P.3 Polynomials and Special Products

What you should learn

- Write polynomials in standard form.
- Add, subtract, and multiply polynomials.
- Use special products to multiply polynomials.
- Use polynomials to solve real-life problems.

Why you should learn it

Polynomials can be used to model and solve real-life problems. For instance, in Exercise 106 on page 34, polynomials are used to model the cost, revenue, and profit for producing and selling hats.

David Noton/Masterfile

Polynomials

The most common type of algebraic expression is the **polynomial.** Some examples are $2x + 5$, $3x^4 - 7x^2 + 2x + 4$, and $5x^2y^2 - xy + 3$. The first two are *polynomials in x* and the third is a *polynomial in x and y*. The terms of a polynomial in x have the form ax^k, where a is the **coefficient** and k is the **degree** of the term. For instance, the polynomial

$$2x^3 - 5x^2 + 1 = 2x^3 + (-5)x^2 + (0)x + 1$$

has coefficients 2, -5, 0, and 1.

Definition of a Polynomial in *x*

Let $a_0, a_1, a_2, \ldots, a_n$ be real numbers and let n be a nonnegative integer. A polynomial in x is an expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

where $a_n \neq 0$. The polynomial is of **degree** n, a_n is the **leading coefficient,** and a_0 is the **constant term.**

Polynomials with one, two, and three terms are called **monomials, binomials,** and **trinomials,** respectively. In **standard form,** a polynomial is written with descending powers of x.

Example 1 Writing Polynomials in Standard Form

	Polynomial	*Standard Form*	*Degree*	*Leading Coefficient*
a.	$4x^2 - 5x^7 - 2 + 3x$	$-5x^7 + 4x^2 + 3x - 2$	7	-5
b.	$4 - 9x^2$	$-9x^2 + 4$	2	-9
c.	8	$8 \quad (8 = 8x^0)$	0	8

CHECK*Point* Now try Exercise 19.

A polynomial that has all zero coefficients is called the **zero polynomial,** denoted by 0. No degree is assigned to this particular polynomial. For polynomials in more than one variable, the degree of a *term* is the sum of the exponents of the variables in the term. The degree of the *polynomial* is the highest degree of its terms. For instance, the degree of the polynomial $-2x^3y^6 + 4xy - x^7y^4$ is 11 because the sum of the exponents in the last term is the greatest. The leading coefficient of the polynomial is the coefficient of the highest-degree term. Expressions are not polynomials if a variable is underneath a radical or if a polynomial expression (with degree greater than 0) is in the denominator of a term. The following expressions are not polynomials.

$x^3 - \sqrt{3x} = x^3 - (3x)^{1/2}$ The exponent "1/2" is not an integer.

$x^2 + \dfrac{5}{x} = x^2 + 5x^{-1}$ The exponent "-1" is not a nonnegative integer.

Operations with Polynomials

You can add and subtract polynomials in much the same way you add and subtract real numbers. Simply add or subtract the *like terms* (terms having the same variables to the same powers) by adding their coefficients. For instance, $-3xy^2$ and $5xy^2$ are like terms and their sum is

$$\begin{aligned} -3xy^2 + 5xy^2 &= (-3 + 5)xy^2 \\ &= 2xy^2. \end{aligned}$$

WARNING / CAUTION

When an expression inside parentheses is preceded by a negative sign, remember to distribute the negative sign to each term inside the parentheses, as shown.

$$\begin{aligned} -(x^2 - x + 3) \\ = -x^2 + x - 3 \end{aligned}$$

Example 2 Sums and Differences of Polynomials

a. $(5x^3 - 7x^2 - 3) + (x^3 + 2x^2 - x + 8)$

$= (5x^3 + x^3) + (-7x^2 + 2x^2) - x + (-3 + 8)$ Group like terms.

$= 6x^3 - 5x^2 - x + 5$ Combine like terms.

b. $(7x^4 - x^2 - 4x + 2) - (3x^4 - 4x^2 + 3x)$

$= 7x^4 - x^2 - 4x + 2 - 3x^4 + 4x^2 - 3x$ Distributive Property

$= (7x^4 - 3x^4) + (-x^2 + 4x^2) + (-4x - 3x) + 2$ Group like terms.

$= 4x^4 + 3x^2 - 7x + 2$ Combine like terms.

CHECK*Point* Now try Exercise 41.

