

# An Alternating Minimization Algorithm with Trajectory for Direct Exoplanet Detection

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# Exoplanet Imaging





# Exoplanet Imaging

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[Signs of life? James Webb reveals more about exoplanet K2-18 b's atmosphere](#)

By Luke West  
Published on 13/09/2023 - 17:25 • Updated 13/09/2023 - 08:38



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[Exoplanet discovered around neighbouring star](#)

© 14 November 2018

By Paul Rincon  
Science writer, BBC News website





# Exoplanet Imaging

The collage includes:

- A BBC NEWS article titled "Exoplanet discovered around neighbouring star" from The Brussels Times on December 14, 2018.
- An article from "Names chosen for Belgium's official exoplanet and star" by Luke W. published on Tuesday, 17 December 2019, featuring an image of the exoplanet HD 49674 b.
- A news item from "Signs of life? James Webb reveals more about exoplanet K2-18 b's atmosphere" on euronews.next, dated November 2019.
- A BBC NEWS article from 2018 titled "James Webb reveals more about exoplanet K2-18 b's atmosphere" featuring a large image of the exoplanet.

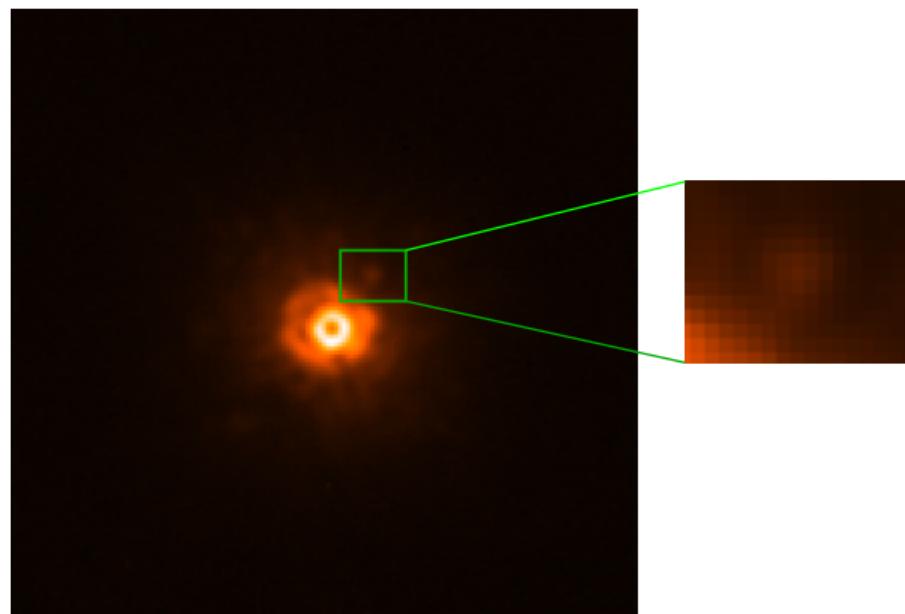
# Exoplanet Imaging

A real image



# Exoplanet Imaging

A real image of exoplanet



# Direct Imaging



Credit: <https://exoplanets.nasa.gov/>

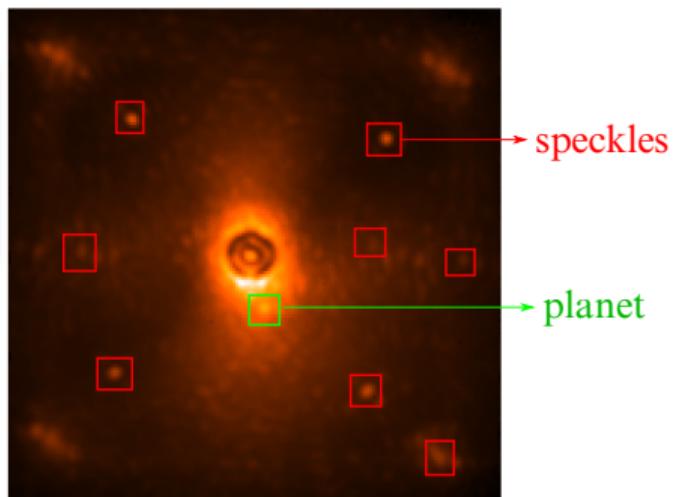
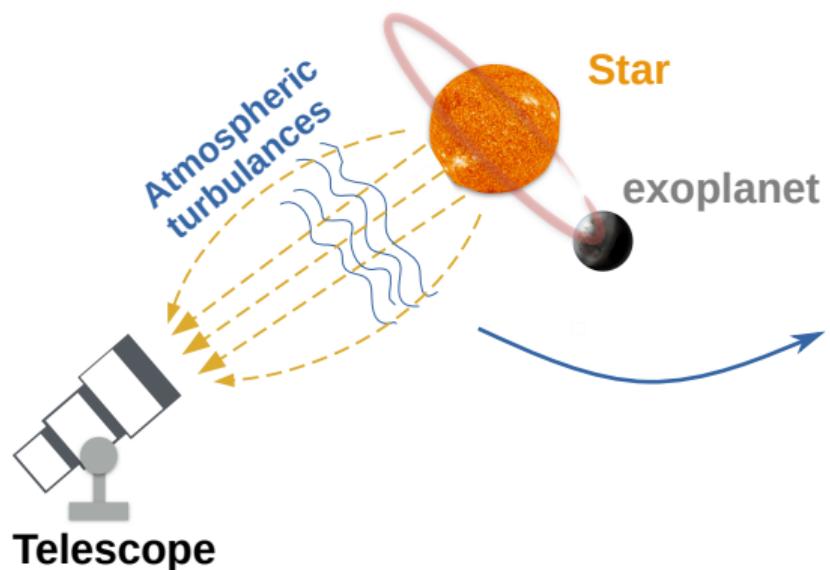
# Direct Imaging



