

# Direct Exoplanet Detection Using L1 Norm Low-Rank Approximation

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# Exoplanet imaging



# Exoplanet imaging

The image displays two news articles side-by-side, both featuring exoplanets.

**euronews.next** article:

**Signs of life? James Webb reveals more about exoplanet K2-18 b's atmosphere**

A large image shows a blue exoplanet against a dark background with a small red star visible.

By Luke Wurst  
Published on 12/08/2023 - 17:35 • Updated 13/08/2023 - 08:38

**BBC NEWS** article:

**Exoplanet discovered around neighbouring star**

A large image shows a brownish exoplanet against a black background.

By Paul Rincon  
Published on 14 November 2010

# Exoplanet imaging



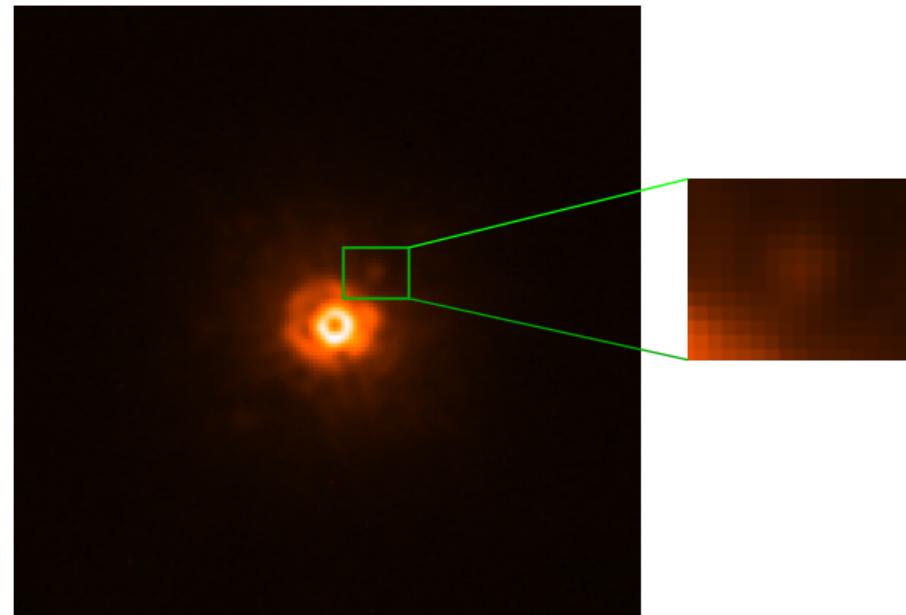
# Exoplanet imaging

A real image



# Exoplanet imaging

A real image of exoplanet



# Direct imaging



Credit: <https://exoplanets.nasa.gov/>

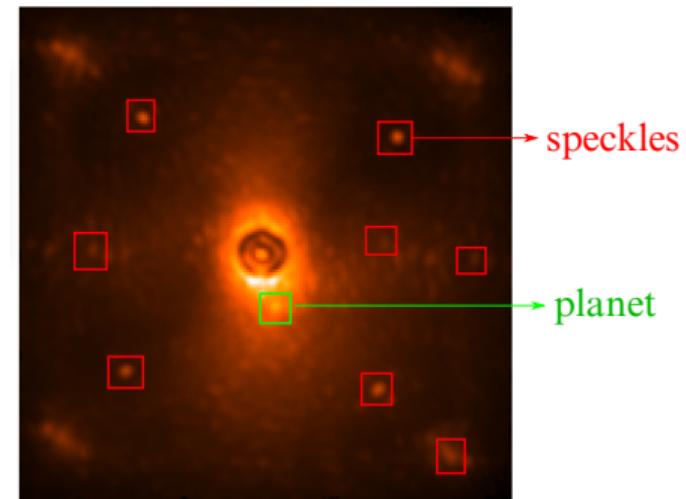
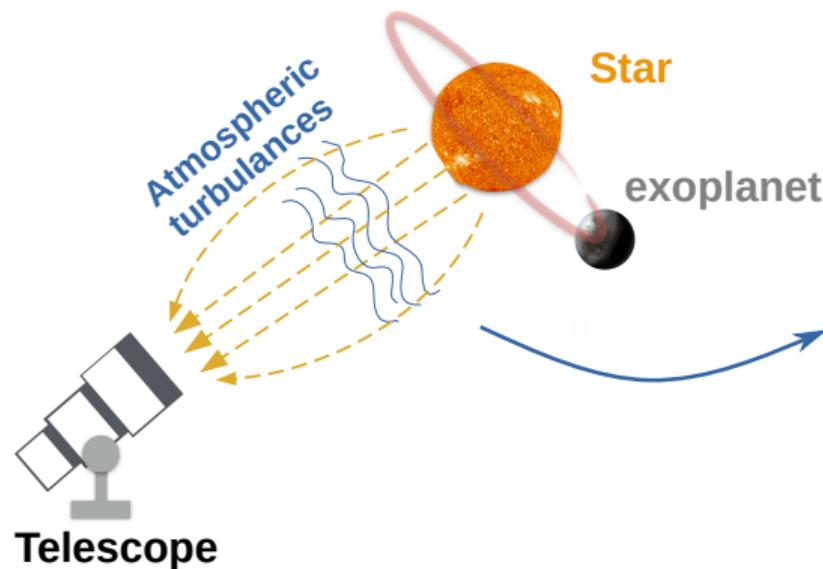
# Direct imaging



Credit: <https://exoplanets.nasa.gov/>

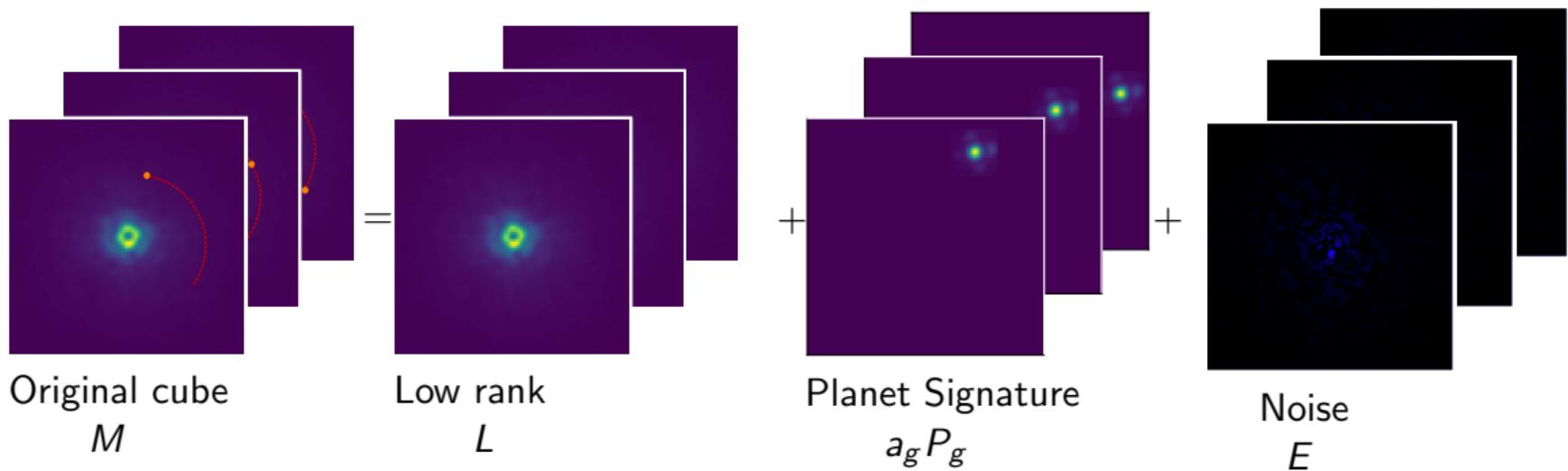
- ▶ firefly → exoplanet
- ▶ lighthouse → star

