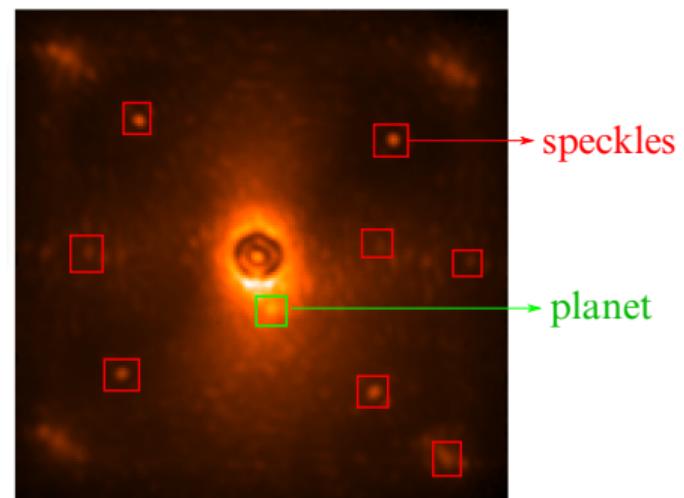
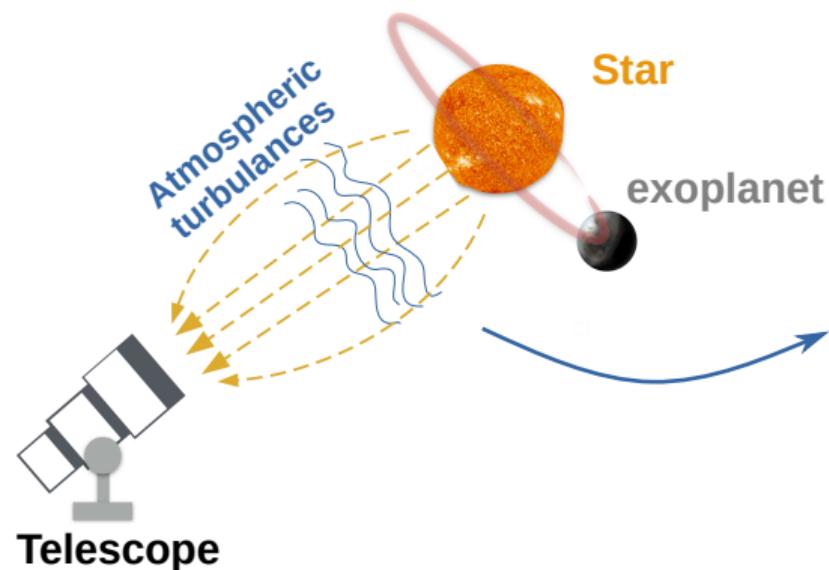


# An Alternating Minimization Algorithm with Trajectory for Direct Exoplanet Detection

Hazan Daglayan, Simon Vary, and P.-A. Absil

ICTEAM/UCLouvain

# Direct Imaging



# Angular Differential Imaging

# Problem Setup & Goal

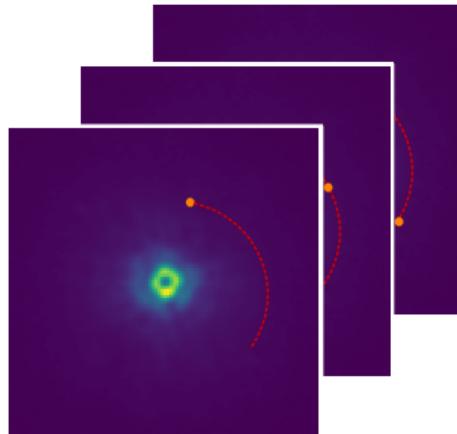
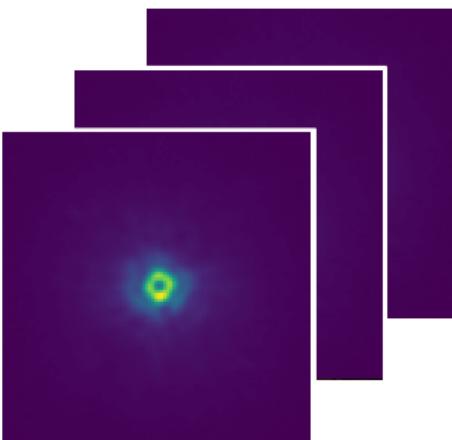


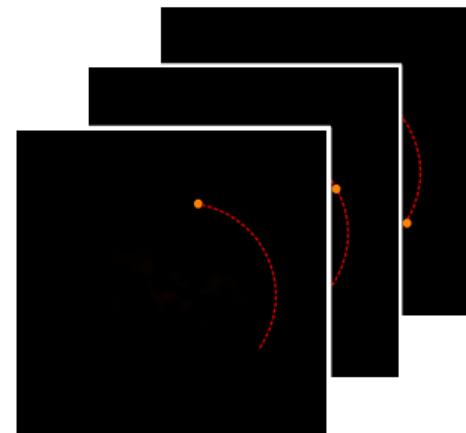
Image sequence

=

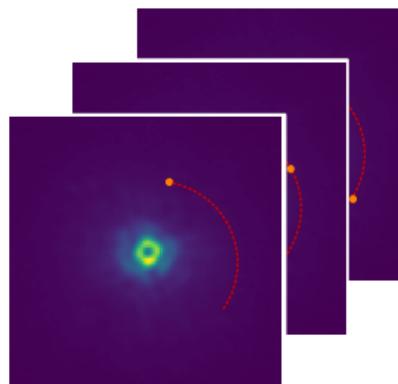


Background  
(star+speckles)

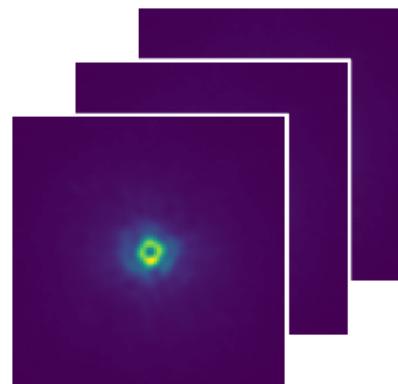
+



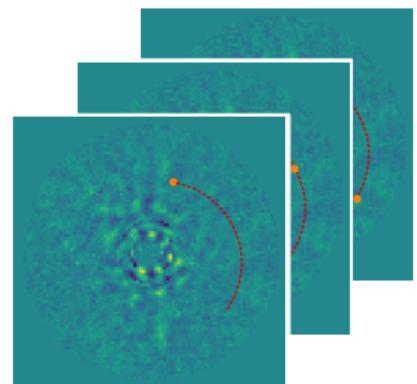
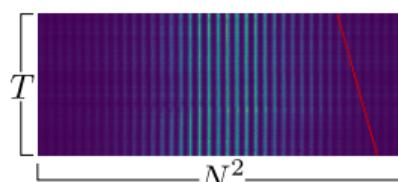
Foreground  
(planet)

