Concept Checking Paper 05

1. What is “math” based on all the math you have learned in your life?

2. List the four properties of the generalized product in a **group**: _____, _____, _____, _____. To become an **abelian group**, we need another property: _____.

3. Given a group G and its two distinct subgroups H_1 and H_2. Check whether the following sentences are true or false:
   i) The identity element in G and H must be the same.
   ii) $H_1 \cup H_2$ is a group.
   iii) $H_1 \cap H_2$ cannot be empty and it is a group.
   iv) A subset in G that is not a subgroup may be a group.

*Comment. Compare to the concept of **vector space**. If necessary, take vv186.*

4. State the Bézout Identity.

5. Give two additive group $S_1 = aZ, S_2 = bZ, a,b \in \mathbb{Z}^+$.
   i) Find $S_1 \cap S_2$ and $S_1 + S_2$.

   ii) Are they $(S_1,S_2,S_1 + S_2,S_1 \cap S_2)$ cyclic?

   iii) Find the identity element, order and generator of S_1.

6. Let $\langle x \rangle$ be a cyclic of order $p \in P$. Find all of its subgroups.

7. Calculate $\varphi(p_1^{k_1}p_2^{k_2})$, where $p_1,p_2 \in \mathbb{P}, k_1,k_2 \in \mathbb{Z}^+$.

8. For a symmetric group $S_3 = \{e,\tau,\tau',\tau'',\sigma,\sigma'\}$. Explain what does the following means:

i) $e = ()$
ii) $\tau = (12)$
iii) $\sigma = (123)$
iv) $\tau\sigma(1)$=1

9. Calculate (1423) (134). Is it an even permutation or odd permutation?