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# On the Efficiency of a Game Theoretic Approach to Sparse Regenerator Placement in WDM Networks

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**Abstract**—In this paper we provide a mathematical ILP model for the Regeneration Placement Problem (RPP) which minimizes the total number of regeneration nodes allocated in a translucent optical network ensuring that all the node pairs can always reach one another via two link-disjoint lightpaths under physical-impairment constraints. Since RPP is NP-complete, large-site design problem can not be solved relying upon exact approaches. We then propose a game-theoretic approach to model RPP as a non-cooperative game and solve it applying the *best response dynamic* concept. Finally, we evaluate the performance of the proposed approach in terms of closeness of the obtained results to these provided by ILP: a MILP formulation is given in order to study the quality of the Nash equilibria by comparison to Price-of-Anarchy and Price-of-Stability bounds.

**Index Terms**—Optical network, WDM, translucent network, regeneration node, ILP, game theory, congestion game, price-of-anarchy, greedy algorithm

## I. INTRODUCTION

Optical networks provide a transport infrastructure with very high capacity, thanks to Wavelength-Division-Multiplexing (WDM) technology. WDM optical networks have been classified in [1] into transparent and opaque based on the network capability to set-up end-to-end communication of data, independent of bit rates, and signal formats. In a transparent optical network a connection bypasses any expensive electronic signal processing at intermediate nodes. However, transparent optical networks can not be practically deployed on a large scale because the negative effect of transmission impairments on the signal quality after a connection travels through several optical components. In the opposite, an opaque optical network incorporates such signal regeneration at every intermediate node along the lightpath. However this kind of network is very expensive and energy consuming because every wavelength needs Optical-Electronic-Optical (OEO) conversions at all the intermediate nodes.

In this paper we investigate an intermediate solution, which benefits of the trade-off between efficiency and cost [2]. In a translucent optical network a signal traverses many links without regenerations and then undergoes an OEO conversion before its quality falls below a threshold value. Because a signal is regenerated only if necessary, we need fewer regeneration resources, that we call Regenerating Nodes (RNs).

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To efficiently place RNs in a translucent network, two optimization approaches are recently presented in literature [3]. One is an exact optimization which minimizes the total number of RNs allocated in a network ensuring that all the node pairs can always reach one another via two Shared Link Risk Group (SRLG)-disjoint transparent or translucent lightpaths. Unfortunately, network design related to this problem, called Regenerator Placement Problem (RPP), is known to be NP-Complete [4] and any exact approach is unapplicable on large network instances. Opposite to exact optimization, the non-cooperative game approach naturally deals with RPP in an approximated but direct manner. According to such method, each pair of nodes behaves like a player of a game which acts selfishly pursuing its specific objective. In order to reduce the overall number of regenerators in the network, the objective set for each node pair has to maximize the OEO sharing degree on the RNs. It is a well-known fact that the resulting Nash equilibrium may not be Pareto efficient [5]. We then address a fundamental issue related to the Nash equilibria reachable by the game theoretic approach: how close are the solutions to the one provided by exact optimization? For this purpose, we derive non-trivial upper bounds for the *Price-of-Anarchy* (PoA) and the *Price-of-Stability* (PoS). PoA is the measure which compares the worst case performance Nash equilibrium to that of the optimal allocation. In other words, PoA is the pessimistic point of view in which the players are guided to play at the worst Nash equilibrium. On the other hand, PoS is the measure which compares the best case performance Nash equilibrium to that of the optimal allocation. So, we propose a mathematical programming model to characterize the PoA and PoS, and we thoroughly comment on the quality of the game equilibria through numerical results.

The rest of this paper is organized as follows. Section II presents a survey of recent research results in planning of translucent networks. In Section III we briefly describe the translucent optical network model utilized in this paper. In section IV we formally state the RPP, and we introduce a mathematical Integer Linear Program (ILP) formulation. In Section V we model the RPP like a non-cooperative game, comment on its properties and propose a Mixed Integer Linear Program (MILP) to determine and characterize its Nash equilibria. Section VI evaluates by simulations the performance of exact optimization compared to the approximated one. Section VII draws some conclusions.

