Concept Checking Paper 06

1. What is “math” based on all the math you have learned in your life?

2. Given two groups $G, G^{'}$, a homomorphism is a map $f{:}G \to G'$, such that: _____.

3. Prove that a homomorphism $f{:}G \to G^{'}$ is injective iff $\ker f = \{1_G\}$.

4. Calculate the left coset of $\langle(13)\rangle$ in S_3, then explain what is $[S_3{:}\langle(13)\rangle]$.

5. Verify Lagrange’s Theorem for the subgroup $H = \{0,3\}$ of $\mathbb{Z}_6$.

6. Check the following statements are true or false:
   i) $A_n \trianglelefteq S_n$
   ii) Every subgroup of an abelian group is normal.
   iii) An abelian group may have a non-abelian subgroup.
   iv) A non-abelian group may have an abelian subgroup.
   v) A non-abelian group may have a non-abelian subgroup.
   vi) Any subgroup of index 2 in a group is a normal subgroup.
   vii) The center of a group is always a normal subgroup.

7. State Fermat's (Little) Theorem and Euler's Theorem.

8. State two methods to find the Arithmetic inverse, e.g. solve $7x \equiv 1 \pmod{31}$.

Note: Since most of the content regarding **Chinese Remainder Theorem** and **RSA** would be covered in homework, I won't cover them here again.