Background: (Annular) PCA<sup>1,2</sup>Image sequence  
 $M$ 

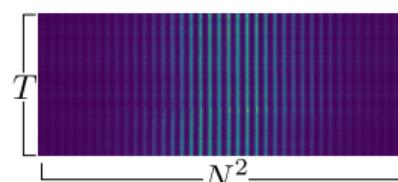
=

Low rank  
 $L$ 

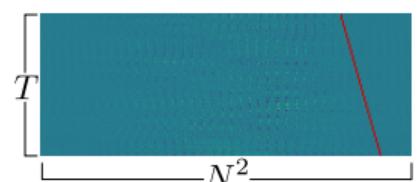
+

Foreground  
 $R$ 

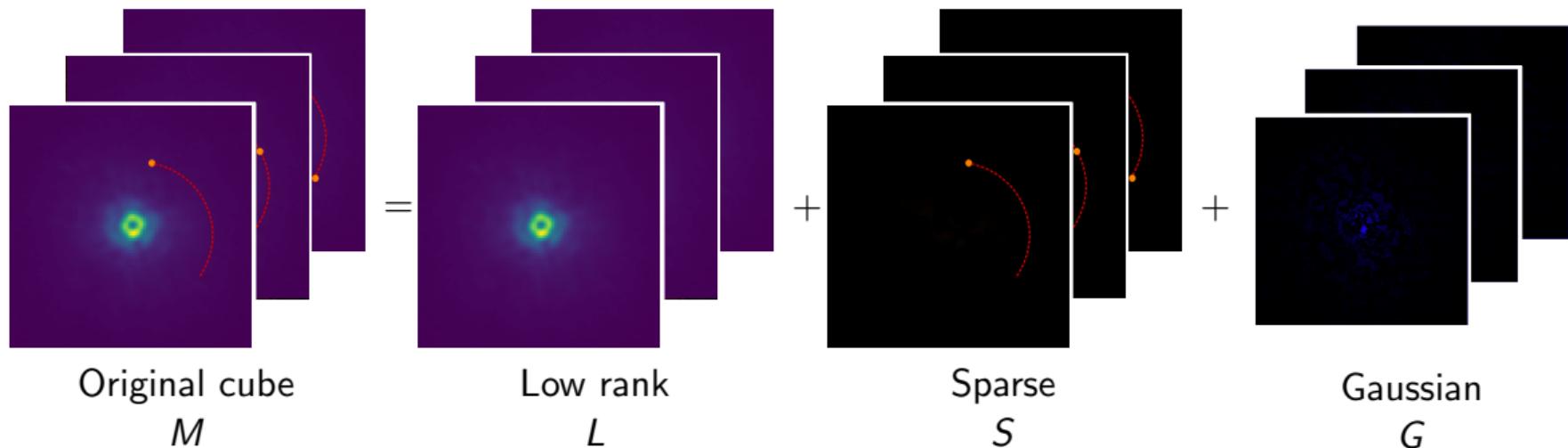
=



+

<sup>1</sup>Amara and Quanz, 2012<sup>2</sup>Soummer, et al., 2012

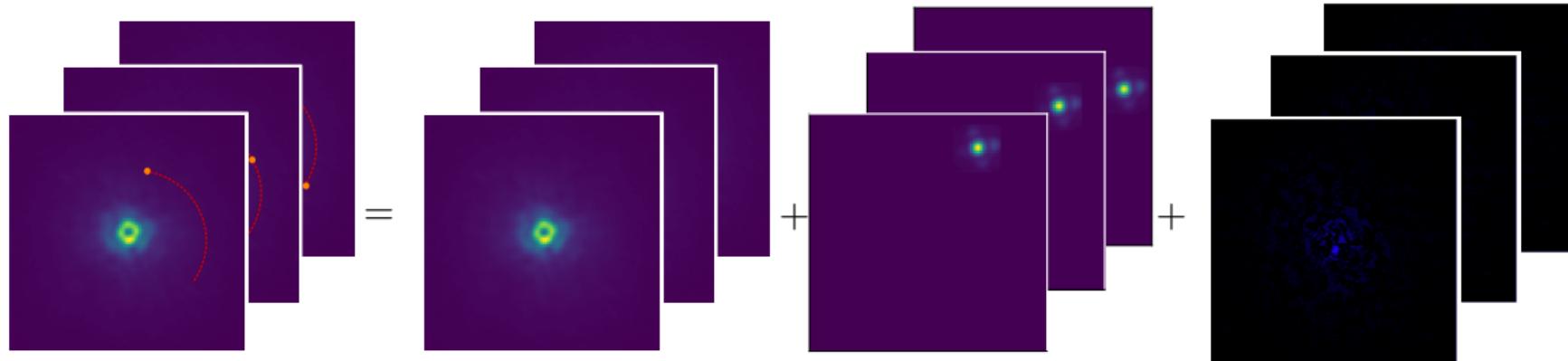
## State of art: LLSG<sup>3</sup>



$$\text{rank}(L) \leq k, \quad \text{card}(S) \leq s$$

<sup>3</sup>Gomez Gonzalez, et al., 2016

## Alternating Minimization Algorithm with Trajectory (AMAT)

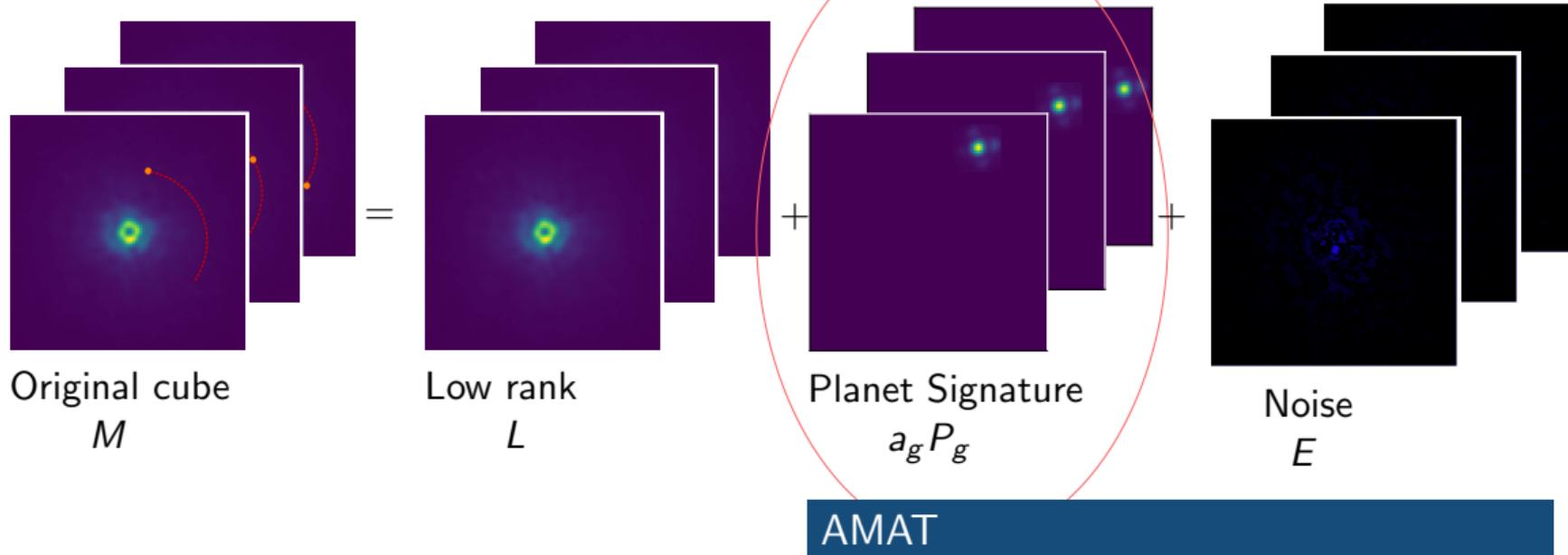
Original cube  
 $M$ Low rank  
 $L$ Planet Signature  
 $a_g P_g$ Noise  
 $E$ 

$$\text{rank}(L) \leq k, \quad P_g \in \Lambda$$

AMAT

$$\begin{array}{ll} \min_{L \in \mathbb{R}^{t \times n}, a_g \in \mathbb{R}} & \|M - L - a_g P_g\| \\ \text{s.t.} & \text{rank}(L) \leq k \end{array}$$