## II. PRIOR WORK

Many factors may degrade optical signal quality in long-reach optical transmission systems to the extent of making the data unrecognizable at the receiver. Recently, intensive researches on inclusion of physical layer effects in network design and routing problems have been performed, e.g., [6] [7]. In details, the degradation of Bit-Error-Ratio (BER) inherent to propagation is mainly due to five physical effects known as Crosstalk (XT), Chromatic Dispersion (CD), Polarization Mode Dispersion (PMD), Amplified Spontaneous Emission (ASE) and Nonlinear Phase Shift (NPS) [8]. A lightpath is said admissible if BER at its destination node remains under a given threshold. In case on BER non-admissibility, signal regeneration is mandatory to re-amplify, reshape, and retune the optical signals. Optical translucent networks use a set of sparsely but strategically placed regenerator for this purpose.

We can broadly classify translucent optical networks into different categories, depending on the architecture and deployment policy of regenerators in the switching nodes. The authors in [2] assume the networks to be characterized by hybrid optical switches which contains both electronic and optical switching cores. Then, each lightpath through a translucent node can be switched either all-optically via the optical module, or through the electronic core module to regenerate the optical signal. However, in the perspective of an optical transport networks evolution, a translucent networks may contain some isolated domains of transparency in which the regeneration processing is reserved only to RNs that are located on the subnetwork boundary while the interior nodes are entirely transparent [9][10]. Moreover, advances in optical technologies will lead the size of the transparent domains to become large and consequently more overlapped. Therefore, instead of a boundary deployment of regenerators, the RNs can be sparsely deployed in the network, increasing their sharing by lightpaths and their cost effectiveness [11]. In [12], the RNs are place on sites which have a higher chance that the signal regeneration capability may be required, assuming that each node pair needs a lightpath routed along the shortest path. Alternatively, the authors in [13] propose a greedy approach which allocates RNs, step-by-step, on sites with higher nodal degrees, until the network is fully connected. The obtained RN placements guarantees multiple transparent segments between RNs which enable survivable routing.

Optical network design have to provide reliability. In [14], algorithms are proposed to compute the minimum-cost subset of nodes which must be provided with regeneration capability to guarantee path protection against any single link failure in the network. To obtain a transport network which offers either resilience against regeneration failures and load balancing benefits during routing, in [15] regenerators placements guarantees the desired number of link-disjoint lightpaths for each connection request (k-degree dedicated protection).

The RPP of large-scale photonic mesh networks cannot be handled by exact approaches with an acceptable level of computational effort. The entire optimization problem may however be solved in approximate manners. To the best of the

authors' knowledge, a game theory model has been developed<sup>2</sup> for translucent networks optimization and design for the first time in [3]. Its strength is the capability of rapidly finding solutions that guarantee reachability for any wavelength path routed in the network under physical impairment constraints. The drawback is that finding the absolute optimum solution is not guaranteed, as locally optimal solutions may be incurred.

## III. TRANSLUCENT OPTICAL NETWORK MODEL

Optical signals propagating in transparency experience a variety of quality-degrading phenomena that introduce different types of signal distortions. These impairments accumulate along the path and limit the maximum transparency reach and the overall network performance. Unfortunately, a complicated analysis is required to solve the impairment aware Regenerator Placement Problem (RPP) because both linear and nonlinear effects should be considered. This paper focuses on the planning method rather than physical modeling of transmission: thus, we can simplify the problem introducing the following basic assumption. The effect of all the physical impairments are accounted for by deterministically-limit the transparency reach of all the lightpaths (independently of their specific paths and system crossed).

