P.1 Review of Real Numbers and Their Properties

What you should learn

- Represent and classify real numbers.
- Order real numbers and use inequalities.
- Find the absolute values of real numbers and find the distance between two real numbers.
- Evaluate algebraic expressions.
- Use the basic rules and properties of algebra.

Why you should learn it

Real numbers are used to represent many real-life quantities. For example, in Exercises 83–88 on page 13, you will use real numbers to represent the federal deficit.

Real Numbers

Real numbers are used in everyday life to describe quantities such as age, miles per gallon, and population. Real numbers are represented by symbols such as

$$-5, 9, 0, \frac{4}{3}, 0.666 \ldots, 28.21, \sqrt{2}, \pi, \text{ and } \sqrt[3]{-32}.$$

Here are some important **subsets** (each member of subset B is also a member of set A) of the real numbers. The three dots, called *ellipsis points*, indicate that the pattern continues indefinitely.

$$\{1, 2, 3, 4, \ldots\} \quad \text{Set of natural numbers}$$

$$\{0, 1, 2, 3, 4, \ldots\} \quad \text{Set of whole numbers}$$

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \quad \text{Set of integers}$$

A real number is **rational** if it can be written as the ratio p/q of two integers, where $q \neq 0$. For instance, the numbers

$$\frac{1}{3} = 0.3333 \ldots = 0.\overline{3}, \frac{1}{8} = 0.125, \text{ and } \frac{125}{111} = 1.126126 \ldots = 1.\overline{126}$$

are rational. The decimal representation of a rational number either repeats $\left(\text{as in } \frac{173}{55} = 3.1\overline{45}\right)$ or terminates $\left(\text{as in } \frac{1}{2} = 0.5\right)$. A real number that cannot be written as the ratio of two integers is called **irrational.** Irrational numbers have infinite nonrepeating decimal representations. For instance, the numbers

$$\sqrt{2} = 1.4142135 \ldots \approx 1.41 \quad \text{and} \quad \pi = 3.1415926 \ldots \approx 3.14$$

are irrational. (The symbol $\approx$ means "is approximately equal to.") Figure P.1 shows subsets of real numbers and their relationships to each other.

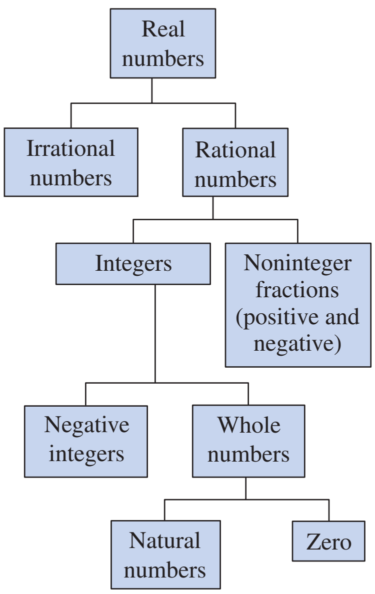


FIGURE P.1 Subsets of real numbers

Example 1 Classifying Real Numbers

Determine which numbers in the set

$$\left\{-13, -\sqrt{5}, -1, -\frac{1}{3}, 0, \frac{5}{8}, \sqrt{2}, \pi, 7\right\}$$

are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.

Solution

a. Natural numbers: $\{7\}$

b. Whole numbers: $\{0, 7\}$

c. Integers: $\{-13, -1, 0, 7\}$

d. Rational numbers: $\left\{-13, -1, -\frac{1}{3}, 0, \frac{5}{8}, 7\right\}$

e. Irrational numbers: $\{-\sqrt{5}, \sqrt{2}, \pi\}$

CHECKPoint Now try Exercise 11.

Real numbers are represented graphically on the **real number line.** When you draw a point on the real number line that corresponds to a real number, you are **plotting** the real number. The point 0 on the real number line is the **origin.** Numbers to the right of 0 are positive, and numbers to the left of 0 are negative, as shown in Figure P.2. The term **nonnegative** describes a number that is either positive or zero.

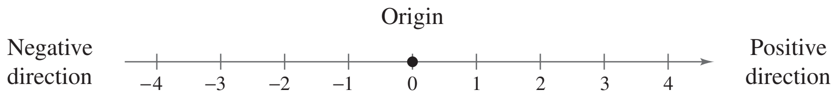


FIGURE P.2 The real number line

As illustrated in Figure P.3, there is a *one-to-one correspondence* between real numbers and points on the real number line.

Every real number corresponds to exactly one point on the real number line.

Every point on the real number line corresponds to exactly one real number.

FIGURE P.3 One-to-one correspondence

Example 2 Plotting Points on the Real Number Line

Plot the real numbers on the real number line.

a. $-\dfrac{7}{4}$

b. 2.3

c. $\dfrac{2}{3}$

d. -1.8

Solution

All four points are shown in Figure P.4.