# Direct Imaging

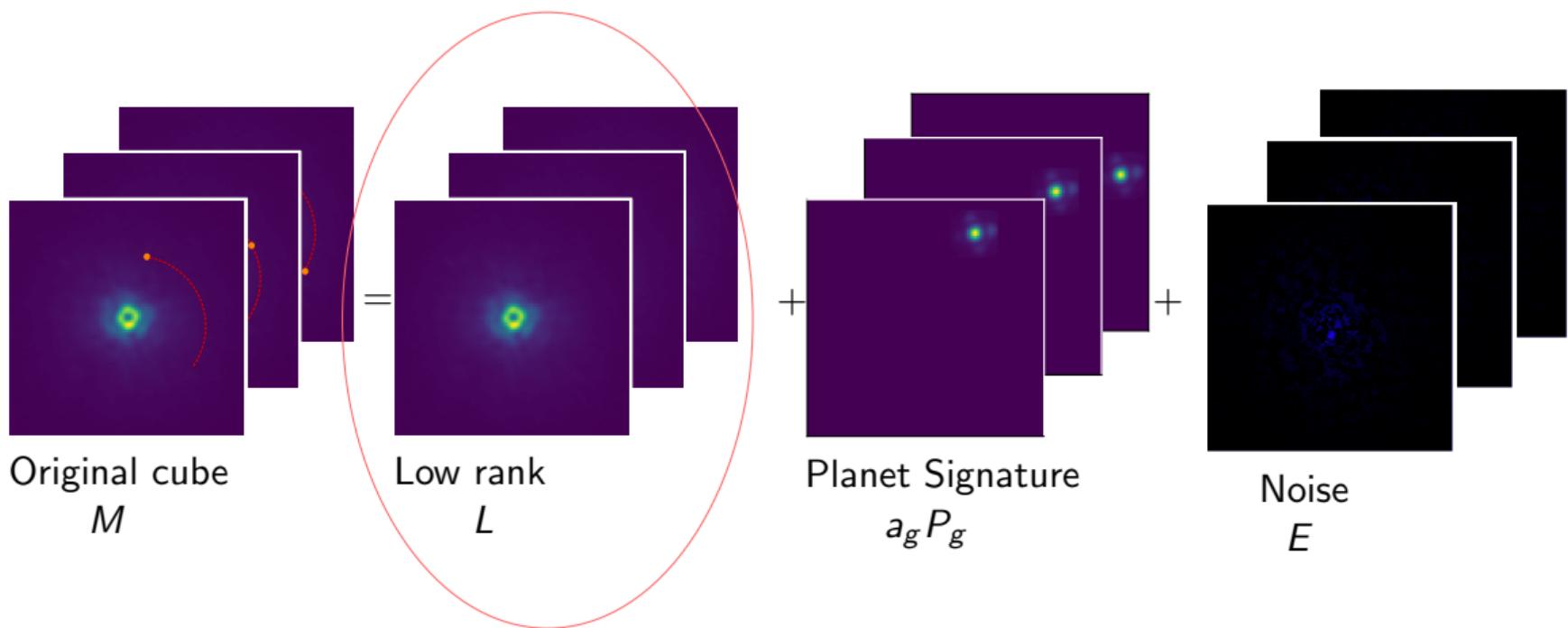


# Angular differential imaging

## Problem setup & goal



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# Background: (annular) PCA<sup>1,2</sup>

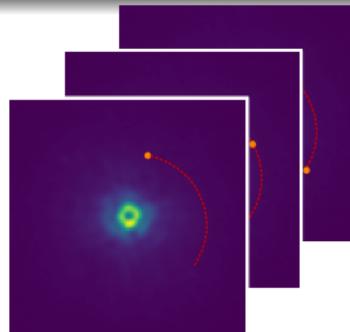
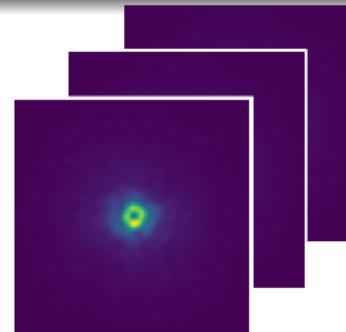


