CSCI 1311: Problem Set 6

Early Deadline: 17 Apr 2023
Final Deadline: 19 Apr 2023

Instructions:

- You will submit your answers to via gradescope.
- Refer to gradescope to determine which questions or sub-questions are submitted individually.
- For each submission of a question or sub-question, you should submit an image of your answer, which can be a picture of your handwriting, a screenshot of a tablet, an image of something you typed, or a scan of a document. The most important point is that it's legible and clearly marked in gradescope.
- Submission by the *Early Deadline* will receive a 10 point bonus. Submission after the *Final Deadline* will not be accepted.
- You may ask to use your *one time amnesty* for this assignment to receive two additional days past the *Final Deadline*.

Question Weighting

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	10	10	5	6	29	20	10	100

1. Is the following relations an equivalences relation? That is, show that it is reflexive, symmetric, and transitive, or provide a counter example in any category.

   (a) **[5 points]** Let $A = \mathbb{Z} \times \mathbb{Z}$, and $(x_1, y_1)\, R\, (x_2, y_2) \iff y_1 = y_2$

   (b) **[5 points]** Let W be all 4 digit numbers, and the relation R is such, for all x and y in W, $x\, R\, y$ if, and only if, the sum of the digits of x is equal to the sum of the digits in y.

2. Are the following partial order relations? That is, show that it is reflexive, antisymmetric, and transitive, or provide a counter example in any category.

   (a) **[5 points]** Let R be a relation on the set $\mathbb{Z}$, where $a\, R\, b$ if, and only if, $a \geq b$.

   (b) **[5 points]** Let R be a relation on the set $\mathbb{R}$, where $a\, S\, b$ if, and only if, $x^2 \leq y^2$.

3. **[10 points]** Consider the partial order relation R on $\mathbb{Z}^+ \times \mathbb{Z}^+$ where

$$(a, b)\, R\, (c, d) \iff a < c \vee (a = c \wedge b \leq d)$$

   is this a total order relation? Prove your result, or provide a counter example.
   (*Hint: consider using a proof by cases to show your result, or find a counter example*)

4. **[5 points]** Prove that following is equivalent under a modulo if $a \equiv c \bmod n$ and $b \equiv d \bmod n$:

$$ab + c \equiv cd + a \mod n$$

5. Find the multiplicative inverse of each of the following modulo 7:
   (a) **[2 points]** 5
   (b) **[2 points]** 6
   (c) **[2 points]** 493

6. For this question, consider 6 digit PINs that contain the numbers 0-9 for each digit.
   (***Writing a number is not sufficient, show your work and the formula you used to reach that conclusion!***)
   (a) **[1 point]** How many 6-digit PINs with repetitions are there?
   (b) **[1 point]** How many 6-digit PINs are there where numbers cannot repeat?
   (c) **[3 points]** How many 6-digit PINs without repetitions are there where the first digit must be a 5 and the last must be a 7 ?
   (d) **[4 points]** How many 6-digit PINs with repetitions are there that have at least one digit is a 7?
   (e) **[10 points]** How many 6-digit PINs with repetition are there that have at least one digit that is a 7 **or** at least one digit that is a 5.
   (f) **[10 points]** How many 6-digit PINs with repetition of digits are there where at least *two* digits are a 7?

7. Consider a urn of balls: 4 balls are blue, 6 balls are red, and 10 balls are yellow. (***Writing a number is not sufficient, show your work and the formula you used to reach that conclusion!***)
   (a) **[5 points]** You draw five balls from the urn, what is the probability of drawing all red balls *or* all yellow balls?
   (b) **[5 points]** If you draw two balls from the urn, what is the probability that you draw at least one yellow ball?
   (c) **[10 points]** Assume you draw three balls, if at least one of the balls were red, what is the probability that the other two balls you've drawn are also red?

8. **[10 points]** You're playing "Let's make a deal, 2.0" and on this version of the game show, there is a game with **4 doors!** Behind two of the doors is a goat, behind one door is a car, and behind a third door is a *sweet* road bike. You're goal is to win a car or the *sweet* road bike.

   Like in the game before, you get to make an initial selection of one of the doors, and for example, let's assume that it is door #1. The game show host than reveals to you the location whats behind one of the doors, always revealing a goat. Let's say that the door revealed is door #2.

   Now, the host asks you a question: Would you like to swap to one of the other remaining unrevealed doors? That is door #3 and door #4. Should you swap or stay?

   Provide an argument, for why you should either swap or stay. Your answer should include concrete probabilities for either swapping to another door or staying with your initial door with respect to winning the car or the *sweet* road bike.