Exponents and Radicals

$$x^m x^n = x^{m+n} \qquad \frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn} \qquad x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x}$$

$$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y} \qquad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$$

Special Products

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Factoring Formulas

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities and Absolute Value

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$.

$|x| < a$ means $-a < x < a$.

$|x| > a$ means $x > a$ or $x < -a$.

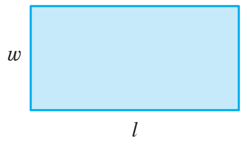
Geometric Formulas

Formulas for area A, perimeter P, circumference C, and volume V:

Rectangle

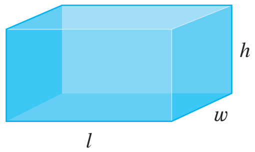
$A = lw$

$P = 2l + 2w$



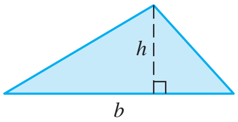
Box

$V = lwh$



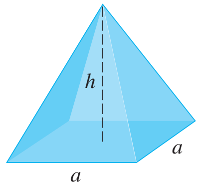
Triangle

$A = \frac{1}{2}bh$



Pyramid

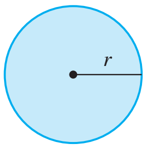
$V = \frac{1}{3}ha^2$



Circle

$A = \pi r^2$

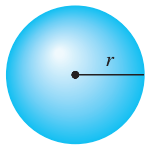
$C = 2\pi r$



Sphere

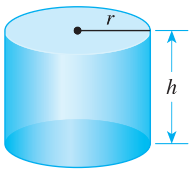
$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$



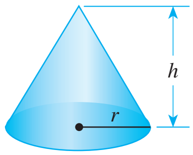
Cylinder

$V = \pi r^2 h$



Cone

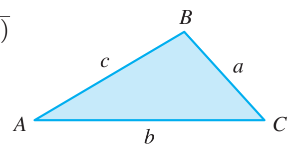
$V = \frac{1}{3}\pi r^2 h$



Heron's Formula

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

$$\text{where } s = \frac{a + b + c}{2}$$



Distance and Midpoint Formulas

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of P_1P_2: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Lines

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m $\qquad y - y_1 = m(x - x_1)$

Slope-intercept equation of line with slope m and y-intercept b $\qquad y = mx + b$

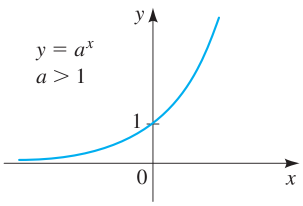
Two-intercept equation of line with x-intercept a and y-intercept b $\qquad \dfrac{x}{a} + \dfrac{y}{b} = 1$

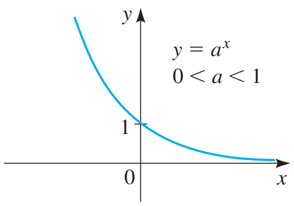
Logarithms

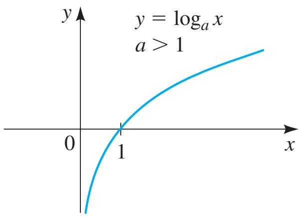
$y = \log_a x \quad \text{means} \quad a^y = x$

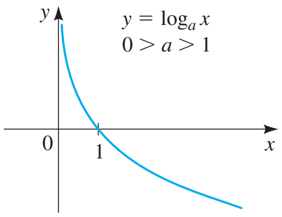
$\log_a a^x = x$	$a^{\log_a x} = x$
$\log_a 1 = 0$	$\log_a a = 1$
$\log x = \log_{10} x$	$\ln x = \log_e x$
$\log_a xy = \log_a x + \log_a y$	$\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$
$\log_a x^b = b \log_a x$	$\log_b x = \dfrac{\log_a x}{\log_a b}$

