# Derivatives

A *derivative* (slope of a line) is measuring the sensitivity of a function's output with respect to a very small change in input. In other words, it measures the *steepness* of the graph of a function. In functions with 2 or more variables, the partial derivative is the derivative of one variable with respect to the others.

If we want to find the partial derivative of the a linear function like below:

$$f(x) = 3x^2$$

Then, given the definition of slope is the change in $y$ divided by the change in $x$, we have the below where $h$ represents an extremely small number like .00001:

$$\partial = \frac{\Delta y}{\Delta x} = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 3x^2}{h}$$

Apply the slope formula:

$$\partial = \frac{(3x^2 + 2xh + 3h^2) - 3x^2}{h}$$

Simplify:

$$\partial = \frac{(3x^2 + 2xh + 3h^2) - 3x^2}{h} = \frac{2xh + 3h^2}{h} = 2x + 3h$$

Next, set $h$ to 0 (the limit of $h$ approaches 0):

$$\partial = 2x + (3 \cdot 0) = 2x$$

Therefore the slope at any point is $2x$ for the function $f(x) = 3x^2$.

More formally, we say that *the derivative of $f(x)$ with respect to $x$ is $3x^2$*:

$$\delta = \frac{\partial f(x)}{\partial x} = 3x^2$$

Other ways of writing this would be:

$$\frac{\partial}{\partial x} f(x) \text{ or } \frac{\partial f}{\partial x}(x) \text{ or } \nabla_x f(x)$$

# Derivatives and Quadratics

For getting the derivative of a quadratic function we will need to calculate it with respect to the tangent line as shown below. Also, since quadratics are complex function and don't yield a simple straight line we only know the partial derivative $\partial$ at any given point since it is constantly changing.

$$\frac{\partial f(x)}{\partial x} \text{ or } \frac{\partial}{\partial x} f(x)$$