Credit: <https://exoplanets.nasa.gov/>

- ▶ firefly → exoplanet
- ▶ lighthouse → star

# Direct Imaging



# Angular Differential Imaging

# Problem Setup & Goal

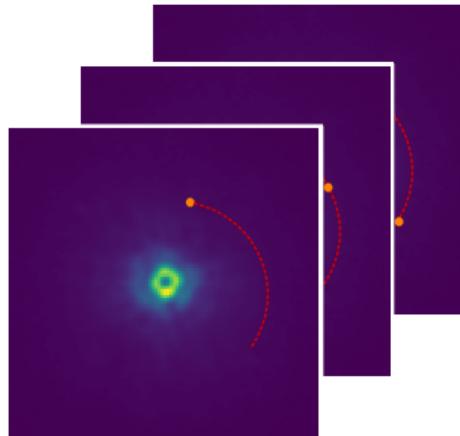
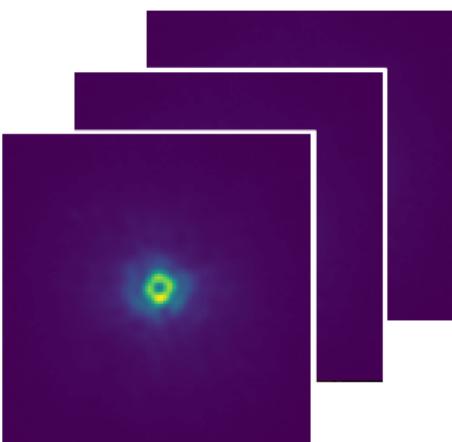


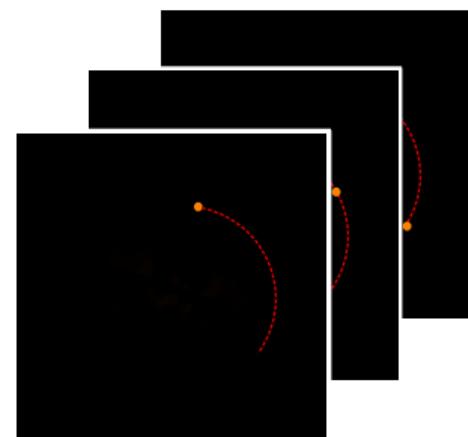
Image sequence

=



Background  
(star+speckles)

+



Foreground  
(planet)

# Background: (Annular) PCA<sup>1,2</sup>

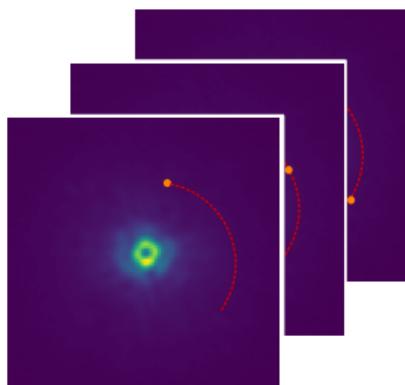
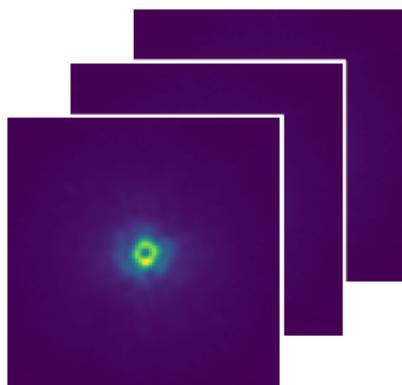


