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Assignment 2: word2vec

1.a Show that: $-\sum_{w\in\text{Vocab}} y_w \log \hat{y}_w = -\log(\hat{y}_o)$, answer should be in one line.

$\mathbf{Answer}$: The ground truth y_w is a one-hot-vector with only the component w.r.t. the outside word being 1, the only term left is $-\log(\hat{y}_o)$

1.b Compute the partial derivative of $J_{naive-softmax}(v_c, o, U)$ w.r.t. v_c.

Write equations with matrices and vectors:

$$
\begin{aligned}
J_{naive-softmax}(v_c, o, U) &= -\log \hat{y}_o = -y^T \cdot \log(\hat{y}) \\
\hat{y} &= p(\circ|v_c) = \frac{\exp(e)}{\sum \exp(e)} \\
e &= U^T v_c
\end{aligned}
$$

Use chain rule of derivatives:

$$\frac{\partial J}{\partial v_c} = \frac{\partial J}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial e} \frac{\partial e}{\partial v_c}$$

.

For the first part and the last part, apply derivative rules w.r.t. vectors and matrices directly:

$$\frac{\partial J}{\partial \hat{y}} = -y \circ \frac{1}{\hat{y}}$$
$$\frac{\partial e}{\partial v_c} = U$$

For the second part, take the derivative element wise:

1. If $i \neq o$, we have:
$$\frac{\partial \hat{y}_i}{\partial e_o} = -\frac{\exp(e_o)\exp(e_i)}{(\sum_{w=1}^{V}\exp(e_w))^2} = \hat{y}_i\hat{y}_o = \hat{y}_i(y_i - \hat{y}_o)$$
2. If $i = o$, we have:
$$\frac{\partial \hat{y}_o}{\partial e_o} = \frac{\exp(e_o)}{\sum_{w=1}^{V}\exp(e_w)} - \frac{(\exp(e_o))^2}{(\sum_{w=1}^{V}\exp(e_w))^2} = \hat{y}_o(y_o - \hat{y}_o)$$
3. Combine the above 2:
$$\frac{\partial \hat{y}}{\partial e} = (y - \hat{y}) \circ \hat{y}$$

Put all 3 parts together:

$$\frac{\partial J}{\partial v_c} = -\frac{1}{\hat{y}} \circ y \circ \hat{y} \circ (y - \hat{y}) \cdot U$$
$$= (\hat{y} - y) \cdot U$$

Since v_c is a D-dimentional vector, the derivative of J w.r.t. v_c is also a D-dimentional vector.

1.c Compute the partial derivative of $J_{naive-softmax}(v_c, o, U)$ w.r.t. u_w

Use the chain rule of derivatives again:

$$\frac{\partial J}{\partial U} = \frac{\partial J}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial e} \frac{\partial e}{\partial U}$$

The chain rule of derivatives will have two terms in common compared to 1.b:

$$\frac{\partial J}{\partial \hat{y}} = -y \circ \frac{1}{\hat{y}}$$
$$\frac{\partial \hat{y}}{\partial e} = (y - \hat{y}) \circ \hat{y}$$

The first two terms are the same, the third term is:

$$\frac{\partial e}{\partial U} = \frac{\partial (U^T v_c)}{\partial U} = v_c$$

Combine the above 2 cases and consider the result is a DxD matrix, we have:

$$\frac{\partial J}{\partial U} = -\frac{1}{\hat{y}} \circ y \circ \hat{y} \circ (y - \hat{y}) \otimes v_c$$
$$\frac{\partial J}{\partial U} = (\hat{y} - y) \otimes v_c$$

Here $\otimes$ denotes outer product.

1.d Derivative of sigmoid function

Given:

$$\begin{aligned}\sigma(x) &= \frac{1}{1+\exp(-x)} \\ &= \frac{\exp(x)}{1+\exp(x)} \\ &= \exp(x)\frac{1}{1+\exp(x)}\end{aligned}$$

The derivative w.r.t. x is:

$$\begin{aligned}\sigma'(x) &= (\exp(x))'\frac{1}{1+\exp(x)} + \exp(x)(\frac{1}{1+\exp(x)})' \\ &= \frac{\exp(x)}{1+\exp(x)} + \exp(x)\frac{\exp(x)}{(1+\exp(x))^2} \\ &= \frac{\exp(x)}{1+\exp(x)}(1-\frac{\exp(x)}{1+\exp(x)}) \\ &= \sigma(x)(1-\sigma(x))\end{aligned}$$

1.e Gradient w.r.t center/output word vectors when using negative sampling loss

$$J_{neg-sample}(v_c, o, U) = -\log(\sigma(u_o^T v_c)) - \sum_{k=1}^{K}\log(\sigma(-u_k^T v_c))$$

$$\sigma(u_o^T v_c) = \frac{1}{1+\exp(-u_o^T v_c)}$$

1. Derivative w.r.t. v_c will contain two terms summed together for the expected and negative each

$$\begin{aligned}\frac{\partial J}{\partial v_c} &= \frac{\partial J}{\partial(-log(\sigma(u_o^T v_c)))}\frac{\partial(-log(\sigma(u_o^T v_c)))}{\partial u_o^T v_c}\frac{\partial u_o^T v_c}{\partial v_c} \\ &+ \frac{\partial J}{\partial(-\sum_{k=1}^{K} log(\sigma(-u_k^T v_c)))}\frac{\partial(-\sum_{k=1}^{K} log(\sigma(-u_k^T v_c)))}{\partial u_o^T v_c}\frac{\partial u_o^T v_c}{\partial v_c} \\ &= -\frac{1}{\sigma(u_o^T v_c)}\sigma(u_o^T v_c)(1-\sigma(u_o^T v_c))u_o \\ &+ \sum_{k=1}^{K}(\frac{1}{\sigma(-u_k^T v_c)}\sigma(-u_k^T v_c)(1-\sigma(-u_k^T v_c))u_k) \\ &= -(1-\sigma(u_o^T v_c))u_o + \sum_{k=1}^{K}(1-\sigma(-u_k^T v_c))u_k\end{aligned}$$

