

WS #10 - Bootstrap t

Monday, October 6, 2025

Your Name: _____

Names of people you worked with: _____

Name the people sitting one table over from you. Tell your partner one fantastic thing from your weekend.

Task:

Put in the back of your head the distribution of: $\frac{\bar{X}-\mu}{s/\sqrt{n}}$ (which, incidentally, we know is distributed according to t_{n-1} if $X_i \stackrel{iid}{\sim} N(\mu, \sigma^2)$.)

Additionally, let

$$\begin{aligned}\hat{\theta}_b^* &= \text{estimate of } \theta \text{ from the } b^{th} \text{ resample} \\ \hat{SE}^* &= \left[\frac{1}{B-1} \sum_{b=1}^B (\hat{\theta}_b^* - \hat{\theta}^*)^2 \right]^{1/2}\end{aligned}$$

1. If you sample B times from a population, how many copies of \bar{X} will there be? How many copies of s/\sqrt{n} will there be?
2. If you re-sample B times from a single dataset, how many copies of $\hat{\theta}_b^*$ will there be? How many copies of \hat{SE}^* ?
3. Gosset realized that s varies from sample to sample. In bootstrapping, we want to mimic the process of sampling from a population. What is the problem with using the bootstrap values given above to produce a bootstrapped test statistic?
4. To address the problem, suggest a way of estimating the SE of $\hat{\theta}$ separately for each b .

Solution:

1. When sampling from a population, there will be B copies each of \bar{X} and s/\sqrt{n} .
2. When re-sampling from a dataset, there will be B copies of $\hat{\theta}_b^*$ and 1 copy of \hat{SE}^* .
3. Somehow we need to create a test statistic where both the numerator and the denominator are random variables.
4. To find $\hat{SE}^*(b)$, we must bootstrap twice. The algorithm is as follows:
 - a. Generate B_1 bootstrap samples (resamples from the original data), and for each sample \underline{X}^{*b} compute the bootstrap estimate $\hat{\theta}_b^*$.
 - b. Take B_2 bootstrap samples (resamples from the bootstrapped data) from \underline{X}^{*b} , and estimate the standard error, $\hat{SE}^*(b)$.
 - c. The resulting distribution will be based on B_1 values for $T^*(b) = \frac{\hat{\theta}_b^* - \hat{\theta}}{\hat{SE}^*(b)}$.