Let us now formally define the RPP. A network is represented by the directed graph  $G = (V, E, L)$ , where  $V$  is the set of nodes,  $E$  is the set of unidirectional fibers (referred to as links), and  $L : E \rightarrow R^+$  is a function that maps the elements in  $E$  to positive real numbers representing the link length. The reach limit  $L_{max}$  is the maximum distance a signal can travel without OEO regeneration. Given the network topology, the final purpose is to place a minimum number of RNs, leaving the others entirely transparent, in order that, between any nodes pair, there are at least two SRLG-disjoint paths feasible under the constraint imposed by propagation impairments. This is equivalent to have a full-mesh connection demand denoted by the set  $C = \{c\}$  of  $|V|(|V| - 1)$  demands. For each  $c^{th}$  demand we define a set  $K_c = \{c^k\}$  which specifies  $|K_c|$  couples of working/backup paths defined in off-line phase using Yen's algorithm [16]. Typically a  $k$ -shortest-path algorithm is applied to compute  $k$  candidate working paths and for each one, a link-disjoint backup path is computed. In this way the connectivity between any node pair is guaranteed against any single link failure. For each  $c^k$  couple of working/backup paths we associate a set  $P_{c,k} = \{c^{k,p}\}$  of all the feasible RNs placements along those paths, where  $c^{k,p}$  denotes the  $p^{th}$  feasible RN placement for working/backup paths couple  $k$  of connection  $c^{th}$ . In particular, a RN placement is feasible if the longest transparent span included in both working and backup paths is shorter than the reach limit  $L_{max}$ .

Two different approaches are used to solve the RPP. The first one involves an exact mathematical formulation (OPT-RPP) which solves the problem at the optimum. The other one is a game theory model (GT-RPP) that solves RPP in approximated fashion by a greedy procedure, which is the *best response dynamic* [17]. Both models rely on the pre-

calculation of working/backup path pairs and all the feasible OEO regenerator placements along them.

#### IV. OPT-RPP: AN EXACT APPROACH

An ILP formulation for the RPP with pre-calculated paths can be found in [3]. Differently to that work, we relax the assumption which constraints the number of RNs along a path to one.

Let us define all the variables involved in this network planning formulation:

- $x_c^{k,p}$  is a boolean variable indicating whenever feasible RN placement  $p$  along the working/backup paths of couple  $k$  is adopted or not;
- $y_v$  is a boolean variable which specifies if the node  $v$  is selected (promoted) as RN or not.

The following additional boolean symbols are also defined:

- $r_v^{c,k,p}$  is set to 1 if connection  $c$ , in configuration  $\{k,p\}$ , requires that node  $v$  is a RN.

Now we can detail a route-based formulation. The cost function to be minimized is the total number of RNs allocated in network

$$\min \sum_{v \in V} y_v$$

The set of constraints is following

$$\sum_{k \in K_c} \sum_{p \in P_{c,k}} x_c^{k,p} = 1 \quad \forall c \in C, \quad (1)$$

$$y_v \geq x_c^{k,p} \cdot r_v^{c,k,p} \quad \forall v \in V. \quad (2)$$

This formulation assigns a working/backup paths couple to each connection demand in respect to the maximal reach distance constraint. Constraints (1) assure that each connection selects one and only one working/backup paths couple with a feasible RN placement. Constraints (2) enforce the RN promotion on sites where at least one connection demand requires OEO regeneration. The previous model has  $|C| + |V|$  constraints and  $|C| \cdot |K| \cdot |P| \cdot (1 + |V|)$  variables.

#### V. GT-RPP: A GAME THEORETICAL APPROACH

In the following we provide a complete description of our game theoretical approach which we will call hereafter GT-RPP. Then, we will propose an analytical exact formulation to estimate the efficiency of GT-RPP.

##### A. Game Theoretic Procedure

Our approach extends the one proposed in [3] in sense of a greater flexibility and generality in a similar way to OPT-RLP. A formal specification of GT-RPP is given in Algorithm 1, in which the game theoretic formulation is used as an optimization procedure. The game is defined assuming that the  $|C|$  connection demands represent the players of the game. Therefore, each pair of nodes behave like a player of a game who acts selfishly to route two end-to-end SRLG-disjoint lightpaths under the propagation impairments constraints. The objective of each player is maximizing the OEO sharing degree

of the RN traversed, in order to reduce the overall number of regenerators allocated in the network.

