VE203 Discrete Mathematics
Spring 2022 — CCP8

January 18, 2022

8.1 A general linear recurrence has the form:

$$f(n) = b_1 f(n-1) + b_2 f(n-2) + \ldots + b_d f(n-d) + g(n)$$

how to get the particular solution if $g(n)$ is a constant? a polynomial? an exponential?

8.2 You may assume that if $f(n) \geq 1$ and $g(n) \geq 1$ for all n, then $f \sim g \Rightarrow f^{\frac{1}{n}} \sim g^{\frac{1}{n}}$. Show that

$$\sqrt[n]{n!} = \Theta(n)$$

8.3 Define three asymptotic notations.

8.4 Which of these symbols

$$\Theta \quad O \quad \Omega \quad o \quad \omega$$

can go in these boxes? (List all that apply.)

$$\begin{array}{rcl} 2n + \log n & = & (n) \\ & & \Theta, O, \Omega \\ \log n & = & (n) \\ & & O, o \\ \sqrt{n} & = & (\log^{300} n) \\ & & \Omega, \omega \\ n2^n & = & (n) \\ & & \Omega, \omega \\ n^7 & = & (1.01^n) \\ & & O, o \end{array}$$

8.5 Write down the master theorem.

8.6

1. Solve $T(n) = 4T(n/2) + n$
2. Solve $T(n) = 4T(n/2) + n^2$
3. Solve $T(n) = 4T(n/2) + n^3$
4. Solve $T(n) = 4T(n/2) + n^2/\lg n$
5. Solve $T(n) = T(n/4) + T(n/2) + n^2$