
Research Article

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A comprehensive survey of the home advantage in American football

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Abstract: The existence and justification to the home advantage – the benefit a sports team receives when playing at home – has been studied across sport. The majority of research on this topic is limited to individual leagues in short time frames, which hinders extrapolation and a deeper understanding of possible causes. Using nearly two decades of data from the National Football League (NFL), the National Collegiate Athletic Association (NCAA), and high schools from across the United States, we provide a uniform approach to understanding the home advantage in American football. Our findings suggest home advantage is declining in the NFL and the highest levels of collegiate football, but not in amateur football. This increases the possibility that characteristics of the NCAA and NFL, such as travel improvements and instant replay, have helped level the playing field.

Keywords: Home Advantage, National Football League, Bayesian Models, Football

1 Introduction

Nearly every fan, player, and coach has tried to reconcile the impact of playing at home in football. In the National Football League (NFL) regular season, and in several high school playoff formats, teams fight tooth and nail for a home advantage (HA) in the postseason, in part because they've assumed it provides some form of a benefit over playing on the road. Likewise, home fans dress up and scream loudly when their team is on defense (and only when their team is on defense), in an effort to rattle the visitors. In the 2023 Wild Card round contest between Detroit and Los Angeles, for example, the noise made by the Detroit home crowd was estimated to reach 118 decibels, roughly the equivalent to an 737 airplane at takeoff [1].

The goal of our paper is to estimate the benefit to all that screaming. Currently, though arguably dwindling, the home advantage in football is considered to be on the margins of 2.5 points per year in the NFL and 3-5 points in National Collegiate Athletic Association (NCAA) play (see Section 2). However, most research into the home advantage is restricted to single leagues, small periods of time, or both single leagues over small periods of time. Additionally, several approaches assessing the home advantage fail to account for team strength, which can bias estimates, especially when better teams are more likely to play at home. Finally, the evolution of the game itself, including instant replay in college and in the NFL, and an increased emphasis on passing, has arguably changed the playing field for visitors. Home advantage has declined in other sports such as basketball (professional [2–4] and collegiate [5, 6]), ice hockey [7], and soccer [8, 9]. As such, it is worth thoroughly assessing the likelihood of the home advantage declining in football, particularly in the larger context of these numerous factors that have changed in recent seasons, with the hope of better understanding possible drivers of home advantage.

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Using three Bayesian models with differing assumptions, we (i) estimate the home advantage, (ii) identify if the home advantage has changed over time, and (iii) apply uniformly across all levels of football, including each of the 50 states, each collegiate level (FBS, FCS, Division II, and Division III), and the NFL. Models are fit in Stan [10], an open-source software for Bayesian inference with Markov-chain Monte Carlo (MCMC) sampling, and compared using leave-one-out cross-validation [11].

We find that the most extreme home advantages in 2023 tend to exist collegiately, with FCS leading the way at 2.49 points/game. Additionally, in FBS, FCS, and Division II, as well as the NFL, a significant decline in the home advantage is more probable than not. On the other hand, for all but a few states, high school home advantage since 2004 has been stagnant, or perhaps increasing. Our code and findings are all provided at https://github.com/ThompsonJamesBliss/comprehensive_survey_american_football_home_adv, for researchers in other sports or fields to build upon.

The paper is laid out as follows: Section 2 reviews research into American football home advantage, Section 3 details our Bayesian framework, Section 4 reviews our data, Section 5 presents results, and finally Section 6 concludes and comments on plausible explanations for our findings.

2 Reviewing the Home Advantage in American Football

Nearly a half century of research has attempted to estimate home advantages in American football. This has been done using various models, scales, and time periods. Findings are universal – that it's better to play at home – but the size and magnitude of that benefit can vary slightly. Table 1 provides an overview of roughly 20 papers across various levels of football, including the league, seasons, statistical model, scale (score differential or home team win percentage), and result.

On a point scale, National Football League home advantage has varied from roughly 2 to 2.5 points [21, 22] in the 1970's, to 2.5 to 3.5 points in more modern game play [14, 16]. This point difference equates to home teams winning at a rate somewhere between 56% and 63% of games against an equal caliber opponent at home [12, 13, 17, 20]. Though their primary focus was on the impact of Covid-19, Higgs and Stavness [12] concluded that the NFL's home advantage had been declining for multiple years prior to the pandemic. In NCAA, results are more extreme, with point differentials as high as five to six points in recent seasons, [23, 24, 26], which equates to home teams winning between 60% and 65% of the time [17].

Typical statistical models for estimating home advantages include Bayesian frameworks (Negative Binomial and State Space paired comparison models), probit regression, hierarchical models, and neural networks. Higgs and Stavness [12] compared three versions of Bayesian models, including Poisson, Negative Binomial, and Normal, and found that Negative Binomial and Normal distributions performed best.

The mechanisms behind the home advantage, including crowd impact and officiating tendencies, travel and rest discrepancies, fan interaction, visiting team playing style, and comfort level for the home team, have long been debated [30]. Several NFL findings tie back to officiating and replay. In the book Scorecasting, Moskowitz and Wertheim [31] identified a connection between the advent of the NFL's instant replay process and the drop in fumble recovery rates for the home team. The implication is that, with additional technology, subjective fumble recovery decisions were eliminated. On the penalty side, Snyder and Lopez [32] studied several common NFL infractions, finding that, for example, the odds of defensive pass interference fouls were 18% higher when the home team was on offense, after accounting for score, time remaining, and team. However, it is unclear if this result was due to players fouling more often or varying tendencies of officials. Vergin and Sosik [33] found a significantly higher home advantage for home teams on Monday night, which corresponds to both additional attention on the game and, potentially, more crowd noise.

Travel has also been linked to home advantage in football. In two seasons of NCAA data, Fullagar et al. [23] identified links between larger crowds and larger home advantage, as well as longer travel and worse performance for the visiting team. In the NFL, Nichols [34] found that NFL home teams received a boost in performance when the visiting team has traveled further, although the effect was too small to turn a profit in betting markets.

Paper	League	Seasons	Method	Scale	Result
Higgs and Stavness [12]	NFL	2016-2019	Bayesian Neg. Binomial	Score Differential	8%
Lopez et al. [13]	NFL	2006-2016	Bayesian State-Space	Home Win %	8.9%
Glickman and Stern [14]	NFL	2006-2014	Bayesian State-Space	Score Differential	2.4 Points
Jones [15]	NFL	1995-2014	Empirical Home Win %	Home Win %	3-12%
David et al. [16]	NFL	2008-2010	Neural Networks	Score Differential	3 Points
Pollard and Gonzalez [17]	NFL	2006-2009	Empirical Home Win %	Home Win %	6.2%
Baker and McHale [18]	NFL	2001-2008	Markov Process	Scoring Intensity	0.1416**
Glickman and Stern [19]	NFL	1998-2003	Bayesian State-Space	Score Differential	3.2 Points
Boulier and Stekler [20]	NFL	1994-2000	Probit Regression	Home Win %	12.7%
Steffani [21]	NFL	1970-1978	Empirical Scores	Score Differential	2 Points
Harville [22]	NFL	1970-1977	Linear Model	Score Differential	2-2.4 Points
Fullagar et al. [23]	NCAA	2013, 2016	Linear Mixed Model	Score Differential	5 Points
Wang et al. [24]	NCAA	2008-2009	Multi-level Model	Score Differential	5.9 Points [⊕]
Pollard and Gonzalez [17]	NCAA	2006-2009	Empirical Home Win %	Home Win %	12.8%
Caudill and Mixon [25]	NCAA*	1974-2005	Linear Probability Model	Home Win %	11.4-22.5%
Gajewski [26]	NCAA [†]	1996-2004	Bayesian Piecewise LM	Score Differential	3-4 Points
Massey [27]	NCAA	1996	Linear Model	Score Differential	3.6 Points
Steffani [21]	NCAA	1970-1978	Empirical Scores	Score Differential	3 Points
Harville [28]	NCAA	1975	Linear Model	Score Differential	3.5 Points
McCutcheon [29]	High School [‡]	1982	Empirical Scores	Score Differential	1-4 Points
Harville [28]	High School ^{††}	1975	Linear Model	Score Differential	—

*SEC; [†]Big 12, Big 10, Pac 10; [‡]Virginia; ^{††}Ohio, theoretical results only; **Hazard ratio scale;

[⊕] Wang et al. [24] report 5.9 points but include an intercept term in their model of -2.8 points indicating a team would beat an equal strength opponent by 3.1 points at home

Tab. 1: Summary of literature examining home advantage in American football across various levels. Papers frequently report the home advantage on two distinct scales, probability of beating an equal caliber opponent at home, or the expected score differential of playing an equal caliber opponent at home. For papers reporting home advantage on the win percentage scale, we report the advantage above 50%. For example, a home advantage of 10% would imply teams have a $50 + 10 = 60\%$ chance of beating an equal caliber opponent at home. Note that while, Higgs and Stavness [12] report results on the score differential scale, they report a home advantage of 8%, which equates to an expected score differential that is 8% of league average scoring if a team were to play an equal caliber opponent at home.

3 Methods

3.1 Modeling Football Outcomes

American football has a unique scoring system scoring where the most common scoring results contribute 3 (field goal), 7 (touchdown and extra point), 8 (touchdown and 2-point conversion), 6 (touchdown without any extra point or 2-point conversion), or 2 (safety) points towards a team’s overall score. Despite the fact that a team’s score is a sum of these discrete set of numbers, the difference between two teams’ scores in any given game reasonably follows a Normal distribution [14].

Let Y_{ijkt} be the score differential in a game between team i and team j in league k during year t . We assume that

$$Y_{ijkt} \sim N(\mu_{ijkt}, \sigma_k^2)$$

We consider the following three models, which differ in how home advantage is modeled.



$$\begin{aligned}\mu_{ijkt} &= \theta_{ikt} - \theta_{jkt} + \alpha_k && (\text{Model 1, Constant HA}) \\ \mu_{ijkt} &= \theta_{ikt} - \theta_{jkt} + \beta_{0k} + \beta_{1k}(t - t_0) && (\text{Model 2, Linear HA}) \\ \mu_{ijkt} &= \theta_{ikt} - \theta_{jkt} + \gamma_{kt} && (\text{Model 3, Time-Varying HA})\end{aligned}$$

In the above three models, θ_{ikt} and θ_{jkt} represent team strength parameters for teams i and j in season t , respectively.

In Model 1, α_k is a league specific home advantage parameter that is constant over time. In Model 2, home advantage is modeled as a linear trend over time where β_{0k} denotes the home advantage in league k during year t_0 , the earliest year examined, and β_{1k} denotes the year rate of change in home advantage in points/year. Finally, in Model 3, home advantage is denoted by a league-season specific term γ_{kt} . These three models, which we refer to as constant HA, linear HA, and time-varying HA, respectively, represent increases in model flexibility going from Model 1 to Model 3. The choice of a linear home advantage trend in Model 2 is designed to capture trends where there have been small changes in HA over time.

We chose to assume that team strength parameters (θ_{ikt}) are independent season to season rather than pursuing a dynamic state-space model, as has been used when analyzing the NFL in isolation [13, 14, 19]. This choice was made in part due to large amounts of roster turnover at the high school and college levels between seasons, and the fact that some teams play very few games in a given season at the high school level. Additionally, some high school teams appear, disappear, and then appear again (see Section 4 for more details). More importantly, we are not interested in conducting inference on team strength. Rather, we are only interested in team strength insofar as properly accounting for team strength is necessary for accurately estimating HA term(s) of interest and adequately characterizing trends in home advantage over time.

