A. Notations

A.1 Notation of data and variables

The notation of data and variables is now quite well normalised. We present in tables A.1 and A.2 the set of notations used throughout the book for the data and the variables respectively.

A.2 Usual notation of single criterion scheduling problems

Two notations exist to refer to scheduling problems. The older has been proposed by [Conway et al., 1967]. But as it is not the more used in the literature we present the one introduced by [Graham et al., 1979] and later detailled by [Blazewicz et al., 1996]. The notation is decomposed into three fields: $\alpha|\beta|\gamma$.
The field α directly refers to the typology presented in figure 1.1 and presents the structure of the scheduling problem (table A.3).
The field β contains the set of constraints of the problem (table A.4).
At last, in the field γ we put the criteria to optimise (table A.5). Concerning a more detailled presentation of the different classical criteria in scheduling, the reader is referred to [Rinnooy Kan, 1976].

Table A.1. Notation of data

Data of problems	
Notation	Meaning
n	number of jobs.
m	number of machines.
J_i	job number i, $i = 1, ..., n$.
n_i	number of operations of job J_i, we often have $n_i = m, \forall i$, $i = 1, ..., n$.
$m^{(\ell)}$ (or $M^{(\ell)}$)	number of machines at stage ℓ.
M_j	machine number j, $j = 1, ..., m$.
$O_{i,j}$	operation j of job J_i.
r_i ($r_{i,j}$)	release time of job J_i (respectively of operation $O_{i,j}$).
s_i ($s_{i,j}$)	desired start time of job J_i (respectively of operation $O_{i,j}$).
$p_{i,j}$ (or p_i^j)	processing time of operation $O_{i,j}$. When there is only one operation per job we use the notation p_i.
$\underline{p}_{i,j}$ ($\overline{p}_{i,j}$)	minimum processing time (respectively maximum) of operation $O_{i,j}$. When there is only one operation per job we use $\underline{p}_i$ (respectively $\overline{p}_i$). This data is generaly used in problems in which the processing times are variables to determine.
d_i ($d_{i,j}$)	due date of job J_i (respectively of operation $O_{i,j}$)
$\widetilde{d}_i$ ($\widetilde{d}_{i,j}$)	deadline of job J_i (respectively of operation $O_{i,j}$). The job J_i (resp. the operation $O_{i,j}$) cannot complete after this date.
w_i ($w_{i,j}$)	weight associated to job J_i (respectively to operation $O_{i,j}$).
k_j	production rate associated to machine M_j. This data is generaly used in uniform parallel machines scheduling problems.
$k_{i,j}$	production rate associated to the processing of job J_i on machine M_j. This data is generally used in unrelated parallel machines scheduling problems.
$S_{i,j}$	non sequence dependent setup time required before the processing of operation $O_{i,j}$.
$R_{i,j}$	non sequence dependent removal time required after the processing of operation $O_{i,j}$.

Table A.2. Notation of variables

Variables of problems	
Notation	Meaning
$t_{i,j}$	start time of operation $O_{i,j}$. When there is only one operation per job, we use the notation t_i.
$C_{i,j}$	completion time of operation $O_{i,j}$.
C_i	completion time of job J_i. $C_i = \max_{j=1,\ldots,n_i}(C_{i,j})$.
T_i	tardiness of job J_i. We have $T_i = \max(0; C_i - d_i)$.
E_i	earliness of job J_i. We have $E_i = \max(0; d_i - C_i)$.
L_i	lateness of job J_i. We have $L_i = C_i - d_i$.
U_i	is equal to 1 if $C_i > d_i$ and 0 otherwise.

Table A.3. The field α

Field $\alpha = \alpha_1\alpha_2$			
sub-field α_1		sub-field α_2	
Value	Meaning	Value	Meaning
$\emptyset$	single machine.	$\emptyset$	the number of machines or pools is not fixed.
P, Q, R	identical, proportionnal or unrelated parallel machines.	$1, 2, 3$, *etc.*	the number of machines or stages is fixed and equal to $1, 2, 3$, *etc.*
F, J, O, X	flowshop, jobshop, openshop, mixed shop.	m	the number of machines or stages is unknown but fixed.
HF	hybrid flowshop.		
GO	general openshop.		
GJ	general jobshop.		
$\{P, Q, R\}MPM$	parallel machines (of type P or Q or R) with a general assignment problem.		
$GMPM$	shop problem with a general assignment problem.		
$OMPM$	openshop problem with a general assignment problem.		

