Concept Checking Paper

The idea of *Concept Checking Paper* (CCP) comes from an extraordinary math TA Leyang Zhang. He created CCP in the Honors Mathematics courses (Vv186 - Vv285 - Vv286) to help student to solidify the new concepts/knowledge learned in the course. All the questions are purely about concepts. Students can do them right after the class each week to see what they should focus in the RC class, or as review material before exams. Generally, the content in CCP covers more material than will be tested in exams, so it should be able to help you prepare for exams, or simply have a sound understanding of the course.

Here I imitate his idea and create CCP for Discrete Mathematics. I also hope this would be helpful for your learning and reviewing. I won't provide answers for CCP because I want you to find the answer on slides, and fill in the blanks with your own understandings. If you are familiar with what you learn, you should be able to fill in each concept checking paper in 20 minutes.

Concept Checking Paper 01

1. What is "math" based on all the math you have learned in your life?

2. Write down the interpretation of the following set operations in the table, along with their notations (list all if possible).

Operations	Notation	Interpretation
cardinality	$\|A\|, \#A, cardA$	The size of a set A
superset		
proper subset		
set difference		
symmetric difference		
Cartesian product		
Power Set		

3. Prove the following properties:
   i) $C - (A \cup B) = (C - A) \cap (C - B)$ (De Morgan's Laws)
   ii) Let $A, B \subset M$, then $(M - A)\,\Delta\,(M - B) = A\,\Delta\,B$

4. **Logical Operator**

Please write down the logical operator for negation, conjunction (and), disjunction (or), implies, iff.

5. Given $\varphi = A \to (B \to C)$. Express it in DNF and CNF, try to explain your method.

6. Briefly explain the following concepts and give examples

(i) contraposition

(ii) tautology

(iii) vacuous truth

7. Use **structural induction** to prove that for any logical proposition φ using the connectives $\{\neg, \wedge, \vee, \to, \leftrightarrow\}$, there exists a proposition using only $\{\neg, \vee\}$ that is logically equivalent to φ.

8. Write down the recursive definition of **Linked List** and **Binary Tree**. Try to implement them in C++ language. You may refer to Ve280 Project 2.