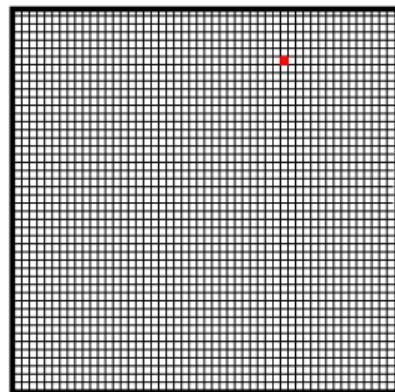
## Alternating Minimization Algorithm with Trajectory (AMAT)



$$\text{rank}(L) \leq k, \quad P_g \in \Lambda$$

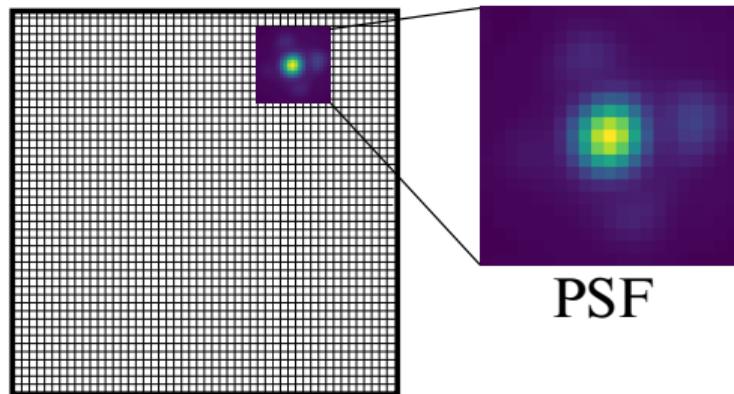
$$\min_{L \in \mathbb{R}^{t \times n}, a_g \in \mathbb{R}} \|M - L - a_g P_g\| \\ \text{s.t. } \text{rank}(L) \leq k$$

# Planet Signature



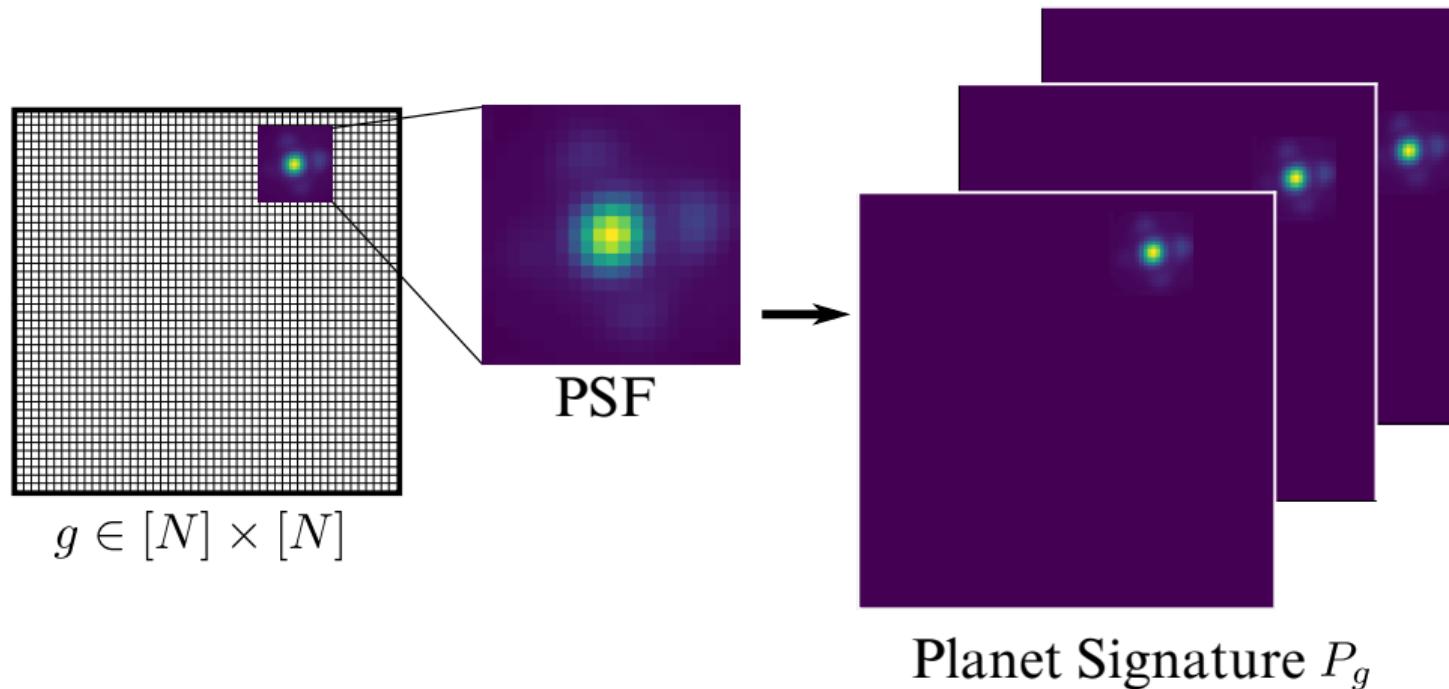
$$g \in [N] \times [N]$$

# Planet Signature

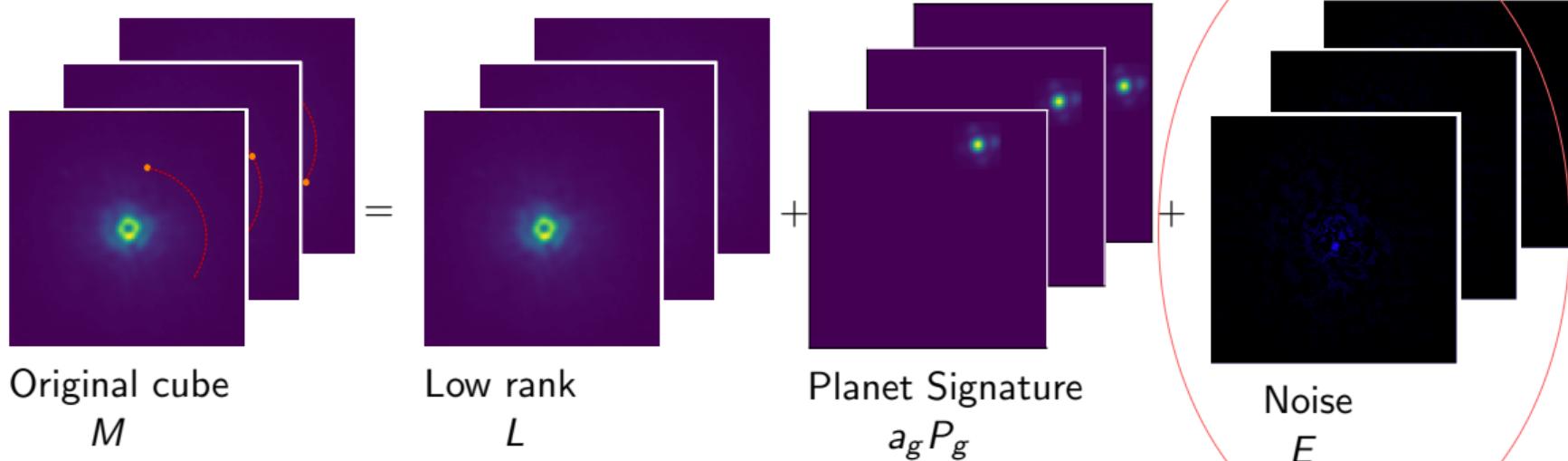


$$g \in [N] \times [N]$$

# Planet Signature



## Alternating Minimization Algorithm with Trajectory (AMAT)



Original cube  
 $M$

Low rank  
 $L$

Planet Signature  
 $a_g P_g$

Noise  
 $E$

$$\text{rank}(L) \leq k, \quad P_g \in \Lambda$$

AMAT

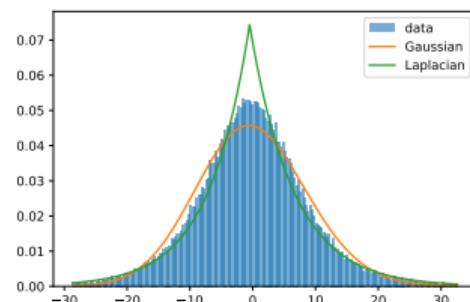
$$\begin{aligned} & \min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - a_g P_g\| \\ & \text{s.t. } \text{rank}(L) \leq k \end{aligned}$$

