PS8 From Language to Logic

Problem 4: Logical Inference

a.

Given the KB:

$$\mathrm{KB} = \{(A \vee B) \to C, A\}$$

Derive C.

$$\begin{aligned}(A \vee B) \to C &= \neg(A \vee B) \vee C \\ &= \neg A \wedge \neg B \vee C \\ &= \neg A \vee C \wedge \neg B \\ &= A \to C \wedge \neg B\end{aligned}$$

The converted CNF form KB:

$$\mathrm{KB} = \{A \to C \wedge \neg B, A\}$$

Since $A \to C \wedge \neg B \Rightarrow A \to C$, together with A, and apply Modus ponens, we can derive C from the KB.

b.

Given the KB, convert the knowledge base into CNF and apply the resolution rule repeatedly to derive D.

$$\mathrm{KB} = \{A \vee B, B \to C, (A \vee C) \to D\} \Rightarrow \mathrm{KB} = \{A \vee B, \neg B \vee C, \neg A \wedge \neg C \vee D\}$$

From the first two apply resolution rule: $A \vee B, \neg B \vee C \Rightarrow A \vee C$,
Together with the third and apply modus ponnes: $(A \vee C) \to D, A \vee C \Rightarrow D$

Problem 5: Odd and even integers

b.

Prove that there is no finite, non-empty model for which the resulting set of 7 constraints is consistent.

Suppose there are 2 different values (x,y) in the model, that satisfies all the 6 constraints in previous section.

1. From rule 1, because Each number should have a successor and it is unique, we have:
   Successor(x, y) = Successor(y, x) = True,
   Not(Equals(x,y)) = True

1. From rule 5:
   Larger(Successor(x), x)=True, which can be just written as Larger(y,x)=True;
   Larger(Successor(y), y)=True, which can just be written as Larger(x,y)=True

1. From rule 6 and above 2, we have: Implies(And(Larger(y,x), Larger(x,y)), Larger(y,y))=True, which contradicts with the last rule: Not(Larger(y,y))=True

Suppose there are 3 different values (x,y,z), similarily:

1. From rule 1, because Each number should have a successor and it is unique, we have:
   Successor(x, y) = Successor(y, z) = Successor(z, x) = True,
   Not(Equals(x,y)) = True, Not(Equals(y,z)) = True, Not(Equals(z,x)) = True

1. From rule 5:
   Larger(Successor(x), x)=True, which can be just written as Larger(y,x)=True;
   Larger(Successor(y), y)=True, which can just be written as Larger(z,y)=True
   Larger(Successor(z), z)=True, which can just be written as Larger(x,z)=True

1. From rule 6 and above 2, we have: Implies(AndList(Larger(y,x), Larger(z,y), Larger(x,z)), Larger(x,x))=True,
   which contradicts with the last rule: Not(Larger(x,x))=True

We can use induction to any finite, non-empty model in the similar way as the above 2,3 cases, and find out that finally the result contradicts the last rule. So there is no finite, non-empty model for which the resulting set of 7 constraints is consistent.