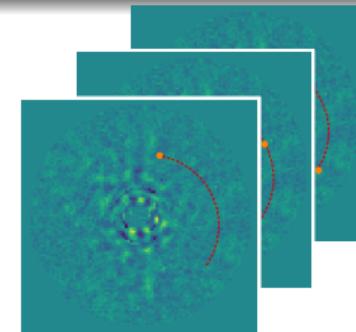
Image sequence  
 $M$

=

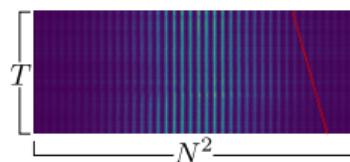


Low rank  
 $L$

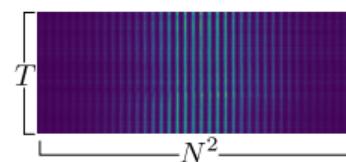
+



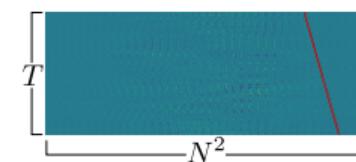
Foreground  
 $R$



=



+



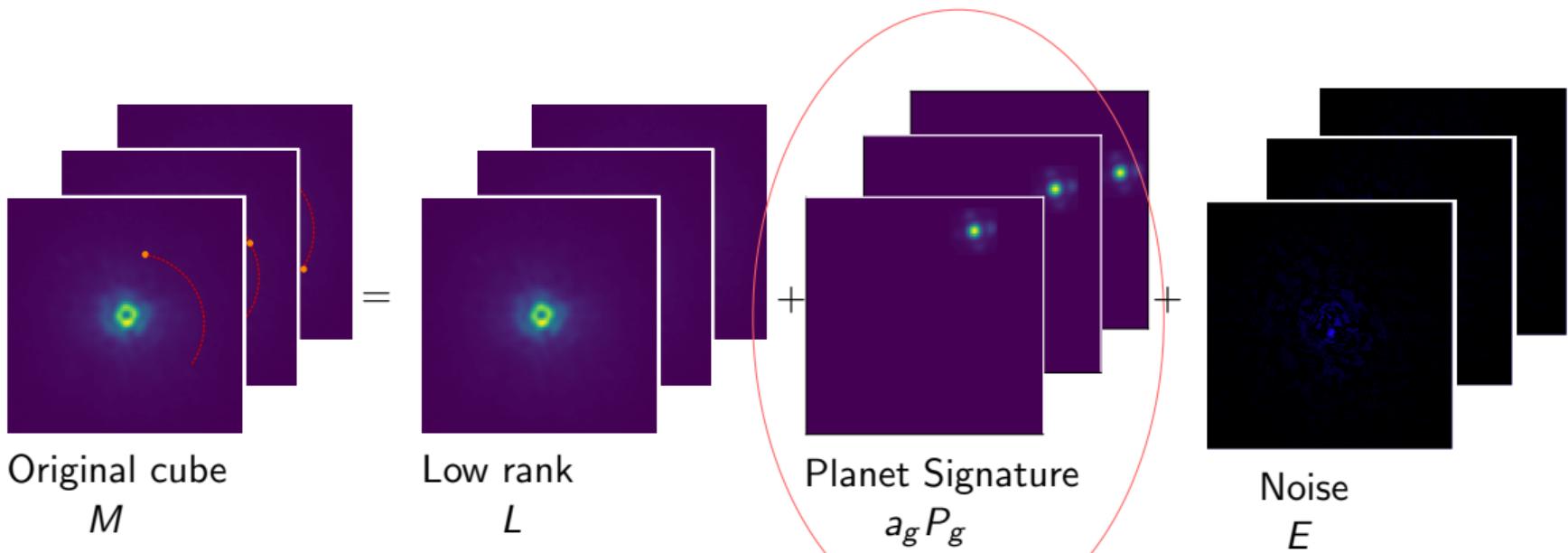
$$\hat{L} = \arg \min_L \|M - L\|_F \quad \text{subject to} \quad \text{rank}(L) \leq k$$

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<sup>1</sup>Amara and Quanz, 2012

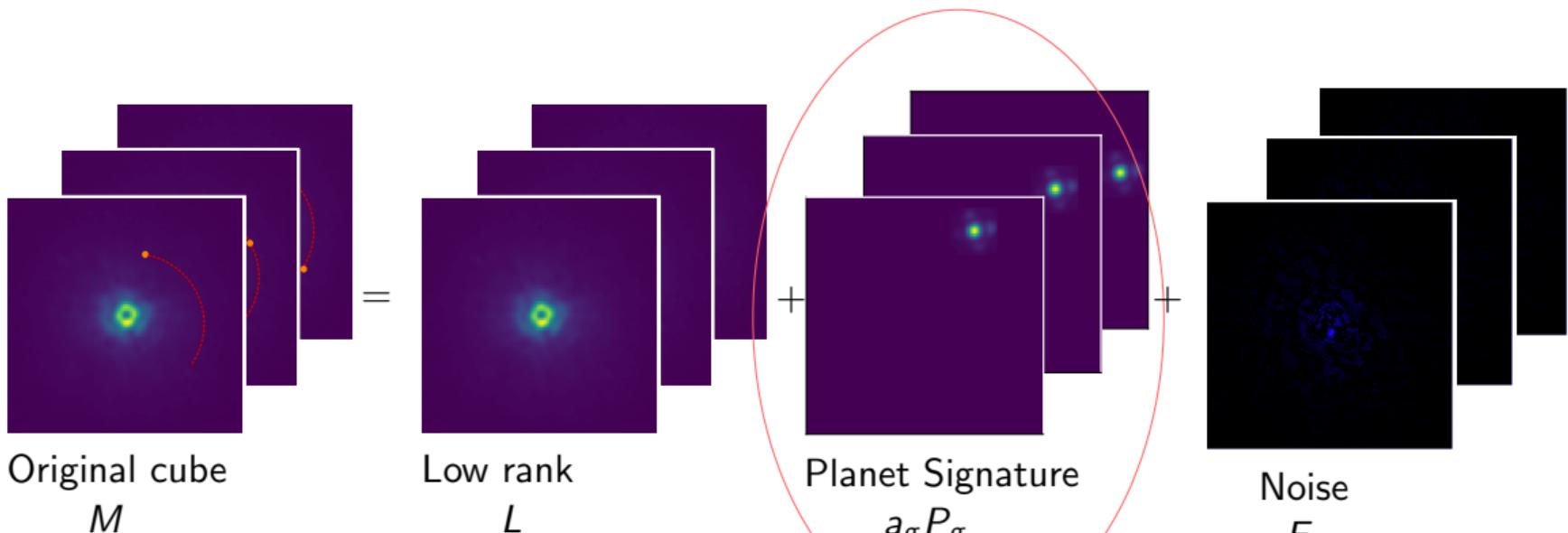
<sup>2</sup>Soummer, et al., 2012

# The intensity of planet signature



$$\hat{a}_g = \arg \min_{a_g > 0} \|M - \hat{L} - a_g P_g\|_2,$$

# The intensity of planet signature



$$\hat{a}_g = \arg \min_{a_g > 0} \|M - \hat{L} - a_g P_g\|_2,$$

$$\hat{a}_g = \frac{\langle P_g, M - \hat{L} \rangle}{\|P_g\|_F^2}$$

# The intensity of planet signature

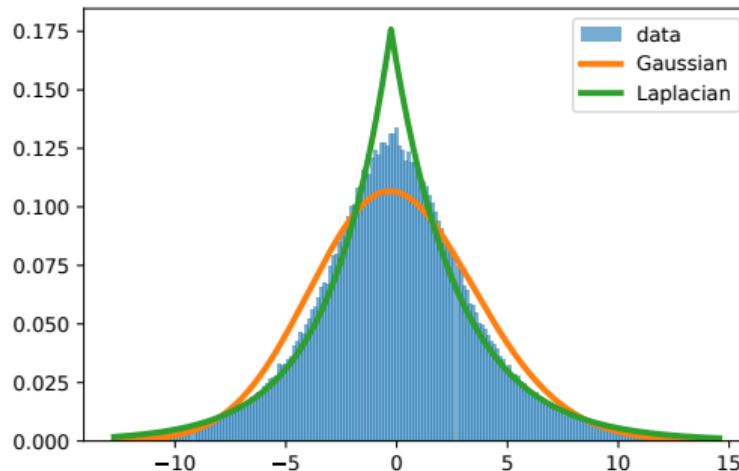


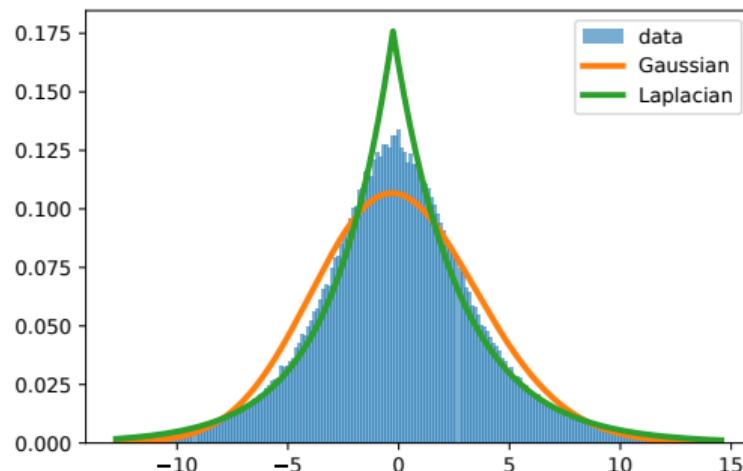
Figure: Residual data at  $10\lambda/D$ <sup>3,4</sup>

