

Concept Checking Paper 04

1. What is “math” based on all the math you have learned in your life?
2. If $d \in \mathbb{Z}$ and $d \mid 2021$, then $d =$ _____.
3. Prove the necessary condition for **Mersenne Prime**, i.e. $2^n - 1 \in P \Rightarrow n \in P$.
4. Special cases for **Dirichlet’s Theorem**:
 - (i) Prove that there are infinitely many primes of the form $4m + 1$, $m \in \mathbb{N}$.
 - (ii) Prove that there are infinitely many primes of the form $4m - 1$, $m \in \mathbb{N}$.
5. Prove the **Division Algorithm**: Given $m, n \in \mathbb{N} \setminus \{0\}$, there exist unique integers q and r with $q \geq 0$ and $0 \leq r < m$ so that $n = qm + r$.
 - (i) Existence:
 - (ii) Uniqueness:
6. Let p be a prime, $m, n \in \mathbb{N} \setminus \{0\}$. Prove that:
 - (i) If $p \mid m$ and $p \mid n$, then $\gcd(m, n) = p \gcd(\frac{m}{p}, \frac{n}{p})$
 - (ii) If $p \mid m$ and $p \nmid n$, then $\gcd(m, n) = \gcd(\frac{m}{p}, n)$



7. Let F_n be Fermat numbers, i.e. $F_n = 2^{2^n} + 1$. Prove that they are pairwise coprime, namely $\gcd(F_n, F_m) = 1$.

8. Use the **Euclidean Algorithm** to find a integer pair (x, y) that $111x - 321y = 75$.