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#### Algorithm 1 GT-RPP: game description

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*Input:*  $G = (V, E, L)$ ; the set of connections (players)  $C = \{c\}$ ; for each player  $c$ ,  $K_c = \{c^k\}$  specifies the set of pre-computed working/backup paths couples; for each couple  $c^k$ ,  $P_{c,k} = \{c^{k,p}\}$  defines the set of all the feasible regenerator placements along the working/backup paths which permit to respect the reach limit  $L_{max}$ .

*Output:* A set of actions  $S^* = \{c^{k^*,p^*}\}$ , one for each player  $c$ , which achieves a Nash equilibrium situation.

*Set of actions:*  $S^{-1}$  (Actions at previous stage),  $S$  (Actions at current stage)

- 1) Assign a random action  $\{k,p\}$  to every player  $c$  and store them in  $S^{-1}$ ;
- 2) Generate a random-ordered list  $C_r$  of players,  $i = 1$ ;
- 3) Select the best action  $\{k',p'\}$  for the  $i^{th}$  connection  $c$  in  $C_r$  which minimize the current player's utility  $u_c$ :

$$u_c = \min_{k,p} \{u_c^{k,p}\} \quad (3)$$

where  $u_c^{k,p}$  is the utility associated to the generic action  $\{k,p\}$ , which corresponds to the maximum value of the traversed RN cost in configuration  $\{k,p\}$ :

$$u_c^{k,p} = \max_{v \in V} \{\$v \cdot r_v^{c,k,p}\}. \quad (4)$$

The cost  $\$v$  associated to the generic node  $v$  is inversely proportional to the number of paths  $U_v$  which currently select  $v$  like RN:

$$\$v = \begin{cases} \frac{1}{U_v} & \text{if } U_v \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Insert action  $\{k',p'\}$  in  $S$ .

- 4) If  $i < |C_r|$ ,  $i++$  and go to step 3; else if  $S \neq S^{-1}$  then  $S^{-1} = S$  and go to step 2; else a Nash equilibrium is reached with the action set  $S^* = S$ .
- 

Players update their action in round robin fashion, following the simple rule referred to in the game theory literature as the best response dynamic, i.e., they choose the action that maximize their utility function, on base of the actions of other players. At each step of the game, the best action of a player  $c$  corresponds always to selecting the couple  $\{k',p'\}$  which, in the current network state, minimizes the maximum cost of the RNs utilized along the chosen working/backup paths. The cost of an RN is inversely proportional to the number of routed lightpaths that require OEO capability on that node, while a transparent node cost is equal to 0. In other words, players should share as much regenerator as they possibly can, in order to reduce the overall number of RNs in the network. The game terminates when no player has interest any more in taking further actions to change the current solution, i.e., a Nash Equilibrium (NE) is reached [17].

The game described in Algorithm 1 falls into the category of congestion games. Then, RPP admits at least one NE pure-strategy and the proof comes directly from the consideration

that every congestion game with player-specific pay-off function always admits a pure-strategy NE [18]. However, provided that a game admits Nash equilibria, it becomes fundamental to find such equilibria and to characterize their quality. In the next section we comment on the quality of the Nash equilibria of GT-RPP, and we provide an operational method to find them.

### B. Characterizing GT-RPP Equilibria

Deriving from the mathematical formulation OPT-RPP in Sec. IV, we can now introduce a MILP model to enforce Nash equilibria situations. Let us define the following simple variables:

- $U_v$  is an integer variable which represents the number of paths which promote  $v$  to RN
- $\mathcal{L}_v$  is a continuous variable representing the cost associated to node  $v$ ;  $\mathcal{L}_v$  is a linearization of  $\$v$  (Eqn.5) and it is bounded:

$$0 \leq \mathcal{L}_v \leq 1 - \frac{1}{M_1} \quad \forall v \in V \quad (6)$$

where  $M_1$  is a big number ( $M_1 \gg U_v, \forall v \in V$ ).