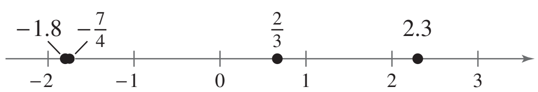


FIGURE P.4

a. The point representing the real number $-\frac{7}{4} = -1.75$ lies between -2 and -1, but closer to -2, on the real number line.

b. The point representing the real number 2.3 lies between 2 and 3, but closer to 2, on the real number line.

c. The point representing the real number $\frac{2}{3} = 0.666 \ldots$ lies between 0 and 1, but closer to 1, on the real number line.

d. The point representing the real number -1.8 lies between -2 and -1, but closer to -2, on the real number line. Note that the point representing -1.8 lies slightly to the left of the point representing $-\frac{7}{4}$.

CHECK*Point* Now try Exercise 17.

Ordering Real Numbers

One important property of real numbers is that they are *ordered*.

Definition of Order on the Real Number Line

If a and b are real numbers, a is less than b if $b - a$ is positive. The **order** of a and b is denoted by the **inequality** $a < b$. This relationship can also be described by saying that b is *greater than* a and writing $b > a$. The inequality $a \le b$ means that a is *less than or equal to* b, and the inequality $b \ge a$ means that b is *greater than or equal to* a. The symbols $<$, $>$, $\le$, and $\ge$ are *inequality symbols*.

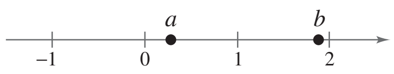


FIGURE P.5 $a < b$ if and only if a lies to the left of b.

Geometrically, this definition implies that $a < b$ if and only if a lies to the *left* of b on the real number line, as shown in Figure P.5.

Example 3 Ordering Real Numbers

Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

a. $-3, 0$ **b.** $-2, -4$ **c.** $\frac{1}{4}, \frac{1}{3}$ **d.** $-\frac{1}{5}, -\frac{1}{2}$

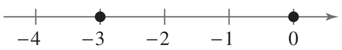


FIGURE P.6

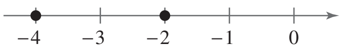


FIGURE P.7

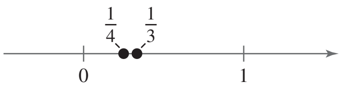


FIGURE P.8

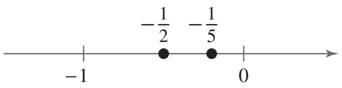


FIGURE P.9

Solution

a. Because -3 lies to the left of 0 on the real number line, as shown in Figure P.6, you can say that -3 is *less than* 0, and write $-3 < 0$.

b. Because -2 lies to the right of -4 on the real number line, as shown in Figure P.7, you can say that -2 is *greater than* -4, and write $-2 > -4$.

c. Because $\frac{1}{4}$ lies to the left of $\frac{1}{3}$ on the real number line, as shown in Figure P.8, you can say that $\frac{1}{4}$ is *less than* $\frac{1}{3}$, and write $\frac{1}{4} < \frac{1}{3}$.

d. Because $-\frac{1}{5}$ lies to the right of $-\frac{1}{2}$ on the real number line, as shown in Figure P.9, you can say that $-\frac{1}{5}$ is *greater than* $-\frac{1}{2}$, and write $-\frac{1}{5} > -\frac{1}{2}$.

CHECK*Point* Now try Exercise 25.

Example 4 Interpreting Inequalities

Describe the subset of real numbers represented by each inequality.

a. $x \le 2$ **b.** $-2 \le x < 3$

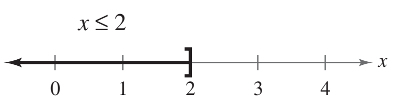


FIGURE P.10

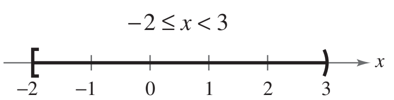


FIGURE P.11

Solution

a. The inequality $x \le 2$ denotes all real numbers less than or equal to 2, as shown in Figure P.10.

b. The inequality $-2 \le x < 3$ means that $x \ge -2$ *and* $x < 3$. This "double inequality" denotes all real numbers between -2 and 3, including -2 but not including 3, as shown in Figure P.11.

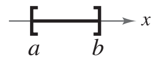
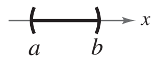
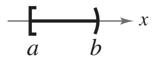
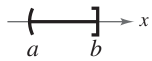
CHECK*Point* Now try Exercise 31.

Inequalities can be used to describe subsets of real numbers called **intervals.** In the bounded intervals below, the real numbers a and b are the **endpoints** of each interval. The endpoints of a closed interval are included in the interval, whereas the endpoints of an open interval are not included in the interval.