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<sup>3</sup>Pairet, et al., 2019

<sup>4</sup>Daglayan, et al., 2022

# The intensity of planet signature



$$\hat{a}_g = \arg \min_{a_g > 0} \| M - \hat{L} - a_g P_g \|_1$$

Figure: Residual data at  $10\lambda/D$ <sup>3,4</sup>

<sup>3</sup>Pairet, et al., 2019

<sup>4</sup>Daglayan, et al., 2022

# The intensity of planet signature

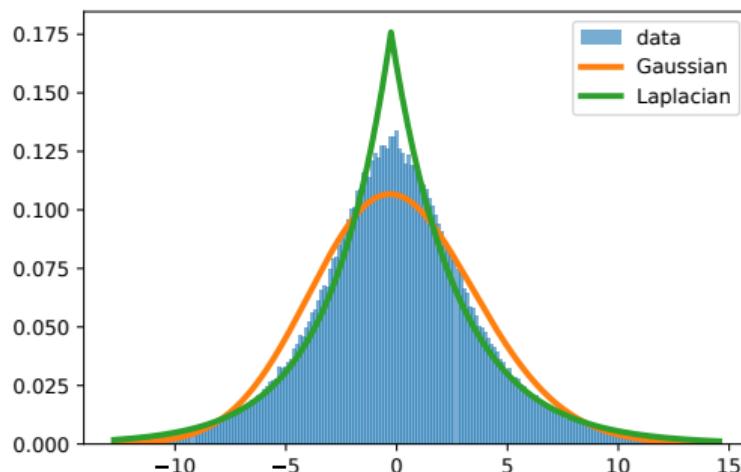


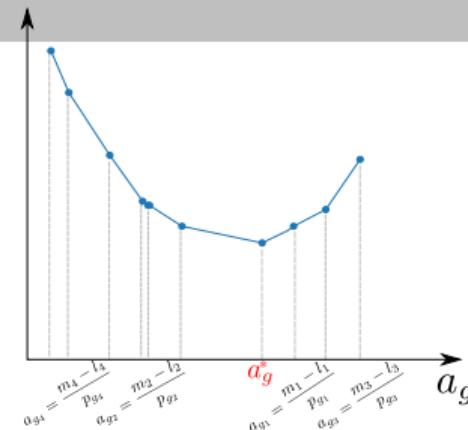
Figure: Residual data at  $10\lambda/D$ <sup>3,4</sup>