Moreover, we have to add more complex variables which follow all the possible network changes caused by a player's unilaterally deviation:

- $\mathcal{L}_v^{c,k,p}$  is a continuous variable which express the cost associated to node  $v$  when player  $c$  chooses the alternative action  $\{k, p\}$ ; it is bounded:

$$0 \leq \mathcal{L}_v^{c,k,p} \leq 1 - \frac{1}{M_1} \quad \forall v \in V; \quad (7)$$

- $y_v^{c,k,p}$  is a boolean variable which represents if  $v$  is promoted or not to RN by some lightpaths when player  $c$  chooses the alternative action  $\{k, p\}$
- $H_v^{c,k,p}$  is a boolean variable indicating the node with the higher cost for player  $c$  when it chooses the alternative action  $\{k, p\}$

Constraints (1) of the formulation in Sec. IV, which guarantees that exactly one feasible action  $\{k, p\}$  is adopted by every player, are applied unmodified. However, constraints (2) is not required in this formulation.

As previously stated, the estimation cost of a generic node  $v$  needs the knowledge of the number of lightpaths which promote that node  $v$  to RN

$$U_v = \sum_{c \in C} \sum_{k \in K_c} \sum_{p \in P_{c,k}} x_c^{k,p} \cdot r_v^{c,k,p} \quad \forall v \in V. \quad (8)$$

Unfortunately, being  $U_v$  a variable of our problem, the computation of node cost as reported in Eqn.5 is not linear because contains an hyperbolic function and a jump discontinuity. In constraints (9) we linearize it:

$$\mathcal{L}_v = y_v - \frac{U_v}{M_1} \quad \forall v \in V. \quad (9)$$

If any selected action does not promote  $v$  to RN ( $U_v = 0$ ), then cost  $\mathcal{L}_v = 0$ . As a matter of fact,  $y_v$  will be set automatically to 0 because  $\mathcal{L}_v$  is constrained by Eqn.(6) between  $[0, 1 - 1/M_1]$ .

Otherwise, in case of  $U_v > 0$  and  $M_1 \gg U_v$ ,  $y_v$  will be set to 1 preventing that  $\mathcal{L}_v$  assumes negative values. Then, when required by at least one lightpath, the enforcement of RN placement is guaranteed without constraints (2). In conclusion,  $\mathcal{L}_v = 0$  when no lightpath promotes  $v$  to RN, it assumes low values if a high number of selected actions requires regeneration capability on  $v$ , and higher if the regeneration demands on  $v$  decrease.

We now define additional constraints to introduce the players' selfish behavior into the problem. A NE is reached when no selected action  $\{k^*, p^*\}$  deviates towards another action  $\{k, p\}$ . Then, constraints (10) force each user to choose the action  $\{k^*, p^*\}$  which minimizes the maximum cost of RNs traversed. It ensures that if the single user unilaterally changes his strategy, the change does not improve his own payoff (Eqn.3)

$$\begin{aligned} & \overbrace{\mathcal{L}_v + (x_c^{k^*, p^*} - 1)M_2}^{(a)} \leq \overbrace{\mathcal{L}_n^{c,k,p} + (1 - H_n^{c,k,p})M_2}^{(b)} \\ & \quad \forall v, n \in V; \quad c \in C; \quad k^*, k \in K_c; \\ & \quad \forall p^* \in P_{c,k^*} : r_v^{c,k^*,p^*} \neq 0; \quad p \in P_{c,k} : r_n^{c,k,p} \neq 0. \end{aligned} \quad (10)$$

In the previous expression, (a) represents the cost of node  $v$  when the player  $c$  chooses the action  $\{k^*, p^*\}$ . The identity (4) can be translated in the left side of constraints (10) verifying the inequality for all the nodes that in the action  $\{k^*, p^*\}$  are promoted to RN ( $\forall v : r_v^{c,k^*,p^*} \neq 0$ ). The factor  $(x_c^{k^*, p^*} - 1)M_2$  is used to activate the constraint only when  $x_c^{k^*, p^*} = 1$ .  $M_2$  is a big number ( $M_2 \gg \mathcal{L}_v, \forall v \in V$ ).