# Noise

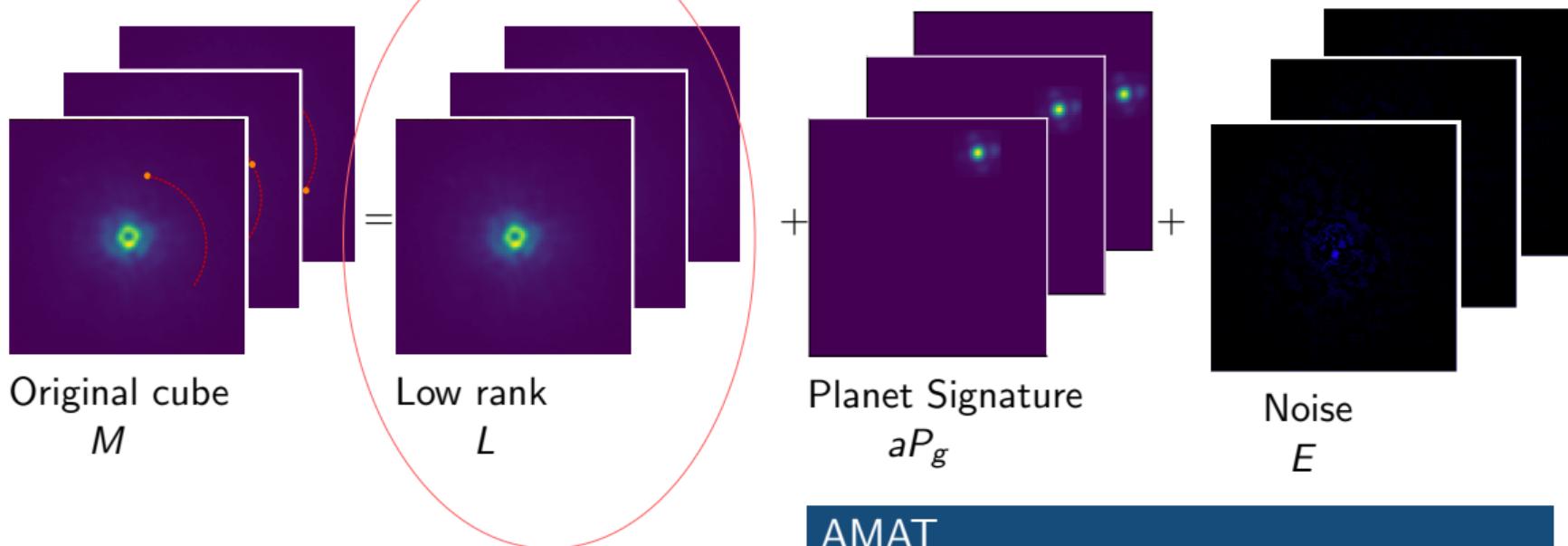
- ▶ Gaussian
- ▶ Laplacian

# Noise

- ▶ Gaussian
- ▶ Laplacian



## Alternating Minimization Algorithm with Trajectory (AMAT)



$$\text{rank}(L) \leq k, \quad P_g \in \Lambda$$

AMAT

$$\begin{array}{ll} \min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} & \|M - L - a_g P_g\| \\ \text{s.t.} & \text{rank}(L) \leq k \end{array}$$

# Low Rank Approximation

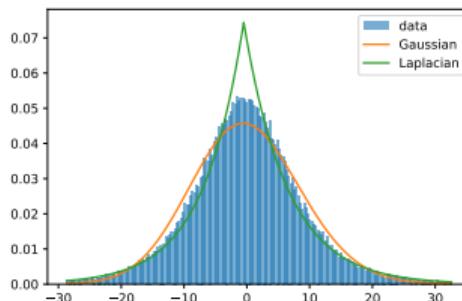
- ▶ PCA
- ▶ L1LRA<sup>2</sup>
  - ▶ an exact block cyclic coordinate descent method

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<sup>2</sup>Gillis, N., Vavasis, S.A.: On the complexity of robust pca and  $\ell_1$ -norm low-rank matrix approximation. Mathematics of Operations Research 43(4), 1072–1084 (2018)

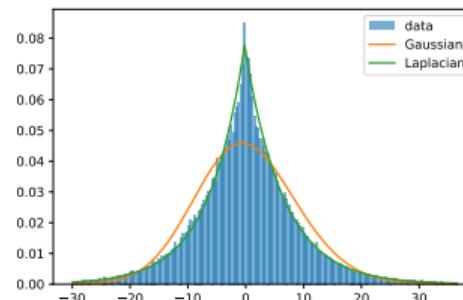
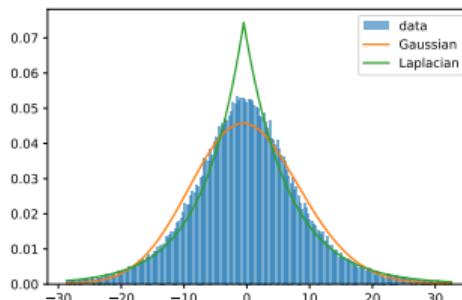
## Noise

- ▶ Gaussian
- ▶ Laplacian



## Noise

- ▶ Gaussian
- ▶ Laplacian



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Daglayan H., et al. Direct Exoplanet Detection Using L1 Norm Low-Rank Approximation. BNAIC/BeNeLearn 2023, (Accepted).

## AMAT

$$\min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - a_g P_g\| \quad \text{s.t.} \quad \text{rank}(L) \leq k$$

## AMAT

$$\min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - a_g P_g\| \quad \text{s.t.} \quad \text{rank}(L) \leq k$$

## Alternating Minimization

$$L^{(i)} = \arg \min_{L \in \mathbb{R}^{t \times n}} \|M - L - a_g^{(i-1)} P_g\| \quad (1)$$

$$a_g^{(i)} = \arg \min_{a_g \in \mathbb{R}} \|M - L^{(i)} - a_g P_g\| \quad (2)$$

