Neuron Model

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Neuron process unit:

$$z_l(t) = \sum_{m=1}^{M} w_m \cdot (y_m * h_{\tau_m})(t), \tag{1}$$

with $h_{\tau_m}(t) \triangleq h(t - \tau_m)$.

Spiking neurons:

$$y_m(t) = \sum_{t_f \in T_m} \delta(t - t_f) \tag{2}$$

So Equation (1) is equivalent to

$$z_l(t) = \sum_{m=1}^{M} w_m \cdot \sum_{t_f \in T_m} h_{t_f + \tau_m}(t) = \sum_{m=1}^{M} w_m \cdot \tilde{h}_{\tau_m}(t) \tag{3}$$

with $\tilde{h}_{\tau_m}(t) \triangleq \sum_{t_f \in T_m} h_{t_f+\tau_m}(t)$.

Now, we assume the model is in continuous time but firing occurs on a grid with resolution T_g:

$$T_m \subset \{kT_g : k \in \mathbb{Z}_n\}. \tag{4}$$

Without loss of generality, we assume $T_g = 1$.

For a fixed sequence to learn and fixed synaptic delays, $\tilde{h}_{\tau_{l,m}}$ is fixed for every neuron $l = 1, \ldots, L$ and every synapse $m = 1, \ldots, M_l$. Moreover, z_l is also fixed.

We define $\tilde{h}_l = \left(\tilde{h}_{\tau_{l,1}}, \ldots, \tilde{h}_{\tau_{l,M}}\right)$

So, for each neuron the weights $w_l \in \mathbb{R}^M$ can be learned by minimizing the cost

$$\ell_f(w_l) = \frac{1}{n} \sum_{k=1}^{n} \left| \left\langle w_l, \tilde{h}_l[k] \right\rangle - \theta_l \right| + \lim_{b \to \infty} \left| \left\langle w_l, \tilde{h}_l[k] \right\rangle + (-1)^{y[k]} \cdot b \right| - \lim_{b \to \infty} \left| \theta_l + (-1)^{y[k]} \cdot b \right| \tag{5}$$

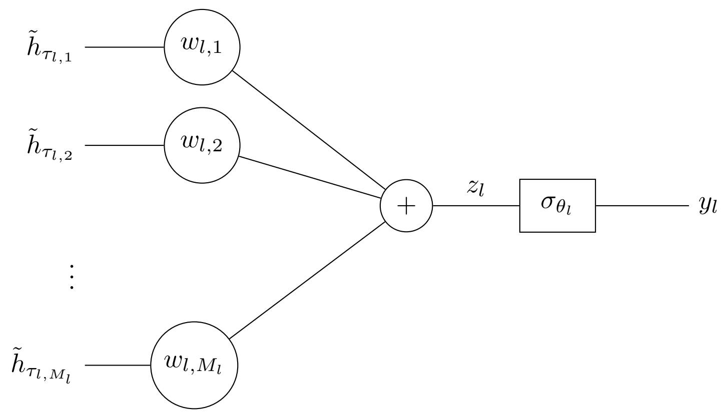

Figure 1: Spiking neuron model

by a gradient descent learning rule:

$$w_l \leftarrow w_l - \eta \cdot \nabla_{w_l} \ell_f(w_l), \tag{6}$$

with

$$\frac{\partial \ell_f(w_l)}{\partial w_{l,m}} = \frac{1}{n} \sum_{k=1}^{n} \left(\operatorname{sgn} \left(\left\langle w_l, \tilde{h_l}[k] \right\rangle - \theta_l \right) + (-1)^{y[k]} \right) \cdot \tilde{h}_{l,\tau_m}[k] \tag{7}$$

To enforce the weights being in the range $[a, b]$, we use the loss function

$$\ell_b(w_l) = \frac{1}{n} \sum_{m=1}^{M} |w_i - a| + |w_i - b| - |b - a| \tag{8}$$

with partial derivatives

$$\frac{\partial \ell_b(w_l)}{\partial w_{l,m}} = \operatorname{sgn}(w_i - a) + \operatorname{sgn}(w_i - b) \tag{9}$$

The new gradient descent update rule becomes

$$w_l \leftarrow w_l - \eta \cdot \nabla_{w_l} \ell(w_l), \tag{10}$$

with $\ell(w_l) = \ell_f(w_l) + \alpha \cdot \ell_b(w_l)$, for some $\alpha \geq 0$.