The right side (b) represents the utility for the player  $c$  when it deviates on action  $\{k, p\}$ . The factor  $(1 - H_n^{c,k,p})M_2$  is used to activate the constraint only when the node  $n$  ( $n : r_n^{c,k,p} \neq 0$ ) assumes the maximum cost in respect to the other nodes promoted RN by choice  $\{k, p\}$ . As a matter of fact, the action changes impose nodes cost updating. When the player selects the alternative action  $\{k, p\}$ , the new cost  $\mathcal{L}_n^{c,k,p}$  of the generic node  $n$  is obtained subtracting the contribute  $(x_c^{k^*, p^*} \cdot r_n^{c,k^*,p^*})$  of the action  $\{k^*, p^*\}$  and adding that one ( $r_n^{c,k,p}$ ) due to action  $\{k, p\}$ . Similar to constraints (9), constraints (11) adopt an auxiliary boolean variable  $y_n^{c,k,p}$  to introduce the jump discontinuity of the nodes cost function which is defined between  $[0, 1 - 1/M_1]$  too. Therefore, when the new action is chosen and there is no lightpath that requires regenerator on  $n$ ,  $\mathcal{L}_n^{c,k,p}$  will be set to 0; otherwise the cost will be increased decreasing the number of lightpaths which promote that node to RN

$$\begin{aligned} \mathcal{L}_n^{c,k,p} &= y_n^{c,k,p} - \frac{U_n - x_c^{k^*, p^*} \times r_n^{c,k^*,p^*} + r_n^{c,k,p}}{M_1} \\ & \quad \forall c \in C; \quad k^*, k \in K_c; \quad p^* \in P_{c,k^*}; \quad p \in P_{c,k}. \end{aligned} \quad (11)$$

If the new selected action  $\{k, p\}$  does not require regenerator capability on a generic node  $n$  ( $r_n^{c,k,p} = 0$ ), the associate cost will be set automatically to 0. In this case, the following constraint needs to be satisfied rather than constraint 10

$$\mathcal{L}_v + (x_c^{k^*, p^*} - 1)M_2 \leq (1 - H_n^{c,k,p})M_2$$

$$\forall v, n \in V; \quad c \in C; \quad k^*, k \in K_c;$$

$$\forall p^* \in P_{c,k^*} : r_v^{c,k^*,p^*} \neq 0; \quad p \in P_{c,k} : r_n^{c,k,p} = 0. \quad (12)$$

In conjunction to the inequalities 10 and 12, constraints (13) identify the node  $n$  which with its updated cost  $\bar{f}_n^{c,k,p}$  gives the utility value to alternative action  $\{k, p\}$

$$\sum_{n \in V} H_n^{c,k,p} = 1 \quad \forall c \in C; k \in K_c; p \in P_{c,k}; \quad (13)$$

As previously stated, Nash equilibria can be multiple and of different quality. Pareto optimality is one of the most frequently adopted criteria in game theory to classify equilibria. An equilibrium identified by a given strategy profile is said to be Pareto optimal if no other strategy profile leads to an enhancement in the payoffs of all the users [17].

To find Pareto optimal NE in MILP model, we have to utilize merely the objective function already proposed for OPT-ILP in Sec. IV, which minimizes the total number of RNs allocated in the network. Moreover, besides determining the optimal equilibria, we may be interested to derive and characterize the worst equilibria too. To this end, it is sufficient to overturn the objective function of the MILP formulation, maximizing the total number of RNs allocated in the network ( $\max \sum_{v \in V} y_v$ ).

