On Firing Signals

Hugo Aguettaz

Institut für Signal- und Informationsverarbeitung
ETH Zürich

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Outline

Section 1

Introduction

Firing sequences

Definition (Firing sequence)

A binary sequence of length L is a firing sequence if and only if there at least T_r 0s between any two consecutive 1s, and considering periodic boundaries.

We denote by $\mathcal{F}$ the set of all firing sequences of length L and refractory period T_r (with implicit L and T_r).

Example

With $L = 10$ and $T_r = 2$, $(0, 0, 1, 0, 0, 0, 0, 1, 0, 0)$ is a firing signal but $(0, 0, 1, 0, 1, 0, 0, 0, 1, 0)$ and $(1, 0, 0, 1, 0, 0, 0, 0, 1, 0)$ are not.

Firing sequences

Cardinality

Theorem (Cardinality)

Assume $T_r > 0$ and let $\ell^L_{T_r}$ denotes the number of firing signals of length L and refractory period T_r. Then:

$$\ell^L_{T_r} = \begin{cases} 1 & 1 \leq L \leq T_r \\ L+1 & T_r + 1 \leq L \leq 2T_r \\ \sum_{i=1}^{T_r+1} \frac{\varphi^{T_r+1+L}_{T_r+1,i}}{\varphi^{T_r+1}_{T_r+1,i} + T_r} - 2T_r^2 + LT_r & 2T_r + 1 \leq L \leq 3T_r + 1 \\ \sum_{i=1}^{T_r+1} \varphi^{L}_{T_r+1,i} & L > 3T_r + 1 \end{cases},$$

where $\varphi_{p,i}, i = 1, \ldots, p$ denotes the roots of the polynomials $x^p - x^{p-1} - 1$ with $p \geq 2$.

Firing sequences
Reasonable

Definition (Reasonable firing sequence)

A firing sequence $(x_1, \ldots, x_L)$ is said to be T_h-*reasonable* if for any $i \in \{1, \ldots, L\}$, x_i is fully determined by $(x_{i-T_h}, \ldots, x_{i-1})$.

Example

With $T_h = 4$ $(0,0,0,0,0,0,0,0,0,0)$ and $(1,0,0,0,0,1,0,0,0,0)$ are T_h-reasonable but $(0,1,0,0,0,1,0,0,1,0)$ is not.

Firing sequences

Autonomous (energy)

Definition (Autonomous firing sequence)

A firing sequence $(x_1, \ldots, x_L)$ is said to be T_h-*autonomous* if for any $1 \leq i \leq L$ such that $x_i = \cdots = x_{i+T_h} = 0$, $x_{i+T_h+1} = 0$.

Example

With $T_h = 4$, $(0,0,0,0,0,0,0,0,0,0)$ and $(0,1,0,0,0,1,0,0,1,0)$ are T_h-autonomous but $(1,0,0,0,0,1,0,0,0,0)$ is not.

Firing sequences
Reducible

Definition (Reducible firing sequence)

A firing sequence $(x_1, \ldots, x_L)$ is said to be *reducible* (with order d) if for any $1 \leq i \leq L$ $x_{i+kT} = x_i$ with $T = L/d$ and d as large as possible.

Example

$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ and $(1, 0, 0, 0, 0, 1, 0, 0, 0, 0)$ are reducible with orders $d = 6$ and $d = 2$ respectively.