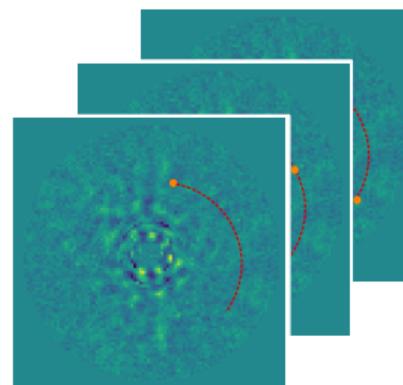
Image sequence  
 $M$

=

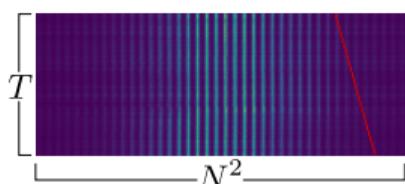


Low rank  
 $L$

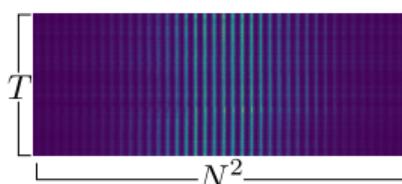
+



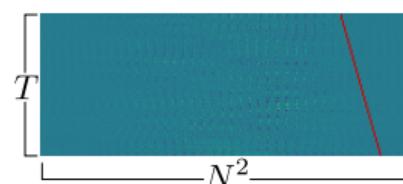
Foreground  
 $R$



=

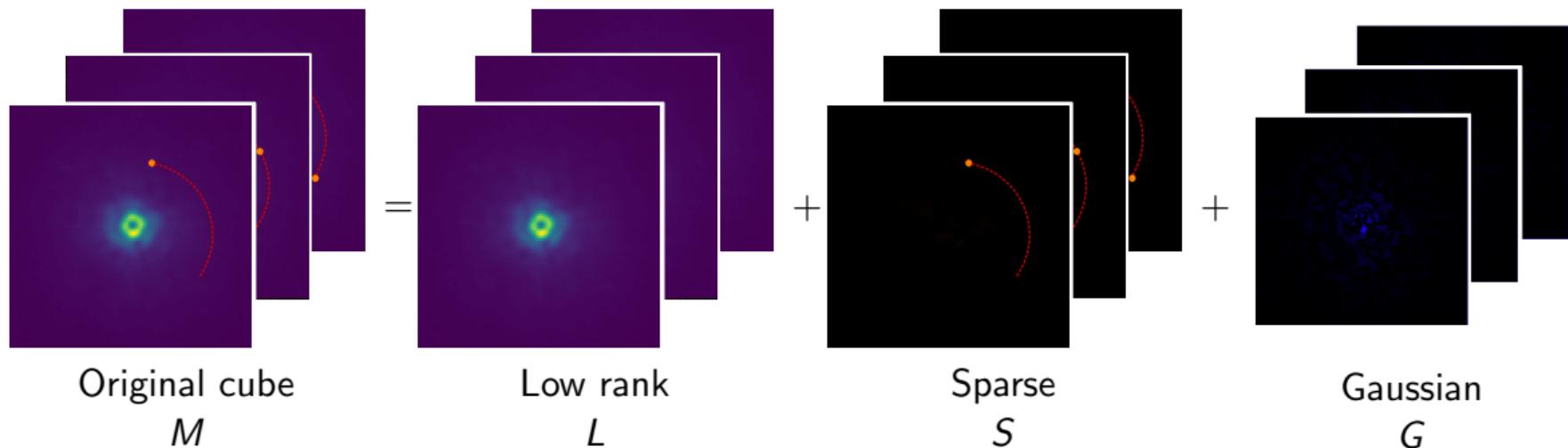


+



<sup>1</sup>Amara and Quanz, 2012

<sup>2</sup>Soummer, et al., 2012

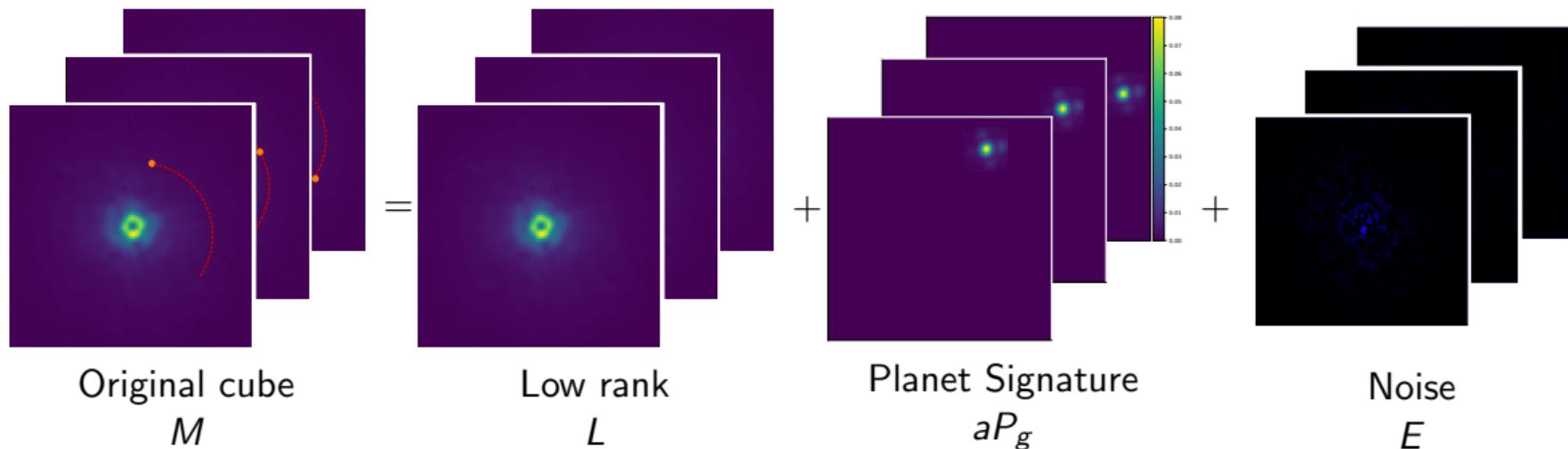
State of art: LLSG<sup>3</sup>

$$\text{rank}(L) \leq k, \quad \text{card}(S) \leq s$$

---

<sup>3</sup>Gomez Gonzalez, et al., 2016

## Alternating Minimization Algorithm with Trajectory (AMAT)

Original cube  
 $M$ Low rank  
 $L$ Planet Signature  
 $aP_g$ Noise  
 $E$ 

$$\text{rank}(L) \leq k, \quad P_g \in \Lambda$$

$$\begin{aligned} & \min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - aP_g\| \\ \text{s.t. } & \text{rank}(L) \leq k \end{aligned}$$

## AMAT

$$\min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - aP_g\| \quad \text{s.t.} \quad \text{rank}(L) \leq k$$