## VI. RESULT

We now quantitatively compare the performance of GT-RPP versus OPT-RPP. We first test the effectiveness of GT-RPP in a common network scenario implementing the game theoretical procedure reported in Algorithm 1 and applying it repeatedly. Afterwards we prove the quality of GT-RPP Nash equilibria, on simple networks, utilizing the MILP formulation proposed in Sec.V-B. All simulated networks are loaded by static traffic; each node pair requires one single connection demanding a couple of working/backup paths (dedicated path protection). The set up of the two lightpaths is constrained by the physical impairments. The transparent reach limit  $L_{max}$  is set to 2600 km, inline with results found in [19]. In our analysis we employ four metrics to highlight the performance of our approach: density distribution of GT-RPP Nash equilibria ( $f_{NE}$ ), Computational Time ( $CT$ ), Price-of-Anarchy ( $PoA$ ) and Price-of-Stability ( $PoS$ ).  $f_{NE}$  is the density distribution of the total RNs consumption obtained by repeating the GT-RPP game<sup>1</sup>.  $CT$  is the time in seconds to solve RPP by OPT-RPP and GT-RPP approaches.  $PoA$  and  $PoS$  define the ratio between the value of the GT-RPP best/worst equilibrium, respectively, and the OPT-RPP optimal solution. Moreover one main parameter is varied in the following analysis:  $K$ , the number of alternate working/backup paths that are explored for each node pair. OPT-RPP solutions and  $PoA/PoS$  estimations are all obtained adopting CPLEX 11.0.1 solver.

### A. GT-RPP vs. OPT-RPP

Now let us consider the network topology in Fig.1. To estimate the efficiency of GT-RPP, the game theoretical procedure described in Algorithm 1 was adopted for different  $K$  values. For each  $K$  case, 1000 single game was applied. Any single game is characterized by a starting random-order list of

<sup>1</sup>In other words, we repeat the best response dynamic procedure (presented in Algorithm 1)  $N$  times; we then count the occurrences of each reached NE; finally, we divided the numbers of occurrences by  $N$ .

players and the starting random actions of players. Single game is terminated only when a Nash equilibrium is reached. For different  $K$  values, in Tab.I we reported the Nash equilibria distributions,  $f_{NE}$ . It shows the NE occurrences in percentage over 1000 reached Nash equilibria.

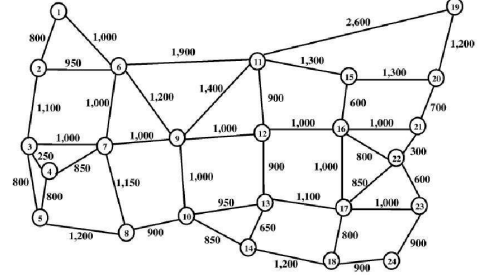


Fig. 1. A carriers US nationwide backbone network topology.

Exploiting the GT-RPP approach, we can observe that the Nash equilibria are characterized by a compact density distribution function. The lowest occurring for the best NE value is almost equal to 12%, which is experimented under  $K = 2$  case. This means that repeating that game for a restricted number of times, e.g. 40 times, we reach the best NE with a high probability ( $1 - 0.88^{40} \approx 0.99$ ). For this reason, we limited the repeating of GT-RPP to 40 times, and for each  $K$  case we selected the best reached NE. In Fig.2, the minimum number of total RNs allocated over 40 runs by GT-RPP and the corresponding optimal exact solution OPT-RPP are reported. It is interesting to note that increasing the  $K$  value, the distribution tends to narrow down on the best reached NE.

	Number of RNs					
	11	12	13	14	15	16
$K = 2$	0	0	0	11.9	86.6	1.5
$K = 4$	0	0	0	62.4	37.6	0
$K = 6$	0	0	0	98.5	1.5	0
$K = 8$	0	99.5	0.5	0	0	0
$K = 10$	0	99.6	0.4	0	0	0
$K = 12$	100	0	0	0	0	0