To find the *product* of two polynomials, use the left and right Distributive Properties. For example, if you treat $5x + 7$ as a single quantity, you can multiply $3x - 2$ by $5x + 7$ as follows.

$$\begin{aligned} (3x - 2)(5x + 7) &= 3x(5x + 7) - 2(5x + 7) \\ &= (3x)(5x) + (3x)(7) - (2)(5x) - (2)(7) \\ &= 15x^2 + 21x - 10x - 14 \end{aligned}$$

Product of First terms | Product of Outer terms | Product of Inner terms | Product of Last terms

$$= 15x^2 + 11x - 14$$

Note in this **FOIL Method** (which can only be used to multiply two binomials) that the outer (O) and inner (I) terms are like terms and can be combined.

Example 3 Finding a Product by the FOIL Method

Use the FOIL Method to find the product of $2x - 4$ and $x + 5$.

Solution

$$\begin{aligned} (2x - 4)(x + 5) &= \overset{F}{2x^2} + \overset{O}{10x} - \overset{I}{4x} - \overset{L}{20} \\ &= 2x^2 + 6x - 20 \end{aligned}$$

CHECK*Point* Now try Exercise 59.

When multiplying two polynomials, be sure to multiply each term of one polynomial by *each* term of the other. A vertical arrangement is helpful.

Example 4 A Vertical Arrangement for Multiplication

Multiply $x^2 - 2x + 2$ by $x^2 + 2x + 2$ using a vertical arrangement.

Solution

$$\begin{array}{rrrrrl} & x^2 & -2x & +2 & & \text{Write in standard form.} \\ \times & x^2 & +2x & +2 & & \text{Write in standard form.} \\ \hline x^4 - 2x^3 + 2x^2 & & & & & x^2(x^2 - 2x + 2) \\ 2x^3 - 4x^2 + 4x & & & & & 2x(x^2 - 2x + 2) \\ 2x^2 - 4x + 4 & & & & & 2(x^2 - 2x + 2) \\ \hline x^4 + 0x^3 + 0x^2 + 0x + 4 = x^4 + 4 & & & & & \text{Combine like terms.} \end{array}$$

So, $(x^2 - 2x + 2)(x^2 + 2x + 2) = x^4 + 4$.

CHECKPoint Now try Exercise 61.

Special Products

Some binomial products have special forms that occur frequently in algebra. You do not need to memorize these formulas because you can use the Distributive Property to multiply. However, becoming familiar with these formulas will enable you to manipulate the algebra more quickly.

Special Products

Let u and v be real numbers, variables, or algebraic expressions.

Special Product	*Example*
Sum and Difference of Same Terms	
$(u + v)(u - v) = u^2 - v^2$	$(x + 4)(x - 4) = x^2 - 4^2$ $= x^2 - 16$
Square of a Binomial	
$(u + v)^2 = u^2 + 2uv + v^2$	$(x + 3)^2 = x^2 + 2(x)(3) + 3^2$ $= x^2 + 6x + 9$
$(u - v)^2 = u^2 - 2uv + v^2$	$(3x - 2)^2 = (3x)^2 - 2(3x)(2) + 2^2$ $= 9x^2 - 12x + 4$
Cube of a Binomial	
$(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$	$(x + 2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3$ $= x^3 + 6x^2 + 12x + 8$
$(u - v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$	$(x - 1)^3 = x^3 - 3x^2(1) + 3x(1^2) - 1^3$ $= x^3 - 3x^2 + 3x - 1$

Example 5 Sum and Difference of Same Terms

Find the product of $5x + 9$ and $5x - 9$.

Solution

The product of a sum and a difference of the *same* two terms has no middle term and takes the form $(u + v)(u - v) = u^2 - v^2$.

$$(5x + 9)(5x - 9) = (5x)^2 - 9^2 = 25x^2 - 81$$

CHECK*Point* Now try Exercise 67.

Study Tip

When squaring a binomial, note that the resulting middle term is always *twice* the product of the two terms.

Example 6 Square of a Binomial

Find $(6x - 5)^2$.

Solution

The square of a binomial has the form $(u - v)^2 = u^2 - 2uv + v^2$.

$$(6x - 5)^2 = (6x)^2 - 2(6x)(5) + 5^2 = 36x^2 - 60x + 25$$

CHECK*Point* Now try Exercise 71.

Example 7 Cube of a Binomial

Find $(3x + 2)^3$.