Exponential and Logarithmic Functions



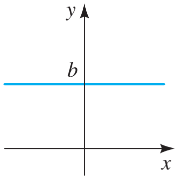




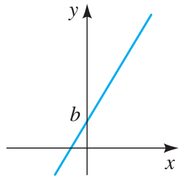


Graphs of Functions

Linear functions: $f(x) = mx + b$

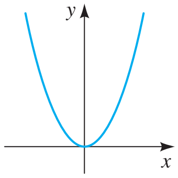


$f(x) = b$

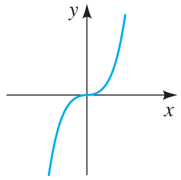


$f(x) = mx + b$

Power functions: $f(x) = x^n$

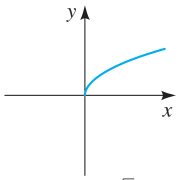


$f(x) = x^2$

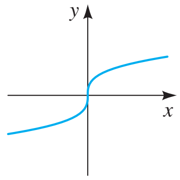


$f(x) = x^3$

Root functions: $f(x) = \sqrt[n]{x}$

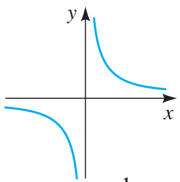


$f(x) = \sqrt{x}$

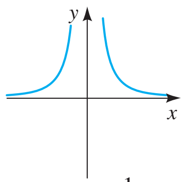


$f(x) = \sqrt[3]{x}$

Reciprocal functions: $f(x) = 1/x^n$

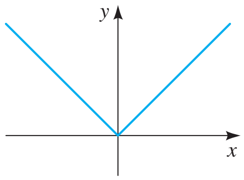


$f(x) = \dfrac{1}{x}$



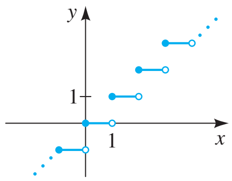
$f(x) = \dfrac{1}{x^2}$

Absolute value function



$f(x) = |x|$

Greatest integer function



$f(x) = [\![x]\!]$

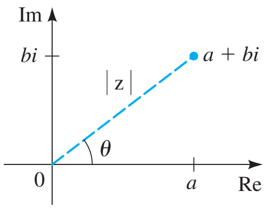
Complex Numbers

For the complex number $z = a + bi$

the **conjugate** is $\bar{z} = a - bi$

the **modulus** is $|z| = \sqrt{a^2 + b^2}$

the **argument** is θ, where $\tan \theta = b/a$



Polar form of a complex number

For $z = a + bi$, the **polar form** is

$$z = r(\cos \theta + i \sin \theta)$$

where $r = |z|$ is the modulus of z and θ is the argument of z

De Moivre's Theorem

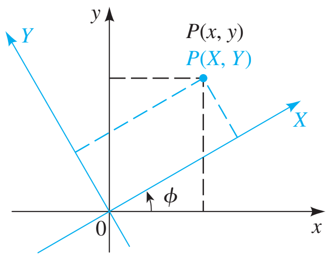
$$z^n = [r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$$

$$\sqrt[n]{z} = [r(\cos \theta + i \sin \theta)]^{1/n}$$

$$= r^{1/n}\left(\cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n}\right)$$

where $k = 0, 1, 2, \ldots, n - 1$

Rotation of Axes



Rotation of axes formulas

$$x = X \cos \phi - Y \sin \phi$$

$$y = X \sin \phi + Y \cos \phi$$

Angle-of-rotation formula for conic sections

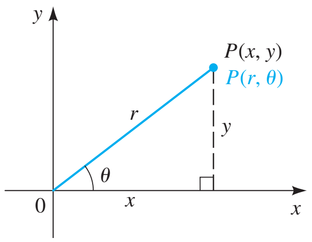
To eliminate the xy-term in the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

rotate the axis by the angle ϕ that satisfies

$$\cot 2\phi = \frac{A - C}{B}$$

Polar Coordinates



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$

Sums of Powers of Integers

$$\sum_{k=1}^{n} 1 = n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

The Derivative

The **average rate of change** of f between a and b is

$$\frac{f(b) - f(a)}{b - a}$$

The **derivative** of f at a is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