3.2 Model Fits in Stan

We used Stan [10], an open-source statistical software designed for Bayesian inference with MCMC sampling, for each league k , and each of the three model options outlined in Section 3.1. We chose to utilize a Bayesian approach when performing inference for several reasons. Of primary interest was obtaining posterior distributions of the change in home advantage [35]. No paper referenced in Table 1 has assessed HA change probabilistically. Additionally, the Bayesian framework allows for more flexibility when building models compared to standard methods like ordinary least squares (OLS) regression, particularly in how team strength are estimated. Finally, the decision to adhere to a Bayesian paradigm aligns our model building framework with those of Glickman and Stern [19] and Lopez et al. [13], two of the seminal works on home advantage in the NFL.

Models 1, 2 and 3 were fit using the following prior distributions. These prior distributions are weakly-informative and do not impose any outside knowledge on parameter estimation.

$$\begin{aligned}\theta_{ikt} &\sim N(0, \zeta_k^2) && (\text{Team Strengths}) \\ \zeta_k &\sim \text{HalfNormal}(0, 5^2) && (\text{Team Strength Variance}) \\ \sigma_k &\sim \text{HalfNormal}(0, 5^2) && (\text{Score Differential Variance}) \\ \\ \alpha_k &\sim \text{Normal}(0, \eta_k^2) && (\text{Model 1 Home Advantage}) \\ \eta_k &\sim \text{HalfNormal}(0, 5^2) && (\text{Model 1 Home Advantage Variance})\end{aligned}$$

$$\begin{aligned}\beta_{0k} &\sim \text{Normal}(0, \lambda_{0k}^2) \quad (\text{Model 2 Home Advantage Intercept}) \\ \beta_{1k} &\sim \text{Normal}(0, \lambda_{1k}^2) \quad (\text{Model 2 Home Advantage Trend}) \\ \lambda_{0k} &\sim \text{HalfNormal}(0, 5^2) \quad (\text{Model 2 Home Advantage Intercept Variance}) \\ \lambda_{1k} &\sim \text{HalfNormal}(0, 5^2) \quad (\text{Model 2 Home Advantage Trend Variance})\end{aligned}$$

$$\begin{aligned}\gamma_{kt} &\sim \text{Normal}(0, \tau_k^2) \quad (\text{Model 3 Home Advantage}) \\ \tau_k &\sim \text{HalfNormal}(0, 5^2) \quad (\text{Model 3 Home Advantage Variance})\end{aligned}$$

Models were fit using 4 parallel chains, each made up of 2000 iterations, and a burn in of 500 draws. To check for model convergence, we examined the \hat{R} statistic [36, 37] for each parameter. If \hat{R} statistics are near 1, that indicates convergence [38], which was the case for all parameters in our model. To check for the informativeness of a parameter's posterior distribution, we also examined effective sample size (ESS, [38]), which uses the relative independence of draws to equate the posterior distribution to the level of precision achieved in a simple random sample. Tables summarizing \hat{R} statistics and effective sample sizes are available in the Supplementary Materials.

3.3 Posterior Probability of Decline

We assessed the likelihood that home advantage has declined by computing the posterior probability $P(\beta_{1k} < 0)$, where β_{1k} represents the average annual change in HA in points/year. If this posterior probability is close to 1, there is strong evidence to suggest the HA in a given league has been declining significantly (at least in the statistical sense) in a linear fashion over a long period of time. If this posterior probability is close to 0 (analogously, $P(\beta_{1k} > 0)$ is close to 1) there is strong evidence to suggest the HA in a given league has been increasing significantly over a long period of time. On the other hand, more moderate values of $(\beta_{1k} < 0)$ suggest there is less evidence to favor Model 2 to Model 1.

3.4 Model Comparison

For more formal model comparison, we computed expected log pointwise predictive density (ELPD) estimated via the leave-one-out cross-validation (LOO) approach of Vehtari et al. [11]. Operationalizing this approach entails computing the log-likelihood of each observation Y_{ijkt} under each posterior sample, and supplying the resulting $n_k \times m$ matrix to `loo()` [39] in R, where n_k denotes the total number of games analyzed in league k and m denotes the number of posterior samples. The primary motivation for model comparison via ELPD is that we were able to obtain associated standard error estimates, which enable comparisons between the difference in ELPD between models relative to the size of the associated standard error.

4 Data

The data used for this analysis were comprised of games from the NFL, four divisions of NCAA, and all 50 US states. For ease, we refer to each of these entities as leagues, even though in practice there are several divisions within a single state for high school sports. Data collected include the season, the teams playing, the location (home or neutral site) and the scores of each team. NFL data were collected from an internal database (though such information is publicly available [40]), while NCAA and high school data were scraped from MasseyRatings.com [41] and Max Preps [42] respectively.

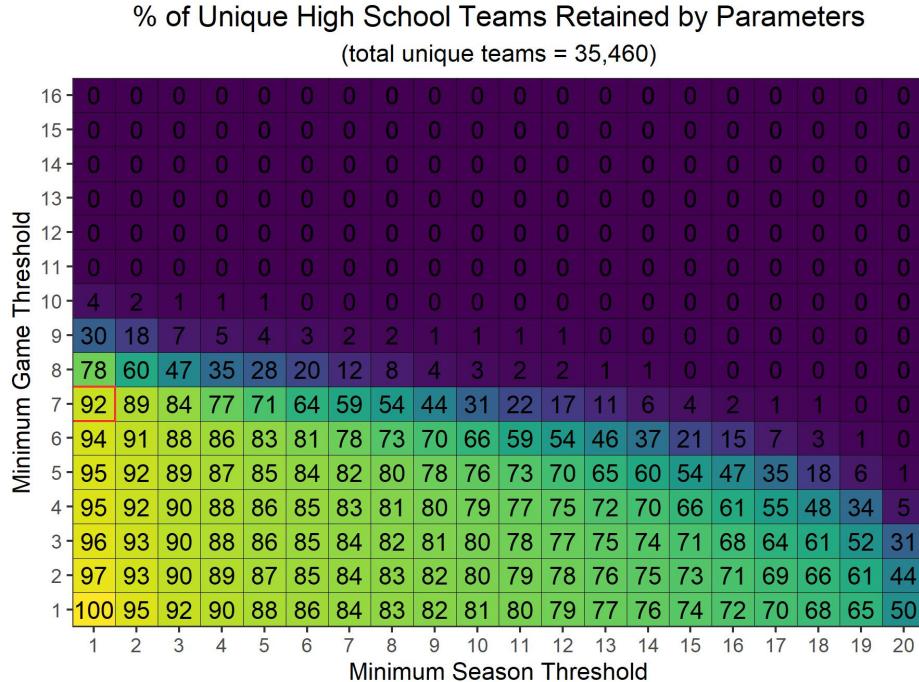


Fig. 1: The percent of unique high school teams retained after an iterative filtering process for a given minimum games/seasons and minimum # of seasons threshold. Teams that do not reach either the game or seasons threshold are removed from the data. Teams no longer reaching the threshold due to removal of opponents after the previous filtering are filtered out themselves. This process continues until no new teams are removed. The red outline indicates the resulting threshold choices for this work.

MasseyRatings.com is a website created by Kenneth Massey and has model-based team ratings for nearly every sport [27]. According to the website, data is collected electronically from a variety of publicly available domains with basic consistency checks run on multiple independent sources to verify the data's accuracy. Additionally, corrections and hard-to-find scores are entered manually. Max Preps is a news and data source for high school sports including information on players, teams and games. Game results are entered by a team's head coach and coaching staff.

We examined a nearly 20 year sample from 2004 - 2023, with the 2020 season excluded due to irregularities in play, travel, fan restrictions, and home advantage due to the COVID-19 pandemic [12, 35]. 2004 was chosen to be t_0 across all leagues as that was the earliest year Max Preps had public data available. Exceptions to $t_0 = 2004$ were Alaska and North Dakota (2005), New Mexico (2006), and Wyoming (2007) where data that met the thresholds in Figure 1 weren't available until later years, as well as Oregon (2007) and Maryland (2008), where data quality was poor prior to the 2007 and 2008 season respectively. The high school data were filtered to include in-state games only, in order to more precisely measure the effect of home advantage in a particular state and remove travel effects that may arise from playing a one-off game on the other side of the country.

One difficulty with the high school data was the presence of missing game results across seasons and teams. This is likely due to inaccuracy from the crowd-sourced collection, and that some high school football programs have folded or combined with other school(s) during the 20 year period we chose to examine. Considering this missing data, it seemed ideal to remove teams with few observations, as a sufficient number of games for team i in season t is needed to properly estimate team strength parameter θ_{it} . Of course, removing one team's game(s) from the dataset reduces the number of games for their opponents in the dataset as well. Thus, to reach our final high school sample, we applied the following iterative procedure.

- Pick thresholds for minimum # of games per season and # of seasons present in data.

2. Remove team-seasons below these thresholds.
3. Update # of games/seasons a team appears in the data.
4. Repeat steps 2 and 3 until no more teams are removed.

League	Number of Team/Seasons	Number of Games
NFL	640	5,395
NCAA	12,377	64,345
High School	247,402	1,283,531

Tab. 2: Counts of observations (games) and unique team/seasons by league.

Figure 1 displays the percentage of high school teams retained under an array of cutoffs for games/season and # of seasons appearing in the data. As is apparent along the x-axis, it is quite uncommon for a team to have more than 2-3 observations across all 18 seasons of data. Additionally, it is quite uncommon to have greater than 7 observations in any given season. Ultimately, given the inconsistent coverage across seasons, we decided to treat each team-season independently without any prior data from previous iterations of the team informing the estimates, as in a dynamic state-space model [13, 14, 19]. Given the extreme year-to-year roster turnover in high school relative to professional sports, and the fact that we are only interested in θ_{it} as a necessary adjustment towards estimating our suite of target home advantage parameters, this decision seems justifiable.

After applying the iterative filtering algorithm, we restricted analysis to team-seasons with at least 7 games played, retaining 92% of teams. Final sample sizes are available in Table 2.

5 Results

5.1 Model Performance and Overall Home Advantage

Results from Models 1, 2, and 3 are shown in Table 3. Of 55 leagues examined, the constant home advantage model (Model 1) had the best expected log predictive density in 29 leagues, the linear home advantage model (Model 2) had the best ELPD in 22 leagues, and the time-varying home advantage model (Model 3) was preferred in the remaining 6 leagues. Of note, Model 2 was preferred for the NFL, and the top 3 NCAA (FBS, FCS, and Division II), but not in Division III. Out of 24 leagues where Model 1 did not have the best ELPD, none had an ELPD difference that was 4 standard errors worse than the preferred model, a common rule of thumb [43], suggesting that differences from a constant HA model are small. This is not to say that home advantage is not changing, but rather the rate at which it is changing is small in comparison to the absolute magnitude of HA itself.

The estimated home advantage during the 2023 season from the best model in each respective state/league is shown in the last column in Table 3. Posterior distributions of HA estimates for the 2023 season are displayed in Figure 2. Across all 50 high school states, Wyoming had the greatest estimated home advantage in 2023, of 2.40 points (95% credible interval 1.86, 2.93). Wyoming was one of only nine states where the home advantage was estimated to be 2 points or greater along with Alaska [2.01 (0.91, 3.12)], Delaware [2.01 (0.91, 3.12)], Massachusetts [2.27 (1.93, 2.61)], Montana [2.33 (1.93, 2.74)], New Mexico [2.12 (1.72, 2.52)], Pennsylvania [2.12 (1.82, 2.42)], Washington [2.08 (1.32, 2.84)], and West Virginia [2.12 (1.32, 3.00)]. At the other extreme, the lowest estimated home advantage in 2023 belonged to New Hampshire with an estimated HA of just 0.65 points (-0.01, 1.30), making it one of just two states with an estimated home advantage under 1 point, along with Florida [0.99 (0.47, 1.51)]. Other states with small home advantages included Maryland [1.05 (0.81, 1.29)], Michigan [1.23 (1.09, 1.37)], and Idaho [1.25 (0.30, 2.16)].