<sup>3</sup>Pairet, et al., 2019

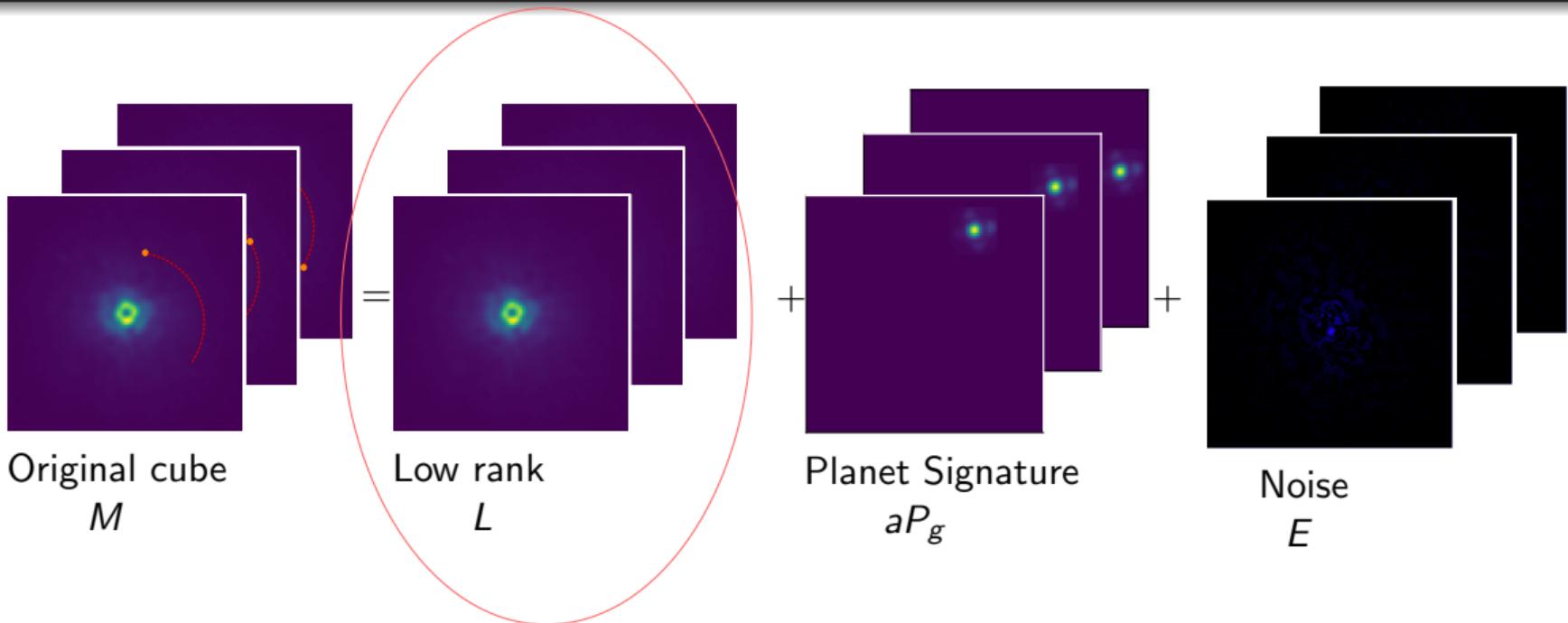
<sup>4</sup>Daglayan, et al., 2022

$$\hat{a}_g = \arg \min_{a_g > 0} \| M - \hat{L} - a_g P_g \|_1$$

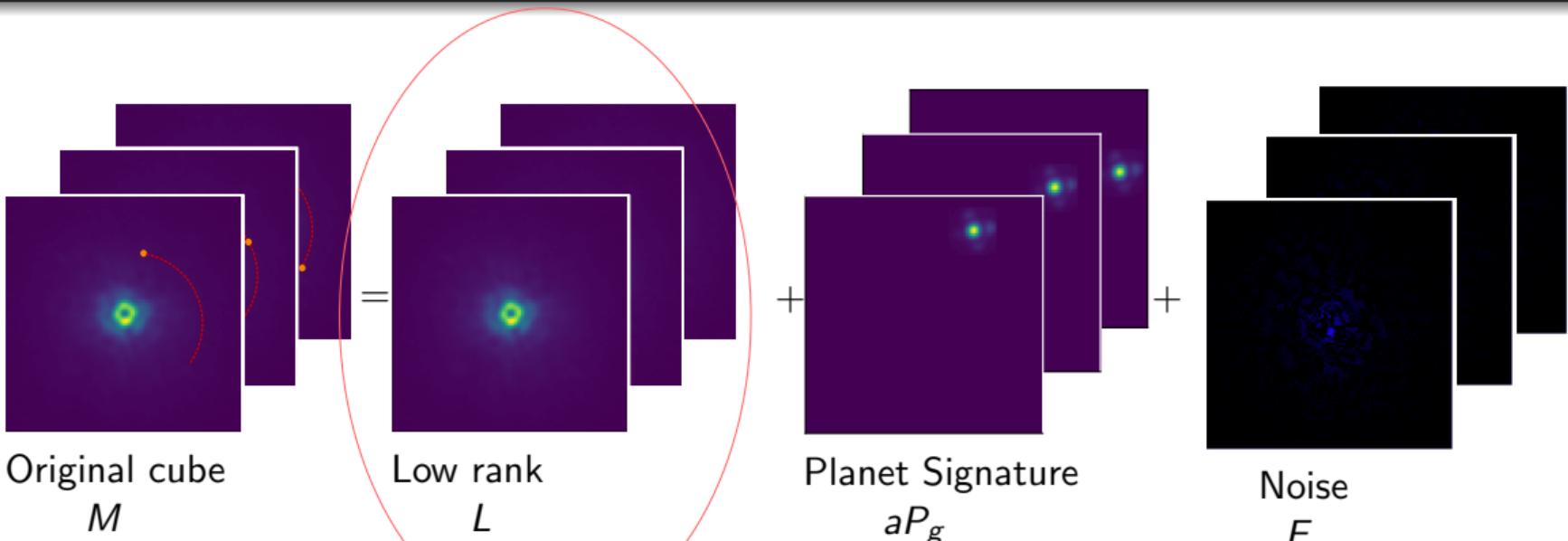
$$\hat{a}_g = \arg \min_{a_g \in \mathbb{R}} \sum_j |m_j - l_j - a_g p_{gj}|$$



# Background: (annular) L1-LRA



## Background: (annular) L1-LRA



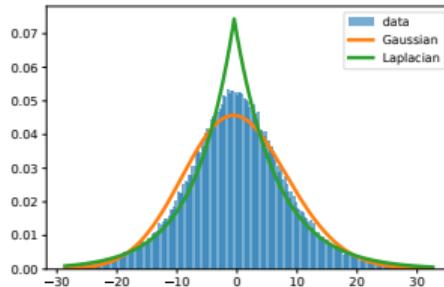
$$\hat{L} = \arg \min_L \|M - L\|_1 \quad \text{subject to} \quad \text{rank}(L) \leq k$$

- Solved using an exact block-cyclic coordinate descent method<sup>5</sup>.

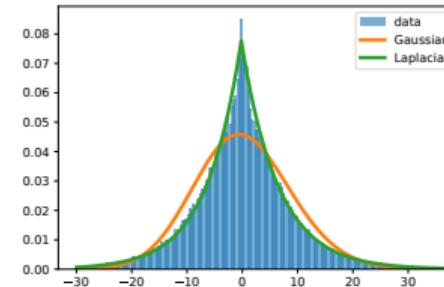
<sup>5</sup>Gillis and Plemmons, 2011

# Data distributions of residual matrix after PCA/L1-LRA

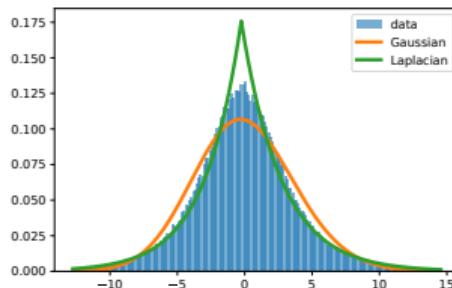
PCA  
at  $4\lambda/D$



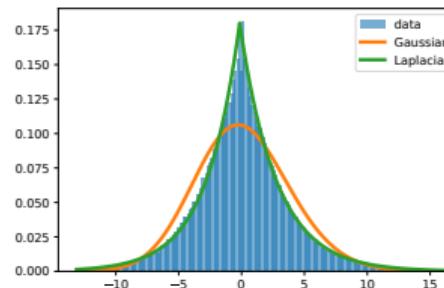
L1-LRA  
at  $4\lambda/D$



PCA  
at  $10\lambda/D$



L1-LRA  
at  $10\lambda/D$



# Coefficient of determination

$$\rho^2 = \frac{(\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}))^2}{(\sum_{i=1}^n (x_i - \bar{x})^2)(\sum_{i=1}^n (y_i - \bar{y})^2)}$$

- ▶  $x_i$  : the height of the bins of the data histogram
- ▶  $y_i$  : the values of the probability density function within these bins
- ▶  $\bar{x}$  and  $\bar{y}$  : mean of  $x_i$  and  $y_i$ , respectively

Table: The coefficient of determination  $\rho^2$  for large separation  $10\lambda/D$

Rank	PCA		L1-LRA	
	Gaussian	Laplacian	Gaussian	Laplacian
5	0.9866	0.9920	0.9894	<b>0.9940</b>
10	0.9859	0.9936	0.9872	<b>0.9948</b>
15	0.9912	0.9954	0.9922	<b>0.9960</b>

# Likelihood ratio map

L1 Norm<sup>6</sup>

$$\log \Lambda_g(R) = - \sum_{(t,r) \in \Omega_g} \frac{|R(t,r) - \hat{a}_g P_g(t,r)| - |R(t,r)|}{\sigma_{R(r)}} \quad (1)$$

L2 Norm

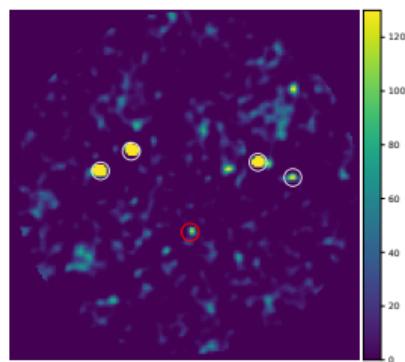
$$\log \Lambda_g(R) = - \frac{1}{2} \sum_{(t,r) \in \Omega_g} \frac{|R(t,r) - \hat{a}_g P_g(t,r)|^2 - |R(t,r)|^2}{\sigma_{R(r)}^2} \quad (2)$$