Study Tip

The reason that the four types of intervals at the right are called *bounded* is that each has a finite length. An interval that does not have a finite length is *unbounded* (see below).

Bounded Intervals on the Real Number Line

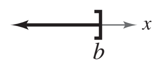
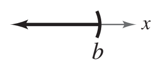
Notation	*Interval Type*	*Inequality*	*Graph*
$[a, b]$	Closed	$a \le x \le b$	
(a, b)	Open	$a < x < b$	
$[a, b)$		$a \le x < b$	
$(a, b]$		$a < x \le b$	

WARNING / CAUTION

Whenever you write an interval containing ∞ or $-\infty$, always use a parenthesis and never a bracket. This is because ∞ and $-\infty$ are never an endpoint of an interval and therefore are not included in the interval.

The symbols ∞, **positive infinity,** and $-\infty$, **negative infinity,** do not represent real numbers. They are simply convenient symbols used to describe the unboundedness of an interval such as $(1, \infty)$ or $(-\infty, 3]$.

Unbounded Intervals on the Real Number Line

Notation	*Interval Type*	*Inequality*	*Graph*
$[a, \infty)$		$x \ge a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$		$x \le b$	
$(-\infty, b)$	Open	$x < b$	
$(-\infty, \infty)$	Entire real line	$-\infty < x < \infty$	

Example 5 Using Inequalities to Represent Intervals

Use inequality notation to describe each of the following.

a. c is at most 2. **b.** m is at least -3. **c.** All x in the interval $(-3, 5]$

Solution

a. The statement "c is at most 2" can be represented by $c \le 2$.

b. The statement "m is at least -3" can be represented by $m \ge -3$.

c. "All x in the interval $(-3, 5]$" can be represented by $-3 < x \le 5$.

CHECK*Point* Now try Exercise 45.

Example 6 Interpreting Intervals

Give a verbal description of each interval.

a. $(-1, 0)$ **b.** $[2, \infty)$ **c.** $(-\infty, 0)$

Solution

a. This interval consists of all real numbers that are greater than -1 and less than 0.

b. This interval consists of all real numbers that are greater than or equal to 2.

c. This interval consists of all negative real numbers.

CHECKPoint Now try Exercise 41.

Absolute Value and Distance

The **absolute value** of a real number is its *magnitude*, or the distance between the origin and the point representing the real number on the real number line.

Definition of Absolute Value

If a is a real number, then the absolute value of a is

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}.$$

Notice in this definition that the absolute value of a real number is never negative. For instance, if $a = -5$, then $|-5| = -(-5) = 5$. The absolute value of a real number is either positive or zero. Moreover, 0 is the only real number whose absolute value is 0. So, $|0| = 0$.

Example 7 Finding Absolute Values

a. $|-15| = 15$ **b.** $\left|\frac{2}{3}\right| = \frac{2}{3}$

c. $|-4.3| = 4.3$ **d.** $-|-6| = -(6) = -6$

CHECKPoint Now try Exercise 51.

Example 8 Evaluating the Absolute Value of a Number

Evaluate $\dfrac{|x|}{x}$ for (a) $x > 0$ and (b) $x < 0$.

Solution

a. If $x > 0$, then $|x| = x$ and $\dfrac{|x|}{x} = \dfrac{x}{x} = 1$.

b. If $x < 0$, then $|x| = -x$ and $\dfrac{|x|}{x} = \dfrac{-x}{x} = -1$.

CHECKPoint Now try Exercise 59.

The **Law of Trichotomy** states that for any two real numbers a and b, *precisely* one of three relationships is possible:

$a = b,\quad a < b,\quad \text{or}\quad a > b.$ Law of Trichotomy

Example 9 Comparing Real Numbers

Place the appropriate symbol (<, >, or =) between the pair of real numbers.

a. $|-4| \;\square\; |3|$ **b.** $|-10| \;\square\; |10|$ **c.** $-|-7| \;\square\; |-7|$

Solution

a. $|-4| > |3|$ because $|-4| = 4$ and $|3| = 3$, and 4 is greater than 3.

b. $|-10| = |10|$ because $|-10| = 10$ and $|10| = 10$.

c. $-|-7| < |-7|$ because $-|-7| = -7$ and $|-7| = 7$, and -7 is less than 7.

CHECKPoint Now try Exercise 61.

Properties of Absolute Values

1. $|a| \geq 0$ **2.** $|-a| = |a|$

3. $|ab| = |a||b|$ **4.** $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|},\quad b \neq 0$

Absolute value can be used to define the distance between two points on the real number line. For instance, the distance between -3 and 4 is

$$\begin{aligned}|-3 - 4| &= |-7|\\ &= 7\end{aligned}$$

as shown in Figure P.12.

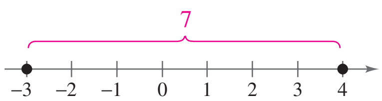


FIGURE P.12 The distance between -3 and 4 is 7.