TABLE I  
GT-RPP NE DISTRIBUTION  $f_{NE}$  OVER 1000 RUNS

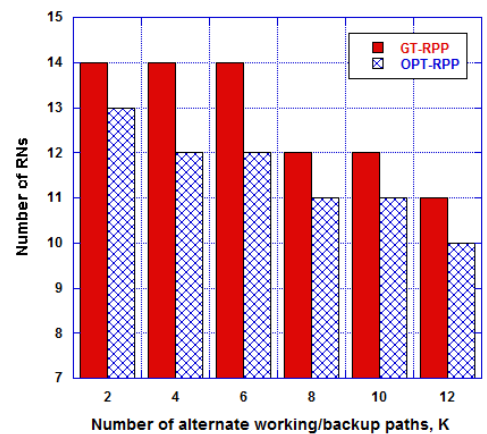


Fig. 2. RN consumption comparison between OPT-RPP and the best GT-RPP NE over 40 runs

Finally, Tab.II reports the Computational Time,  $CT$ , required by OPT-RPP and 40 runs of GT-RPP to solve the same

RPP instance: we obtained savings ranging to almost 72%. Clearly, larger instances can not be solved by OPT-RPP approach because the amount of variables generated. However, a solution (i.e., a Nash equilibrium) is always found by GT-RPP due to the greedy nature of the algorithm. Therefore, GT-RPP procedure repeated for a small number of times is able to reach the best NE with low computational efforts.

	OPT-RPP CT (s.)	GT-RPP CT (s.)
$K = 2$	23	20
$K = 4$	90	50
$K = 6$	120	67
$K = 8$	216	103
$K = 10$	428	187
$K = 12$	769	217

TABLE II

CT COMPARISON BETWEEN OPT-RPP AND 40 RUNS OF GT-RPP

### B. Estimating GT-RPP Equilibria

Although GT-RPP achieves a solution in briefly time, the reached equilibrium in a single game may not be Pareto efficient. Thus, let us now examine how efficient the solution represented by an equilibrium may be, relatively to the optimal one. In other words, we examine whether the solution represented by an equilibrium point can approximate the optimal solution. For this purpose we exactly estimate the Nash equilibria bounds on  $PoA$  and  $PoS$  for a small network (Fig.3) adopting the MILP formulation proposed in Sec.V-B. They represent the worst case and the best case performance NE, respectively, for a single game of GT-RPP.

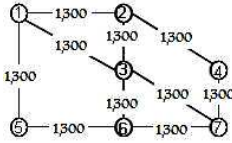


Fig. 3. A carrier's simple backbone network topology.

Our observations, summarized in Tab.III, indicate that the best GT-RPP NE always allow RN allocations equal to the corresponding optimal solutions OPT-RPP ( $PoA = 1$ ).

	OPT-RPP	GT-RPP		$PoA$	$PoS$
		Best NE	Worst NE		
$K = 1$	3	3	5	1	0.6
$K = 2$	3	3	5	1	0.6
$K = 3$	3	3	4	1	0.75
$K = 4$	2	2	4	1	0.5
$K = 5$	2	2	3	1	0.66
$K = 6$	2	2	3	1	0.66
$K = 7$	2	2	3	1	0.66
$K = 8$	1	1	3	1	0.33

TABLE III

QUALITY OF GT-RPP NASH EQUILIBRIA COMPARED TO THE OPTIMUM OPT-RPP SOLUTIONS

Conversely, the worst one finds configurations which are not always close to optimum ( $PoS = 0.33$ ). However, we noted that the GT-RPP procedure applied repeatedly 40 times reached in every case the best NE with high probability. For this simple scenario, the lowest occurring for the best NE value is above to 91%. We conclude that GT-RPP approach is a methodology which can solve efficiently the RPP.

## VII. CONCLUSION

In this paper, we have first proposed a new ILP formulation for an efficient planning to design a resilient and translucent network. We have introduced an approximated approach based on a game theory model, which is able to solve the Regeneration Placement Problem (RPP) much more quickly obtaining very good performances, close to the exact solutions. To prove the efficiency of the model we have determined the distribution of its Nash equilibria on large network instances and characterized its best/worst Nash equilibria by mathematical programming on smaller scenarios. The numerical results have highlighted the fact that, even if the game is non-cooperative, it features Nash equilibria close to optimal solutions.

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