Background subtraction	Planet detection	
	L1 (2)	L2 (1)
L1-LRA	L1L1	L1L2
PCA	L2L1	L2L2

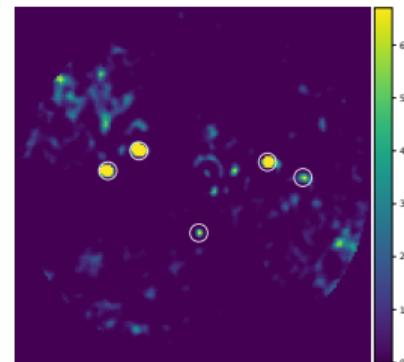
<sup>6</sup>Daglayan, et al., 2022

# Likelihood ratio map results

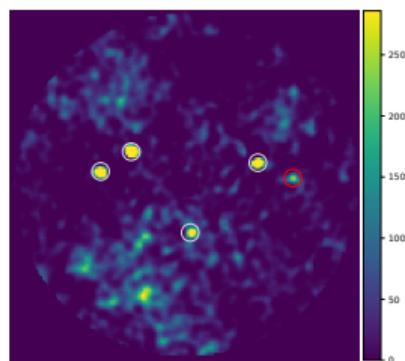
L2L2



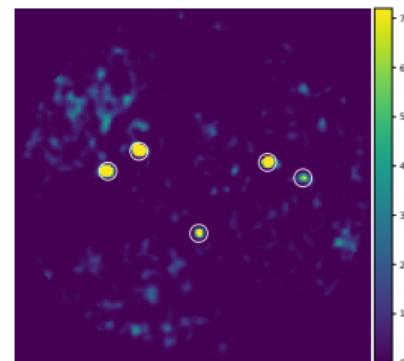
L2L1



L1L2

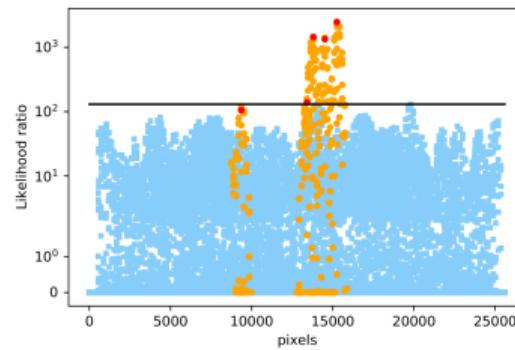


L1L1

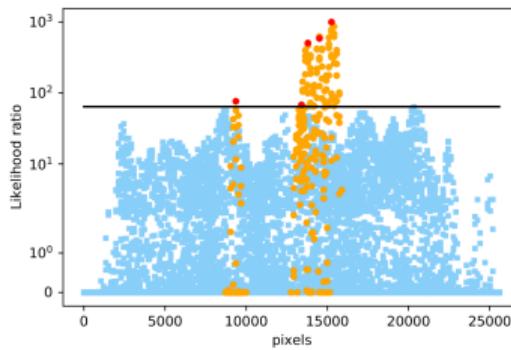


# Likelihood ratio map results

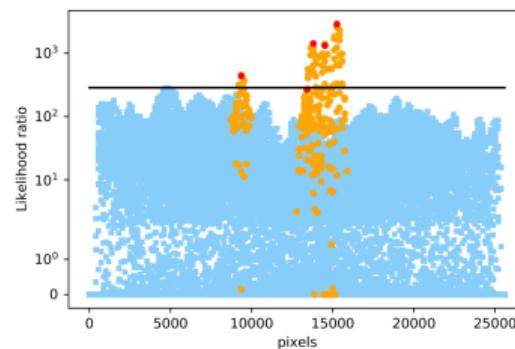
L2L2



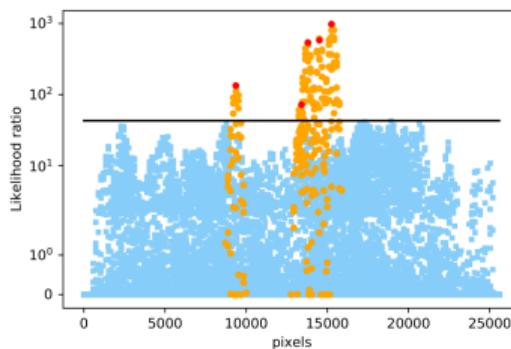
L2L1



L1L2



L1L1



# ROC curve results

- ▶  $\sqrt{\text{TPR}}$  &  $\sqrt{\text{FPR}}$  are used instead of TPR & FPR.
- ▶ 50 different datasets are used by injecting two synthetic planets in each, 180 degrees apart.
- ▶ Planets are placed at the separation  $10\lambda/D$ .

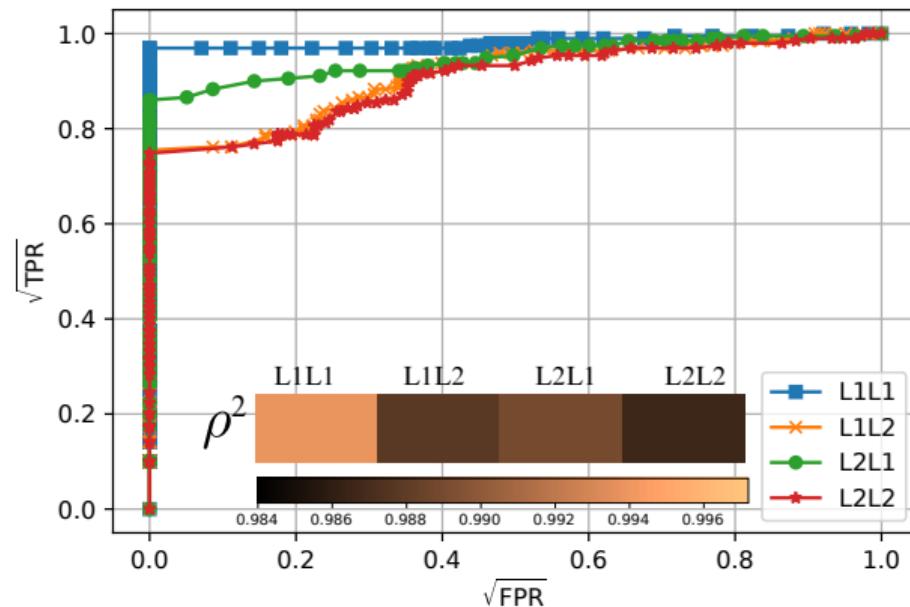


Figure: ROC Curves - Likelihood ratio map

# ROC curve results

- ▶  $\sqrt{\text{TPR}}$  &  $\sqrt{\text{FPR}}$  are used instead of TPR & FPR.
- ▶ 50 different datasets are used by injecting two synthetic planets in each, 180 degrees apart.
- ▶ Planets are placed at the separation  $4\lambda/D$ .