Solution

The cube of a binomial has the form

$$(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3.$$

Note the *decreasing* powers of $u = 3x$ and the *increasing* powers of $v = 2$.

$$\begin{aligned}(3x + 2)^3 &= (3x)^3 + 3(3x)^2(2) + 3(3x)(2^2) + 2^3 \\ &= 27x^3 + 54x^2 + 36x + 8\end{aligned}$$

CHECK*Point* Now try Exercise 73.

Example 8 The Product of Two Trinomials

Find the product of $x + y - 2$ and $x + y + 2$.

Solution

By grouping $x + y$ in parentheses, you can write the product of the trinomials as a special product.

$$\begin{aligned}(x + y - 2)(x + y + 2) &= [(x + y) \underset{\text{Difference}}{-} 2][(x + y) \underset{\text{Sum}}{+} 2] \\ &= (x + y)^2 - 2^2 && \text{Sum and difference of same terms} \\ &= x^2 + 2xy + y^2 - 4\end{aligned}$$

CHECK*Point* Now try Exercise 81.

Application

Example 9 Volume of a Box

An open box is made by cutting squares from the corners of a piece of metal that is 16 inches by 20 inches, as shown in Figure P.13. The edge of each cut-out square is x inches. Find the volume of the box when $x = 1$, $x = 2$, and $x = 3$.

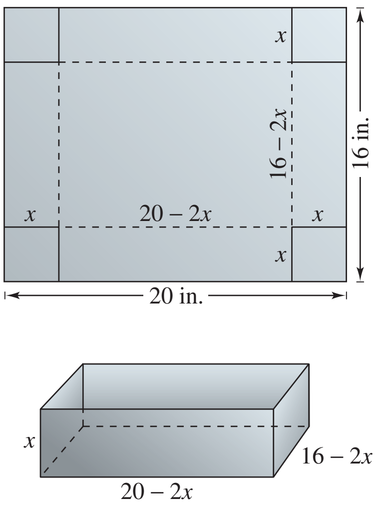


FIGURE P.13

Solution

The volume of a rectangular box is equal to the product of its length, width, and height. From the figure, the length is $20 - 2x$, the width is $16 - 2x$, and the height is x. So, the volume of the box is

$$\begin{aligned}\text{Volume} &= (20 - 2x)(16 - 2x)(x)\\ &= (320 - 72x + 4x^2)(x)\\ &= 320x - 72x^2 + 4x^3.\end{aligned}$$

When $x = 1$ inch, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(1) - 72(1)^2 + 4(1)^3\\ &= 252 \text{ cubic inches.}\end{aligned}$$

When $x = 2$ inches, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(2) - 72(2)^2 + 4(2)^3\\ &= 384 \text{ cubic inches.}\end{aligned}$$

When $x = 3$ inches, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(3) - 72(3)^2 + 4(3)^3\\ &= 420 \text{ cubic inches.}\end{aligned}$$

CHECKPoint Now try Exercise 109.

Classroom Discussion

Mathematical Experiment In Example 9, the volume of the open box is given by

$$\text{Volume} = 320x - 72x^2 + 4x^3.$$

You want to create a box that has as much volume as possible. From Example 9, you know that by cutting one-, two-, and three-inch squares from the corners, you can create boxes whose volumes are 252, 384, and 420 cubic inches, respectively. What are the possible values of x that make sense in this problem? Write your answer as an interval. Try several other values of x to find the size of the squares that should be cut from the corners to produce a box that has maximum volume. Write a summary of your findings.

P.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY

In Exercises 1–5, fill in the blanks.

1. For the polynomial $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, $a_n \neq 0$, the degree is ________, the leading coefficient is ________, and the constant term is ________.

2. A polynomial in x in standard form is written with ________ powers of x.

3. A polynomial with one term is called a ________, while a polynomial with two terms is called a ________, and a polynomial with three terms is called a ________.

4. To add or subtract polynomials, add or subtract the ________ ________ by adding their coefficients.

5. The letters in "FOIL" stand for the following. F ________ O ________ I ________ L ________

In Exercises 6–8, match the special product form with its name.

6. $(u + v)(u - v) = u^2 - v^2$ (a) A binomial sum squared

7. $(u + v)^2 = u^2 + 2uv + v^2$ (b) A binomial difference squared

8. $(u - v)^2 = u^2 - 2uv + v^2$ (c) The sum and difference of same terms

SKILLS AND APPLICATIONS

In Exercises 9–14, match the polynomial with its description. [The polynomials are labeled (a), (b), (c), (d), (e), and (f).]