Relative to high school, estimated NCAA home advantages were higher. Greatest in 2023 was FCS [2.49 (1.99, 2.96)], followed by FBS [2.39 (1.89, 2.88)], Division III [2.37 (2.18, 2.55)], Division II [2.40 (2.20, 2.64)]. HA in all of the NCAA divisions were larger than our 2023 HA estimate for the NFL of 1.73 (1.07, 2.39).

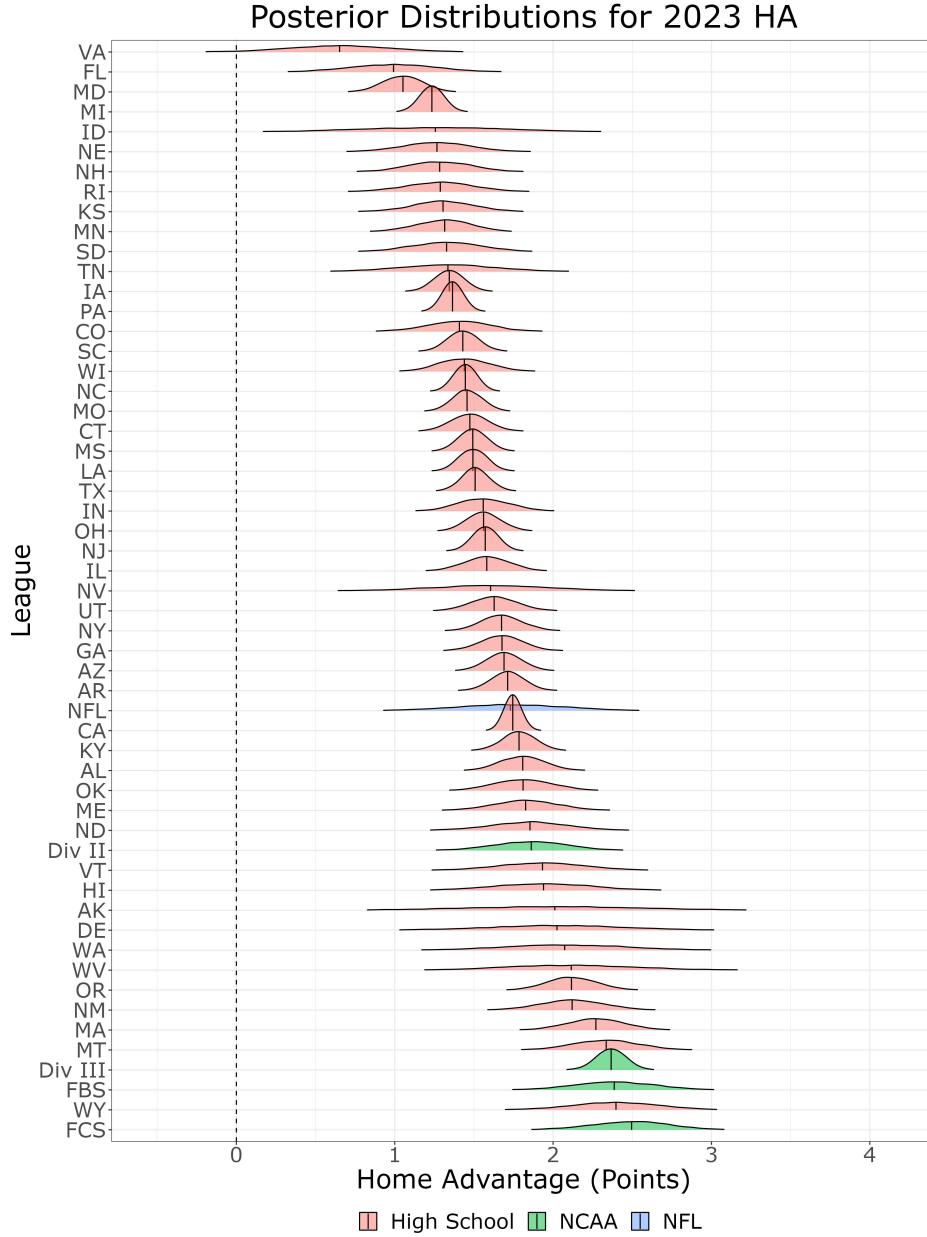


Fig. 2: Posterior distributions of 2023 HA estimates from the best model for each league on the basis of ELPD. Posterior means and 95% credible intervals are reported in Table 3.

5.2 Trends in Home Advantage

Figure 3 displays posterior distribution for β_{1k} , the linear change in home advantage. Among the 55 leagues, FBS exhibited the largest estimated linear decline in HA, with a drop of roughly $\hat{\beta}_{1k} = -0.097$ points/year between 2004-2023 (roughly a point drop per decade). Related, the associated probability of

decline was $P(\beta_{1k} < 0) = 1.000$. The other 3 NCAA divisions also had $\hat{\beta}_{1k} < 0$, though only Division II [$\hat{\beta}_{1k} = -0.058, P(\beta_{1k} < 0) = 0.999$] presented strong evidence of HA decline, while FCS [$\hat{\beta}_{1k} = -0.014, P(\beta_{1k} < 0) = 0.743$] and Division III [$\hat{\beta}_{1k} = -0.009, P(\beta_{1k} < 0) = 0.707$] exhibited less obvious linear HA changes over time.

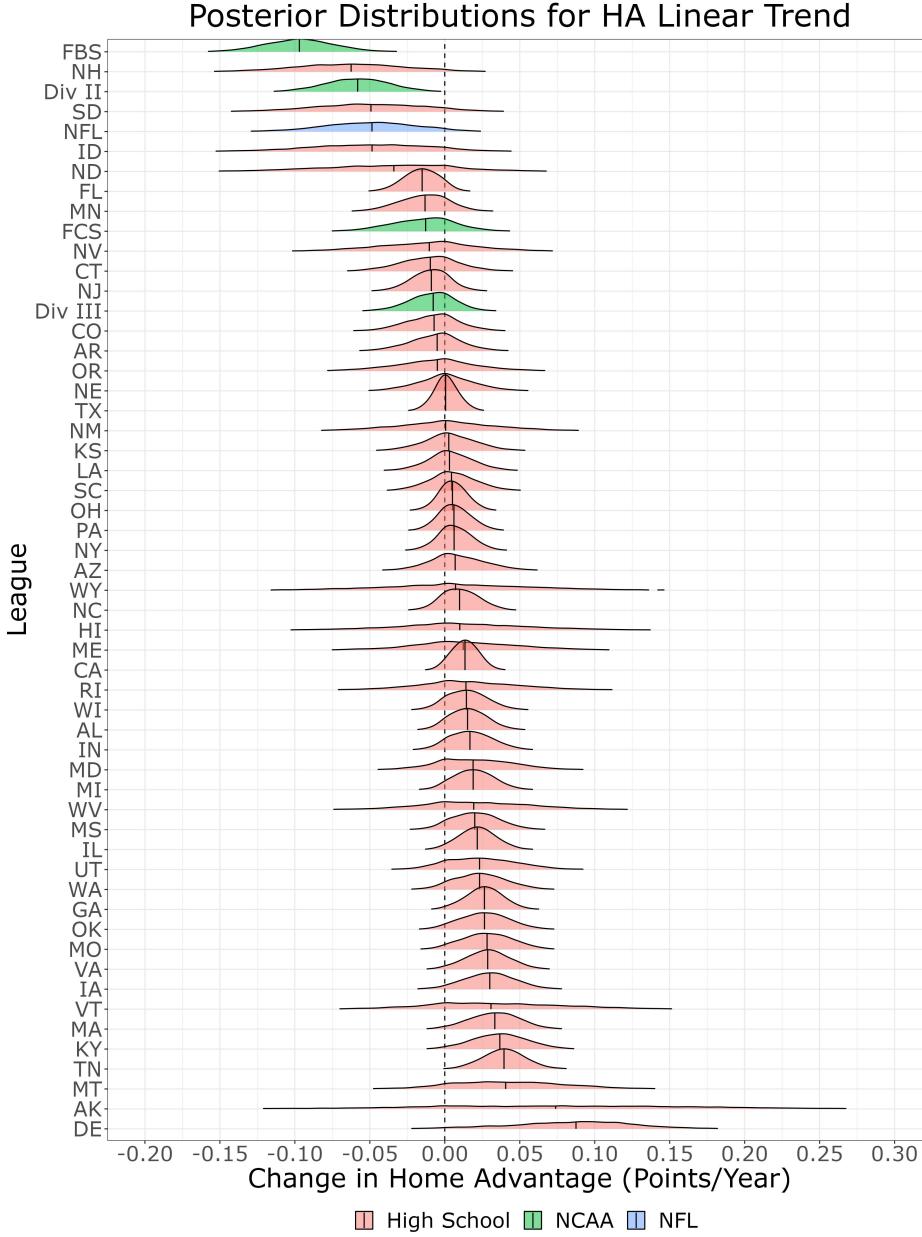


Fig. 3: Left: Posterior distributions for β_{1k} , the slope of the linear trend for HA in Model 2, which denotes the change in home advantage in points/year. Negative values of β_{1k} denote a decline in HA while positive values of β_{1k} denote an increase in HA.

Given both the preference of Model 2 and $\hat{\beta}_{1k}$ which significantly differed from 0, there is evidence to suggest that while the two tiers of Division I football have similar 2023 estimated home advantages, the temporal trends in HA are distinct, with stronger evidence that home advantage in FBS has been on the decline for the past 2 decades than in FCS. Despite the fact that Model 2 was preferred in the NFL, and the



likelihood of a linear decline was deemed relatively high, with $P(\beta_{1k} < 0) = 0.857$, such a trend is weaker than that observed in FBS, with $\hat{\beta}_{1k} = -0.032$ points/year (roughly 0.65 points in our twenty year sample).

As observed in Figure 3, greater heterogeneity in home advantage trends was observed at the high school level with posterior means $\hat{\beta}_{1k}$ ranging from -0.063 points/year (New Hampshire) to 0.084 points/year (Delaware). While $\hat{\beta}_{1k} < 0$ in the NFL and all four NCAA divisions, $\hat{\beta}_{1k} > 0$ in 38 of the 50 high school states and < 0 in the remaining 12.

The probability of HA increase $P(\beta_{1k} > 0)$ exceeded 90% in 14 states, albeit to varying degrees of practical significance. When fitting models on 50 states, however, we would expect a few significant states by chance. With this in mind, the most notable states with likely HA increase, on the basis of both statistical significance and practical significance were Delaware [$\hat{\beta}_{1k} = 0.084, P(\beta_{1k} > 0) = 0.968$], Tennessee [$\hat{\beta}_{1k} = 0.040, P(\beta_{1k} > 0) = 0.999$], Kentucky [$\hat{\beta}_{1k} = 0.038, P(\beta_{1k} > 0) = 0.977$] and Massachusetts [$\hat{\beta}_{1k} = 0.033, P(\beta_{1k} > 0) = 0.977$]. In the other direction, New Hampshire [$\hat{\beta}_{1k} = -0.063, P(\beta_{1k} < 0) = 0.956$] and South Dakota [$\hat{\beta}_{1k} = -0.049, P(\beta_{1k} < 0) = 0.908$] were noteworthy examples of states where HA has potentially been in decline.