- ▶ Solve (1) by SVD or L1-LRA<sup>4</sup>
- ▶ Solve (2) by

$$a_i = \frac{\langle P_g, M - L_i \rangle}{\|P_g\|_F^2}$$

or minimum of the points  
 $(M - L_i)/P_g$

---

<sup>4</sup>Gillis and Vavasis, 2018

## AMAT

$$\min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - aP_g\| \quad \text{s.t.} \quad \text{rank}(L) \leq k$$

## Alternating Minimization

$$L_i = \arg \min_{L \in \mathbb{R}^{t \times n}} \|M - L - a_{i-1}P_g\| \quad (1)$$

$$a_i = \arg \min_{a \in \mathbb{R}} \|M - L_i - aP_g\| \quad (2)$$

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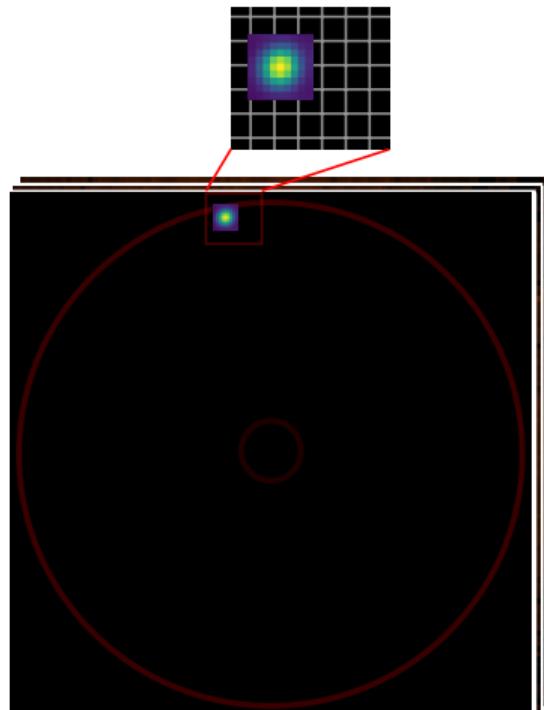
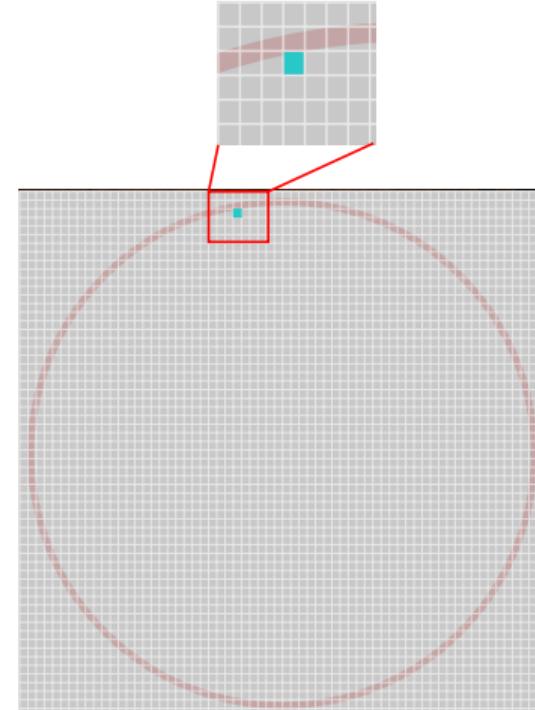
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or minimum of the points  
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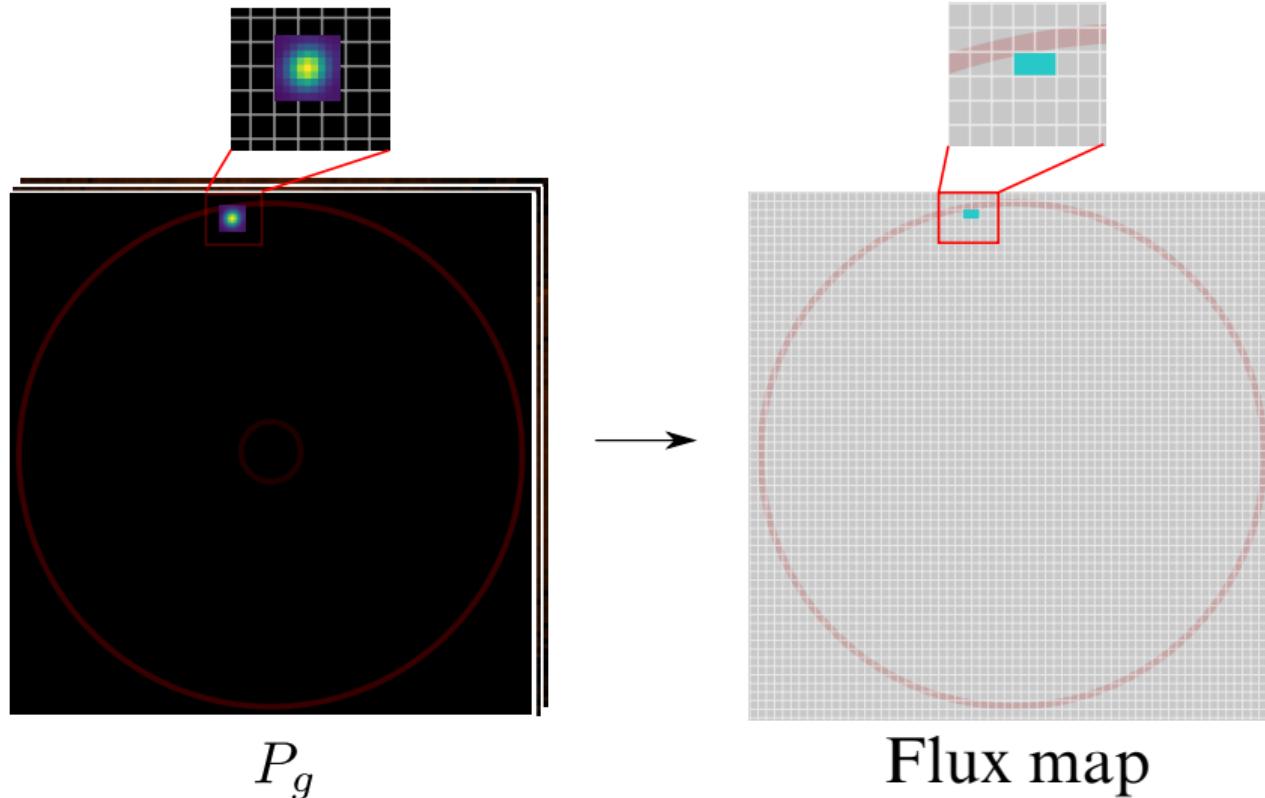
<sup>4</sup>Gillis and Vavasis, 2018