Distance Between Two Points on the Real Number Line

Let a and b be real numbers. The **distance between *a* and *b*** is

$$d(a, b) = |b - a| = |a - b|.$$

Example 10 Finding a Distance

Find the distance between -25 and 13.

Solution

The distance between -25 and 13 is given by

$|-25 - 13| = |-38| = 38.$ Distance between -25 and 13

The distance can also be found as follows.

$|13 - (-25)| = |38| = 38$ Distance between -25 and 13

CHECKPoint Now try Exercise 67.

Algebraic Expressions

One characteristic of algebra is the use of letters to represent numbers. The letters are **variables,** and combinations of letters and numbers are **algebraic expressions.** Here are a few examples of algebraic expressions.

$$5x, \qquad 2x - 3, \qquad \frac{4}{x^2 + 2}, \qquad 7x + y$$

Definition of an Algebraic Expression

An **algebraic expression** is a collection of letters **(variables)** and real numbers **(constants)** combined using the operations of addition, subtraction, multiplication, division, and exponentiation.

The **terms** of an algebraic expression are those parts that are separated by *addition*. For example,

$$x^2 - 5x + 8 = x^2 + (-5x) + 8$$

has three terms: x^2 and $-5x$ are the **variable terms** and 8 is the **constant term.** The numerical factor of a term is called the **coefficient.** For instance, the coefficient of $-5x$ is -5, and the coefficient of x^2 is 1.

Example 11 Identifying Terms and Coefficients

Algebraic Expression	*Terms*	*Coefficients*
a. $5x - \frac{1}{7}$	$5x, -\frac{1}{7}$	$5, -\frac{1}{7}$
b. $2x^2 - 6x + 9$	$2x^2, -6x, 9$	$2, -6, 9$
c. $\frac{3}{x} + \frac{1}{2}x^4 - y$	$\frac{3}{x}, \frac{1}{2}x^4, -y$	$3, \frac{1}{2}, -1$

CHECK*Point* Now try Exercise 89.

To **evaluate** an algebraic expression, substitute numerical values for each of the variables in the expression, as shown in the next example.

Example 12 Evaluating Algebraic Expressions

Expression	*Value of Variable*	*Substitute*	*Value of Expression*
a. $-3x + 5$	$x = 3$	$-3(3) + 5$	$-9 + 5 = -4$
b. $3x^2 + 2x - 1$	$x = -1$	$3(-1)^2 + 2(-1) - 1$	$3 - 2 - 1 = 0$
c. $\frac{2x}{x + 1}$	$x = -3$	$\frac{2(-3)}{-3 + 1}$	$\frac{-6}{-2} = 3$

Note that you must substitute the value for *each* occurrence of the variable.

CHECK*Point* Now try Exercise 95.

When an algebraic expression is evaluated, the **Substitution Principle** is used. It states that "If $a = b$, then a can be replaced by b in any expression involving a." In Example 12(a), for instance, 3 is *substituted* for x in the expression $-3x + 5$.

Basic Rules of Algebra

There are four arithmetic operations with real numbers: *addition*, *multiplication*, *subtraction*, and *division*, denoted by the symbols $+$, $\times$ or $\cdot$, $-$, and $\div$ or /. Of these, addition and multiplication are the two primary operations. Subtraction and division are the inverse operations of addition and multiplication, respectively.

Definitions of Subtraction and Division

Subtraction: Add the opposite. **Division:** Multiply by the reciprocal.

$$a - b = a + (-b) \qquad \text{If } b \neq 0, \text{ then } a/b = a\left(\frac{1}{b}\right) = \frac{a}{b}.$$

In these definitions, $-b$ is the **additive inverse** (or opposite) of b, and $1/b$ is the **multiplicative inverse** (or reciprocal) of b. In the fractional form a/b, a is the **numerator** of the fraction and b is the **denominator.**

Because the properties of real numbers below are true for variables and algebraic expressions as well as for real numbers, they are often called the **Basic Rules of Algebra.** Try to formulate a verbal description of each property. For instance, the first property states that *the order in which two real numbers are added does not affect their sum*.

Basic Rules of Algebra

Let a, b, and c be real numbers, variables, or algebraic expressions.