(a) $3x^2$ (b) $1 - 2x^3$

(c) $x^3 + 3x^2 + 3x + 1$ (d) 12

(e) $-3x^5 + 2x^3 + x$ (f) $\frac{2}{3}x^4 + x^2 + 10$

9. A polynomial of degree 0

10. A trinomial of degree 5

11. A binomial with leading coefficient -2

12. A monomial of positive degree

13. A trinomial with leading coefficient $\frac{2}{3}$

14. A third-degree polynomial with leading coefficient 1

In Exercises 15–18, write a polynomial that fits the description. (There are many correct answers.)

15. A third-degree polynomial with leading coefficient -2

16. A fifth-degree polynomial with leading coefficient 6

17. A fourth-degree binomial with a negative leading coefficient

18. A third-degree binomial with an even leading coefficient

In Exercises 19–30, (a) write the polynomial in standard form, (b) identify the degree and leading coefficient of the polynomial, and (c) state whether the polynomial is a monomial, a binomial, or a trinomial.

19. $14x - \frac{1}{2}x^5$ **20.** $2x^2 - x + 1$

21. $x^2 - 4 - 3x^4$ **22.** $7x$

23. $3 - x^6$ **24.** $-y + 25y^2 + 1$

25. 3 **26.** $-8 + t^2$

27. $1 + 6x^4 - 4x^5$ **28.** $3 + 2x$

29. $4x^3y$ **30.** $-x^5y + 2x^2y^2 + xy^4$

In Exercises 31–36, determine whether the expression is a polynomial. If so, write the polynomial in standard form.

31. $2x - 3x^3 + 8$ **32.** $5x^4 - 2x^2 + x^{-2}$

33. $\dfrac{3x + 4}{x}$ **34.** $\dfrac{x^2 + 2x - 3}{2}$

35. $y^2 - y^4 + y^3$ **36.** $y^4 - \sqrt{y}$

In Exercises 37–54, perform the operation and write the result in standard form.

37. $(6x + 5) - (8x + 15)$

38. $(2x^2 + 1) - (x^2 - 2x + 1)$

39. $-(t^3 - 1) + (6t^3 - 5t)$

40. $-(5x^2 - 1) - (-3x^2 + 5)$

41. $(15x^2 - 6) - (-8.3x^3 - 14.7x^2 - 17)$

42. $(15.6w^4 - 14w - 17.4) - (16.9w^4 - 9.2w + 13)$

43. $5z - [3z - (10z + 8)]$

44. $(y^3 + 1) - [(y^2 + 1) + (3y - 7)]$

45. $3x(x^2 - 2x + 1)$ **46.** $y^2(4y^2 + 2y - 3)$

47. $-5z(3z - 1)$ **48.** $(-3x)(5x + 2)$

49. $(1 - x^3)(4x)$ **50.** $-4x(3 - x^3)$

51. $(1.5t^2 + 5)(-3t)$ **52.** $(2 - 3.5y)(2y^3)$

53. $-2x(0.1x + 17)$ **54.** $6y(5 - \frac{3}{8}y)$

In Exercises 55–62, perform the operation.

55. Add $7x^3 - 2x^2 + 8$ and $-3x^3 - 4$.

56. Add $2x^5 - 3x^3 + 2x + 3$ and $4x^3 + x - 6$.

57. Subtract $x - 3$ from $5x^2 - 3x + 8$.

58. Subtract $-t^4 + 0.5t^2 - 5.6$ from $0.6t^4 - 2t^2$.

59. Multiply $(x + 7)$ and $(2x + 3)$.

60. Multiply $(3x + 1)$ and $(x - 5)$.

61. Multiply $(x^2 + 2x + 3)$ and $(x^2 - 2x + 3)$.

62. Multiply $(x^2 + x - 4)$ and $(x^2 - 2x + 1)$.

In Exercises 63–100, multiply or find the special product.