Table A.4. The field β - (1)

Field β			
Value	Meaning	Value	Meaning
prec	there is general precedence constraints between operations.	*tree*	there is precedence constraints, which forms a tree, between operations.
chains	there is precedence constraints, which form a set of chains, between operations.	$in-tree$	there is precedence constraints, which forms an in-tree, between operations.
$out-tree$	there is precedence constraints, which forms an out-tree, between operations.	$sp-graph$	There is precedence constraints, which forms a serie-parallel graph, between operations.
r_i	jobs have distinct realease times.	s_i	jobs have desired start time.
$p_{i,j} = p$	jobs have a common processing time.	$p_i \in [\underline{p}_i;\overline{p}_i]$	the processing times of jobs are variables to determine and belong to the interval $[\underline{p}_i;\overline{p}_i]$.
d_i	jobs have a due date.	$d_i = d$	jobs have a common due date.
d_i *unknown*	jobs have a due date which is to be determined.	$d_i = d$ *unknown*	jobs have a common due date which is to be determined.
$\tilde{d}_i$	jobs have a deadline.	a_{j_1,j_2}	there is a minimum time lag to satisfy between the last operation of job J_{j_1} and the first operation of job J_{j_2}.
split	the splitting of an operation into parts is allowed and several parts can be processed simultaneously.	*over*	the overlapping of two consecutive operations of a job is allowed.
pmtn	the operations can be interrupted and resumed later on any machine.	$no-wait$	for each job, when an operation completes the next one must start.
			to follow

Table A.5. The field β - (2)

Field β *(second part)*			
Value	Meaning	Value	Meaning
block	the shop has storage areas, with a limited capacity, between the machines, which may leads an operation to be stored on a machine.	*recrc*	a job can be processed several times by the same machine.
batch	the operations can be gathered into batches during their processing.	$s - batch$	the operations can be gathered into batches and are processed in series in each batch.
$p - batch$	the operations can be gathered into batches and are processed in parallel in each batch.	*permu* or *prmu*	we consider the set of permutation schedules (only available for flowshop scheduling problems).
blcg	the machines must complete their processing at the same time.	$unavail_j$	machine M_j can have unavailability periods, known in advance.
R_{sd} (R_{nsd})	there is a removal time after the processing of an operation. This one depends (respectively does not depend) on the sequence of operations on each machine.	S_{sd} (S_{nsd})	there is a setup time before the processing of an operation. This time depends (respectively does not depend) on the sequence of operations on each machine.
nmit	when a machine has started its processing, no idle time between operations is allowed.	$no - idle$	on each machine the processing of the operations is performed without idle time.

Table A.5. The field γ

Field γ		
Criterion	Expression	Meaning
C_{max}	$\max_{i=1,\ldots,n}(C_i)$	Makespan, or maximum completion time.
T_{max}	$\max_{i=1,\ldots,n}\big(\max(C_i - d_i; 0)\big)$	Maximum tardiness of jobs.
L_{max}	$\max_{i=1,\ldots,n}(C_i - d_i)$	Maximum lateness of jobs.
E_{max}	$\max_{i=1,\ldots,n}\big(\max(d_i - C_i; 0)\big)$	Maximum earliness of jobs.
F_{max}	$\max_{i=1,\ldots,n}(C_i - r_i)$	Maximum flow time of jobs.
P_{max}	$\max_{i=1,\ldots,n}\big(\max(s_i - t_i; 0)\big)$	Maximum promptness of jobs.
f_{max}	$\max_{i=1,\ldots,n}(f_i)$	Generic maximum cost function. Generaly, it is assumed to be an increasing function of the completion times of jobs.
$\overline{C}$ ($\overline{C}^w$)	$\sum_{i=1}^{n} C_i$ ($\sum_{i=1}^{n} w_i C_i$)	(Weighted) Average completion time of jobs, or (weighted) average work-in-process.
$\overline{T}$ ($\overline{T}^w$)	$\sum_{i=1}^{n} T_i$ ($\sum_{i=1}^{n} w_i T_i$)	(Weighted) Average tardiness of jobs.
$\overline{U}$ ($\overline{U}^w$)	$\sum_{i=1}^{n} U_i$ ($\sum_{i=1}^{n} w_i U_i$)	(Weighted) Number of late jobs.
$\overline{E}$ ($\overline{E}^w$)	$\sum_{i=1}^{n} E_i$ ($\sum_{i=1}^{n} w_i E_i$)	(Weighted) Average earliness of jobs.
–		No criterion, it is a feasibility problem.