5.3 Visualizing Model Results

Figure 4 displays model-based estimates of home advantage over time for the NFL, the four NCAA divisions, and select high school states. For comparison, we also present $\hat{\gamma}_{kt}^{\text{empirical}}$, the mean home minus away score-differential of all games in league k during season t , unadjusted for team strength. This estimator is of interest as a point of comparison due to its use (or its analogue, empirical home win percentage) in previous works [15, 17, 21, 29].

One notable result is the difference between model-based home advantage estimates and unadjusted empirical estimates, as $\hat{\gamma}_{kt}^{\text{empirical}}$ nearly always exceeded the value of the analogous time-varying home advantage estimates, $\hat{\gamma}_{kt}$. This is likely because better teams more frequently play at home; in several high school playoff formats, better teams host playoff games, and in the NCAA, top programs pay worse opponents to visit and get blown out.

In many cases differences between $\hat{\gamma}_{kt}^{\text{empirical}}$ and $\hat{\gamma}_{kt}$ exceeded 3 points, which itself was larger than the model-based home advantage estimates in every league. In some states, such as Louisiana, North Carolina, and Texas, $\hat{\gamma}_{kt}^{\text{empirical}}$ increased over time, while $\hat{\gamma}_{kt}$ remained fairly constant. This was further reflected by the fact that for each of these states $P(\beta_{1k} > 0)$ was not close to 1: Louisiana (0.593), North Carolina (0.794), Texas (0.535).

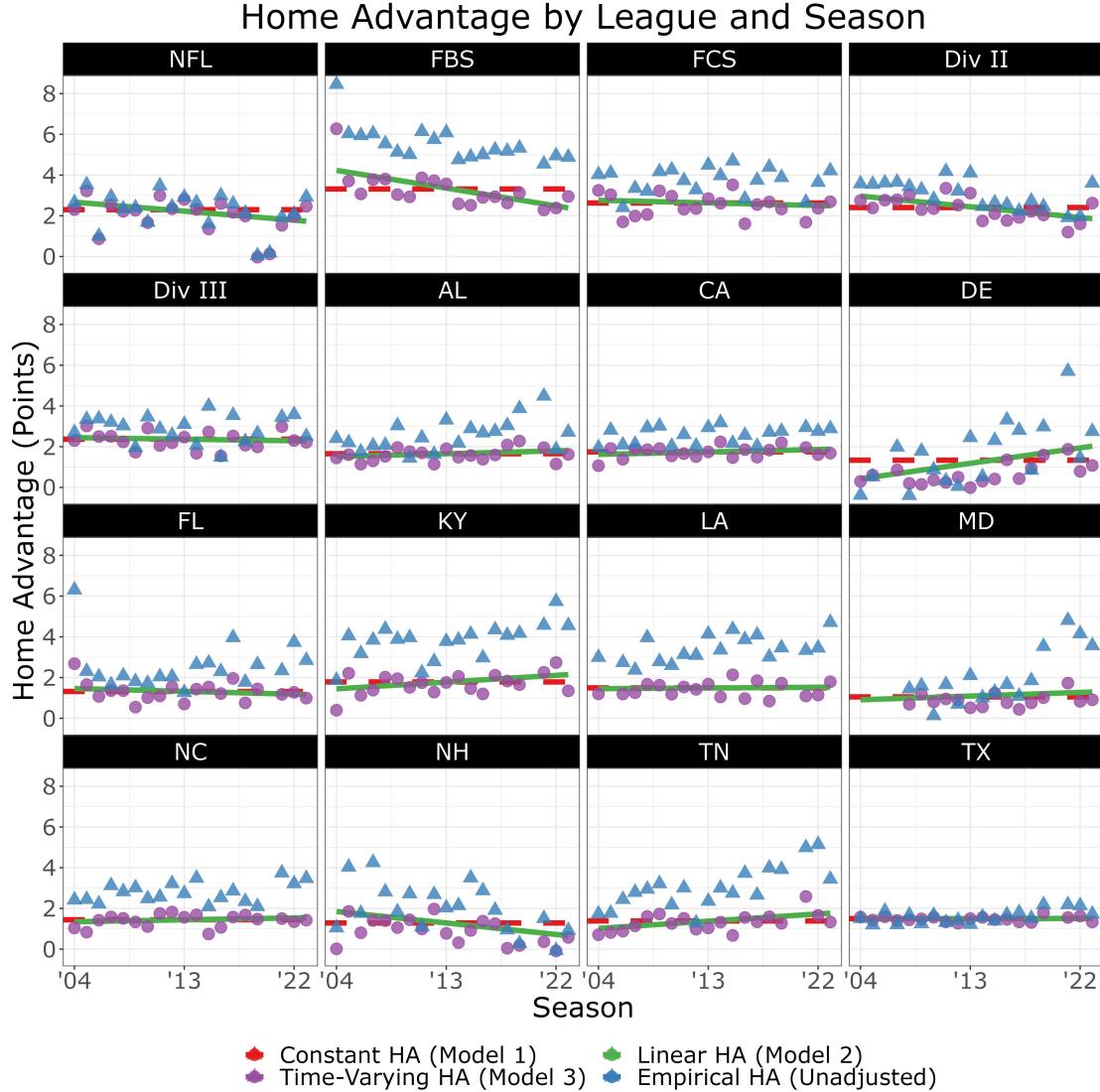


Fig. 4: Posterior mean home advantage over for select leagues. In addition to model-based estimates $\hat{\alpha}_k$ (Model 1), $\hat{\beta}_{0k} + \hat{\beta}_{1k}(t - t_0)$ (Model 2), and $\hat{\gamma}_{kt}$ (Model 3), we include $\hat{\gamma}_{kt}^{\text{empirical}}$, the mean home minus away score differential in season t for league k , unadjusted for team strength. The leagues were selected to reflect heterogeneity in their HA trends over time. Of note is the big difference between $\hat{\gamma}_{kt}^{\text{empirical}}$ and model-based estimates in the majority of leagues, particularly in high school and the NCAA, where better teams are more likely to host home games.

In Figure 4, Texas is an outlier amongst high school states in that there is very little difference between empirical and model-based home advantage estimates. The likely reason for this is that in Texas, playoff games after the first round are conducted at neutral sites equidistant from the two contending schools [44]. Most other states, however, feature a playoff system where games are hosted by stronger teams. When this occurs, empirical estimates can be biased, by attributing part of a game's outcome to the home advantage, when in fact it's due to better teams playing at home.



League	Model 1 (Constant HA)			Model 2 (Linear HA)			Model 3 (Time-Varying HA)			2023 HA
	ΔELPD	SE	# SE	ΔELPD	SE	# SE	ΔELPD	SE	# SE	
NFL	-1.31	1.75	0.75	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-4.93	4.83	1.02	1.73 (1.07, 2.39)
FBS	-11.03	4.75	2.32	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-2.47	5.34	0.46	2.39 (1.89, 2.88)
FCS	-0.30	1.63	0.19	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-7.34	4.61	1.59	2.49 (1.99, 2.96)
Div II	-3.90	3.56	1.09	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-10.06	4.68	2.15	1.86 (1.41, 2.31)
Div III	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-5.45	2.28	2.39	-13.11	5.52	2.38	2.37 (2.18, 2.55)
AK	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.04	0.99	0.04	-1.50	3.08	0.49	2.01 (0.91, 3.12)
AL	-0.68	3.55	0.19	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-14.66	6.76	2.17	1.81 (1.54, 2.08)
AR	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-6.15	2.05	3.00	-13.25	4.61	2.87	1.71 (1.51, 1.92)
AZ	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-2.55	2.21	1.15	-12.94	5.37	2.41	1.69 (1.48, 1.90)
CA	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-4.91	4.98	0.99	-9.45	7.87	1.20	1.75 (1.65, 1.84)
CO	-1.09	2.24	0.49	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-16.07	4.92	3.27	1.40 (0.99, 1.80)
CT	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.14	1.91	0.07	-5.88	4.54	1.29	1.47 (1.25, 1.70)
DE	-0.10	2.35	0.04	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-7.98	3.24	2.46	2.02 (1.14, 2.88)
FL	-6.67	8.38	0.80	-1.66	8.10	0.21	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	0.99 (0.47, 1.51)
GA	-0.72	3.86	0.19	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-5.77	5.79	1.00	1.68 (1.41, 1.93)
HI	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.72	1.14	0.63	-4.62	3.73	1.24	1.94 (1.34, 2.55)
IA	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-2.57	3.16	0.81	-16.58	5.03	3.29	1.34 (1.16, 1.52)
ID	-0.23	1.68	0.14	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-4.72	3.78	1.25	1.25 (0.30, 2.16)
IL	-3.10	4.13	0.75	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-11.32	5.58	2.03	1.58 (1.31, 1.85)
IN	-4.66	3.00	1.56	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-17.79	4.28	4.16	1.56 (1.26, 1.89)
KS	-4.06	2.22	1.83	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-6.12	4.92	1.24	1.31 (0.91, 1.70)
KY	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-2.54	3.07	0.83	-1.59	5.74	0.28	1.79 (1.58, 1.99)
LA	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-2.32	2.33	1.00	-7.96	5.12	1.56	1.49 (1.33, 1.66)
MA	-2.12	3.14	0.68	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-6.84	4.91	1.39	2.27 (1.93, 2.61)
MD	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-2.80	2.31	1.21	-2.50	4.01	0.62	1.05 (0.81, 1.29)
ME	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-4.31	1.18	3.65	-9.27	4.40	2.11	1.83 (1.43, 2.23)
MI	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-3.33	3.75	0.89	-2.96	6.02	0.49	1.23 (1.09, 1.37)
MN	-2.39	2.60	0.92	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-6.42	5.49	1.17	1.31 (0.96, 1.64)
MO	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-3.45	3.06	1.13	-16.10	4.84	3.32	1.46 (1.28, 1.64)
MS	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.22	2.59	0.09	-13.46	4.33	3.11	1.49 (1.33, 1.67)
MT	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.27	1.71	0.16	-6.46	4.79	1.35	2.33 (1.93, 2.74)
NC	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-1.47	2.78	0.53	-4.50	5.39	0.83	1.45 (1.31, 1.58)
ND	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.45	1.40	0.32	-9.02	3.76	2.40	1.85 (1.35, 2.34)
NE	-1.40	1.93	0.72	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-10.04	4.03	2.49	1.27 (0.83, 1.72)
NH	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.50	2.24	0.22	-8.70	4.00	2.18	1.29 (0.89, 1.67)
NJ	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.97	2.80	0.34	-10.76	5.29	2.03	1.57 (1.42, 1.72)
NM	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-4.85	1.33	3.64	-11.48	3.09	3.72	2.12 (1.72, 2.52)
NV	-0.80	1.19	0.67	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-15.01	3.72	4.04	1.61 (0.81, 2.37)
NY	-3.95	3.67	1.08	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-5.17	6.80	0.76	1.68 (1.43, 1.93)
OH	-1.42	3.98	0.36	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-10.35	5.69	1.82	1.56 (1.36, 1.77)
OK	-1.73	2.94	0.59	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-9.53	5.44	1.75	1.81 (1.46, 2.17)
OR	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.76	1.55	0.49	-10.19	3.60	2.83	2.12 (1.82, 2.42)
PA	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-7.18	3.44	2.08	-21.42	5.14	4.17	1.37 (1.25, 1.48)
RI	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-1.58	0.97	1.62	-11.64	3.48	3.35	1.28 (0.85, 1.71)
SC	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-3.11	2.43	1.28	-16.93	4.81	3.52	1.43 (1.25, 1.62)
SD	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-1.30	2.84	0.46	-6.93	4.59	1.51	1.33 (0.90, 1.74)
TN	-3.45	6.65	0.52	-1.79	5.89	0.30	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	1.34 (0.74, 1.94)
TX	-5.92	5.04	1.18	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-29.86	6.12	4.88	1.51 (1.35, 1.68)
UT	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-1.04	1.95	0.54	-14.24	4.09	3.48	1.63 (1.35, 1.91)
VA	-0.48	7.33	0.07	-0.84	7.01	0.12	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	0.65 (-0.01, 1.30)
VT	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-0.03	1.13	0.02	-9.74	3.84	2.54	1.93 (1.38, 2.47)
WA	-0.66	6.54	0.10	-3.53	6.35	0.56	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	2.08 (1.32, 2.84)
WI	-2.58	3.45	0.75	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-8.59	5.85	1.47	1.44 (1.14, 1.76)
WV	-2.05	3.08	0.67	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-4.31	5.12	0.84	2.12 (1.32, 3.00)
WY	<i>0.00</i>	<i>0.00</i>	<i>0.00</i>	-1.02	0.91	1.13	-11.12	3.19	3.49	2.40 (1.86, 2.93)