Property		*Example*
Commutative Property of Addition:	$a + b = b + a$	$4x + x^2 = x^2 + 4x$
Commutative Property of Multiplication:	$ab = ba$	$(4 - x)x^2 = x^2(4 - x)$
Associative Property of Addition:	$(a + b) + c = a + (b + c)$	$(x + 5) + x^2 = x + (5 + x^2)$
Associative Property of Multiplication:	$(ab)c = a(bc)$	$(2x \cdot 3y)(8) = (2x)(3y \cdot 8)$
Distributive Properties:	$a(b + c) = ab + ac$	$3x(5 + 2x) = 3x \cdot 5 + 3x \cdot 2x$
	$(a + b)c = ac + bc$	$(y + 8)y = y \cdot y + 8 \cdot y$
Additive Identity Property:	$a + 0 = a$	$5y^2 + 0 = 5y^2$
Multiplicative Identity Property:	$a \cdot 1 = a$	$(4x^2)(1) = 4x^2$
Additive Inverse Property:	$a + (-a) = 0$	$5x^3 + (-5x^3) = 0$
Multiplicative Inverse Property:	$a \cdot \frac{1}{a} = 1, \quad a \neq 0$	$(x^2 + 4)\left(\frac{1}{x^2 + 4}\right) = 1$

Because subtraction is defined as "adding the opposite," the Distributive Properties are also true for subtraction. For instance, the "subtraction form" of $a(b + c) = ab + ac$ is $a(b - c) = ab - ac$. Note that the operations of subtraction and division are neither commutative nor associative. The examples

$$7 - 3 \neq 3 - 7 \quad \text{and} \quad 20 \div 4 \neq 4 \div 20$$

show that subtraction and division are not commutative. Similarly

$$5 - (3 - 2) \neq (5 - 3) - 2 \quad \text{and} \quad 16 \div (4 \div 2) \neq (16 \div 4) \div 2$$

demonstrate that subtraction and division are not associative.

Example 13 Identifying Rules of Algebra

Identify the rule of algebra illustrated by the statement.

a. $(5x^3)2 = 2(5x^3)$

b. $\left(4x + \frac{1}{3}\right) - \left(4x + \frac{1}{3}\right) = 0$

c. $7x \cdot \frac{1}{7x} = 1, \quad x \neq 0$

d. $(2 + 5x^2) + x^2 = 2 + (5x^2 + x^2)$

Solution

a. This statement illustrates the Commutative Property of Multiplication. In other words, you obtain the same result whether you multiply $5x^3$ by 2, or 2 by $5x^3$.

b. This statement illustrates the Additive Inverse Property. In terms of subtraction, this property simply states that when any expression is subtracted from itself the result is 0.

c. This statement illustrates the Multiplicative Inverse Property. Note that it is important that x be a nonzero number. If x were 0, the reciprocal of x would be undefined.

d. This statement illustrates the Associative Property of Addition. In other words, to form the sum

$$2 + 5x^2 + x^2$$

it does not matter whether 2 and $5x^2$, or $5x^2$ and x^2 are added first.

CHECK*Point* Now try Exercise 101.

Properties of Negation and Equality

Let a, b, and c be real numbers, variables, or algebraic expressions.

Property	*Example*
1. $(-1)a = -a$	$(-1)7 = -7$
2. $-(-a) = a$	$-(-6) = 6$
3. $(-a)b = -(ab) = a(-b)$	$(-5)3 = -(5 \cdot 3) = 5(-3)$
4. $(-a)(-b) = ab$	$(-2)(-x) = 2x$
5. $-(a + b) = (-a) + (-b)$	$-(x + 8) = (-x) + (-8)$ $= -x - 8$
6. If $a = b$, then $a \pm c = b \pm c$.	$\frac{1}{2} + 3 = 0.5 + 3$
7. If $a = b$, then $ac = bc$.	$4^2 \cdot 2 = 16 \cdot 2$
8. If $a \pm c = b \pm c$, then $a = b$.	$1.4 - 1 = \frac{7}{5} - 1 \Rightarrow 1.4 = \frac{7}{5}$
9. If $ac = bc$ and $c \neq 0$, then $a = b$.	$3x = 3 \cdot 4 \Rightarrow x = 4$

Study Tip

Notice the difference between the *opposite of a number* and a *negative number*. If a is already negative, then its opposite, $-a$, is positive. For instance, if $a = -5$, then

$$-a = -(-5) = 5.$$

Study Tip

The "or" in the Zero-Factor Property includes the possibility that either or both factors may be zero. This is an **inclusive or,** and it is the way the word "or" is generally used in mathematics.

Properties of Zero

Let a and b be real numbers, variables, or algebraic expressions.

1. $a + 0 = a$ and $a - 0 = a$
2. $a \cdot 0 = 0$
3. $\dfrac{0}{a} = 0, \quad a \neq 0$
4. $\dfrac{a}{0}$ is undefined.
5. **Zero-Factor Property:** If $ab = 0$, then $a = 0$ or $b = 0$.

Properties and Operations of Fractions

Let a, b, c, and d be real numbers, variables, or algebraic expressions such that $b \neq 0$ and $d \neq 0$.