# Trajectories

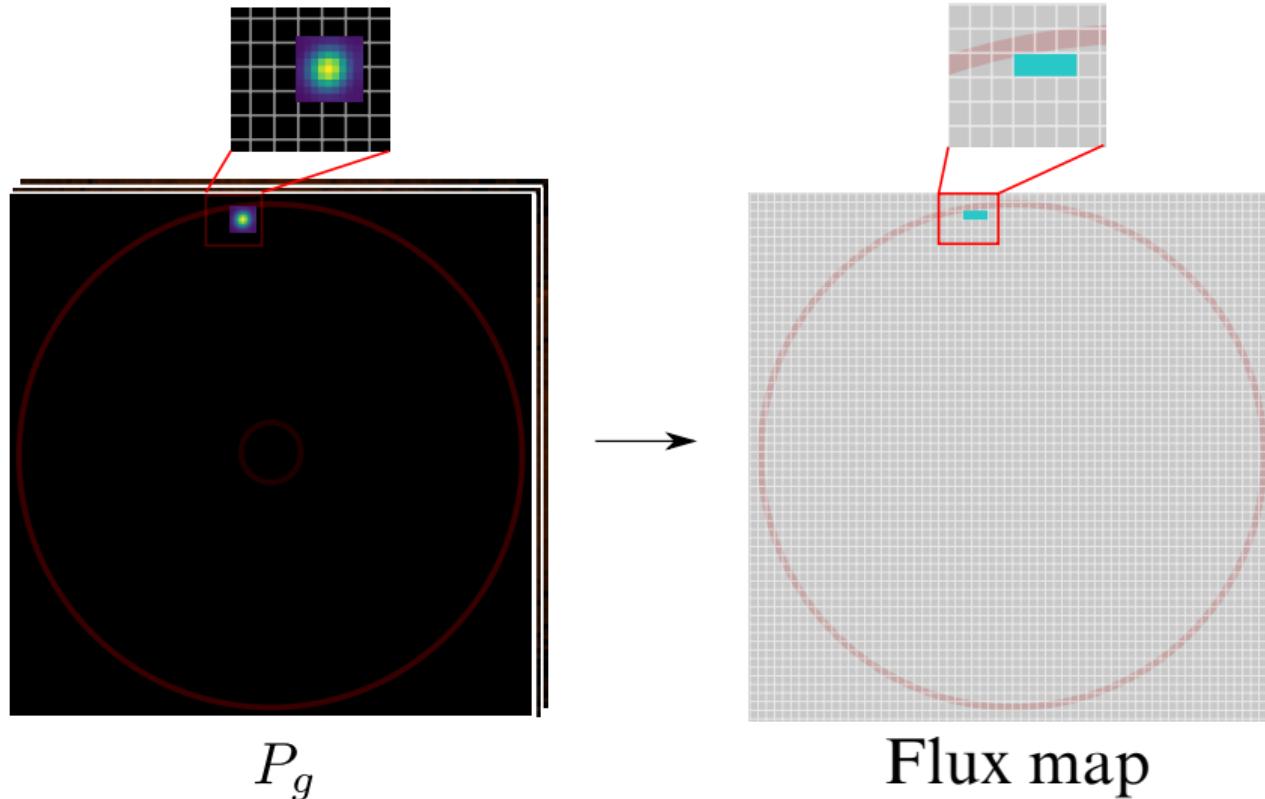
 $P_g$ 

Flux map

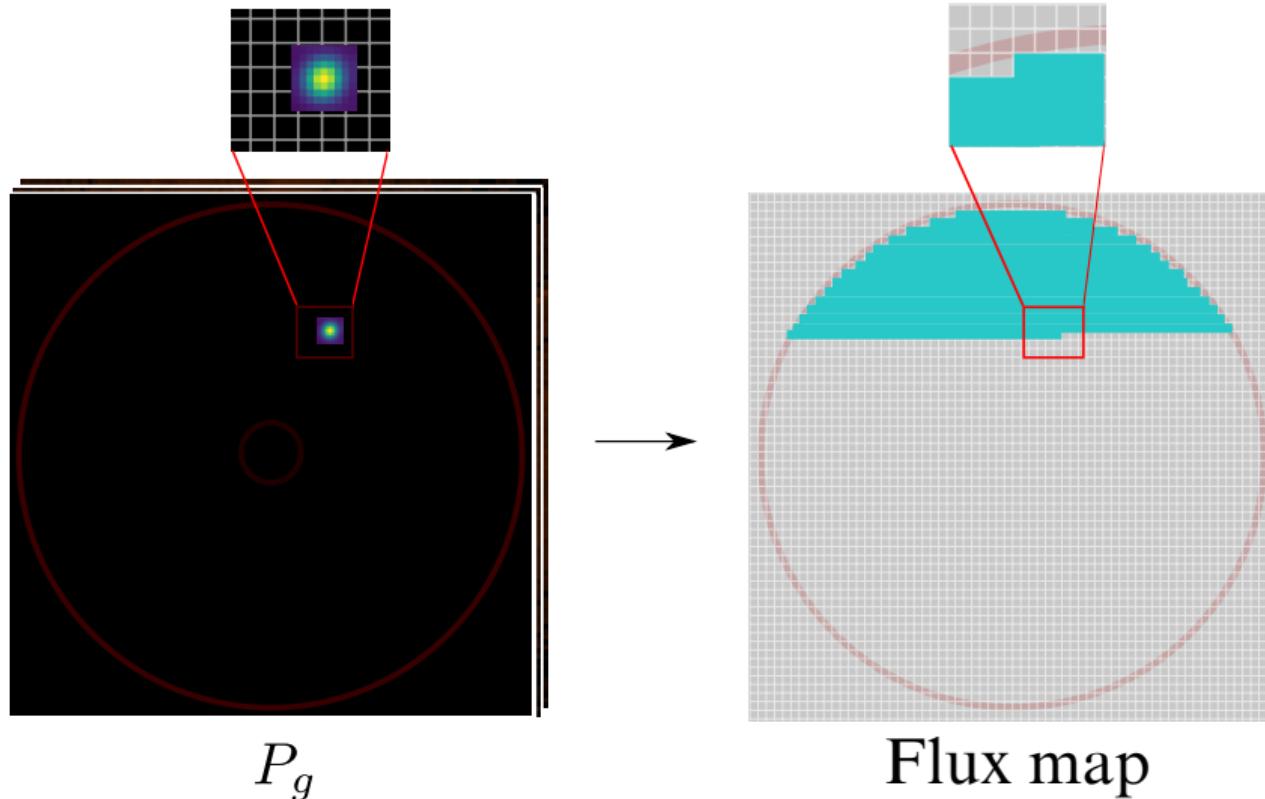
# Trajectories



# Trajectories



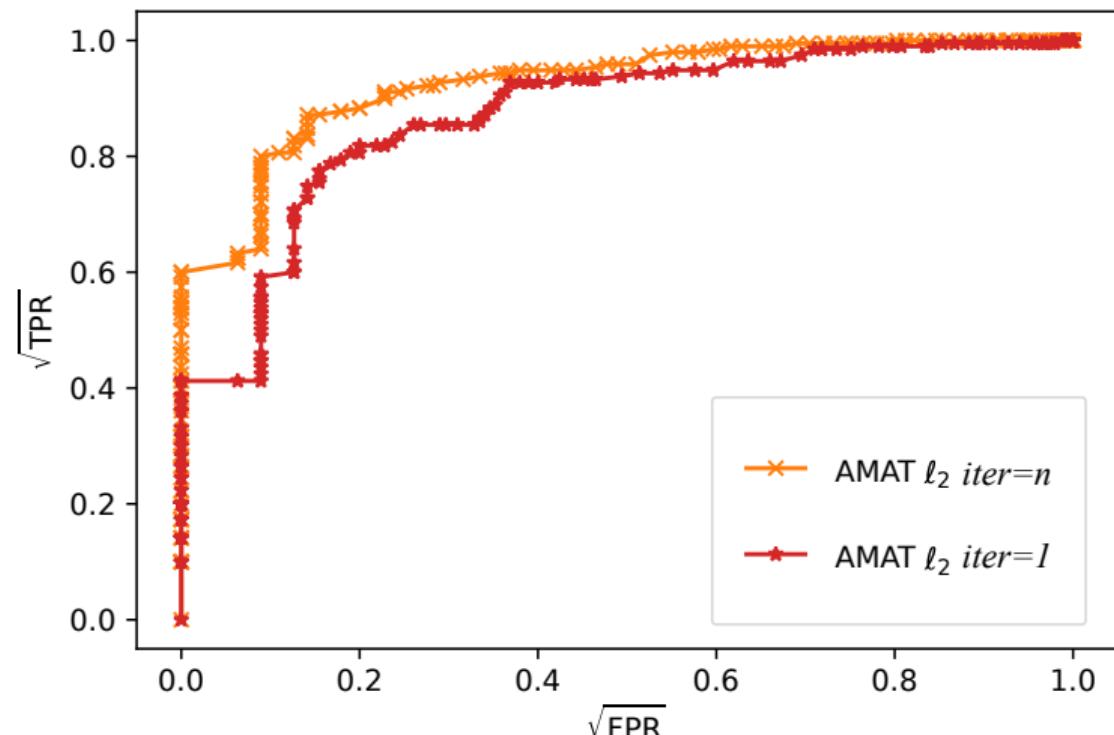