Area Under the Graph of *f*

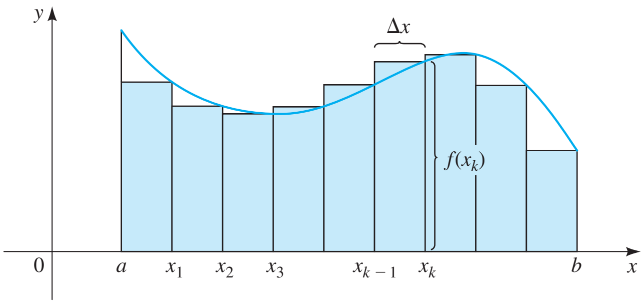
The **area under the graph of *f*** on the interval $[a, b]$ is the limit of the sum of the areas of approximating rectangles

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

where

$$\Delta x = \frac{b - a}{n}$$

$$x_k = a + k\,\Delta x$$



Sequences and Series

Arithmetic

$a, a + d, a + 2d, a + 3d, \ldots$ or $a_n = a + (n - 1)d$

$$S_n = \sum_{k=1}^{n} a_k = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$$

Geometric

$a, ar, ar^2, ar^3, \ldots$ or $a_n = ar^{n-1}$

$$S_n = \sum_{k=1}^{n} a_k = a\frac{1 - r^n}{1 - r}$$

Infinite geometric series

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \cdots$$

If $|r| < 1$ the series converges and its sum is

$$S = \frac{a}{1 - r}$$

If $|r| \geq 1$ the series diverges.

The Binomial Theorem

Binomial Theorem

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

Binomial coefficients

$$\binom{n}{r} = \frac{n!}{r!(n - r)!} \; (r \leq n), \text{ where } n! = 1 \cdot 2 \cdot 3 \cdots (n - 1)n$$

Finance

Compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A is the amount after t years, P is the principal, r is the interest rate, and the interest is compounded n times per year.

Continuously compounded interest

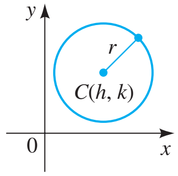
$$A = Pe^{rt}$$

where A is the amount after t years, P is the principal, r is the interest rate, and the interest is compounded continuously.

Conic Sections

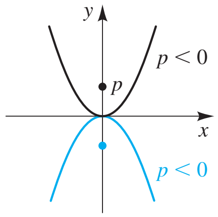
Circles

$(x - h)^2 + (y - k)^2 = r^2$



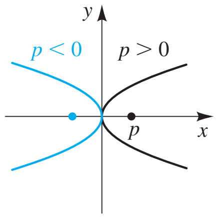
Parabolas

$x^2 = 4py$

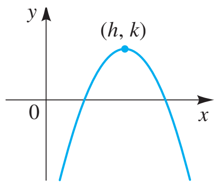


Focus $(0, p)$, directrix $y = -p$

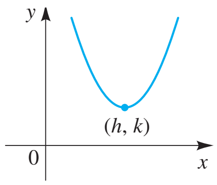
$y^2 = 4px$



Focus $(p, 0)$, directrix $x = -p$



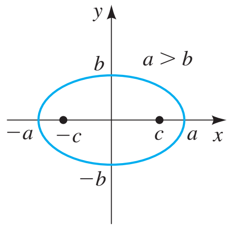
$y = a(x - h)^2 + k,$
$a < 0, \quad h > 0, \quad k > 0$



$y = a(x - h)^2 + k,$
$a > 0, \quad h > 0, \quad k > 0$

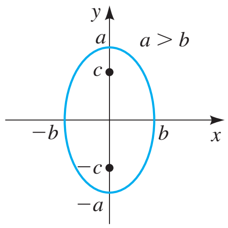
Ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$

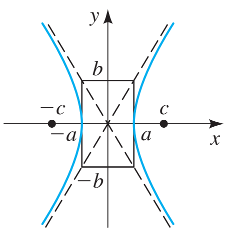
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