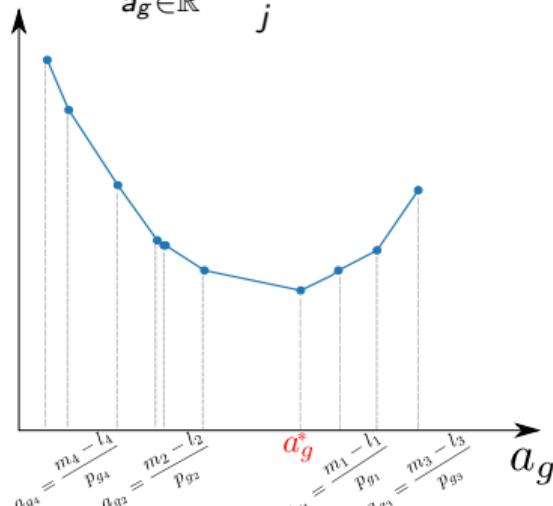
Intensity  $a_g$ 

- ▶ L2 norm

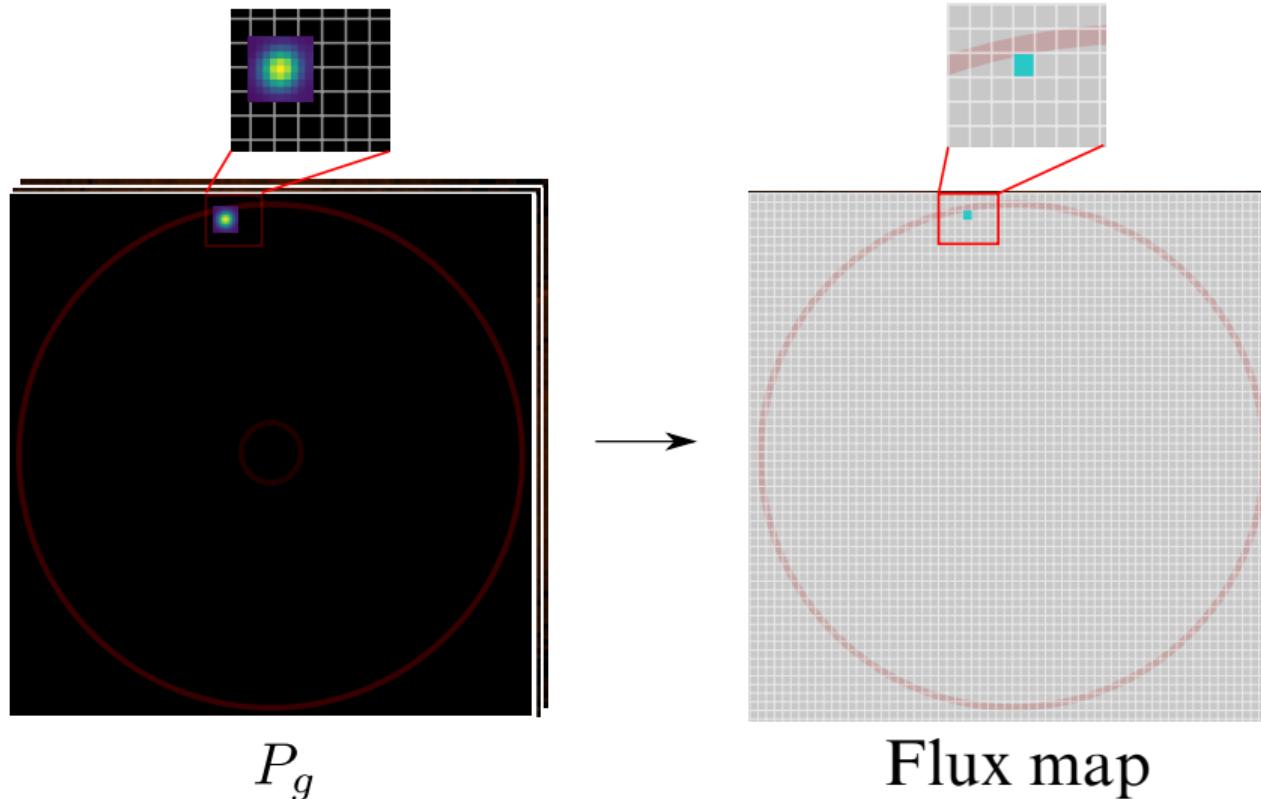
$$a_g^{(i)} = \arg \min_{a_g \in \mathbb{R}} \sum_j (m_j - l_j - a_g p_{gj})^2 = \frac{\langle P_g, M - L_i \rangle}{\|P_g\|_F^2}$$