# Trajectories



# Numerical Experiments

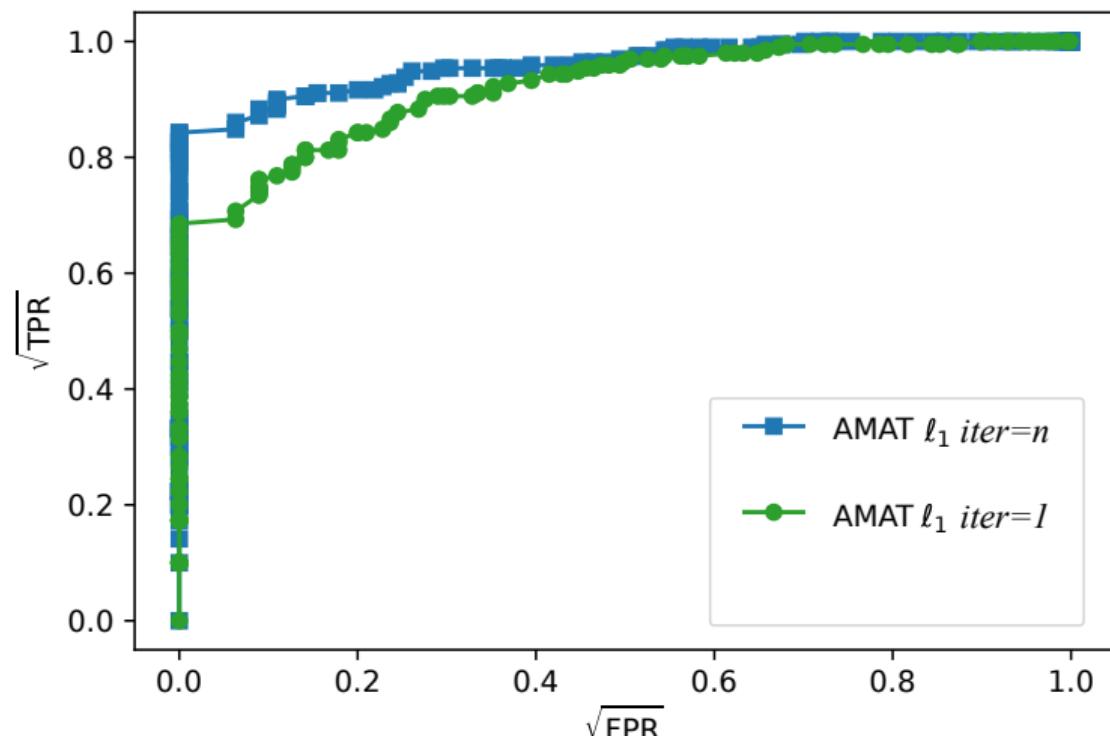
## Numerical Experiments - ROC Curves

- ▶ Synthetic planets are injected.
- ▶  $\sqrt{\text{TPR}}$  &  $\sqrt{\text{FPR}}$  are used instead of TPR & FPR.



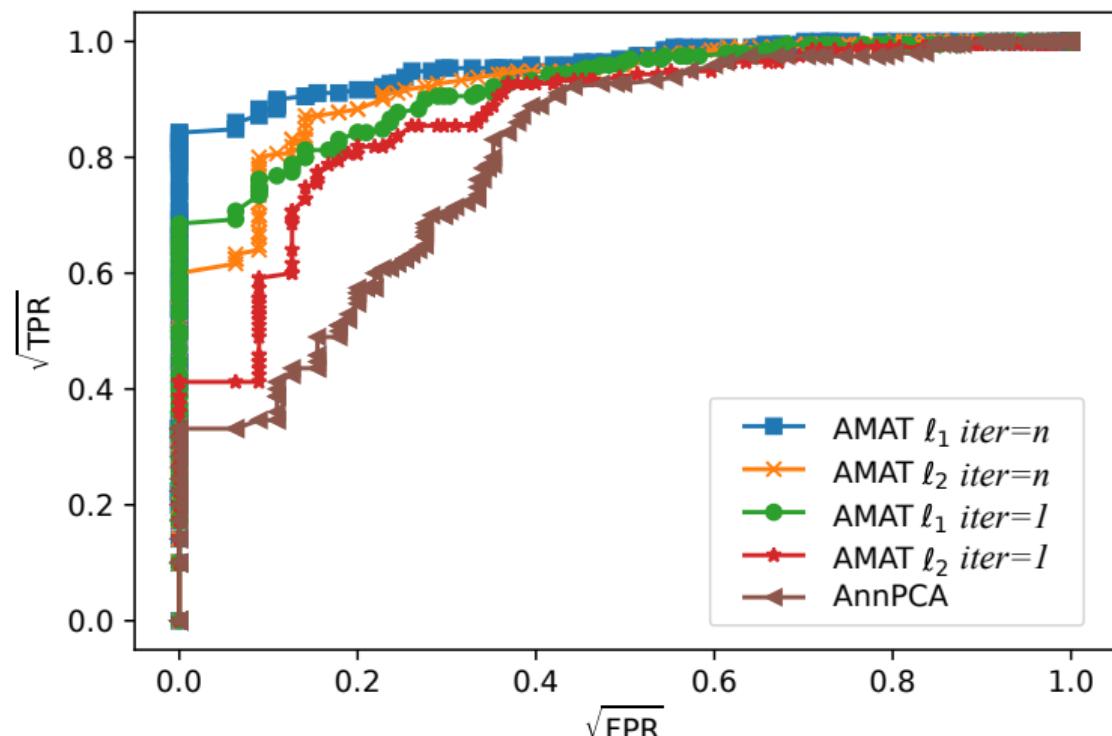
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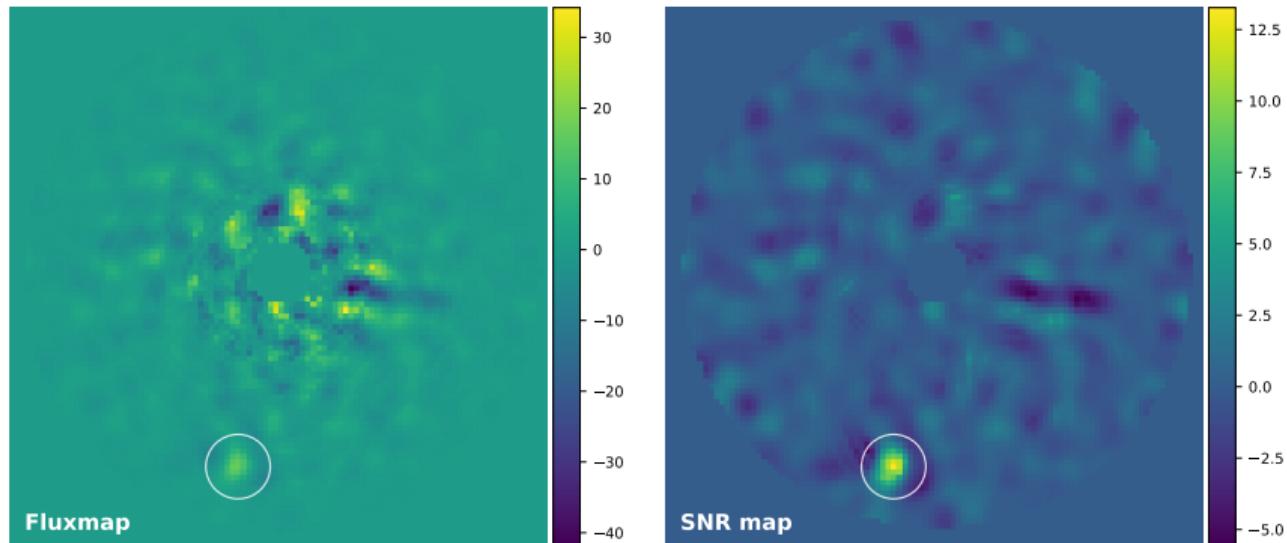


