

CSCI 1311: Problem Set 1

Early Deadline: 26 Jan 2023

Final Deadline: 28 Jan 2023

Instructions:

- You will submit your answers to via gradescope.
- Refer to gradescope to determine which questions or sub-questions are submitted individually.
- For each submission of a question or sub-question, you should submit an image of your answer, which can be a picture of your handwriting, a screenshot of a tablet, an image of something you typed, or a scan of a document. The most important point is that it's legible and clearly marked in gradescope.
- Submission by the *Early Deadline* will receive a 10 point bonus. Submission after the *Final Deadline* will not be accepted.
- You may ask to use your *one time amnesty* for this assignment to receive two additional days past the *Final Deadline*.

Question Weighting

Question:	1	2	3	4	5	6	7	Total
Points:	12	16	8	4	4	20	36	100

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1. Define the following sets used in the next few questions:

$$A = \{1, 3, 6, 10, 15, 21\}$$

$$B = \{x \in \mathbb{Z} \mid x \in (-2, 10]\}$$

$$C = \{x \in \mathbb{Z}^+ \mid \frac{x}{3} \in \mathbb{Z}\}$$

$$D = \{(a, b) \in A \times B \mid a < 4\}$$

Fill in the \square with the most appropriate symbol (\in , \notin , \subseteq , $\not\subseteq$) for each of the formulas below

- (a) [2 points] $8 \square A$
(b) [2 points] $8 \square B$
(c) [2 points] $\{3, 6, 15\} \square C$
(d) [2 points] $\{(3, 6)\} \square D$
(e) [2 points] $D \square A \times B$

- (f) [2 points] $A \square \mathcal{P}(B)$
2. Using the same sets defined in Q1, evaluate the following sets, writing your answers in set roster notation:
- [2 points] $A \cup B$
 - [2 points] $A \cap B$
 - [2 points] $B - A$
 - [2 points] $C \cap \emptyset$
 - [4 points] $(A \cap C) \times A$
 - [4 points] $\mathcal{P}(A \cap C)$
3. Using the same sets defined in Q1,, what are sizes (cardinality) indicated below (show some work)?
- [2 points] $|D|$
 - [2 points] $|A \cap B \cap C|$
 - [2 points] $|\mathcal{P}(A)|$
 - [2 points] $|C|$
4. Recall that two sets are described as **disjoint** if, and only if, they have no elements in common. Using the same sets defined in Q1, answer the following questions:
- [2 points] Are any two sets from A, B , or C disjoint?
 - [2 points] Which set is always disjoint from all other sets? Why?
5. Recall that a collection of sets $\{N_i\}$ is a partition of another set N , if:
- they are mutually disjoint from each other, that is
for any $N_j \in \{N_i\}$ and $N_k \in \{N_i\}$, $N_i \cap N_j = \emptyset$
 - the union of those set equals N , that is $\bigcup_i N_i = N$.
- [2 points] Describe two sets that are a partition of C (defined in Q1) using set roster notation.
 - [2 points] Describe three sets that are a partition of \mathbb{Z} using set roster notation.
6. Which of the following relationships are true and which are false? Provide a one sentence explanation for your answer.
- [2 points] $\mathbb{Z}^+ \subseteq \mathbb{Q}$
 - [2 points] $\mathbb{Z} \subseteq \mathbb{Q}$
 - [2 points] $\mathbb{Z} \supseteq \mathbb{Q}$
 - [2 points] $\mathbb{Z}^+ \cap \mathbb{Z}^- \neq \emptyset$
 - [2 points] $\mathbb{Z}^+ \cup \mathbb{Z}^- = \mathbb{Z}$
 - [2 points] $\mathbb{Z}^+ \cup \mathbb{Q} = \mathbb{Z}^+$
 - [2 points] $\mathbb{Z} \cap \mathbb{Q} = \mathbb{Z}$
 - [2 points] $\mathbb{Z} \cup \mathbb{R} = \mathbb{R}$
 - [2 points] $\mathbb{Z} \cap \mathbb{R} = \mathbb{R}$
 - [2 points] $\mathbb{Z} - \mathbb{Q} = \emptyset$

7. Let's define the following sets:

- D_h = the set of Democrats in the house of representatives
- D_s = the set of Democrats in the senate
- R_h = the set of Republicans in the house of representatives
- R_s = the set of Republicans in the senate
- H_n = the set of representatives of age n , or below
- S_n = the set of senators of age n , or below
- M = the set of male identifying representatives and senators
- F = the set of female identifying representatives and senators
- J = the set of representatives or senators who are currently serving in their first term

There are 100 senators in the senate, and 435 representatives in the house. For the purpose of simplification (without prejudice), let's say all senators and representatives identify as either male or female and are either a Democrat or a Republican. To be a senator, you must be 35 years or older, and to be a representative, you must be 25 years or older.

- (a) Describe in understandable English, each of the following sets:
 - i. [4 points] $M \cap D_h \cap H_{35}$
 - ii. [4 points] $(J^c \cap (D_h \cup R_h)) \cap F$
 - iii. [4 points] $(J \cap F) \cap (H_{50} \cup S_{50})^c$
 - iv. [4 points] $M \cap D_s \cap (H_{45} \cup S_{45})$
- (b) For each of the sentences, convert to set notation using the described sets, \cup , \cap , $-$, \square^c (complement), and $|\square|$ (cardinality), as well as standard comparison relations $=$, \neq , $<$, $>$, \leq , \geq .
 - i. [4 points] The number of senators of age 45 or younger is greater than the number of representatives over the age of 75.
 - ii. [4 points] The set of senators who are in their second term or greater.
 - iii. [4 points] The number of Democrats in congress who identify as male.
 - iv. [4 points] There are no senators under the age of 35.
 - v. [4 points] The number of non-first-term representatives that can run for senate.