- ▶ L1 norm

$$a_g^{(i)} = \arg \min_{a_g \in \mathbb{R}} \sum_j |m_j - l_j - a_g p_{gj}|$$

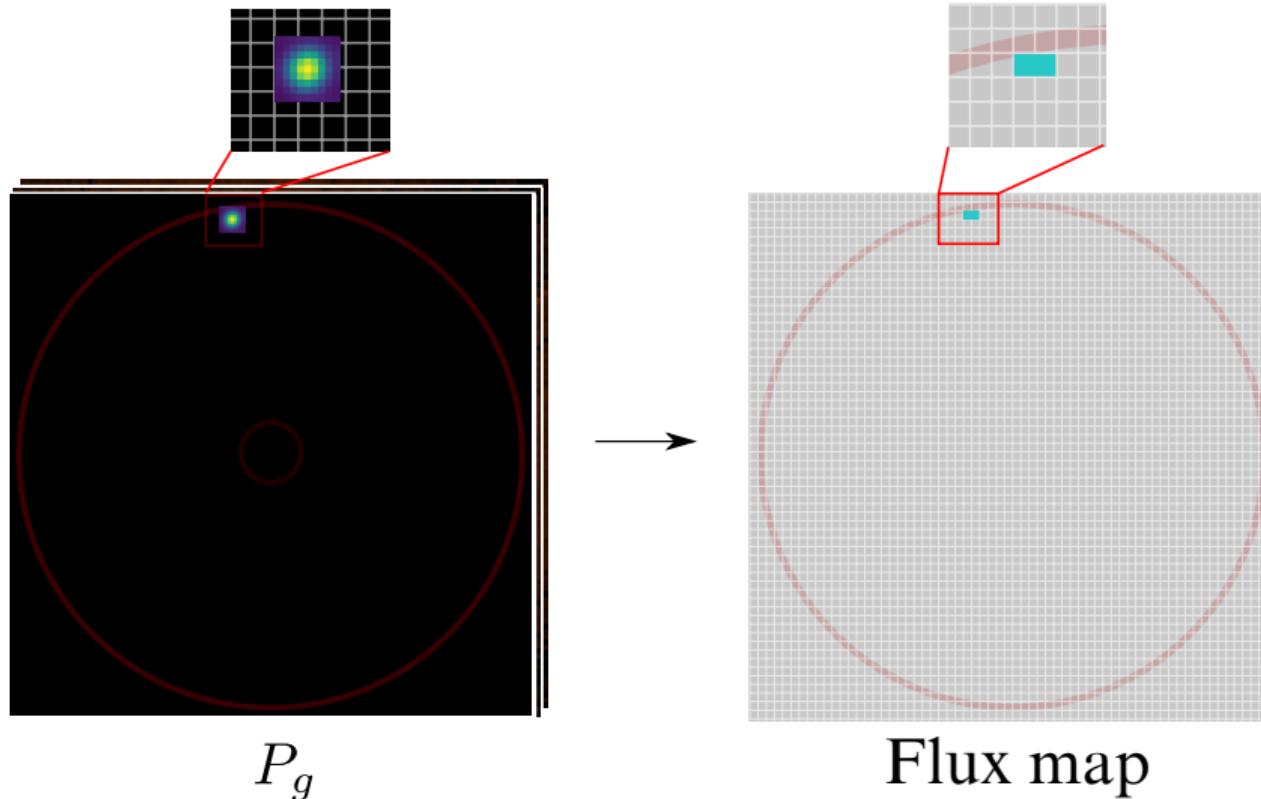


# Trajectories



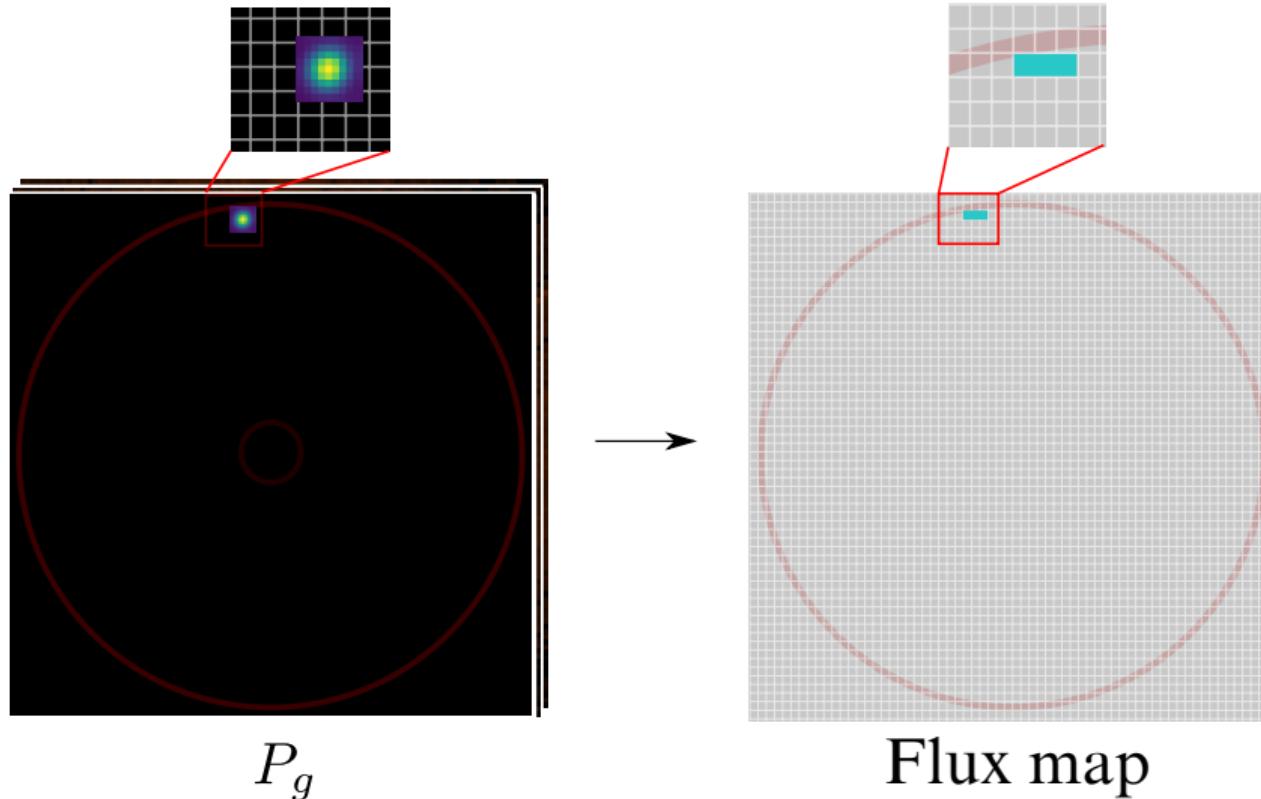
Flux map

# Trajectories



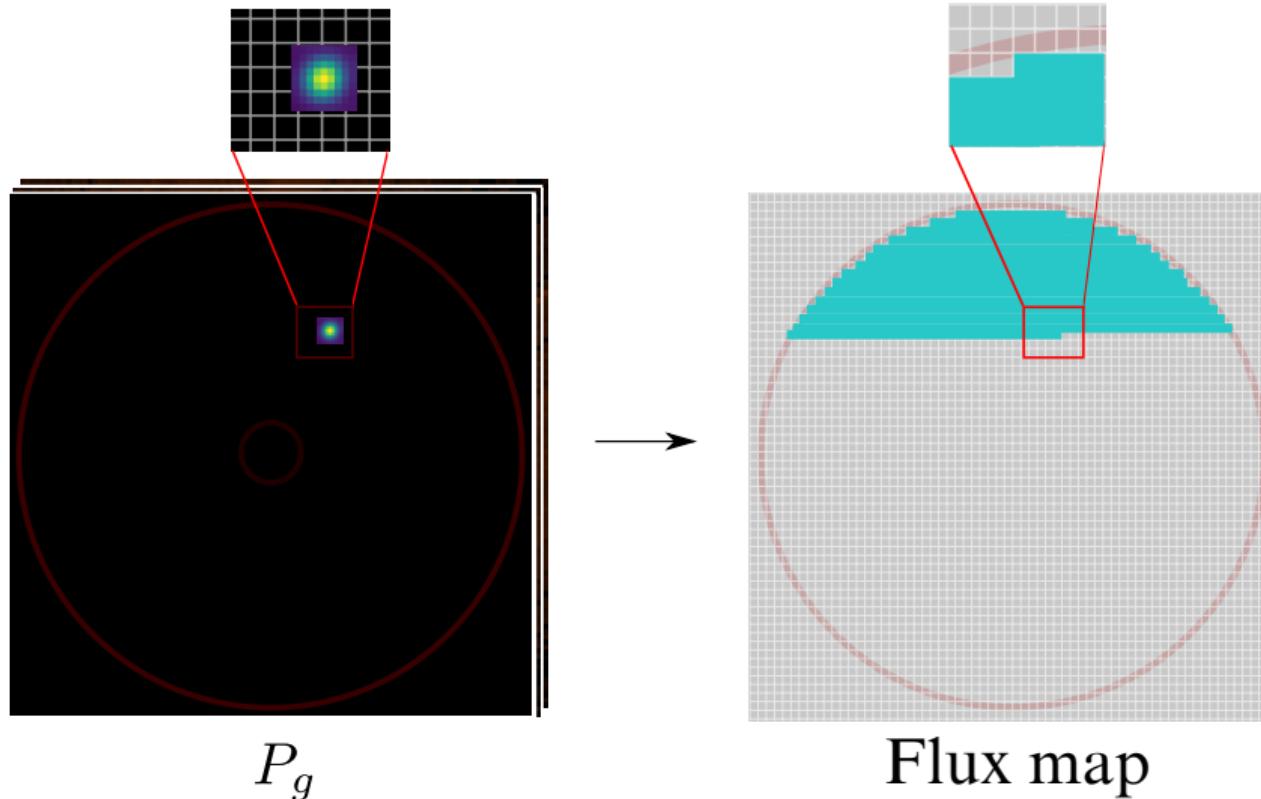
Flux map

# Trajectories

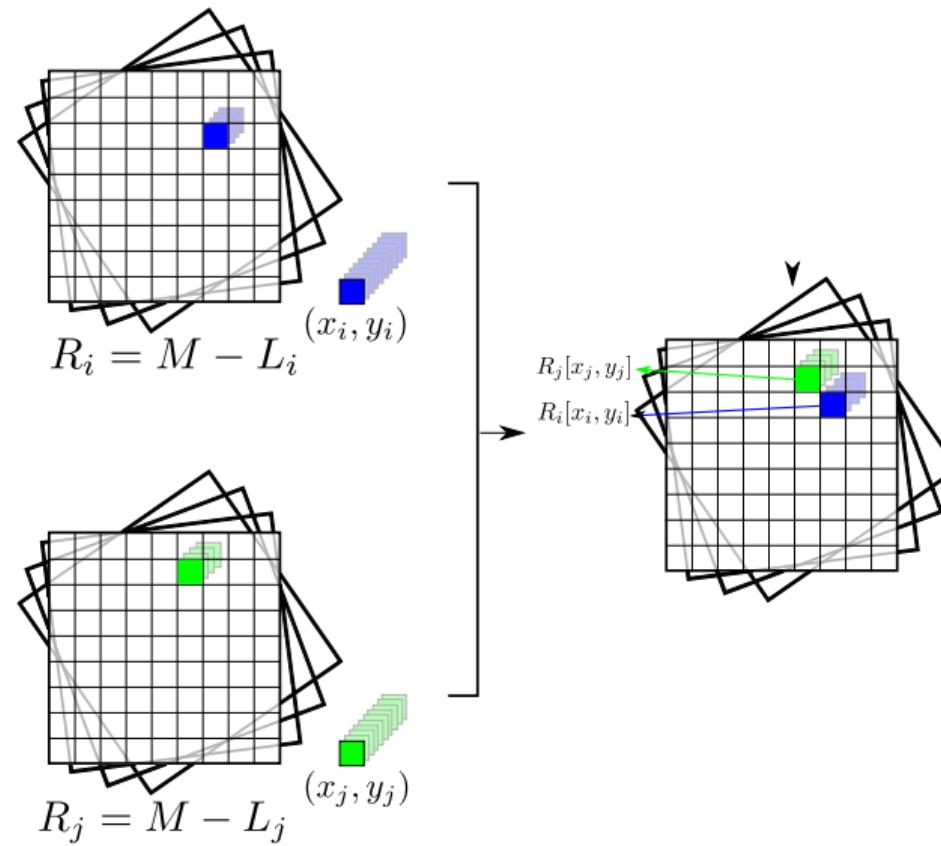


Flux map

# Trajectories



## Residual cube



State of art  
○○○○

AMAT  
○○○○○○○○○○○○

Numerical Experiments  
●○○○○

# Numerical Experiments

## Numerical Experiments - Full vs Annular version

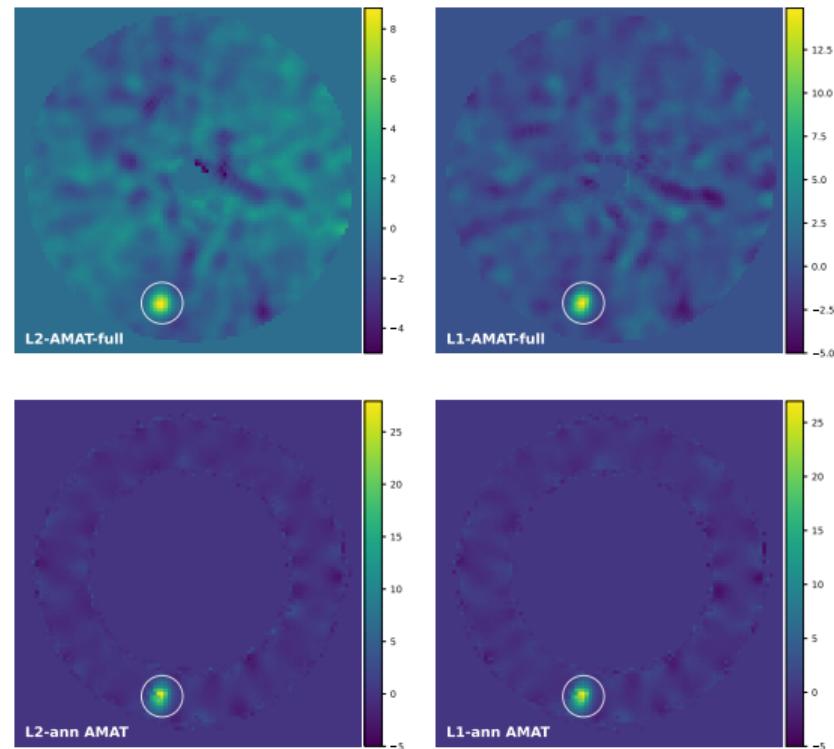