1. **Equivalent Fractions:** $\dfrac{a}{b} = \dfrac{c}{d}$ if and only if $ad = bc$.
2. **Rules of Signs:** $-\dfrac{a}{b} = \dfrac{-a}{b} = \dfrac{a}{-b}$ and $\dfrac{-a}{-b} = \dfrac{a}{b}$
3. **Generate Equivalent Fractions:** $\dfrac{a}{b} = \dfrac{ac}{bc}, \quad c \neq 0$
4. **Add or Subtract with Like Denominators:** $\dfrac{a}{b} \pm \dfrac{c}{b} = \dfrac{a \pm c}{b}$
5. **Add or Subtract with Unlike Denominators:** $\dfrac{a}{b} \pm \dfrac{c}{d} = \dfrac{ad \pm bc}{bd}$
6. **Multiply Fractions:** $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$
7. **Divide Fractions:** $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}, \quad c \neq 0$

Study Tip

In Property 1 of fractions, the phrase "if and only if" implies two statements. One statement is: If $a/b = c/d$, then $ad = bc$. The other statement is: If $ad = bc$, where $b \neq 0$ and $d \neq 0$, then $a/b = c/d$.

Example 14 Properties and Operations of Fractions

a. Equivalent fractions: $\dfrac{x}{5} = \dfrac{3 \cdot x}{3 \cdot 5} = \dfrac{3x}{15}$

b. Divide fractions: $\dfrac{7}{x} \div \dfrac{3}{2} = \dfrac{7}{x} \cdot \dfrac{2}{3} = \dfrac{14}{3x}$

c. Add fractions with unlike denominators: $\dfrac{x}{3} + \dfrac{2x}{5} = \dfrac{5 \cdot x + 3 \cdot 2x}{3 \cdot 5} = \dfrac{11x}{15}$

CHECK*Point* Now try Exercise 119.

If a, b, and c are integers such that $ab = c$, then a and b are **factors** or **divisors** of c. A **prime number** is an integer that has exactly two positive factors—itself and 1—such as 2, 3, 5, 7, and 11. The numbers 4, 6, 8, 9, and 10 are **composite** because each can be written as the product of two or more prime numbers. The number 1 is neither prime nor composite. The **Fundamental Theorem of Arithmetic** states that every positive integer greater than 1 can be written as the product of prime numbers in precisely one way (disregarding order). For instance, the *prime factorization* of 24 is $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

P.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. A real number is ________ if it can be written as the ratio $\frac{p}{q}$ of two integers, where $q \neq 0$.
2. ________ numbers have infinite nonrepeating decimal representations.
3. The point 0 on the real number line is called the ________.
4. The distance between the origin and a point representing a real number on the real number line is the ________ ________ of the real number.
5. A number that can be written as the product of two or more prime numbers is called a ________ number.
6. An integer that has exactly two positive factors, the integer itself and 1, is called a ________ number.
7. An algebraic expression is a collection of letters called ________ and real numbers called ________.
8. The ________ of an algebraic expression are those parts separated by addition.
9. The numerical factor of a variable term is the ________ of the variable term.
10. The ________ ________ states that if $ab = 0$, then $a = 0$ or $b = 0$.

SKILLS AND APPLICATIONS

In Exercises 11–16, determine which numbers in the set are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.

11. $\left\{-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, 2, -11\right\}$
12. $\left\{\sqrt{5}, -7, -\frac{7}{3}, 0, 3.12, \frac{5}{4}, -3, 12, 5\right\}$
13. $\{2.01, 0.666 \ldots, -13, 0.010110111 \ldots, 1, -6\}$
14. $\{2.3030030003 \ldots, 0.7575, -4.63, \sqrt{10}, -75, 4\}$
15. $\left\{-\pi, -\frac{1}{3}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -1, 8, -22\right\}$
16. $\left\{25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 7, -11.1, 13\right\}$

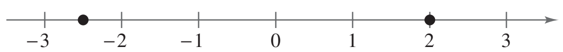
In Exercises 17 and 18, plot the real numbers on the real number line.

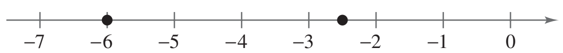
17. (a) 3 (b) $\frac{7}{2}$ (c) $-\frac{5}{2}$ (d) -5.2
18. (a) 8.5 (b) $\frac{4}{3}$ (c) -4.75 (d) $-\frac{8}{3}$

In Exercises 19–22, use a calculator to find the decimal form of the rational number. If it is a nonterminating decimal, write the repeating pattern.

19. $\frac{5}{8}$
20. $\frac{1}{3}$
21. $\frac{41}{333}$
22. $\frac{6}{11}$

In Exercises 23 and 24, approximate the numbers and place the correct symbol (< or >) between them.

23.


24.


In Exercises 25–30, plot the two real numbers on the real number line. Then place the appropriate inequality symbol (< or >) between them.