63. $(x + 3)(x + 4)$ **64.** $(x - 5)(x + 10)$

65. $(3x - 5)(2x + 1)$ **66.** $(7x - 2)(4x - 3)$

67. $(x + 10)(x - 10)$ **68.** $(2x + 3)(2x - 3)$

69. $(x + 2y)(x - 2y)$ **70.** $(4a + 5b)(4a - 5b)$

71. $(2x + 3)^2$ **72.** $(5 - 8x)^2$

73. $(x + 1)^3$ **74.** $(x - 2)^3$

75. $(2x - y)^3$ **76.** $(3x + 2y)^3$

77. $(4x^3 - 3)^2$ **78.** $(8x + 3)^2$

79. $(x^2 - x + 1)(x^2 + x + 1)$

80. $(x^2 + 3x - 2)(x^2 - 3x - 2)$

81. $(-x^2 + x - 5)(3x^2 + 4x + 1)$

82. $(2x^2 - x + 4)(x^2 + 3x + 2)$

83. $[(m - 3) + n][(m - 3) - n]$

84. $[(x - 3y) + z][(x - 3y) - z]$

85. $[(x - 3) + y]^2$ **86.** $[(x + 1) - y]^2$

87. $(2r^2 - 5)(2r^2 + 5)$ **88.** $(3a^3 - 4b^2)(3a^3 + 4b^2)$

89. $\left(\frac{1}{4}x - 5\right)^2$ **90.** $\left(\frac{3}{5}t + 4\right)^2$

91. $\left(\frac{1}{5}x - 3\right)\left(\frac{1}{5}x + 3\right)$ **92.** $\left(3x + \frac{1}{6}\right)\left(3x - \frac{1}{6}\right)$

93. $(2.4x + 3)^2$

94. $(1.8y - 5)^2$

95. $(1.5x - 4)(1.5x + 4)$

96. $(2.5y + 3)(2.5y - 3)$

97. $5x(x + 1) - 3x(x + 1)$

98. $(2x - 1)(x + 3) + 3(x + 3)$

99. $(u + 2)(u - 2)(u^2 + 4)$

100. $(x + y)(x - y)(x^2 + y^2)$

In Exercises 101–104, find the product. (The expressions are not polynomials, but the formulas can still be used.)

101. $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right)$

102. $\left(5 + \sqrt{x}\right)\left(5 - \sqrt{x}\right)$

103. $\left(x - \sqrt{5}\right)^2$

104. $\left(x + \sqrt{3}\right)^2$

105. COST, REVENUE, AND PROFIT An electronics manufacturer can produce and sell x MP3 players per week. The total cost C (in dollars) of producing x MP3 players is $C = 73x + 25{,}000$, and the total revenue R (in dollars) is $R = 95x$.

(a) Find the profit P in terms of x.

(b) Find the profit obtained by selling 5000 MP3 players per week.

106. COST, REVENUE, AND PROFIT An artisan can produce and sell x hats per month. The total cost C (in dollars) for producing x hats is $C = 460 + 12x$, and the total revenue R (in dollars) is $R = 36x$.

(a) Find the profit P in terms of x.

(b) Find the profit obtained by selling 42 hats per month.

107. COMPOUND INTEREST After 2 years, an investment of \$500 compounded annually at an interest rate r will yield an amount of $500(1 + r)^2$.

(a) Write this polynomial in standard form.

(b) Use a calculator to evaluate the polynomial for the values of r shown in the table.

r	$2\frac{1}{2}\%$	3%	4%	$4\frac{1}{2}\%$	5%
$500(1 + r)^2$					

(c) What conclusion can you make from the table?

108. COMPOUND INTEREST After 3 years, an investment of \$1200 compounded annually at an interest rate r will yield an amount of $1200(1 + r)^3$.

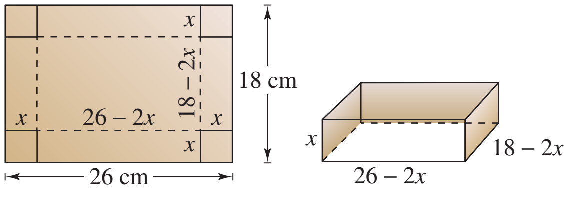
(a) Write this polynomial in standard form.

(b) Use a calculator to evaluate the polynomial for the values of r shown in the table.

r	2%	3%	$3\frac{1}{2}\%$	4%	$4\frac{1}{2}\%$
$1200(1 + r)^3$					

(c) What conclusion can you make from the table?