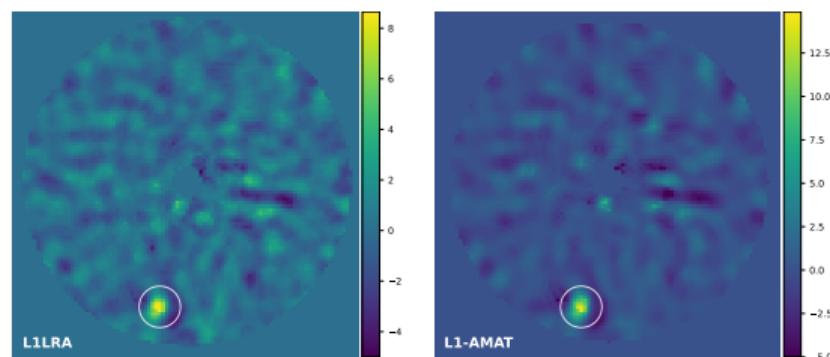
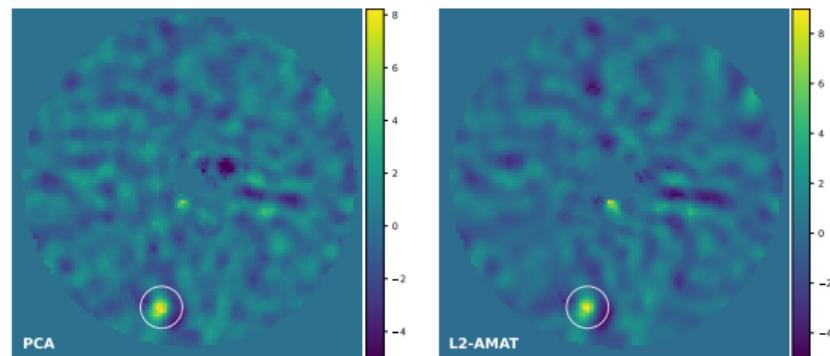


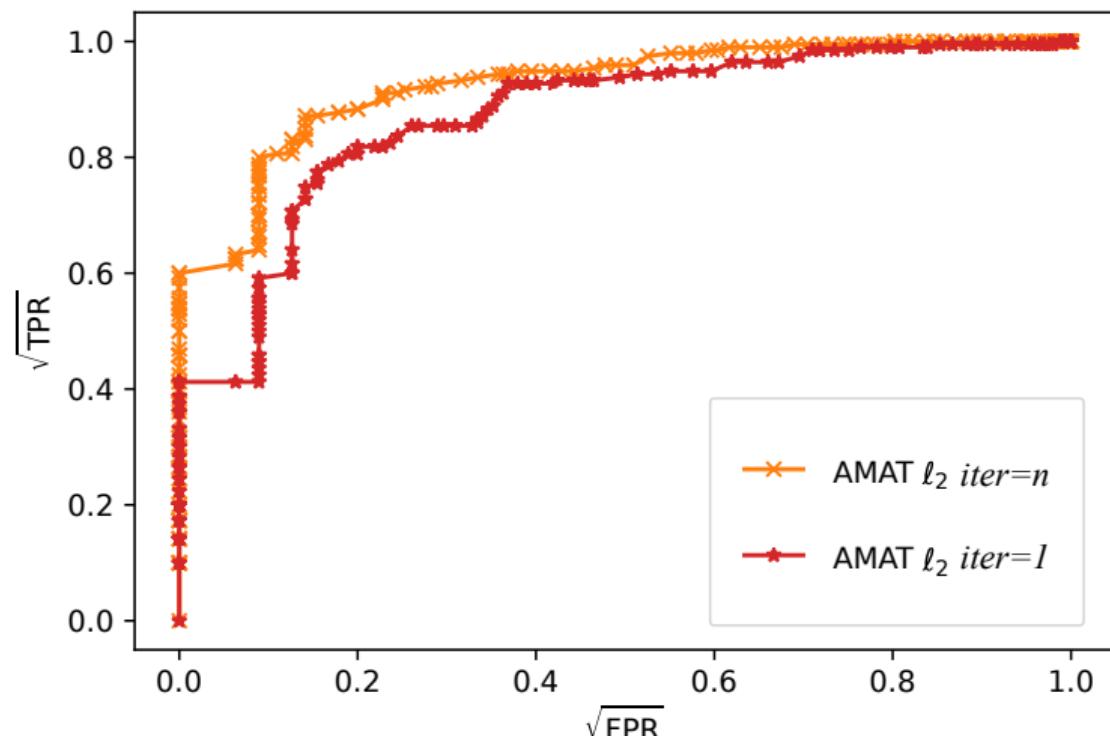
Figure: SNR map after applying full and annular AMAT using both norms. In these maps,  $P_g$  is located in the planet pixels.

## Numerical Experiments - PCA/L1LRA vs AMAT



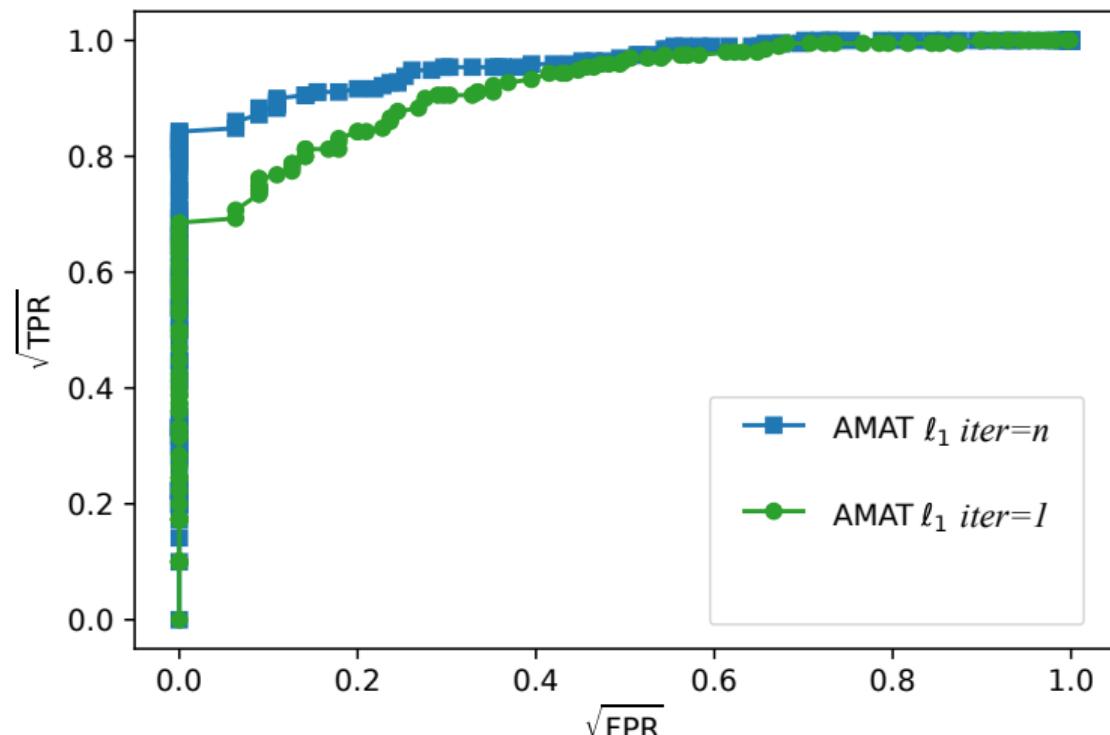
## Numerical Experiments - ROC Curves

- ▶ Synthetic planets are injected.
- ▶  $\sqrt{\text{TPR}}$  &  $\sqrt{\text{FPR}}$  are used instead of TPR & FPR.