1. Derivative w.r.t. u_o contains only one term because $o \notin 1, 2, \ldots, K$

$$\begin{aligned}\frac{\partial J}{\partial u_o} &= -\frac{1}{\sigma(u_o^T v_c)}\sigma(u_o^T v_c)(1-\sigma(u_o^T v_c))v_c \\ &= -(1-\sigma(u_o^T v_c))v_c\end{aligned}$$

1. Derivative w.r.t. u_k also contains only one term because $k \in 1, 2, \ldots, K$ and $k \neq o$

$$\begin{aligned}\frac{\partial J}{\partial u_k} &= \frac{1}{\sigma(-u_k^T v_c)}\sigma(-u_k^T v_c)(1-\sigma(-u_k^T v_c))v_c \\ &= (1-\sigma(-u_k^T v_c))v_c\end{aligned}$$

Negative sampling is much more effective in that it needs no softmax computation, which requires Vocab vector multiplications while negative sampling only needs K+1 vector multiplication.

1.f Skip-gram loss gradients

1. Gradient w.r.t. U:

$$\frac{\partial J_{skip-gram}}{\partial U} = \sum_{-m \leq j \leq m, j \neq 0} \frac{\partial J(v_c, w_{t+j}, U)}{\partial U}$$

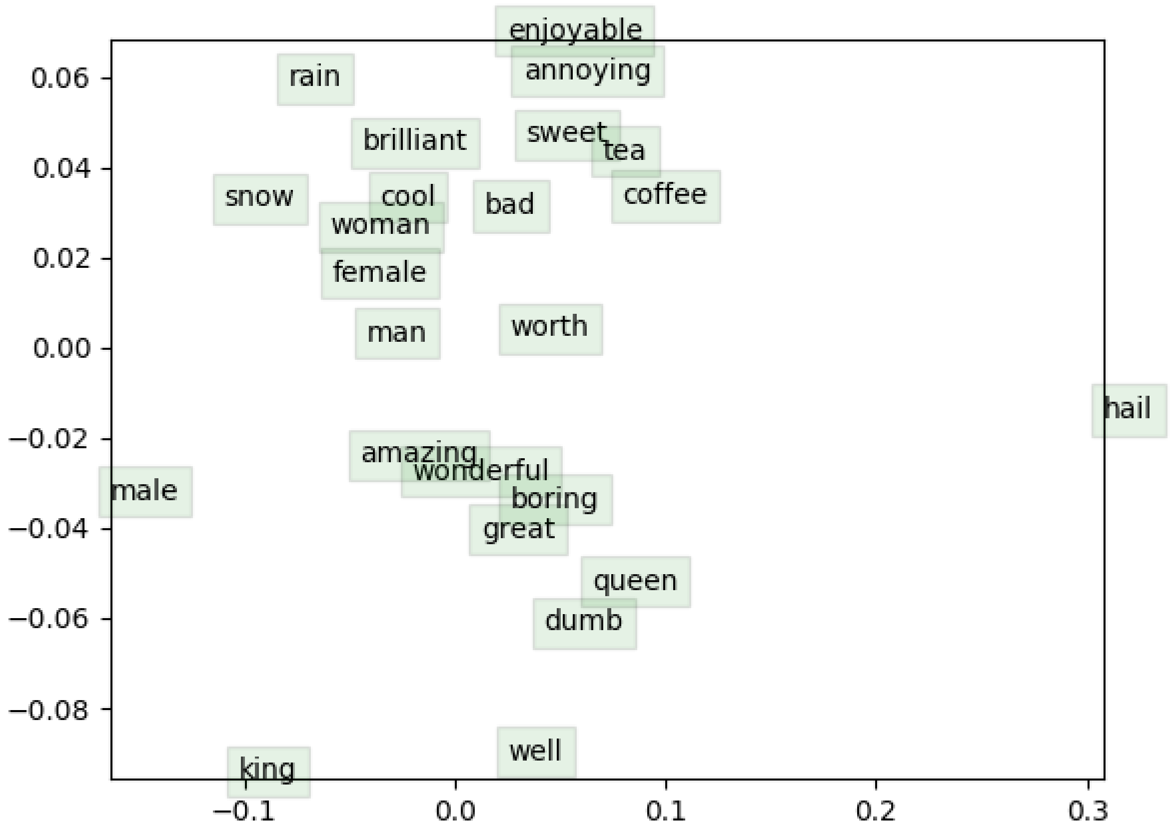
2. Gradient w.r.t. v_c:

$$\frac{\partial J_{skip-gram}}{\partial v_c} = \sum_{-m \leq j \leq m, j \neq 0} \frac{\partial J(v_c, w_{t+j}, U)}{\partial v_c}$$

3. Gradient w.r.t. v_w:

$$\frac{\partial J_{skip-gram}}{\partial v_w} = \sum_{-m \leq j \leq m, j \neq 0} \frac{\partial J(v_c, w_{t+j}, U)}{\partial v_w} = \vec{0}$$

2.c Plot of trained word vectors.



1. Words with similar meaning can cluster together, such as ['amazing', 'wonderful', 'great'], [tea, coffee];
2. The word vectors exhibit some analogy, such as "male : king :: female : queen";
3. The skip-gram model isn't good enough to cluster antinyms correctly, such as 'annoying', 'boring' are not clustered correctly.