Tab. 3: Model comparison via expected log predictive density (ELPD). Shown are the difference in EPLD relative to the best model (ΔELPD), the standard error of EPLD difference (SE), and the number of standards error worse than the best model the EPLD difference is (# SE). For each league, the best model is represented by an EPLD difference and associated standard errors of 0, which are italicised for reference. Using the best model in each league, we report the posterior mean and 95% credible interval for home advantage during the 2023 season.

6 Discussion

Home advantage in sports is a phenomenon whose existence is unequivocal yet whose drivers are poorly understood. Using a 20-year sample two-fold larger than any previously conducted in the literature (Table 1), we set out to understand temporal trends in the home advantage across all levels of American football in order to better understand its drivers. Our findings suggest that home advantage has declined significantly at the FBS level over the past twenty years, and perhaps to a lesser extent at other levels of college and professional football. This pattern, however, is not universal, and home advantage at the high school level remains largely unchanged (and perhaps slightly increasing in parts of the country) during that same time period. A natural question to ask is why does such heterogeneity exist, and why haven't the declines in home advantage at higher levels of American football trickled down to the high school level. Towards positing an answer to this question, it is necessary to understand what has changed across all levels of football in the past twenty years and what hasn't.

One possibility is use of replay and challenges to overturn incorrect calls, mitigating the impact of referee subjectivity by ensuring more eventual rulings are correct. In the NFL, coach's challenges have offered the ability for teams to request review of controversial plays since 1999, preceding the beginning of our sample in 2004 [45]. Since 2014, all challenges have been reviewed at a centralized league office in New York [45]. During the course of our sample, the NFL also added new rules to automatically review all scoring plays (beginning in 2011) and plays resulting in a turnover (beginning in 2012). Finally, beginning in 2022, the NFL now utilizes an expedited review system which allows offsite officials to quickly overrule on-field officials and overturn clear and obvious errors without the need for a coaches challenge or full review. Perhaps unsurprisingly, the rate of successful coaches challenges has nearly doubled over the course of our sample, from 31% in 2004 to 58% in 2022 [46].

While the NFL has seen expanded use of replay review over the course of our sample, such improvements likely have less of an effect on changes in possible home advantage compared to the introduction of replay review to a league that didn't use it previously, as is the case in NCAA football. Experimental utilization of instant replay review in NCAA football began in 2004 in the Big 10 sample, and 2005 in other FBS conferences. It was not until 2006, however, that the NCAA Football Rules Committee officially approved use of replay review and published guidelines governing its use in games [45]. Nevertheless, this did not require teams to adopt the use of replay review at their home stadiums [47] and smaller conferences did not initially adopt replay review [45], owing to insufficient technology needed to run such a system. For analogous reasons, FCS schools were likely later to adopt widespread replay review compared to larger FBS schools, while even today, the majority of Division II/III games do not have access to sufficient angles in a timely manner necessary to run a replay review system. Finally, replay review was explicitly prohibited in high school football until 2019 [45]. Even today's its use is almost non-existent outside of perhaps state championship games played at large college or NFL stadiums.

Another potential driver of home advantage differences between levels of American football is changes in travel. Undoubtedly, NFL teams haven't seen substantial changes in travel accommodations in the past two decades. While the jets teams fly on have probably gotten more luxurious, flying to games has been the norm for all years in our sample. Similarly, high school and low end college teams (e.g. Division III) also have not experienced many changes in their modality of travel in the last two decades, with primary reliance on bus travel. On the other hand, top tier FBS teams have likely seen the biggest improvements towards how they travel between games compared to 20 years ago. While elite programs may have still flown between far games 20 years ago, plane travel is ubiquitous now, and the majority of FBS teams are flying chartered jets as opposed to commercial airfare. While plane travel exists in FCS and Division II, it's rarer [48]. If indeed implicit referee bias and travel are two possible drivers of home advantage, and FBS football has experienced the most substantial changes in both of these areas between 2004-2023, the sample considered in this paper, such reasons could potentially explain why FBS experienced the strongest decline in HA.

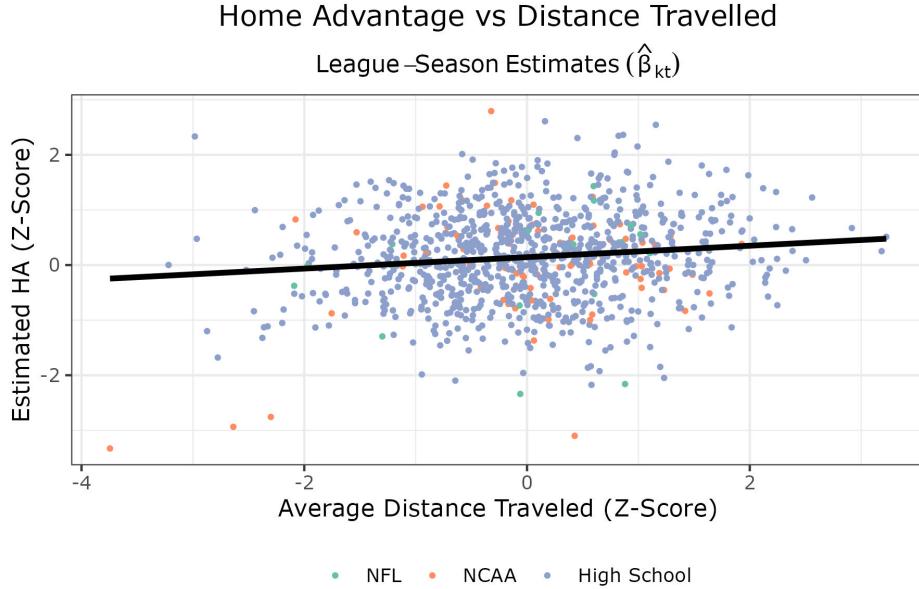


Fig. 5: Standardized mean distance travelled by the away team vs. standardized home advantage estimate $z_{kt}^{(\gamma)}$ across all league-seasons in the study. There is at best a weak association between average distance travelled and estimated home advantage, with line of best fit different by roughly 0.5 SD in estimated home advantage between the extremes in average distance travelled. Overall, this suggests that changes in distance travelled between games is not the primary driver of the trends in home advantage over the past 20 years, at least not in the absence of also knowing method of travel.

Closely related to type of travel is distance travelled between games. Distance travelled is of interest because FBS teams will be required to travel significantly more miles in the coming years following conference realignments that broke traditional geographical ties. Unfortunately, including distance as a covariate using our data is difficult for two reasons. First, it might be easier for an FBS team to fly 500 miles to an away game than a Division III team to drive 250 miles to an away game. Furthermore, due to the lack of granularity about location in our data, we only have longitude and latitude of each high school city, which yields a disproportionate number of games with incorrect estimates of 0 distance travelled. That is, there is heterogeneity in the true distance of these labelled 0-distance games, and missing out on such information could lead to inaccurate modeling of the relationship between distance and home advantage.

In order to understand the relationship between distance travelled and home advantage in our dataset, Figure 5 displays standardized versions of $\hat{\gamma}_{kt}$ from Model 3 against standardized average distance travelled in each league-season. Standardized home advantage estimates are defined as

$$z_{kt}^{(\gamma)} = \frac{\hat{\gamma}_{kt} - \bar{\gamma}}{\sigma_\gamma}$$

where $\bar{\gamma}$ denotes the mean home advantage $\hat{\gamma}_{kt}$ across all leagues and seasons, and σ_γ similarly denotes the standard deviation of home advantage posterior means across all leagues and seasons. To compute standardized distance travelled, we compute the average distance travelled by the away team in each league-season, and then compute the z -score of those quantities across all leagues/seasons in our sample. There is at best a weak association between average distance travelled and estimated home advantage ($R^2 = 0.016$), suggesting that any change in distance between games is not the main factor driving these observed results. Adding further nuance to the idea that distance travelled is perhaps secondary in driving changes in home advantage is the fact that no consistent pattern exists among rural states at the high school level. While Alaska, Montana, Wyoming, and Vermont had some of the largest estimated HA for the 2023 season, South Dakota, Idaho and Vermont's twin sister New Hampshire had some of the lowest home advantages in 2023.

While an even larger sample would be helpful to better test out some of these hypotheses further, doing so is not possible for lack of reliable high school scores. Furthermore, going back before 2004 would likely introduce additional confounding by play style, where changes in home advantage could also be attributed to rule changes that generally favor increases in offensive production. This is particularly important if one believes HA for football outcomes can model modeled on the multiplicative scale [12]. Such changes also likely explain why works examining older seasons [21, 22] have reported smaller HA estimates than more recent works [14, 16], including our own work.

Comparison to other works in Table 1 is imperfect because many papers use data outside the years of our sample and/or report home advantage on a win probability scale as opposed to a point differential scale. Using our best NFL home advantage estimated of 1.73 points and associated temporal trend $\hat{\beta}_{1k} = -0.032$, we can obtain estimated of NFL home advantage in 2006 and 2014 of 2.27 points and 2.01 points, respectively, in line with the constant estimate of 2.4 points reported by Glickman and Stern [14]. A similar computation for home advantage in FBS yields estimates of 3.65 and 3.85 during the 2008 and 2010 seasons respectively, slightly larger than the implied 3.1 points reported by Wang et al. [24]. These estimates are both a full point lower than the estimated NCAA home advantage during the 2013 and 2016 seasons reported by Fullagar et al. [23], although that work did not exclude FBS vs. FCS games, which are nearly always hosted by superior FBS teams and generally result in a large margin of victory for the hosting team.

