Lec 01:
Sets I

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What is a set?

Definition

A set is a well defined collection of objects that are described as members or elements of the set.

Example

The set of the first 5 prime numbers

$$A = \{2, 3, 5, 7, 11\}$$

The numbers 5 is a member of the set A, or $5 \in A$.

Example

The set of odd numbers between 11 and 17, inclusive.

$$B = \{11, 13, 15, 17\}$$

The number 12 is *not* an element of the Set B, or $12 \notin A$.

Set Roster Notation

The set is described by using braces { } with all elements seperated by commas.

Example

- $\{red, blue, orange\}\{1\}\{a, b, d\}$

You can also specify a set to contain another set using nested braces:

Example

- $\{1, \{2, 3\}\}$
- The set containing 1 and the set $\{2, 3\}$

We can use ellipses "..." (read "as so forth") to indicate sets that continue on infinitely.

Example

- $\{0, 1, 2, \ldots\}$ (set of positive integers)
- $\{\ldots, -3, -2, -1\}$ (set of negative integers)

Set Equality

Definition

Two sets A and B are equal (written as $A = B$) when they contain the same elements. (the order of the elements is irrelevant)

Is A equal to B?

- (1) $A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$
  - yes, $A = B$
- (2) $A = \{1, 2, 3\}$ $B = \{2, 1, 3\}$
  - yes, $A = B$ even though elements presented in different orders
- (3) $A = \{1, 2, 3\}$ $B = \{3, 3, 1, 2, 2, 3, 3\}$
  - yes, $A = B$ even with multiple repetitions.
- (4) $A = \{1, 2, 3\}$ $B = \{1, 1, 3\}$
  - No, $A \neq B$ because $2 \in A$ but $2 \notin B$

Common Numeric Sets

- $\mathbb{R}$: The set of real numbers
- $\mathbb{Z}$: The set of integers
- $\mathbb{Q}$: The set of rational numbers
- $\mathbb{N}$: The set of natural numbers
- $\mathbb{C}$: The set of complex numbers

$\mathbb{R}$: Real numbers

$\mathbb{Z}$: $-\infty, \ldots, -1, 0, 1, \ldots \infty$

$\mathbb{Q}$: $\frac{a}{b}$ reduced

$\mathbb{N}$: $0, 1, 2, 3, \ldots \infty$

$\mathbb{W}$: $1, 2, 3 \ldots \infty$

$\mathbb{C}$: $i = \sqrt{-1}$ $\quad a + bi$

when $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$

Set Builder Notation

Set Builder Notation

Describe a set where some condition is met.

$$\{x \in S \mid P(x)\}$$

"The set of all elements x in S such that some property/proposition $P(x)$ is true"

Example

$$\mathbb{Z}^+ = \{x \in \mathbb{Z} \mid x > 0\}$$

but sometimes we also write it this way

$$\mathbb{Z}^+ = \{x \mid x \in \mathbb{Z} \text{ and } x > 0\}$$

Exercises

Describe all positive even numbers using set-builder notation.

Describe $\mathbb{Q}$ (the rational numbers) using set-builder notation.

Describe $\{x \mid x = (-1)^k \text{ where } k \in \mathbb{Z}^+\}$ using set roster notation.

Interval Notation

It's common to want to describe an interval within a numeric set, such as $\mathbb{Z}$ or $\mathbb{R}$. This is easily done using the following notation:

$$\{x \in \mathbb{R} \,|\, a < x < b\}$$

In plain language, this is the set of real numbers between a and b.

However, this is cumbersome, so we have the following shorthand.

$$\begin{array}{ll}
(a,b) = \{x \in \mathbb{R} \,|\, a < x < b\} & [a,b] = \{x \in \mathbb{R} | a \leq x \leq b\} \\
(a,b] = \{x \in \mathbb{R} \,|\, a < x \leq b\} & [a,b) = \{x \in \mathbb{R} \,|\, a \leq x < b\} \\
(a,\infty) = \{x \in \mathbb{R} \,|\, x > a\} & [a,\infty) = \{x \in \mathbb{R} \,|\, x \geq a\} \\
(-\infty,a) = \{x \in \mathbb{R} \,|\, x < a\} & (-\infty,a] = \{x \in \mathbb{R} \,|\, x \leq a\}
\end{array}$$

Cardinality

Definition

The size of the set, or its cardinality, is the number of elements in the set.

Example

For the set $A = \{5, 4, 22\}$ the cardinality of A is 3, written $|A| = 3$

Definition

If a set has cardinality of 0, or $|B| = 0$, then we describe it as the empty set and denote it with special symbol $\emptyset$ or sometimes written simply as $\{\}$.

Exercises

Can you construct an argument to show that $\{\} = \emptyset$ based on the equality rule from before?

Recall that "two sets A and B are equal when they contain the same elements"

Is $\{\{\}\} = \emptyset$?

What is $|\{\{\}, 3, \emptyset, \{1, 2, 3\}\}|$?

If two sets A and B have the same cardinality, $|A| = |B|$, is it the case that $A = B$?

If two sets A and B are equal, $A = B$, is it the case that they have the same cardinality, $|A| = |B|$?