25. $-4, -8$
26. $-3.5, 1$
27. $\frac{3}{2}, 7$
28. $1, \frac{16}{3}$
29. $\frac{5}{6}, \frac{2}{3}$
30. $-\frac{8}{7}, -\frac{3}{7}$

In Exercises 31–42, (a) give a verbal description of the subset of real numbers represented by the inequality or the interval, (b) sketch the subset on the real number line, and (c) state whether the interval is bounded or unbounded.

31. $x \leq 5$
32. $x \geq -2$
33. $x < 0$
34. $x > 3$
35. $[4, \infty)$
36. $(-\infty, 2)$
37. $-2 < x < 2$
38. $0 \leq x \leq 5$
39. $-1 \leq x < 0$
40. $0 < x \leq 6$
41. $[-2, 5)$
42. $(-1, 2]$

In Exercises 43–50, use inequality notation and interval notation to describe the set.

43. y is nonnegative.
44. y is no more than 25.
45. x is greater than -2 and at most 4.
46. y is at least -6 and less than 0.
47. t is at least 10 and at most 22.
48. k is less than 5 but no less than -3.
49. The dog's weight W is more than 65 pounds.
50. The annual rate of inflation r is expected to be at least 2.5% but no more than 5%.

In Exercises 51–60, evaluate the expression.

51. $|-10|$

52. $|0|$

53. $|3 - 8|$

54. $|4 - 1|$

55. $|-1| - |-2|$

56. $-3 - |-3|$

57. $\dfrac{-5}{|-5|}$

58. $-3|-3|$

59. $\dfrac{|x+2|}{x+2}, \quad x < -2$

60. $\dfrac{|x-1|}{x-1}, \quad x > 1$

In Exercises 61–66, place the correct symbol ($<$, $>$, or $=$) between the two real numbers.

61. $|-3| \quad \square \quad -|-3|$

62. $|-4| \quad \square \quad |4|$

63. $-5 \quad \square \quad -|5|$

64. $-|-6| \quad \square \quad |-6|$

65. $-|-2| \quad \square \quad -|2|$

66. $-(-2) \quad \square \quad -2$

In Exercises 67–72, find the distance between *a* and *b*.

67. $a = 126, b = 75$

68. $a = -126, b = -75$

69. $a = -\frac{5}{2}, b = 0$

70. $a = \frac{1}{4}, b = \frac{11}{4}$

71. $a = \frac{16}{5}, b = \frac{112}{75}$

72. $a = 9.34, b = -5.65$

In Exercises 73–78, use absolute value notation to describe the situation.

73. The distance between x and 5 is no more than 3.

74. The distance between x and -10 is at least 6.

75. y is at least six units from 0.

76. y is at most two units from a.

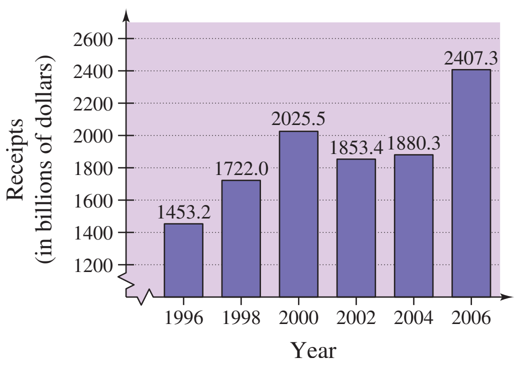
77. While traveling on the Pennsylvania Turnpike, you pass milepost 57 near Pittsburgh, then milepost 236 near Gettysburg. How many miles do you travel during that time period?

78. The temperature in Bismarck, North Dakota was 60°F at noon, then 23°F at midnight. What was the change in temperature over the 12-hour period?

BUDGET VARIANCE In Exercises 79–82, the accounting department of a sports drink bottling company is checking to see whether the actual expenses of a department differ from the budgeted expenses by more than \$500 or by more than 5%. Fill in the missing parts of the table, and determine whether each actual expense passes the "budget variance test."

	Budgeted Expense, b	*Actual Expense, a*	$\lvert a - b\rvert$	$0.05b$
79. Wages	\$112,700	\$113,356		
80. Utilities	\$9,400	\$9,772		
81. Taxes	\$37,640	\$37,335		
82. Insurance	\$2,575	\$2,613		

FEDERAL DEFICIT In Exercises 83–88, use the bar graph, which shows the receipts of the federal government (in billions of dollars) for selected years from 1996 through 2006. In each exercise you are given the expenditures of the federal government. Find the magnitude of the surplus or deficit for the year. (Source: U.S. Office of Management and Budget)



	Year	*Receipts*	*Expenditures*	$\lvert$*Receipts − Expenditures*$\rvert$
83.	1996		\$1560.6 billion	
84.	1998		\$1652.7 billion	
85.	2000		\$1789.2 billion	
86.	2002		\$2011.2 billion	
87.	2004		\$2293.0 billion	
88.	2006		\$2655.4 billion	

In Exercises 89–94, identify the terms. Then identify the coefficients of the variable terms of the expression.

