac{25}{\sqrt{3}} \text{ cm/sec}$$



75.  $1 + \sin^4 x = \cos^2 3x$

$$\sin x = 0 \text{ and } \cos 3x = 1$$

$$0, 2\pi, -2\pi, -\pi, \pi$$

76.  $e^y = xy = e$  differentiate w.r.t. x

$$e^y \frac{dy}{dx} + x \frac{dy}{dx} + y = 0$$

$$e^y \frac{dy}{dx} + x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} (x + e^y) = -y, \frac{dy}{dx} \Big|_{(0,1)} = -\frac{1}{e} \text{ again differentiate w.r.t.x}$$

$$e^y \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot e^y \cdot \frac{dy}{dx} + x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$(x + e^y) \frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^2 \cdot e^y + 2 \frac{dy}{dx} = 0$$

$$e \frac{d^2y}{dx^2} + \frac{1}{e^x} e + 2 \left( -\frac{1}{e} \right) = 0$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{e^2}$$

77.  $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$   
 $\sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$   
 $= \sin^{-1}\left(\frac{33}{65}\right) = \cos^{-1}\left(\frac{56}{65}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

78.  $3x^2 + 4y^2 = 12$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$x = 2 \cos \theta, y = \sqrt{3} \sin \theta$$

$$\text{Equation of normal is } \frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$$

$$2x \sin \theta - \sqrt{3} \cos \theta y = \sin \theta \cos \theta$$

$$\text{Slope } \frac{2}{\sqrt{3}} \tan \theta = -2 \quad \therefore \tan \theta = -\sqrt{3}$$

Equation of tangent is it passes through (4, 4)

$$3x \cos \theta + 2\sqrt{3} \sin \theta y = 6$$

$$12 \cos \theta + 8\sqrt{3} \sin \theta = 6$$

$$\cos \theta = -\frac{1}{2}, \sin \theta = \frac{\sqrt{3}}{2} \quad \therefore \theta = 120^\circ$$

Hence point P is  $(2 \cos 120^\circ, \sqrt{3} \sin 120^\circ)$

$$P\left(-1, \frac{2}{2}\right), Q(4, 4)$$

$$PQ = \frac{5\sqrt{5}}{2}$$

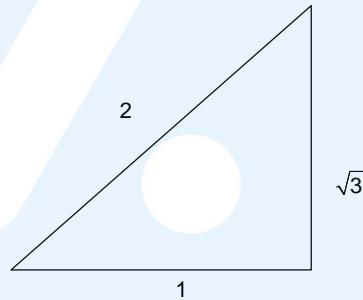
79.  $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$$= 2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 2 & 2 \end{vmatrix}$$

$$= 2(8\hat{i} - 8\hat{j} + 4\hat{k})$$

$$\text{Required vector} = \pm 12 \frac{(2\hat{i} - 2\hat{j} - \hat{k})}{3}$$

$$= \pm 4(2\hat{i} - 2\hat{j} - \hat{k})$$



80.  $2y = 2 \sin x \sin(x+2) - 2 \sin^2(x+1)$   
 $2y = \cos 2 - \cos(2x+2) - (1 - \cos(2x+2))$   
 $= \cos 2 - 1$   
 $2y = -2 \sin^2 \frac{1}{2}$   
 $y = -\sin^2 \frac{1}{2} \leq 0$

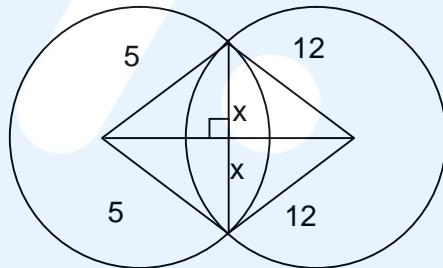
81.  $f(x) = \sqrt{x}, g(x) = \tan x, h(x) = \frac{1-x^2}{1+x^2}$   
 $fog(x) = \sqrt{\tan x}$   
 $hofog(x) = h(\sqrt{\tan x}) = \frac{1-\tan x}{1+\tan x}$   
 $= -\tan\left(\frac{\pi}{4} - x\right)$   
 $\phi(x) = \tan\left(\frac{\pi}{4} - x\right)$   
 $\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$   
 $= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$