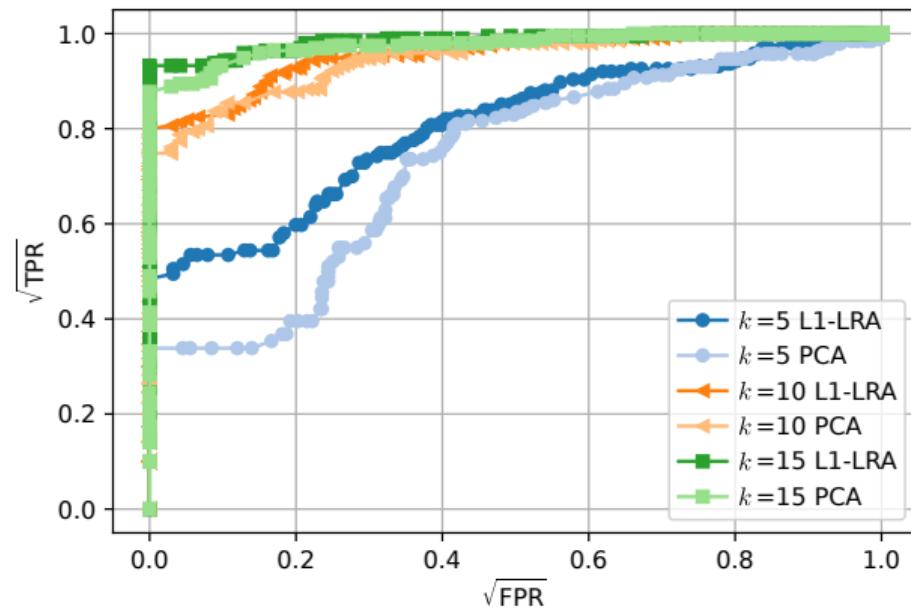


Figure: ROC Curves - SNR map

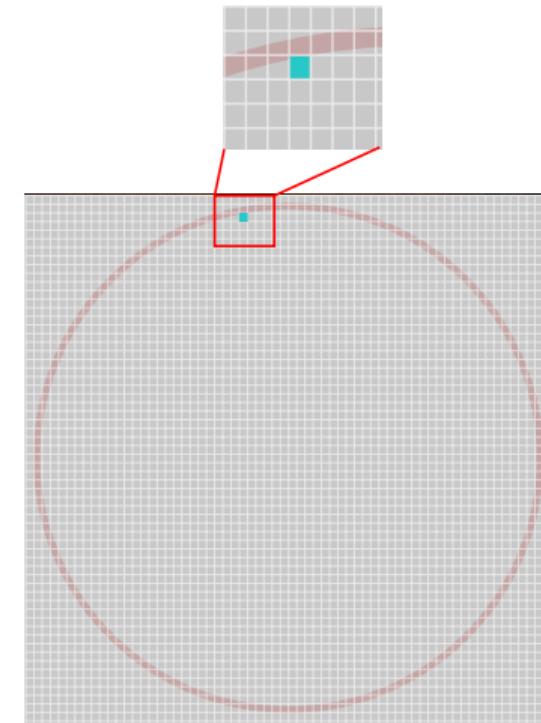
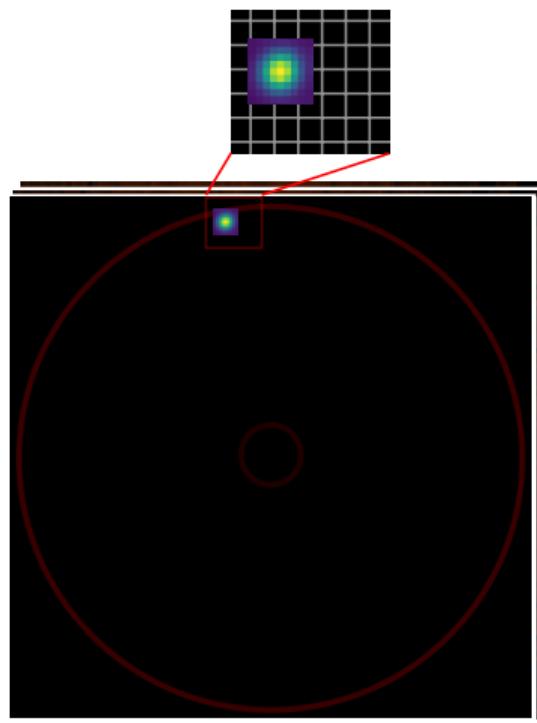
Thank you for your attention!  
Any questions?

[hazan.daglayan@uclouvain.be](mailto:hazan.daglayan@uclouvain.be)

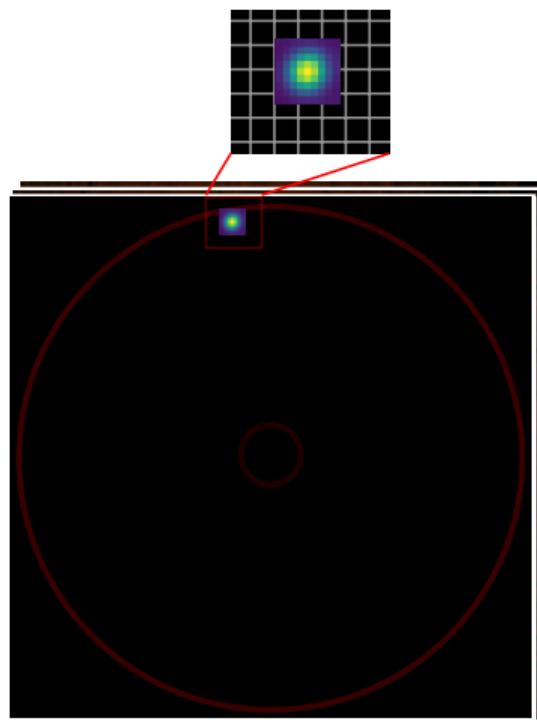
GitHub: [hazandaglayan/l1lra\\_for\\_exoplanets](https://github.com/hazandaglayan/l1lra_for_exoplanets)