Foci $(0, \pm c)$, $c^2 = a^2 - b^2$

Hyperbolas

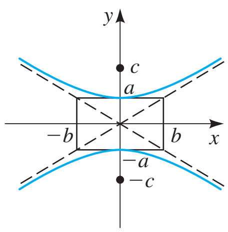
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$

Asymptotes: $y = \pm\frac{b}{a}x$

$$-\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



Foci $(0, \pm c)$, $c^2 = a^2 + b^2$

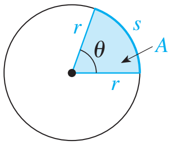
Asymptotes: $y = \pm\frac{a}{b}x$

Angle Measurement

π radians $= 180°$

$1° = \dfrac{\pi}{180}$ rad $\qquad 1$ rad $= \dfrac{180°}{\pi}$

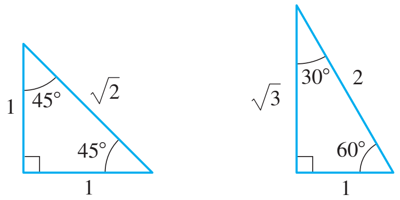
$s = r\theta \qquad A = \frac{1}{2}r^2\theta \quad$ (θ in radians)



To convert from degrees to radians, multiply by $\dfrac{\pi}{180}$.

To convert from radians to degrees, multiply by $\dfrac{180}{\pi}$.

Special Triangles

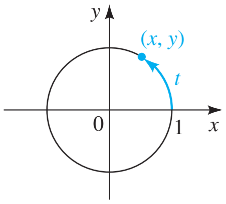


Trigonometric Functions of Real Numbers

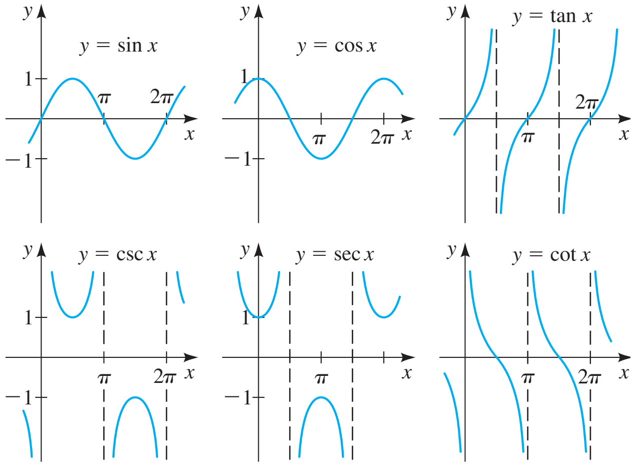
$\sin t = y \qquad \csc t = \dfrac{1}{y}$

$\cos t = x \qquad \sec t = \dfrac{1}{x}$

$\tan t = \dfrac{y}{x} \qquad \cot t = \dfrac{x}{y}$



Graphs of the Trigonometric Functions

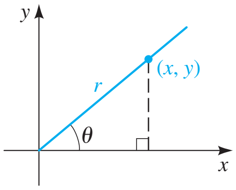


Trigonometric Functions of Angles

$\sin\theta = \dfrac{y}{r} \qquad \csc\theta = \dfrac{r}{y}$

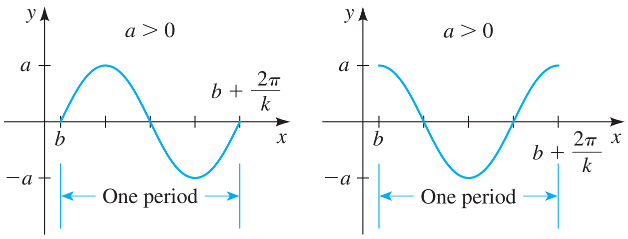
$\cos\theta = \dfrac{x}{r} \qquad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x} \qquad \cot\theta = \dfrac{x}{y}$



