CSCI 1311: Problem Set 4

Early Deadline: 26 Feb 2023

Final Deadline: 28 Feb 2023

Instructions:

- You will submit your answers to via gradescope.
- Refer to gradescope to determine which questions or sub-questions are submitted individually.
- For each submission of a question or sub-question, you should submit an image of your answer, which can be a picture of your handwriting, a screenshot of a tablet, an image of something you typed, or a scan of a document. The most important point is that it's legible and clearly marked in gradescope.
- Submission by the *Early Deadline* will receive a 10 point bonus. Submission after the *Final Deadline* will not be accepted.
- Due to the timing of the midterm **you may not** use your *one time amnesty* on this assignment.

Question Weighting

Question:	1	2	3	Total
Points:	25	25	50	100

There is 120 points out of 100 available on this problems sets. You can do all 120 and earn bonus 120/100, or you can choose not to do one of the 20 point proofs and get a 100/100. Whatever works for you. And, yes, you can still get the 10 point bonus for submitting early. So the max grade on this problem set is 130.

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1. **[25 points]** Using induction, prove that postage of 6-cents or more can be achieved by using only 2-cent and 7-cent stamps.

2. **[25 points]** Using induction, prove that all integers $n \geq 0$, we can write n in base-8. Or put another way, for all integers $n \geq 0$, there exists integers $k \geq 0$ and $d_0 \cdots d_{k-1}$, where each $0 \leq d_i < 8$, such that

$$n = d_{k-1} \cdot 8^{k-1} + d_{k-2} \cdot 8^{k-2} + \ldots + d_1 \cdot 8^1 + d_0 \cdot 8^0.$$

To see why this formulation relates to base-8, let's look at an example. Suppose we wanted to write the base-10 number 245 in base 8. That's the same as

$$245 = \mathbf{3} \cdot 8^2 + \mathbf{6} \cdot 8^1 + \mathbf{5} \cdot 8^0$$

and the base-8 number would be written $365_8 = 245_{10}$, where the subscript indicates the base.

3. For each recurrence, solve the following recurrence relation and provide a proof (using induction) that your solution describes the recurrence relation. That is, generate a formula for the recurrence in terms of n alone, for the n'th term of the sequence. Then prove the statement, "if a_n is described by the recurrence relation, then a_n equals the solution ..."

   Place the answer to each sub-part (a, b and c) on a single page to improve grading when submitting via gradescope

   (a) **[15 points]** $a_n = 5 + a_{n-1}$ where $a_0 = 9$

   (b) **[15 points]** $a_n = n + a_{n-1}$ where $a_0 = 10$

   (c) **[20 points]** $a_n = 2a_{n/2}$ where $a_1 = 1$. (*Hint: you'll need strong induction to prove this result*)

   For this question, you can assume that n is always a power of 2, that is $n = 2^x$ for some $x \geq 0$. However, if you want to better generalize the recurrence, you can define division by 2 as $\lceil n/2 \rceil$ which is well defined for all n and will always reach 1 after successive divisions.