VE203 Discrete Mathematics
Spring 2022 — CCP7

January 17, 2022

7.1 Twelvefold Way

Distribute k balls into n urns. $(f : B \to U, |B| = k, |U| = n)$

Balls (domain)	Urns (codomain)	unrestricted (any function)	≤ 1 (injective)	≥ 1 (surjective)
labeled	labeled	n^k		
unlabeled	labeled			
labeled	unlabeled			
unlabeled	unlabeled			

7.2 What does $\left\{ \begin{array}{c} n \\ k \end{array} \right\}$ and $p_n(k)$ mean? How to calculate them?

7.3 Why is $\left(\begin{array}{c} n \\ k \end{array} \right)$ an integer?

7.4 What is Catalan Number, can you find out its practical meaning?

7.5 Prove that $\left(\left(\begin{array}{c} n \\ k \end{array} \right) \right) = \left(\begin{array}{c} n + k - 1 \\ k \end{array} \right) = \left(\begin{array}{c} n + k - 1 \\ n - 1 \end{array} \right)$

7.6 Consider
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 203$$
What are the number of integer solutions if
1. $x_i > 0$ and $=$ holds;
2. $x_i \geq 0$ and $=$ holds;
3. $x_i > 0$ and $<$ holds;
4. $x_i \geq 0$ and $<$ holds;
5. $x_i \geq 0$.

7.7 State Inclusion-Exclusion Principle.

7.8 Theorem.
$$\sum_{i=0}^{n} \left(\begin{array}{c} k + i \\ k \end{array} \right) = \left(\begin{array}{c} k + n + 1 \\ k + 1 \end{array} \right)$$
We can also prove this theorem using induction. Give such a proof.