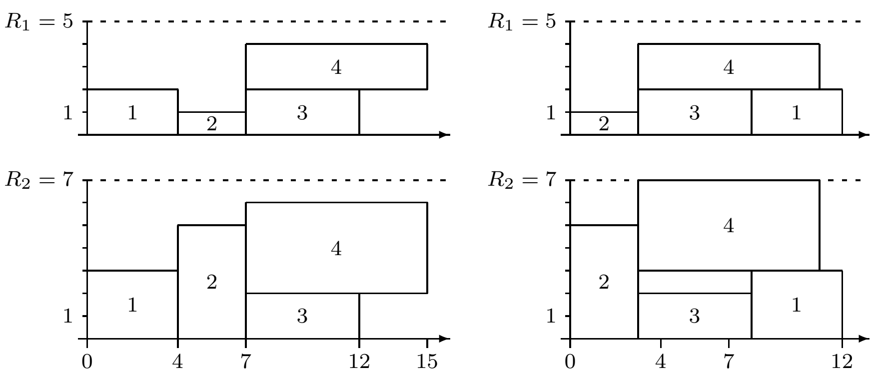
of the project, respectively. The dummy activities need no resources and have processing time 0. In order to impose $0 \to j \to n+1$ for all activities $j = 1, \dots, n$ we set $0 \to j$ for all activities $j$ without any predecessor and $j \to n+1$ for all activities $j$ without any successor. Then $S_0$ is the starting time of the project and $S_{n+1} - S_0$ may be interpreted as the makespan of the project. Usually we set $S_0 := 0$.

If preemption is not allowed, the vector $S = (S_j)$ defines a **schedule** of the project. $S$ is called **feasible** if all resource and precedence constraints are fulfilled.

**Example 1** *Consider a project with $n = 4$ activities, $r = 2$ resources with capacities $R_1 = 5, R_2 = 7$, a precedence relation $2 \to 3$ and the following data:*

| $j$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $p_j$ | 4 | 3 | 5 | 8 |
| $r_{j1}$ | 2 | 1 | 2 | 2 |
| $r_{j2}$ | 3 | 5 | 2 | 4 |




(a) A feasible schedule. (b) An optimal schedule.

FIGURE 1.1: Two feasible schedules for Example 1.

*In Figure 1.1(a) a so-called **Gantt chart** of a feasible schedule with $C_{\max} = 15$ is drawn. This schedule does not minimize the makespan, since by moving activity 1 to the right, a shorter schedule is obtained. An optimal schedule with makespan $C_{\max} = 12$ is shown in (b).*

A precedence relation $i \to j$ with the meaning $S_i + p_i \le S_j$ may be generalized by a start-start relation of the form

$$S_i + d_{ij} \le S_j \tag{1.1}$$