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Ans. 1) Let $P = \sum_K \sum_l |F(k,l) - \hat{F}(k,l)|^2 + \lambda |\hat{F}(k,l) \cdot L(k,l)|^2$

A

where, $\hat{F}(k,l) = W(k,l) \cdot G(k,l)$

and, $G(k,l) = H(k,l) \cdot F(k,l) + N(k,l)$

we can write, $P = \sum_K \sum_l |F - WHF - WN|^2 + \lambda |WHFL + WNL|^2$ — (1)

we know that, $\sum_K \sum_l |a-b|^2 = \sum_K \sum_l |a|^2 + |b|^2$

Put this in (1).

$$P = \sum_K \sum_l |F|^2 |1-HW|^2 + |WN|^2 + \lambda |WL|^2 [|HF|^2 + |N|^2]$$

$$\frac{\partial P}{\partial W} = \cancel{2} \cdot |F|^2 (1-HW)^*(-H) + \cancel{2} \cdot |N|^2 \cdot (W)^* + \cancel{2}\lambda \cdot |L|^2 (W)^* [|HF|^2 + |N|^2] = 0$$

$$-H|F|^2 + W^*(|FH|^2) + W^* |N|^2 + W^* [\lambda \cdot (|HF|^2 + |N|^2) \cdot |L|^2] = 0$$

$$W^* = \frac{H |F|^2}{|FH|^2 + |N|^2 + \lambda |L|^2 (|H \cdot F|^2 + |N|^2)}$$

$$W^* = \frac{H}{|H|^2 + \frac{|N|^2}{|F|^2} + \lambda \cdot |L|^2 \cdot \left(|H|^2 + \frac{|N|^2}{|F|^2}\right)}$$

Conjugate both sides,

$$\boxed{W = \frac{H^*}{|H|^2 + C + \lambda \cdot [C + |H|^2] \cdot |L|^2}}$$ where, $C = \frac{|N|^2}{|F|^2}$

P.T.O.

Ans. 2) A $f(x,y) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 100 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

$$g(\rho,\theta) = \sum_{0}^{N-1}\sum_{0}^{M-1} f(x,y)\cdot \delta(x\cos\theta + y\sin\theta - \rho)$$

For $\theta = 0$.

$$g(\rho,0) = \sum_{-1}^{1}\sum_{-1}^{1} f(x,y)\cdot \delta(x-\rho)$$

$$g(\rho,0) = \sum_{y=-1}^{1} f(\rho,y) \cancel{= f(\rho,-1) + f(\rho,0) + f(\rho,1)}$$

Now, $g(0,0) = 100+2 = 102$

$g(-1,0) = 1+1 = 2$

$g(+1,0) = 1+2+2 = 5$

$$\Rightarrow \begin{bmatrix} 2 & 102 & 5 \\ 2 & 102 & 5 \\ 2 & 102 & 5 \end{bmatrix}$$

For $\theta = \frac{\pi}{2}$.

$$g\left(\rho,\frac{\pi}{2}\right) = \sum_{x=-1}^{1} f(x,\rho)$$

Now, $g\left(0,\frac{\pi}{2}\right) = 100+2 = 102$

$g\left(-1,\frac{\pi}{2}\right) = 2+2+1 = 5$

$g\left(+1,\frac{\pi}{2}\right) = 1+1 = 2$

$$\Rightarrow \begin{bmatrix} 2 & 2 & 2 \\ 102 & 102 & 102 \\ 5 & 5 & 5 \end{bmatrix}$$

For $\theta = \frac{\pi}{4}$.

$$g\left(\rho,\frac{\pi}{4}\right) = \sum_{-1}^{1}\sum_{-1}^{1} f(x,y)\cdot \delta\left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} - \rho\right)$$

$$g\left(\rho,\frac{\pi}{4}\right) = \sum_{y=-1}^{1}\sum_{x=-1}^{1} f(x,y)\cdot \delta\left(\frac{x+y}{\sqrt{2}} - \rho\right)$$

Now, $g\left(0, \frac{\pi}{4}\right) = \sum_{-1}^{1} \sum_{-1}^{1} f(x, y) \cdot \delta\left(\frac{x+y}{\sqrt{2}} - 0\right)$

$\hookrightarrow x+y=0 \Rightarrow \left((-1,1), (0,0), (1,-1) \right)$

$\therefore g\left(0, \frac{\pi}{4}\right) = f(-1,1) + f(0,0) + f(1,-1) = 1 + 100 + 2 = 103$

$g\left(-1, \frac{\pi}{4}\right) = \sum_{-1}^{1} \sum_{-1}^{1} f(x, y) \cdot \delta\left(\frac{x+y}{\sqrt{2}} + 1\right)$

$\hookrightarrow x+y = -\sqrt{2}$ (Not Possible).

$\therefore g\left(-1, \frac{\pi}{4}\right) = 0$

$g\left(1, \frac{\pi}{4}\right) = \sum_{-1}^{1} \sum_{-1}^{1} f(x, y) \cdot \delta\left(\frac{x+y}{\sqrt{2}} - 1\right)$

$\hookrightarrow x+y = \sqrt{2}$ (Not Possible).

$g\left(1, \frac{\pi}{4}\right) = 0$

(B)

$f_\theta(x, y) = g(x\cos\theta + y\sin\theta, \theta) = g(\rho, \theta).$

$$f_0(x, y) = \begin{bmatrix} 2 & 102 & 5 \\ 2 & 102 & 5 \\ 2 & 102 & 5 \end{bmatrix}$$

$$f_{\frac{\pi}{2}}(x, y) = \begin{bmatrix} 2 & 2 & 2 \\ 102 & 102 & 102 \\ 5 & 5 & 5 \end{bmatrix}$$

$$f_{\frac{\pi}{4}}(x, y) = \begin{bmatrix} 103 & 0 & 0 \\ 0 & 103 & 0 \\ 0 & 0 & 103 \end{bmatrix}$$

P.T.O.

(c) $f(x,y) = \sum_{\theta=0}^{\pi} f_\theta(x,y) = f_0(x,y) + f_{\frac{\pi}{4}}(x,y) + f_{\frac{\pi}{2}}(x,y)$.

$$f(x,y) = \begin{bmatrix} 2 & 102 & 5 \\ 2 & 102 & 5 \\ 2 & 102 & 5 \end{bmatrix} + \begin{bmatrix} 103 & 0 & 0 \\ 0 & 103 & 0 \\ 0 & 0 & 103 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 \\ 102 & 102 & 102 \\ 5 & 5 & 5 \end{bmatrix}$$

$$f(x,y) = \begin{bmatrix} 107 & 104 & 7 \\ 104 & 307 & 107 \\ 7 & 107 & 113 \end{bmatrix}$$

Ans $\Rightarrow$