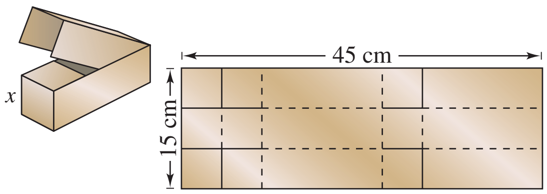
109. VOLUME OF A BOX A take-out fast-food restaurant is constructing an open box by cutting squares from the corners of a piece of cardboard that is 18 centimeters by 26 centimeters (see figure). The edge of each cut-out square is x centimeters.



(a) Find the volume of the box in terms of x.

(b) Find the volume when $x = 1$, $x = 2$, and $x = 3$.

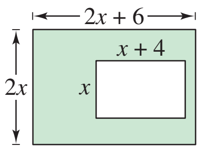
110. VOLUME OF A BOX An overnight shipping company is designing a closed box by cutting along the solid lines and folding along the broken lines on the rectangular piece of corrugated cardboard shown in the figure. The length and width of the rectangle are 45 centimeters and 15 centimeters, respectively.

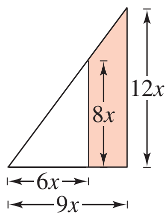


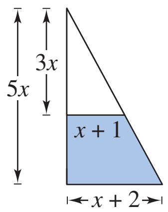
(a) Find the volume of the shipping box in terms of x.

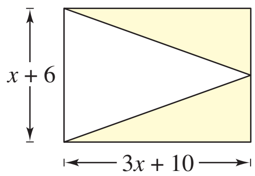
(b) Find the volume when $x = 3$, $x = 5$, and $x = 7$.

111. GEOMETRY Find the area of the shaded region in each figure. Write your result as a polynomial in standard form.

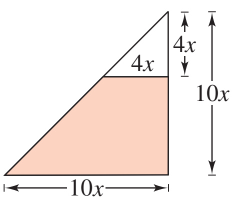
(a)


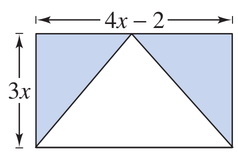
(b)


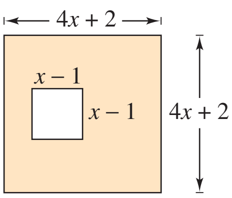
(c)


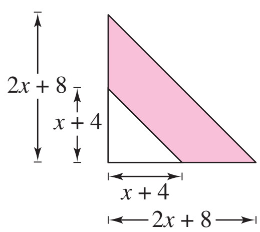
(d)


112. GEOMETRY Find the area of the shaded region in each figure. Write your result as a polynomial in standard form.

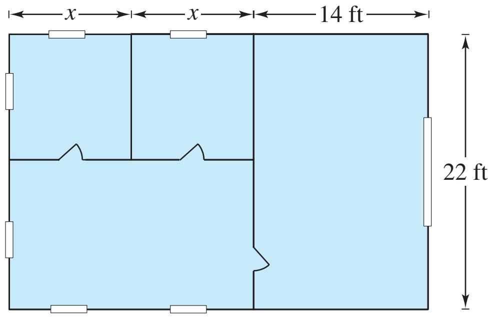
(a)


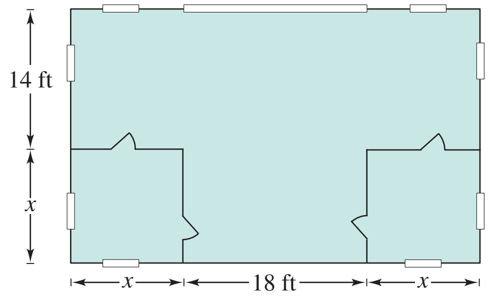
(b)


(c)


(d)


GEOMETRY In Exercises 113 and 114, find a polynomial that represents the total number of square feet for the floor plan shown in the figure.

113.


114.


115. ENGINEERING A uniformly distributed load is placed on a one-inch-wide steel beam. When the span of the beam is x feet and its depth is 6 inches, the safe load S (in pounds) is approximated by

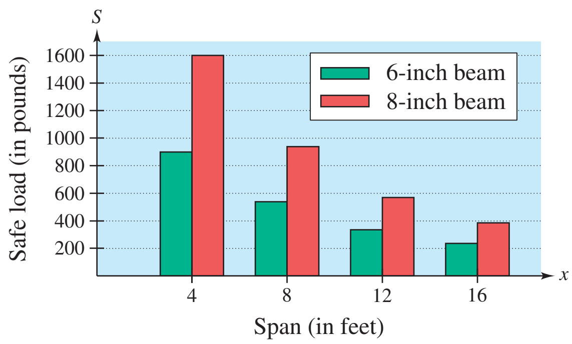
$$S_6 = (0.06x^2 - 2.42x + 38.71)^2.$$

When the depth is 8 inches, the safe load is approximated by

$$S_8 = (0.08x^2 - 3.30x + 51.93)^2.$$

(a) Use the bar graph to estimate the difference in the safe loads for these two beams when the span is 12 feet.

