Calculus: Derivatives

A *derivative* (slope of a line) is measuring the sensitivity of a function's output with respect to a very small change in input. In other words, it measures the *steepness* of the graph of a function. In functions with 2 or more variables, the partial derivative is the derivative of one variable with respect to the others.

If we want to find the partial derivative of the a linear function like below:

> $f(x) = 3x^2$

Then, given the definition of slope is the change in y divided by the change in x, we have the below where h represents an extremely small number like $.00001$:

> $$\partial = \frac{\Delta y}{\Delta x} = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 3x^2}{h}$$

Apply the slope formula:

> $$\partial = \frac{(3x^2 + 2xh + 3h^2) - 3x^2}{h}$$

Simplify:

> $$\partial = \frac{(3x^2 + 2xh + 3h^2) - 3x^2}{h} = \frac{2xh + 3h^2}{h} = 2x + 3h$$

Next, set h to 0 (the limit of h approaches 0):

> $\partial = 2x + (3 \cdot 0) = 2x$

Therefore the slope at any point is $2x$ for the function $f(x) = 3x^2$.

More formally, we say that *the derivative of $f(x)$ with respect to x is $3x^2$*:

> $$\delta = \frac{\partial f(x)}{\partial x} = 3x^2$$

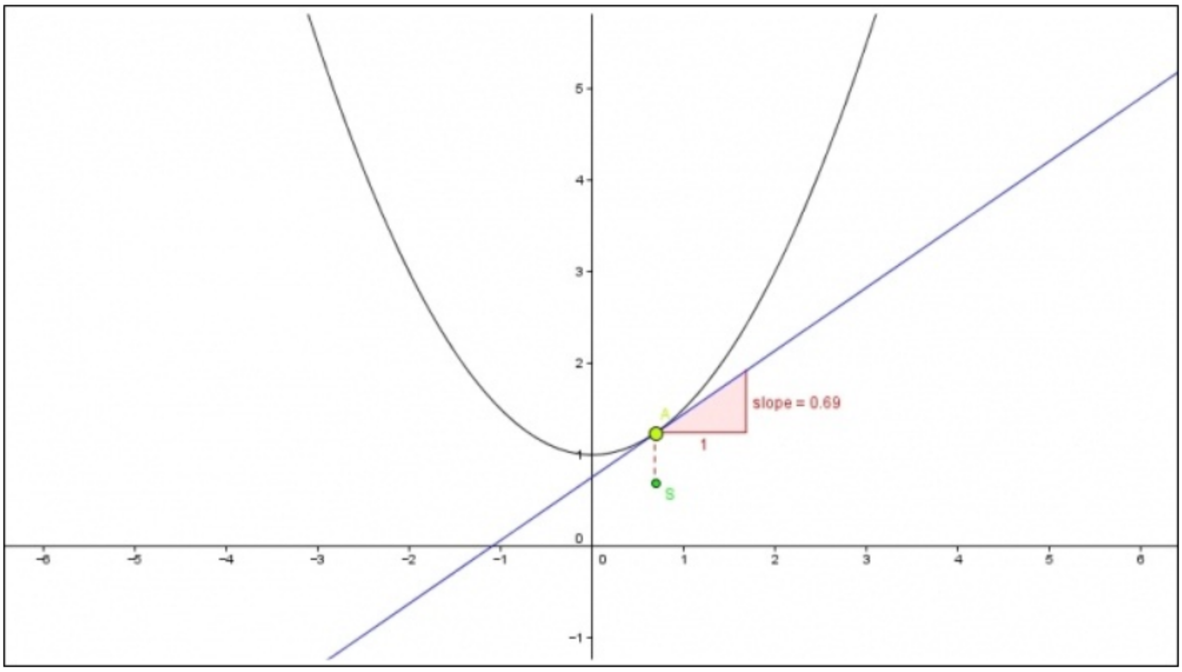
Other ways of writing this would be:

> $\frac{\partial}{\partial x} f(x)$ or $\frac{\partial f}{\partial x}(x)$ or $\nabla_x f(x)$

Derivatives and Quadratics

For getting the derivative of a quadratic function we will need to calculate it with respect to the tangent line as shown below. Also, since quadratics are complex function and don't yield a simple straight line we only know the partial derivative ∂ at any given point since it is constantly changing.

> $\frac{\partial f(x)}{\partial x}$ or $\frac{\partial}{\partial x} f(x)$



slope = 0.69
1
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