Thank you for your attention!  
Any questions?

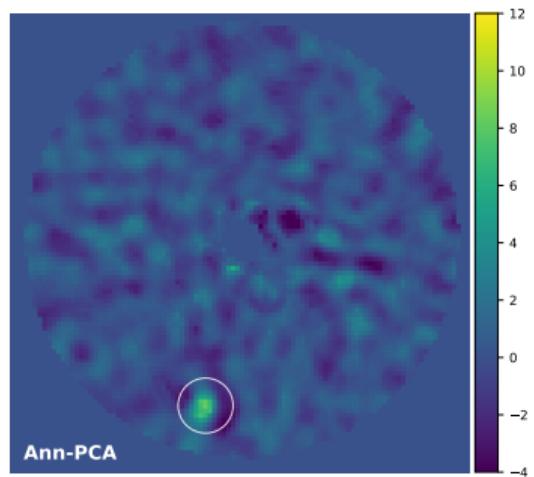
[hazan.daglayan@uclouvain.be](mailto:hazan.daglayan@uclouvain.be)  
GitHub: [hazandaglayan/AMAT](https://github.com/hazandaglayan)



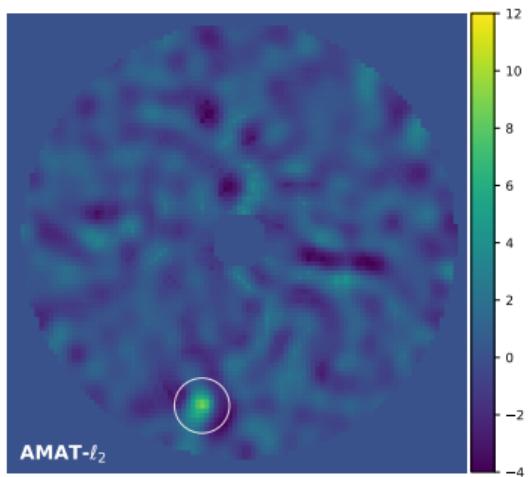
# From Fluxmap to SNR



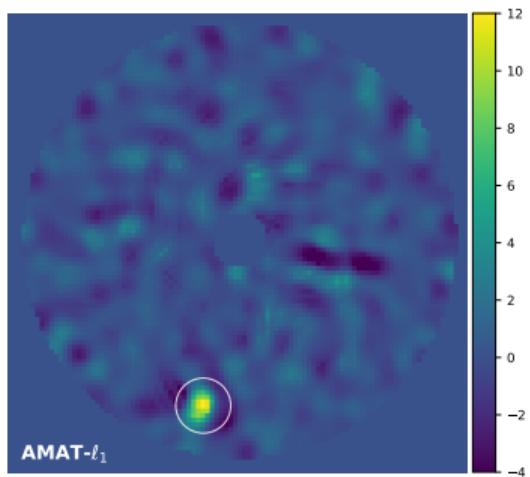
# Detection Maps Comparison



Ann-PCA



AMAT- $l_2$



AMAT- $l_1$