82. Let length of common chord =  $2x$

$$\sqrt{25-x^2} + \sqrt{144-x^2} = 13$$

$$\text{After solving } x = \frac{12 \times 5}{13}$$

$$2x = \frac{120}{13}$$



83.  $\int_0^{\pi/2} \frac{\cot x dx}{\cot x + \operatorname{cosec} x}$

$$\int_0^{\pi/2} \frac{\cos x}{\cos x + 1} dx = \int_0^{\pi/2} \frac{2 \cos^2 \frac{x}{2} - 1}{2 \cos^2 \frac{x}{2}}$$

$$\int_0^{\pi/2} \left(1 - \frac{1}{2} \sec^2 \frac{x}{2}\right) dx$$

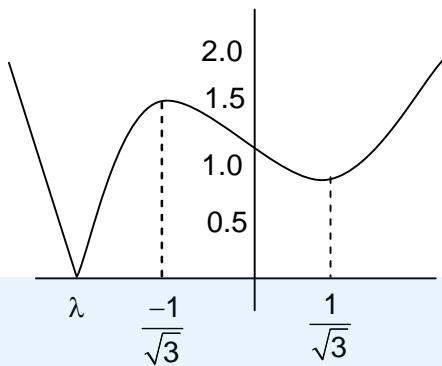
$$\left[ x - \tan \frac{x}{2} \right]_0^{\pi/2}$$

$$\frac{1}{2}[\pi - 2] \quad m = \frac{1}{2}, n = -2 \\ mn = -1$$

84.  $2\{2a + 3d\} = 16$   
 $3(2a + 5d) = -48$   
 $2a + 3d = 8$   
 $2a + 5d = -16$   
 $d = -12$   
 $S_{10} = 5\{44 - 9 \times 12\}$   
 $= -320$
85.  $|B| = 5(-5) - 2\alpha(-\alpha) - 2\alpha$   
 $= 2\alpha^2 - 2\alpha - 25$   
 $1 + |A| = 0$   
 $\alpha^2 - \alpha - 12 = 0$   
 $\text{Sum} = 1$

86.  $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$   
 $x = 3\lambda + 2, y = 2\lambda - 1, z = -\lambda + 1$   
Intersection with plane  $2x + 3y - z + 13 = 0$   
 $2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$   
 $13\lambda + 13 = 0$   
 $\lambda = -1$   
 $\therefore P(-1, -3, 2)$   
Intersection with plane  
 $3x + y + 4z = 16$   
 $3(3\lambda + 2) + (2\lambda - 1) + 4(-\lambda + 1) = 16$   
 $\lambda = 1$   
 $Q(5, 1, 0)$   
 $PQ = \sqrt{6^2 + 4^2 + 2^2} = \sqrt{56} = 2\sqrt{14}$

87. Volume of parallelopiped =  $\begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix}$
- $f(\lambda) = |\lambda^3 - \lambda + 1|$
- 
- Its graphs as follows



where  $\lambda \approx -1.32$

For minimum value of volume of parallelopiped and corresponding value of  $\lambda$ ; the minimum value is zero,  $\therefore$  cubic always has at least one real root.

Hence answer to the question must be root of cubic  $\lambda^3 - \lambda + 1 = 0$ . None of the options satisfies the cubic.

Hence Question must be Bonus.

88.  $A = A', B = -B'$

$$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \quad \dots \dots \dots (1)$$

$$A' - B' = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix} \quad \dots \dots \dots (2)$$

After adding equation (1) and (2)

$$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

89.  $P \rightarrow (\neg q \vee r)$

$$\neg p \vee (\neg q \vee r)$$

$$\neg p \rightarrow F \quad p \rightarrow T$$

$$\neg q \rightarrow F \Rightarrow q \rightarrow T$$

$$\neg r \rightarrow F \quad r \rightarrow F$$

90. 
$$\underbrace{\left[ -\frac{1}{3} \right] + \left[ -\frac{1}{3} - \frac{1}{100} \right]}_{(-1)^{67}} + \dots + \underbrace{\left[ \frac{1}{3} - \frac{66}{100} \right] + \left[ -\frac{1}{3} - \frac{67}{100} \right] + \dots + \left[ -\frac{1}{3} - \frac{99}{100} \right]}_{-2(33)} = -133$$