Our work also illustrates an intuitive but perhaps underappreciated point in the importance of properly adjusting for team strength when estimating home advantage. In lower levels of football, including college and high school, better teams are both more likely to host more home games (perhaps due to better facilities and larger athletic budgets) and win by larger amounts. Additionally, in several leagues (including the NFL), better teams host playoff games, and so even with more balanced schedules, one would expect model-based estimates of HA which account for relative team strengths to be attenuated compared to empirical observations. As evidenced in Figure 4, failure to properly account for confounding by team strength would lead one to not only produce estimates of HA that are severely biased, but also overstate the extent to which strong temporal HA trends exist. Alternative methods that rely on empirical home win % [15, 17] or score differential [21, 29] do not account for relative team strength, and thus provided an incomplete characterization of home advantage. Although the primary purpose of our paper was not to evaluate the ability to estimate team strength, we compared the team ratings θ_{ikt} obtained from the best model for each NCAA division to ratings from MasseyRatings.com [41], obtaining a correlation of 0.92, suggesting that methods utilized in this paper are able to estimate team strengths with well respected publicly available ratings. This is notable because estimation of home advantage is ultimately limited by the ability to model team strengths. Models that better estimate team strength will generally produce more accurate estimates of the home advantage [35].

Though a linear trend appears appropriate for the majority of leagues plotted in Figure 4, our modeling framework does not allow for the possibility of a non-linear relationship between time and changes in the home advantage. For example, low-dimensional polynomial fits (e.g. quadratic or cubic) or a spline term on time t may be more appropriate for a single league of interest. These models would allow for more flexibility than the linear trend assumed by Model 2, while also requiring fewer parameters than Model 3. So while our framework is useful for assessing large-scale temporal trends across leagues, it is plausible one of these other candidate fits would yield improved inference. We leave consideration of additional candidate models as an area of possible exploration for future work.

Another possible limitation of our framework is that no information is shared from state to state or league to league, as we fit the same model structure separately. In certain contexts, however, it may be beneficial to share information across leagues, particularly if there is some justifiable reason why they might be related. To explore this consideration further, we investigated a hierarchical version of Model 2 applied to high school states. Two worked examples, along with comparison to results of Model 2 and a discussion of the hierarchical framing, are presented in the Supplementary Materials.

Finally, another possible extension of this work would consider various correlation structures on γ_{kt} in Model 3, possibly through application of a dynamic state space model [13, 14, 19]. For example, understanding the autocorrelation between γ_{kt} and $\gamma_{k(t-1)}$ could be of interest, or groupings of γ_{kt} across



leagues over time by factors such as replay usage or rule changes. One of the benefits of Model 3 is its ability to capture fast changes in HA in response to massive systematic changes, such as the introduction of replay or return to play following COVID. While effects of this nature are not the primary focus of this work, improved exploration of these correlation structures across both leagues and time could yield improved insight.

Overall, our work shows that conclusions from professional leagues need not generalize to amateur leagues, and provides a simple yet robust framework for creation and evaluation of models to understand the home advantage in American football which scales well with large numbers of teams and high rates of player turnover. This work is the first to consider a comprehensive evaluation of the home advantage of American football across levels of the sport, and does so at a scale far larger than any previous study to date, particularly at the high school level. Home advantage is notoriously difficult to study, and is likely the combination of numerous interdependent factors. In the absence of rare events (e.g. Covid-19 limiting fans in stands [12]), it's often impossible to isolate and study any single factor contributing to home advantage. By considering all levels of American football, not only do we get a more complete understanding of the HA landscape, but also we can better hypothesize which factors are driving home advantage by comparing what is different across levels of the game.

Data and Code Availability

All data and code used for this analysis is publicly available on GitHub at https://github.com/ThompsonJamesBliss/comprehensive_survey_american_football_home_adv.

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Supplementary Materials

Luke Benz[†], Thompson Bliss[†], and Michael Lopez

A comprehensive survey of the home advantage in American football

S1 Hierarchical Modeling Approach

An alternative way to fit Model 2 involves a Bayesian hierarchical model, which has been used in several sports related problems in recent years [1–5]. In the context of our problem, this approach may be appealing when estimating league specific home advantage trends because it shares information across leagues. This assumes that $\beta_{1k} \sim N(\beta_1^*, \lambda_1^2)$ where β_1^* denotes some common trend. Under this framework, β_1^* uses information across all leagues and β_{1k} is shrunk towards β_1^* . Implicitly, these assumptions imply a shared structure between leagues.

In terms of the 4 levels of the NCAA football, we feel that these divisions are sufficiently different from one another that such a model is not justified. That is, differences between Division III (most away teams travel via bus, few NFL players) and FBS (most away teams travel via plane, several NFL players) are stark enough in both travel and team ability, a finding supported by Table 3 in the main body of our paper.

We feel instead that high school football presents a better motivating example to illustrate the use of this model. In Section S1.1 present a mathematical formulation of the hierarchical version Model 2, which we refer to as Model 2H. We present two examples of Model 2H, one on states contributing fewer than 10,000 games to our collective sample (Section S1.2) and one examining all 50 states together (Section S1.3). Finally, in Section S1.4, we offer some discussion on the two examples and some commentary on when a model like Model 2H is well justified.

S1.1 Model Formulation

As in the main paper, let Y_{ijkt} be the score differential in a game between team i and team j in league k during year t . We assume that

$$Y_{ijkt} \sim N(\mu_{ijkt}, \sigma_k^2)$$

Just as in Model 2, we assume

$$\mu_{ijkt} = \theta_{ikt} - \theta_{jkt} + \beta_{0k} + \beta_{1k}(t - t_0)$$

Where Model 2 and Model 2H differ is their priors. Specifically, for Model 2H, we assume the following.

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$$\begin{aligned}
 \theta_{ikt} &\sim N(0, \zeta_k^2) \quad (\text{Team Strengths}) \\
 \zeta_k &\sim \text{HalfNormal}(0, 5^2) \quad (\text{Team Strength Variance}) \\
 \sigma_k &\sim \text{HalfNormal}(0, 5^2) \quad (\text{Score Differential Variance}) \\
 \beta_{0k} &\sim \text{Normal}(0, \lambda_{0k}^2) \quad (\text{League Specific Home Advantage Intercept}) \\
 \beta_{1k} &\sim \text{Normal}(\alpha_1^*, \lambda_1^2) \quad (\text{League Specific Home Advantage Trend}) \\
 \beta_1^* &\sim \text{Normal}(0, 5^2) \quad (\text{Shared Home Advantage Trend}) \\
 \lambda_{0k} &\sim \text{HalfNormal}(0, 5^2) \quad (\text{League Specific Home Advantage Intercept Variance}) \\
 \lambda_{1k} &\sim \text{HalfNormal}(0, 5^2) \quad (\text{League Specific Home Advantage Trend Variance})
 \end{aligned}$$

While we put shared structure on state specific home advantage trends β_{1k} , we choose not to assume a shared structure on intercepts β_{0k} because data begins in different years for different states and thus the reference season indexed by the intercept in each states differs. For example, $\beta_{0,\text{Texas}}$ indexes home advantage in Texas in 2004 while $\beta_{0,\text{Oregon}}$ indexes home advantage in Oregon in 2007. As with our other models, models in each example below were fit using 4 parallel chains, each made up of 2000 iterations, and a burn in of 500 draws.

S1.2 Example 1: Small States

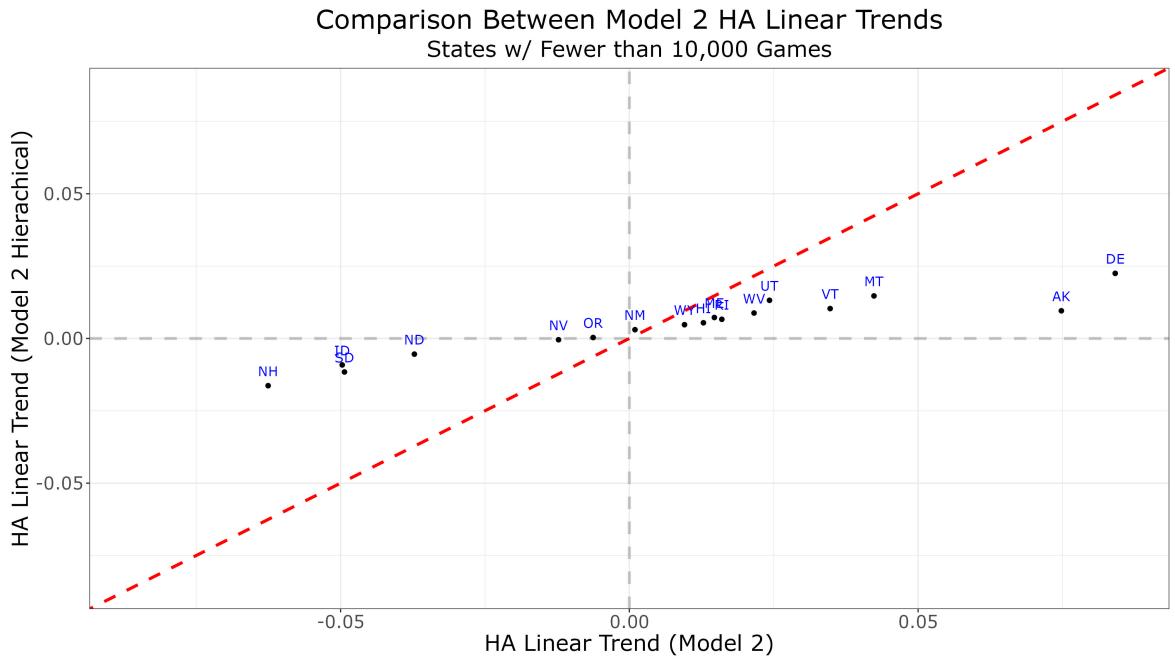


Fig. S1: Comparison of posterior means α_{1k} from Model 2 and Model 2H when Model 2H was fit on states with fewer than 10,000 games

We fit Model 2H on the 17 states which contributed fewer than 10,000 games to our overall sample (Table S1). Though these states are not geographically close, they are all generally rural and not population dense.

Figure S1 compares posterior means $\hat{\beta}_{1k}$ under the original version of Model 2 and the hierarchical version of Model 2. Unsurprisingly, we observe shrinkage towards $\hat{\beta}_1^* = 0.004$, and estimate the associated standard deviation around this shared trend to be $\hat{\lambda}_1 = 0.022$. With the exception of Alaska, the state with the smallest sample size, the order of $\hat{\beta}_{1k}$ is roughly preserved with New Hampshire and Delaware on the respective extremes.

Significance of certain state specific trends are attenuated compared to fitting each league in isolation. For example, under Model 2, $P(\hat{\beta}_{1k} > 0) = 0.968$ for Delaware, while under Model 2H, $P(\hat{\beta}_{1k} > 0) = 0.808$. Though the magnitudes of state specific HA trends may be attenuated, broader conclusions remain the same. Namely, there is not any evidence to suggest HA is in decline in high school school (as is likely the case in higher leagues) and there is a degree of heterogeneity in both home advantage (Figure S2) and home advantage trends at the high school level.