89. $7x + 4$

90. $6x^3 - 5x$

91. $\sqrt{3}x^2 - 8x - 11$

92. $3\sqrt{3}x^2 + 1$

93. $4x^3 + \dfrac{x}{2} - 5$

94. $3x^4 - \dfrac{x^2}{4}$

In Exercises 95–100, evaluate the expression for each value of x. (If not possible, state the reason.)

Expression	*Values*	
95. $4x - 6$	(a) $x = -1$	(b) $x = 0$
96. $9 - 7x$	(a) $x = -3$	(b) $x = 3$
97. $x^2 - 3x + 4$	(a) $x = -2$	(b) $x = 2$
98. $-x^2 + 5x - 4$	(a) $x = -1$	(b) $x = 1$
99. $\dfrac{x+1}{x-1}$	(a) $x = 1$	(b) $x = -1$
100. $\dfrac{x}{x+2}$	(a) $x = 2$	(b) $x = -2$

In Exercises 101–112, identify the rule(s) of algebra illustrated by the statement.

101. $x + 9 = 9 + x$

102. $2\left(\frac{1}{2}\right) = 1$

103. $\dfrac{1}{h+6}(h + 6) = 1, \quad h \neq -6$

104. $(x + 3) - (x + 3) = 0$

105. $2(x + 3) = 2 \cdot x + 2 \cdot 3$

106. $(z - 2) + 0 = z - 2$

107. $1 \cdot (1 + x) = 1 + x$

108. $(z + 5)x = z \cdot x + 5 \cdot x$

109. $x + (y + 10) = (x + y) + 10$

110. $x(3y) = (x \cdot 3)y = (3x)y$

111. $3(t - 4) = 3 \cdot t - 3 \cdot 4$

112. $\frac{1}{7}(7 \cdot 12) = \left(\frac{1}{7} \cdot 7\right)12 = 1 \cdot 12 = 12$

In Exercises 113–120, perform the operation(s). (Write fractional answers in simplest form.)

113. $\frac{3}{16} + \frac{5}{16}$

114. $\frac{6}{7} - \frac{4}{7}$

115. $\frac{5}{8} - \frac{5}{12} + \frac{1}{6}$

116. $\frac{10}{11} + \frac{6}{33} - \frac{13}{66}$

117. $12 \div \frac{1}{4}$

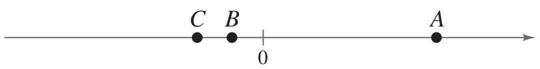
118. $-\left(6 \cdot \frac{4}{8}\right)$

119. $\dfrac{2x}{3} - \dfrac{x}{4}$

120. $\dfrac{5x}{6} \cdot \dfrac{2}{9}$

EXPLORATION

In Exercises 121 and 122, use the real numbers A, B, and C shown on the number line. Determine the sign of each expression.



121. (a) $-A$
(b) $B - A$

122. (a) $-C$
(b) $A - C$

123. CONJECTURE

(a) Use a calculator to complete the table.

n	1	0.5	0.01	0.0001	0.000001
$5/n$					

(b) Use the result from part (a) to make a conjecture about the value of $5/n$ as n approaches 0.

124. CONJECTURE

(a) Use a calculator to complete the table.

n	1	10	100	10,000	100,000
$5/n$					

(b) Use the result from part (a) to make a conjecture about the value of $5/n$ as n increases without bound.

TRUE OR FALSE? In Exercises 125–128, determine whether the statement is true or false. Justify your answer.

125. If $a > 0$ and $b < 0$, then $a - b > 0$.

126. If $a > 0$ and $b < 0$, then $ab > 0$.

127. If $a < b$, then $\dfrac{1}{a} < \dfrac{1}{b}$, where $a \neq 0$ and $b \neq 0$.

128. Because $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$, then $\dfrac{c}{a+b} = \dfrac{c}{a} + \dfrac{c}{b}$.

129. THINK ABOUT IT Consider $|u + v|$ and $|u| + |v|$, where $u \neq 0$ and $v \neq 0$.

(a) Are the values of the expressions always equal? If not, under what conditions are they unequal?

(b) If the two expressions are not equal for certain values of u and v, is one of the expressions always greater than the other? Explain.

130. THINK ABOUT IT Is there a difference between saying that a real number is positive and saying that a real number is nonnegative? Explain.

131. THINK ABOUT IT Because every even number is divisible by 2, is it possible that there exist any even prime numbers? Explain.

132. THINK ABOUT IT Is it possible for a real number to be both rational and irrational? Explain.

133. WRITING Can it ever be true that $|a| = -a$ for a real number a? Explain.

134. CAPSTONE Describe the differences among the sets of natural numbers, whole numbers, integers, rational numbers, and irrational numbers.