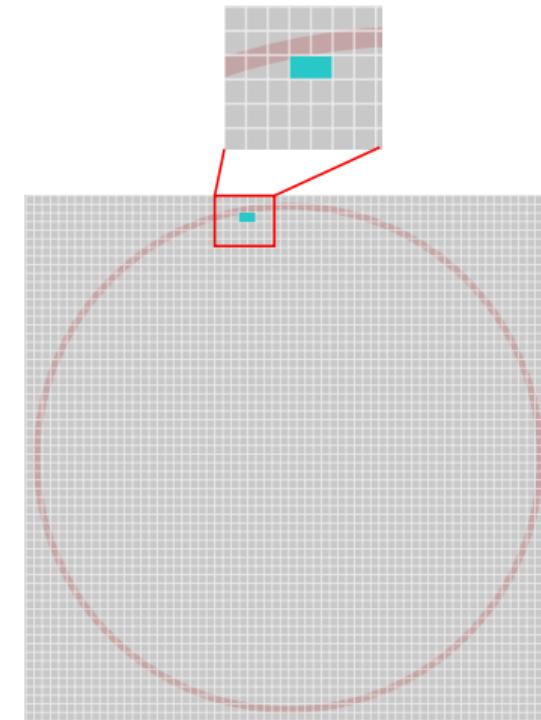
# Trajectories



# Trajectories

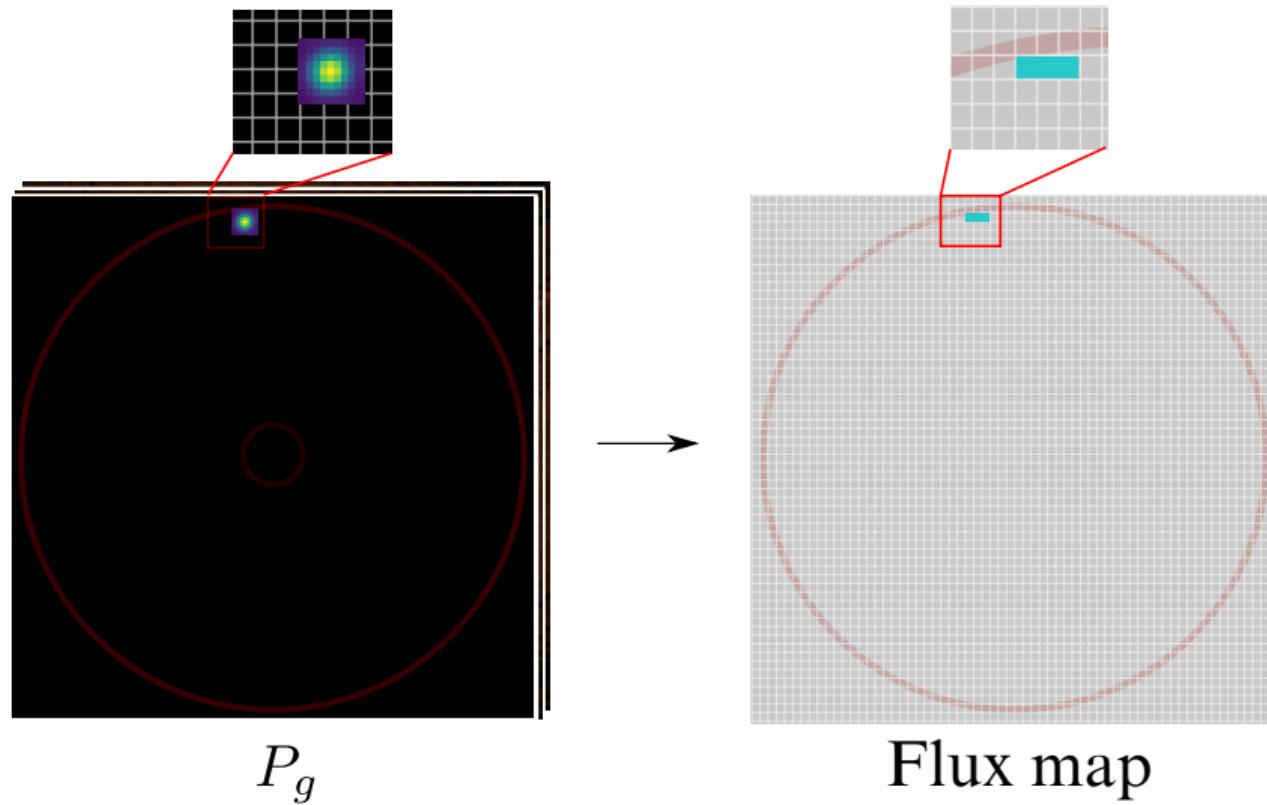


$P_g$

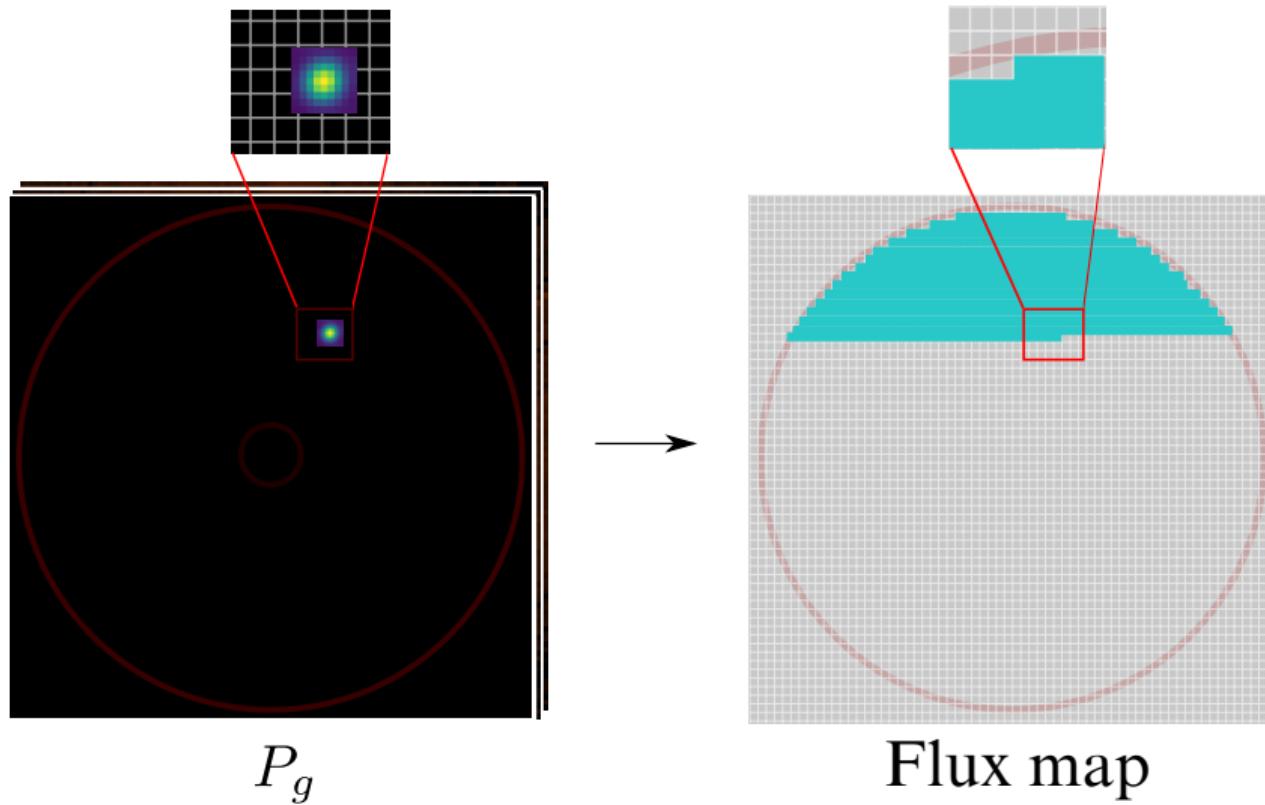


Flux map

# Trajectories



# Trajectories



# More ROC curves