Sine and Cosine Curves

$y = a\sin k(x - b) \quad (k > 0) \qquad y = a\cos k(x - b) \quad (k > 0)$



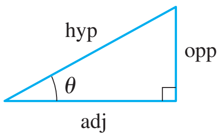
amplitude: $|a|$ $\qquad$ period: $2\pi/k$ $\qquad$ horizontal shift: b

Right Angle Trigonometry

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

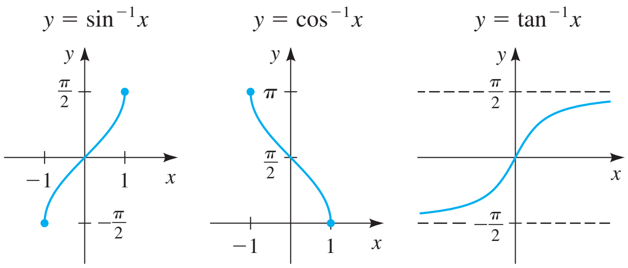
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} \qquad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$



Special Values of the Trigonometric Functions

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—
180°	π	0	-1	0
270°	$3\pi/2$	-1	0	—

Graphs of the Inverse Trigonometric Functions



Fundamental Identities

$\sec x = \dfrac{1}{\cos x}$ $\qquad$ $\csc x = \dfrac{1}{\sin x}$

$\tan x = \dfrac{\sin x}{\cos x}$ $\qquad$ $\cot x = \dfrac{1}{\tan x}$

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $1 + \tan^2 x = \sec^2 x$ $\qquad$ $1 + \cot^2 x = \csc^2 x$

$\sin(-x) = -\sin x$ $\qquad$ $\cos(-x) = \cos x$ $\qquad$ $\tan(-x) = -\tan x$

Cofunction Identities

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$ $\qquad$ $\cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

$\tan\left(\dfrac{\pi}{2} - x\right) = \cot x$ $\qquad$ $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

$\sec\left(\dfrac{\pi}{2} - x\right) = \csc x$ $\qquad$ $\csc\left(\dfrac{\pi}{2} - x\right) = \sec x$

Reduction Identities

$\sin(x + \pi) = -\sin x$ $\qquad$ $\sin\left(x + \dfrac{\pi}{2}\right) = \cos x$

$\cos(x + \pi) = -\cos x$ $\qquad$ $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$

$\tan(x + \pi) = \tan x$ $\qquad$ $\tan\left(x + \dfrac{\pi}{2}\right) = -\cot x$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$\cos(x - y) = \cos x \cos y + \sin x \sin y$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$ $\qquad$ $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-Angle Formulas

$\sin 2x = 2 \sin x \cos x$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

Formulas for Reducing Powers

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$ $\qquad$ $\cos^2 x = \dfrac{1 + \cos 2x}{2}$

$\tan^2 x = \dfrac{1 - \cos 2x}{1 + \cos 2x}$

Half-Angle Formulas

$\sin \dfrac{u}{2} = \pm\sqrt{\dfrac{1 - \cos u}{2}}$ $\qquad$ $\cos \dfrac{u}{2} = \pm\sqrt{\dfrac{1 + \cos u}{2}}$

$\tan \dfrac{u}{2} = \dfrac{1 - \cos u}{\sin u} = \dfrac{\sin u}{1 + \cos u}$

Product-to-Sum and Sum-to-Product Identities

$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$

$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$

$\sin x + \sin y = 2 \sin \dfrac{x + y}{2} \cos \dfrac{x - y}{2}$

$\sin x - \sin y = 2 \cos \dfrac{x + y}{2} \sin \dfrac{x - y}{2}$

$\cos x + \cos y = 2 \cos \dfrac{x + y}{2} \cos \dfrac{x - y}{2}$

$\cos x - \cos y = -2 \sin \dfrac{x + y}{2} \sin \dfrac{x - y}{2}$

The Laws of Sines and Cosines

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$