CSCI 1311: Problem Set 5

Early Deadline: 3 Apr 2023
Final Deadline: 59 Apr 2023

Instructions:

- You will submit your answers to via gradescope.
- Refer to gradescope to determine which questions or sub-questions are submitted individually.
- For each submission of a question or sub-question, you should submit an image of your answer, which can be a picture of your handwriting, a screenshot of a tablet, an image of something you typed, or a scan of a document. The most important point is that it's legible and clearly marked in gradescope.
- Submission by the *Early Deadline* will receive a 10 point bonus. Submission after the *Final Deadline* will not be accepted.
- You may ask to use your *one time amnesty* for this assignment to receive two additional days past the *Final Deadline*.

Question Weighting

Question:	1	2	3	4	5	6	Total
Points:	15	20	15	20	15	15	100

1. Prove the following Big-O's
   (a) **[5 points]** $f(n) = 5n^2 - 3n + 10$ is $O(n^2)$
   (*hint: you don't need induction if you choose your B and b well*)
   (b) **[10 points]** $f(n) = \log_2(n)$ is $O(n)$
   (*hint: you should use induction, and check out your log rules if you've forgotten them*)

2. For the following code-snippets, what is the Big-O? Provide a step function (you can reduce constants to variables) that justifies your answer. It could be a recurrence relation. (Recall that assignment, comparison, subtraction, addition, and function calls all count as one step.)
   (a) **[5 points]**

```
int count=0;
for(int i=0;i<n;i++){
  for(int j=n; j>i; j--){
    count += 1;
  }
}
```

   (b) **[5 points]**

```
int count=0;
for(int i=0;i<n/2;i++){
  for(int j=i; j<=2*n; j++){
    count += 1;
  }
}
```

(c) **[5 points]** Assume called with $n > 1$, and if it helps, you can also assume n is a power of 2.

```
int foo(n){
  if(n=1) return 0;
  return 1 + 2*foo(i,n/2);
}
```

(d) **[5 points]** Assume called with $n > 0$

```
int foo(n){
  if(n<=0) return 0;
  return 1 + foo(n-2) + foo(n-2);
}
```

3. Consider the following sets:

$$A = \{1, 2, 3\}$$
$$B = \{w, x, y, z\}$$

(a) Let $f : A \to \mathbb{Z}$ be defined as $f(x) = x^2 + 2x + 1$.
   i. **[2 points]** What is the co-domain of f?
   ii. **[2 points]** What is the preimage of 16?
   iii. **[3 points]** Is the function one-to-one? Provide a one sentence explanation.
   iv. **[3 points]** Is the function onto? Provide a one sentence explanation.

(b) **[5 points]** How many different functions exists of the form $f : A \to B$? Another way to think about this questions, is how many different ways can you map the elements of A to B such that you get a well-defined function? Provide an explanation of how you calculated this result.

4. For the following functions:

- prove that they are one-to-one, or provide a counter example.
- prove that they are onto, or provide a counter example.

(a) **[10 points]** $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = 2x + 1$

(b) **[10 points]** $f : \mathbb{R} \to \mathbb{R}$, $f(x) = (x + 1)^2$

5. **[15 points]** Show that $\mathbb{Z}^+ \times \{0, 1, 2\}$ is countable. You do not need a formal proof, but you should make a compelling argument for why the set is countable, that is, why there exists a one-to-one correspondence function between the set and the positive integers.

(*Hint: Use the quotient remainder theorem to both define a function and argue that it must be a one-to-one correspondence function.*)

6. **[15 points]** Consider the set of infinite binary strings. This all sequences of 1's and 0's that are infinitely long. For example, you can have the number $1101\ldots$ and the number $0111\ldots$ and so on. Where the $\ldots$ indicates that there are infinitely more 1's and 0's that follow. Prove that the set of all infinite binary strings is uncountable. You should provide a formal proof (via proof by contradiction) based on Cantor's diagonalization technique.