(b) How does the difference in safe load change as the span increases?



116. STOPPING DISTANCE The stopping distance of an automobile is the distance traveled during the driver's reaction time plus the distance traveled after the brakes are applied. In an experiment, these distances were measured (in feet) when the automobile was traveling at a speed of x miles per hour on dry, level pavement, as shown in the bar graph. The distance traveled during the reaction time R was

$$R = 1.1x$$

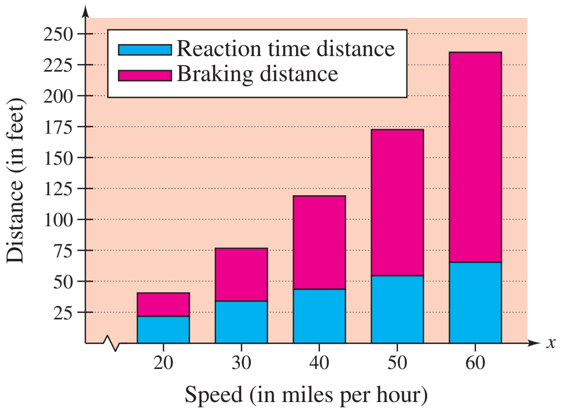
and the braking distance B was

$$B = 0.0475x^2 - 0.001x + 0.23.$$

(a) Determine the polynomial that represents the total stopping distance T.

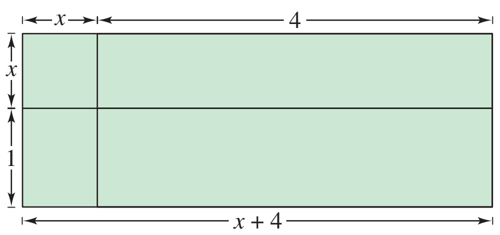
(b) Use the result of part (a) to estimate the total stopping distance when $x = 30$, $x = 40$, and $x = 55$ miles per hour.

(c) Use the bar graph to make a statement about the total stopping distance required for increasing speeds.

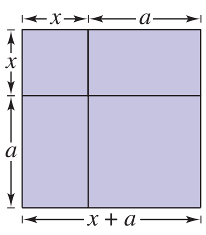


GEOMETRY In Exercises 117 and 118, use the area model to write two different expressions for the area. Then equate the two expressions and name the algebraic property that is illustrated.

117.



118.



EXPLORATION

TRUE OR FALSE? In Exercises 119 and 120, determine whether the statement is true or false. Justify your answer.

119. The product of two binomials is always a second-degree polynomial.

120. The sum of two binomials is always a binomial.

121. Find the degree of the product of two polynomials of degrees m and n.

122. Find the degree of the sum of two polynomials of degrees m and n if $m < n$.

123. WRITING A student's homework paper included the following.

$$\cancel{(x - 3)^2 = x^2 + 9}$$

Write a paragraph fully explaining the error and give the correct method for squaring a binomial.

124. CAPSTONE A third-degree polynomial and a fourth-degree polynomial are added.

(a) Can the sum be a fourth-degree polynomial? Explain or give an example.

(b) Can the sum be a second-degree polynomial? Explain or give an example.

(c) Can the sum be a seventh-degree polynomial? Explain or give an example.

125. THINK ABOUT IT Must the sum of two second-degree polynomials be a second-degree polynomial? If not, give an example.

126. THINK ABOUT IT When the polynomial

$$-x^3 + 3x^2 + 2x - 1$$

is subtracted from an unknown polynomial, the difference is $5x^2 + 8$. If it is possible, find the unknown polynomial.

127. LOGICAL REASONING Verify that $(x + y)^2$ is not equal to $x^2 + y^2$ by letting $x = 3$ and $y = 4$ and evaluating both expressions. Are there any values of x and y for which $(x + y)^2 = x^2 + y^2$? Explain.