CSCI 1311: Problem Set 2

Early Deadline: 6 Feb 2023
Final Deadline: 8 Feb 2023

Instructions:

- You will submit your answers to via gradescope.
- Refer to gradescope to determine which questions or sub-questions are submitted individually.
- For each submission of a question or sub-question, you should submit an image of your answer, which can be a picture of your handwriting, a screenshot of a tablet, an image of something you typed, or a scan of a document. The most important point is that it's legible and clearly marked in gradescope.
- Submission by the *Early Deadline* will receive a 10 point bonus. Submission after the *Final Deadline* will not be accepted.
- You may ask to use your *one time amnesty* for this assignment to receive two additional days past the *Final Deadline*.

Question Weighting

Question:	1	2	3	4	Total
Points:	10	40	30	20	100

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1. Use a truth table to show the following equivalences
   (a) **[5 points]** $p \vee (p \wedge q) \equiv p$
   (b) **[5 points]** $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
2. Using equivalent forms, show the following. Indicate each step and the why you can take each step (e.g., by commutative)
   (a) **[10 points]** $\neg p \vee q \equiv (p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$
   (b) **[10 points]** $p \oplus p \oplus p = p$
   (c) **[10 points]** $p \rightarrow q \;\equiv\; \neg q \rightarrow \neg p$
   (d) **[10 points]** $p \rightarrow (q \vee \neg r) \;\equiv\; (p \wedge r) \rightarrow q$
3. Convert the following either to English sentences, if written using propositional logic, or to propositional logic, if written in English. Try to avoid using English phrases, like "if" and "then" in your formal logic, and instead rely on symbols, like $\implies$.
   (a) **[5 points]** If the square of an integer is greater than 9, then that integer is greater than 3 or it is less than -3.

(b) **[5 points]** For every integer, there exists an integer that is less than that integer.

(c) **[5 points]** Every real number is a complex number.

(d) **[5 points]** $(\forall x, y \in \mathbb{Z})(x > 1 \wedge y > 1) \implies \neg[(\exists z \in \mathbb{Z})(x^3 + y^3 = z^3)]$

(e) **[5 points]** $(\forall x, y \in \mathbb{R})(x < y) \implies (\exists r \in \mathbb{R})(x < r < y)$

(f) **[5 points]** $(\forall x \in \mathbb{Z})[x > 1 \implies (\exists p, q \in Z)(p + q = 2x \wedge \mathsf{Prime}(p) \wedge \mathsf{Prime}(q))]$

4. Which of the following assertions are true or false for the proposition $P(x, y)$. For any false assertion, provide a counterexample statement for which the assertion is false. For any true example, provide a brief explanation for why it is true.

(a) **[4 points]** $\exists x \exists y P(x, y) \equiv \exists y \exists x P(x, y)$

(b) **[4 points]** $\exists x \exists y P(x, y) \implies \exists y \exists x P(x, y)$

(c) **[4 points]** $\exists x \forall y P(x, y) \equiv \forall y \exists x P(x, y)$

(d) **[4 points]** $\exists x \forall y P(x, y) \implies \forall y \exists x P(x, y)$

(e) **[4 points]** $\exists x \forall y P(x, y) \implies \exists y \exists x P(x, y)$