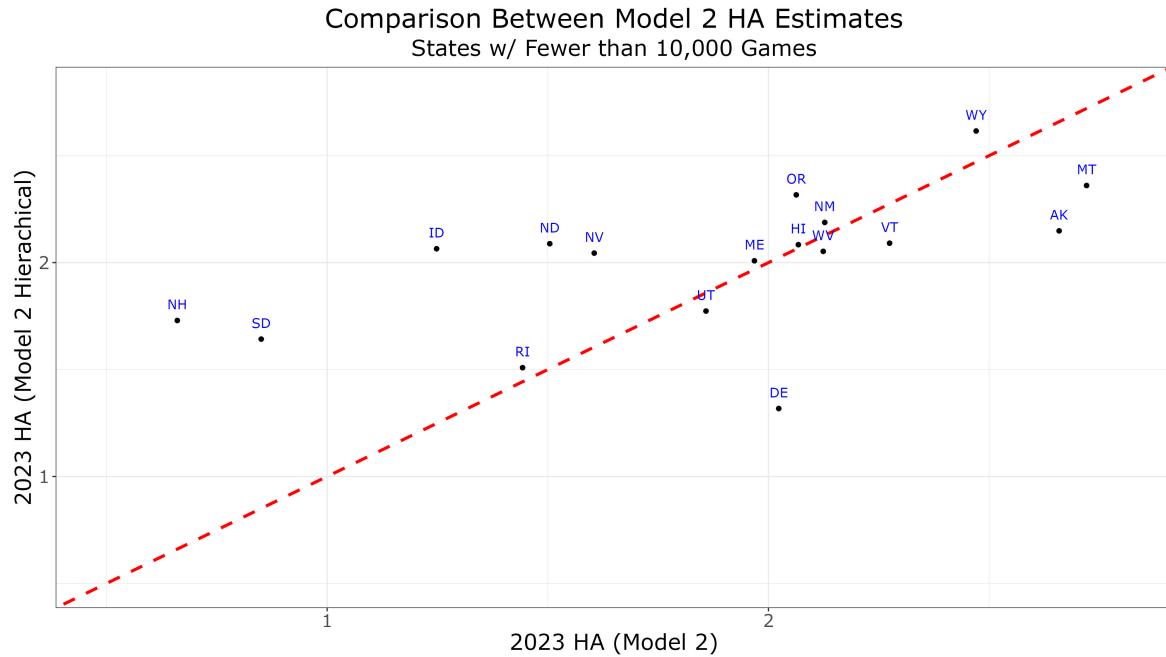


Fig. S2: Comparison of posterior means for home advantage from Model 2 and Model 2H when Model 2H was fit on states with fewer than 10,000 games

S1.3 Example 2: All 50 States

We also fit Model 2H on all 50 states together. Figure S3 compares posterior means $\hat{\beta}_{1k}$ under the original version of Model 2 and the hierarchical version of Model 2. State effects are shrunk substantially closer to $\hat{\beta}_{1*} = 0.012$ than in the previous example. Furthermore, the posterior mean standard deviation of state trends $\hat{\lambda}_1 = 0.006$ seems almost implausibly small. In Figure S3, Texas, California, Pennsylvania, and Ohio, the four states which contributed the largest sample of games (Table S1) are highlighted in blue. Notably, these states do not exhibit much, if any, shrinkage. Collectively, these states contributed roughly 27% of all high school games.

Overall, when fit on all 50 states, Model 2H seems to exhibit over shrinkage. Reasonably, we'd expected β_1^* to be closer to 0 as in the previous example with λ_1 larger than which was estimated in this example. Despite the fact that state specific linear trends $\hat{\beta}_{1k}$ are shrunk considerably towards 0.012, 2023 HA estimates using Model 2H are not substantially different than those using Model 2 (Figure S4), suggesting that state specific intercepts β_{0k} are correcting for some of the observed over shrinkage.



Comparison Between Model 2 HA Linear Trends
All 50 States

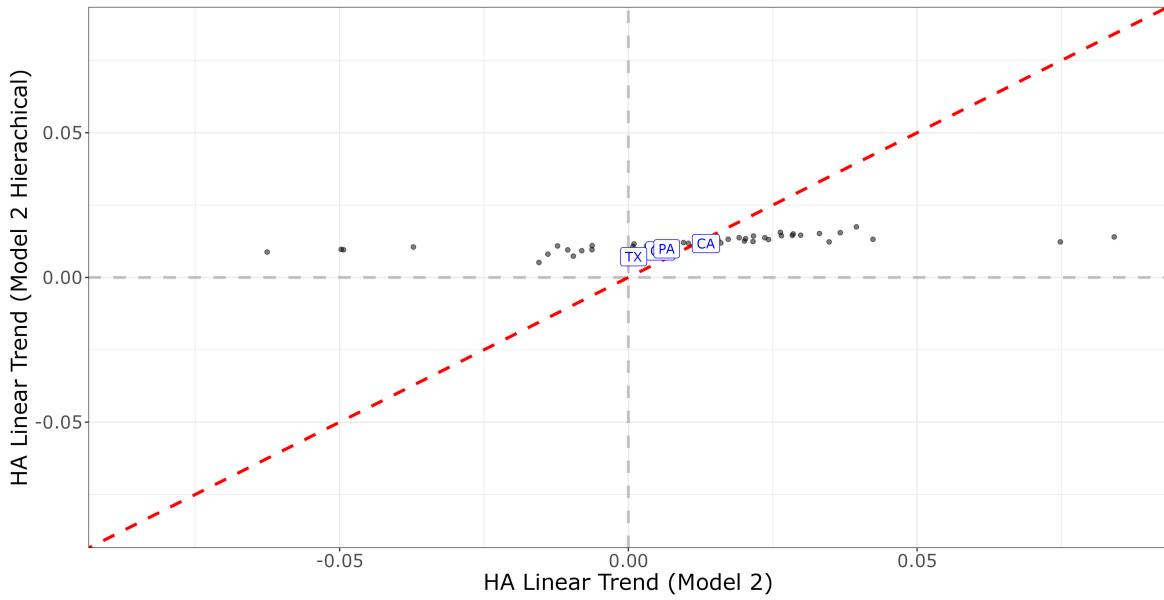


Fig. S3: Comparison of posterior means β_{1k} from Model 2 and Model 2H when Model 2H was fit on all 50 states. States which contributed the four largest sample sizes are highlighted in blue, and do not exhibit much shrinkage.

Comparison Between Model 2 HA Estimates
All 50 States

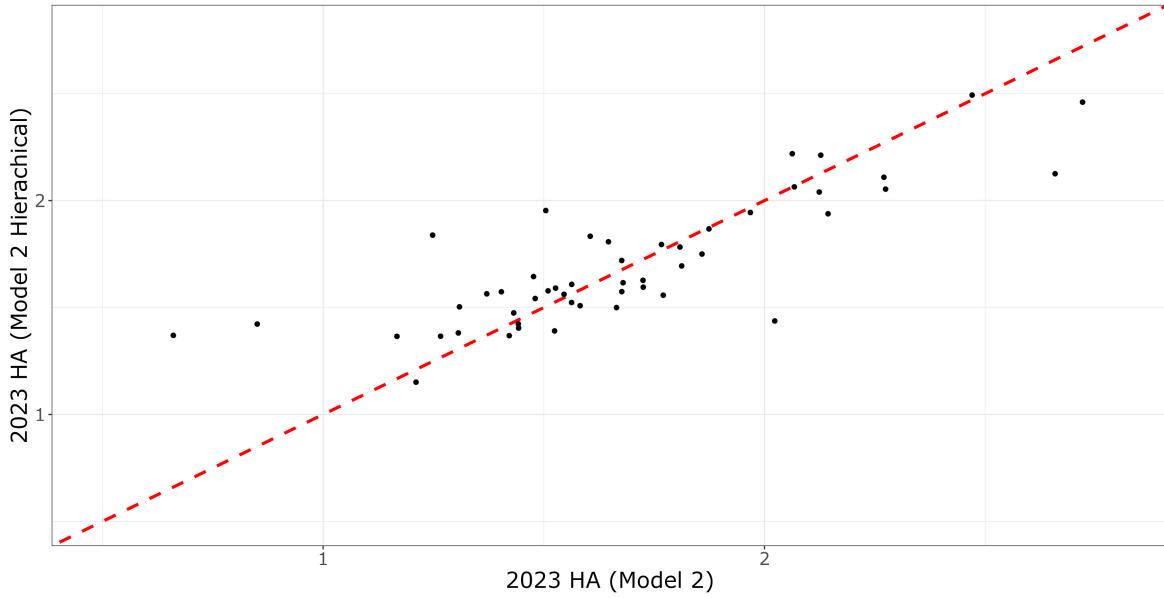


Fig. S4: Comparison of posterior means for home advantage from Model 2 and Model 2H when Model 2H was fit on all 50 states

League	# of Games	League	# of Games
TX	115,261	KY	22,701
CA	105,208	AR	20,487
OH	67,684	KS	20,093
PA	56,720	AZ	19,504
FL	50,519	CO	19,470
NY	46,785	CT	13,997
IL	46,708	MD	13,429
GA	45,343	NE	12,765
MI	44,894	UT	9,946
AL	42,855	OR	8,942
NC	41,586	NM	6,471
WI	35,799	WV	5,320
TN	33,813	ME	5,270
NJ	32,482	MT	4,876
IN	31,242	NH	4,826
VA	30,233	NV	4,761
MO	29,894	SD	4,596
MA	27,751	RI	4,389
LA	27,672	ND	4,158
OK	26,403	HI	3,463
MS	25,885	ID	3,430
MN	25,439	DE	3,122
IA	24,170	VT	2,902
WA	23,272	WY	2,663
SC	23,239	AK	1,093

Tab. S1: # of games in each high school state.

S1.4 Discussion

The two examples in the preceding sections are illustrative of the differences between modeling approaches which fit a separate model for each league, as we did in the main body of our paper, and a hierarchical approach. These new examples suggest that a hierarchical approach, at least in our specification, may yield over shrinkage for estimating trends in home advantage when leagues are extremely different in size.

Estimates of the 2023 home advantage seem to be less affected by over shrinkage than do trends β_{1k} . New Hampshire (Model 2: 0.66 , Model 2H: 1.37) and South Dakota (Model 2: 0.85 , Model 2H: 1.42), for example, are both reasonably pulled towards the overall high school average. One possibility is that because intercepts β_{0k} are not estimated with a shared intercept, state-level HA estimates are not pulled towards a common, high school level HA. In fact, state-level intercept estimates may be reacting to the degree of shrinkage in state trends. Additional hierarchical models with both shared intercepts and shared linear trends, or models that incorporate spatial and/or rules-based similarities between states, are left as future work.

S2 Model Diagnostics

Model diagnostics to assess convergence in the form of \hat{R} statistics and effective sample sizes are presented for Models 1-3 in Tables S2 - S4 respectively. Similar diagnostic information for the hierarchical models discussed in the previous section is available in Table S5. All \hat{R} were very close to 1 indicating good model convergence [6, 7].

League	$\min(\hat{R})$	$\max(\hat{R})$	α_k	η_k	σ_k	ζ_k	θ_k
AK	1.00	1.00	7639	6281	4723	5689	5553
AL	1.00	1.00	8757	7550	4305	4010	5090
AR	1.00	1.01	6772	5798	4138	1627	2602
AZ	1.00	1.02	5404	5079	3899	1275	1245
CA	1.00	1.01	8971	6996	3441	2456	2905
CO	1.00	1.01	5798	5538	3686	1587	1694
CT	1.00	1.01	4382	4866	4615	2357	1284
DE	1.00	1.01	8444	8762	6229	4889	1959
Div II	1.00	1.01	6235	5983	5122	3523	2497
Div III	1.00	1.00	7153	6796	5125	5269	4765
FBS	1.00	1.00	5887	7956	8008	4447	4911
FCS	1.00	1.00	5039	6780	5138	2211	3384
FL	1.00	1.01	10521	9432	4106	2971	4068
GA	1.00	1.01	10256	7518	4586	4261	4521
HI	1.00	1.01	6318	6500	3740	1726	1453
IA	1.00	1.00	6783	5936	4195	6483	6422
ID	1.00	1.01	9125	7599	4284	2923	2007
IL	1.00	1.01	10734	7242	4384	2663	3491
IN	1.00	1.00	6848	6028	4481	6731	6370
KS	1.00	1.00	12518	8315	5064	4064	3711
KY	1.00	1.01	7930	5791	4690	5257	2594
LA	1.00	1.01	10124	6418	5027	4489	4163
MA	1.00	1.00	11382	7744	3594	3393	5556
MD	1.00	1.01	5596	5298	4573	2887	1612
ME	1.00	1.02	4482	4546	4293	2581	872
MI	1.00	1.00	7510	6551	3462	4747	4958
MN	1.00	1.00	6395	5716	4130	6042	6621
MO	1.00	1.00	10748	7811	4349	3718	4125
MS	1.00	1.00	8342	5954	4480	5211	5344
MT	1.00	1.01	6285	5458	5068	4137	929
NC	1.00	1.00	12694	8651	4418	4989	4031
ND	1.00	1.01	8022	6847	6650	3898	1562
NE	1.00	1.01	6339	5449	4962	4102	1477
NFL	1.00	1.00	16221	8515	9909	4960	10489
NH	1.00	1.01	7250	6835	5240	3398	1315
NJ	1.00	1.00	8816	6984	4633	6478	6141
NM	1.00	1.01	6466	6191	2388	1360	810
NV	1.00	1.01	4676	4969	5626	3256	920
NY	1.00	1.01	9052	7607	5950	4414	2310
OH	1.00	1.01	11363	8122	3627	2325	4370
OK	1.00	1.00	9426	7070	4776	5564	5807
OR	1.00	1.01	6590	5797	4555	3176	1669
PA	1.00	1.01	10046	7744	4642	4042	3402
RI	1.00	1.00	10951	7570	6417	3664	2457
SC	1.00	1.01	5129	5442	4138	1603	1969
SD	1.00	1.02	5751	5906	3916	2161	915
TN	1.00	1.00	8425	6462	4822	5623	5185
TX	1.00	1.00	9465	7391	3397	1665	3831
UT	1.00	1.01	6343	6338	4588	1425	1452
VA	1.00	1.01	6359	5159	4168	3229	3175
VT	1.00	1.01	10132	6763	5398	4005	2249
WA	1.00	1.01	6106	6287	3729	4285	3832
WI	1.00	1.00	14213	8640	5481	4388	4304
WV	1.00	1.01	6372	6294	4133	2456	2043
WY	1.00	1.01	6036	4595	4902	3311	990

Tab. S2: Model diagnostics for Model 1. Minimum and maximum \hat{R} and effective sample size (ESS) for parameters of interest are shown. For parameters which are vectors in each league (such as θ_k), mean ESS are shown.

League	$\min(\hat{R})$	$\max(\hat{R})$	β_{0k}	β_{1k}	λ_{0k}	λ_{1k}	σ_k	ζ_k	θ_k
AK	1.00	1.01	1859	3760	2421	6318	3776	2488	1575
AL	1.00	1.01	6793	6474	9089	6848	3469	2380	4024
AR	1.00	1.01	3050	2875	5124	5686	3800	1904	2336
AZ	1.00	1.01	3190	3164	6980	6855	4548	1366	1248
CA	1.00	1.01	5130	4950	7262	6715	3246	2811	3063
CO	1.00	1.00	6023	5883	7870	6332	4108	3304	3176
CT	1.00	1.01	2718	2719	4697	4713	4756	2408	1378
DE	1.00	1.02	2516	2412	3842	5400	5074	2149	1020
Div II	1.00	1.01	2890	3025	6197	5468	6058	3291	2324
Div III	1.00	1.00	6420	6378	8163	7450	5219	4352	4114
FBS	1.00	1.00	2969	3070	7580	6458	6580	3250	4616
FCS	1.00	1.00	3112	2965	6819	6302	5981	2961	4157
FL	1.00	1.00	5459	4900	7970	6422	4155	2695	4111
GA	1.00	1.01	5706	5278	8784	6661	4472	5396	4691
HI	1.00	1.01	2856	2807	5844	5846	3085	1343	1257
IA	1.00	1.01	5336	5502	6340	5560	5077	4256	3574
ID	1.00	1.01	3385	3147	5978	6498	4213	2522	1697
IL	1.00	1.01	4791	4717	7283	6040	4500	3439	3628
IN	1.00	1.00	7786	7500	8561	6417	4935	3218	3489
KS	1.00	1.01	2696	2670	4952	5204	4252	1921	1672
KY	1.00	1.01	3815	3758	7112	6170	5590	4494	2340
LA	1.00	1.00	6626	6121	7542	6018	4835	4856	4356
MA	1.00	1.00	6653	6898	8034	5992	4372	3251	5340
MD	1.00	1.01	3245	3086	5969	5245	3866	4328	1995
ME	1.00	1.01	2165	2215	5185	6418	4575	2928	1050
MI	1.00	1.01	4975	4653	7218	6650	3892	3262	3947
MN	1.00	1.01	3827	3686	6179	5614	4799	3856	3263
MO	1.00	1.00	6115	5911	6777	6264	4676	4311	5102
MS	1.00	1.01	4849	5135	6621	5852	4310	3413	3629
MT	1.00	1.01	2575	2561	6124	6466	5203	3827	938
NC	1.00	1.00	8058	7695	8990	6570	4753	5931	3974
ND	1.00	1.01	3009	2996	6695	5887	5062	3523	1387
NE	1.00	1.01	3073	2873	5362	6333	4798	3344	1198
NFL	1.00	1.00	6639	6562	8633	7641	10177	4286	9451
NH	1.00	1.01	2505	2531	5334	5358	4073	2356	1173
NJ	1.00	1.00	7735	7579	7780	6014	5001	4996	4938
NM	1.00	1.02	2798	2792	5791	5899	2808	1174	796
NV	1.00	1.01	3152	3110	5007	5879	4552	2562	1152
NY	1.00	1.01	4162	4503	6785	6817	6333	4382	1985
OH	1.00	1.01	4926	4807	8341	6298	4591	2424	3675
OK	1.00	1.01	4681	4722	5916	5534	4659	4624	3951
OR	1.00	1.01	2664	2637	4974	5275	4305	2800	1650
PA	1.00	1.01	3649	3522	7878	6796	4651	4098	2848
RI	1.00	1.01	2825	2823	5540	5668	3898	1976	1522
SC	1.00	1.01	3220	3249	5432	5061	4303	1428	1757
SD	1.00	1.02	2830	2848	4943	5897	4070	2333	891
TN	1.00	1.00	7370	6841	7166	5649	4925	5835	5177
TX	1.00	1.01	3843	3833	5784	6639	3197	1613	3308
UT	1.00	1.02	1703	2033	5577	4966	2646	1217	1134
VA	1.00	1.00	6745	6875	6600	5949	5259	5605	4719
VT	1.00	1.00	3545	3781	6298	7149	5046	3605	2115
WA	1.00	1.00	5357	5680	7110	5814	5069	5379	5124
WI	1.00	1.01	4728	4574	6616	6620	5390	3110	3101
WV	1.00	1.01	2437	2454	5592	5486	3819	2186	1682
WY	1.00	1.01	4131	4057	4799	5630	5474	4813	1416

Tab. S3: Model diagnostics for Model 2. Minimum and maximum \hat{R} and effective sample size (ESS) for parameters of interest are shown. For parameters which are vectors in each league (such as θ_k), mean ESS are shown.

League	$\min(\hat{R})$	$\max(\hat{R})$	γ_{kt}	τ_k	σ_k	ζ_k	θ_k
AK	1.00	1.00	6435	1108	4827	4470	3825
AL	1.00	1.01	9208	5819	3778	2254	3256
AR	1.00	1.01	5885	4100	4180	1484	2195
AZ	1.00	1.01	5865	4550	3585	2607	3052
CA	1.00	1.01	8700	6981	3411	2592	3093
CO	1.00	1.01	5935	4133	3316	2242	1924
CT	1.00	1.02	4942	3758	4935	1907	1259
DE	1.00	1.01	6515	1389	5634	4171	1632
Div II	1.00	1.01	6412	4486	5201	4194	2968
Div III	1.00	1.01	6098	4990	4860	3126	2426
FBS	1.00	1.00	7382	5785	5710	3688	4706
FCS	1.00	1.00	6703	4875	4554	2639	3615
FL	1.00	1.00	11243	6704	4500	3675	4140
GA	1.00	1.00	12237	8003	5215	5162	4660
HI	1.00	1.01	5561	2282	4050	1651	1413
IA	1.00	1.00	8621	5487	4735	6142	4927
ID	1.00	1.01	7361	2570	3839	2825	1936
IL	1.00	1.00	7967	5767	4284	3827	4759
IN	1.00	1.00	12961	6592	4090	3260	3695
KS	1.00	1.01	6632	3945	4174	2767	2275
KY	1.00	1.01	6396	5015	5625	3293	1806
LA	1.00	1.00	10279	6107	5381	5392	5127
MA	1.00	1.01	6802	4987	3786	2800	3655
MD	1.00	1.01	4893	3521	4683	3089	1567
ME	1.00	1.01	4469	2456	3911	3142	986
MI	1.00	1.01	10131	6664	4652	3184	4300
MN	1.00	1.00	6935	5007	4723	5470	5545
MO	1.00	1.00	11317	7161	4173	3429	4952
MS	1.00	1.00	7986	5546	4608	4682	5552
MT	1.00	1.01	6331	3889	5463	4136	1003
NC	1.00	1.00	12466	7904	4876	5700	4334
ND	1.00	1.01	6832	3643	5662	3968	1605
NE	1.00	1.02	5004	2460	5278	3567	1170
NFL	1.00	1.00	13948	9185	11446	5404	10783
NH	1.00	1.01	6573	2556	4850	3253	1338
NJ	1.00	1.00	8924	7308	4446	6428	5786
NM	1.00	1.02	5047	2971	3359	1235	713
NV	1.00	1.01	6356	3330	5298	4098	1126
NY	1.00	1.01	8957	6528	4834	4649	2939
OH	1.00	1.00	11147	7157	4266	3740	4444
OK	1.00	1.00	8430	6473	4499	6056	5391
OR	1.00	1.01	5077	3706	4201	2586	1444
PA	1.00	1.01	9493	6421	4766	4010	3340
RI	1.00	1.01	6778	1665	6143	3644	2331
SC	1.00	1.01	5689	4400	4729	2947	2520
SD	1.00	1.01	6989	2929	4426	2821	1106
TN	1.00	1.00	13272	9891	5095	5730	4583
TX	1.00	1.01	8695	5901	3388	1747	3723
UT	1.00	1.01	6149	4077	3470	1199	1426
VA	1.00	1.00	9841	5726	4095	5583	4084
VT	1.00	1.00	7691	4758	4837	4315	2927
WA	1.00	1.01	6262	4508	4518	4802	4143
WI	1.00	1.00	12594	7848	4706	4080	4132
WV	1.00	1.01	6015	2630	4373	2711	1937
WY	1.00	1.01	6813	3389	4961	3772	1179

Tab. S4: Model diagnostics for Model 3. Minimum and maximum \hat{R} and effective sample size (ESS) for parameters of interest are shown. For parameters which are vectors in each league (such as θ_k and γ_{kt}), mean ESS are shown.

Model	$\min(\hat{R})$	$\max(\hat{R})$	β_{0k}	β_{1k}	β_1^*	λ_{0k}	λ_1	σ_k	ζ_k	θ_k
States < 10,000 Games	1.00	1.02	1987	1384	1050	4564	313	3069	2005	1079
All 50 States	1.00	1.02	4240	2319	1332	7134	115	5074	3257	2847

Tab. S5: Model diagnostics for hierarchical models. Minimum and maximum \hat{R} and effective sample size (ESS) for parameters of interest are shown. For parameters which are vectors in each model, mean ESS are shown.



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