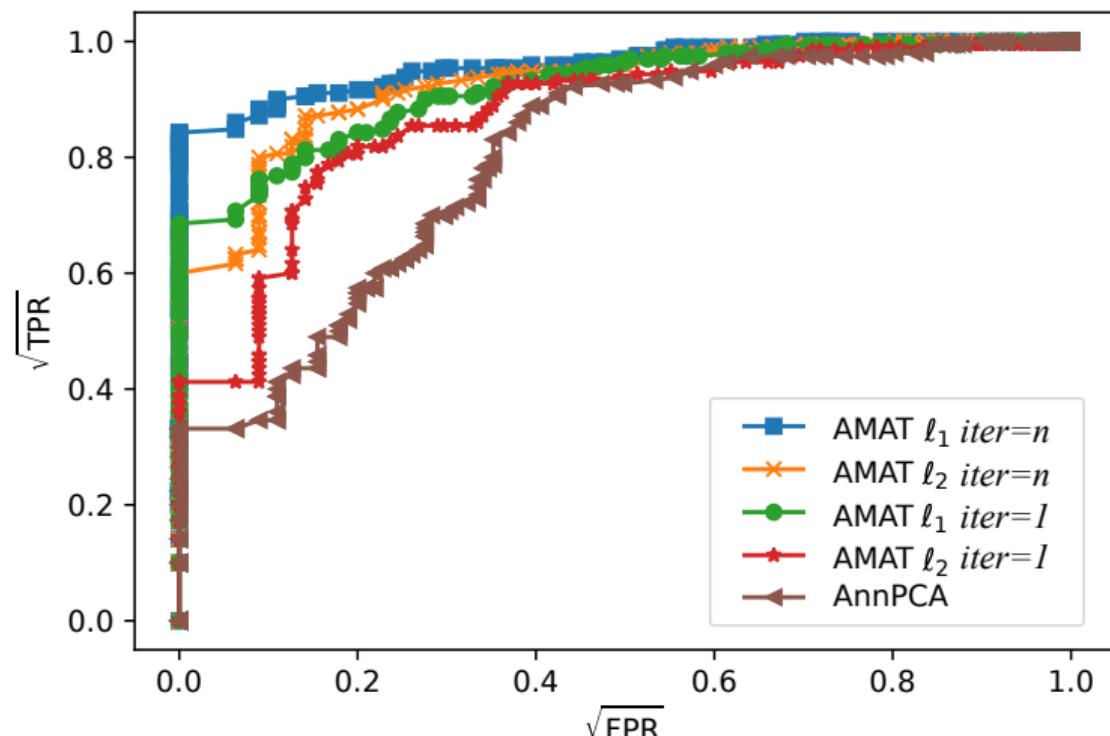
## Numerical Experiments - ROC Curves

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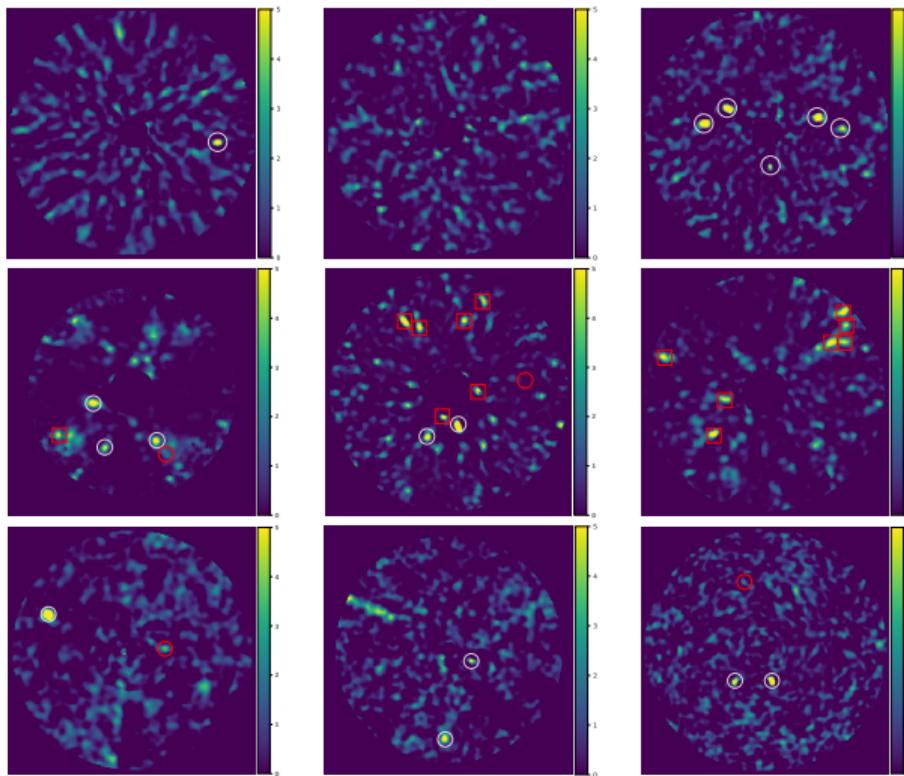


## Numerical Experiments - ROC Curves

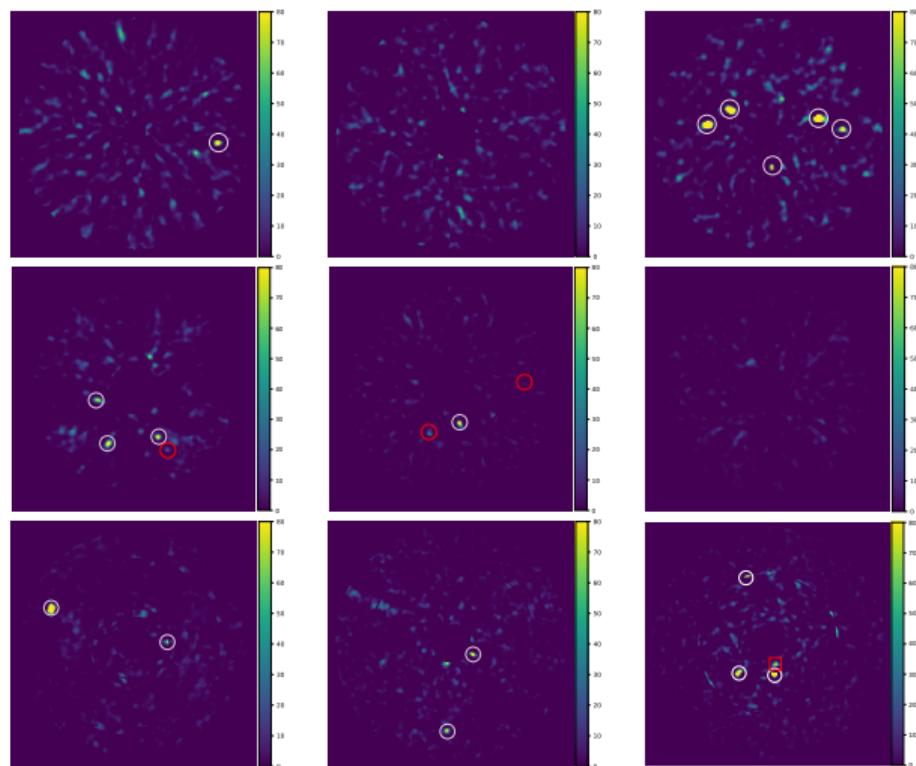
- ▶ Synthetic planets are injected.
- ▶  $\sqrt{\text{TPR}}$  &  $\sqrt{\text{FPR}}$  are used instead of TPR & FPR.



## EIDC - SNR



## EIDC - Likelihood Ratio Map

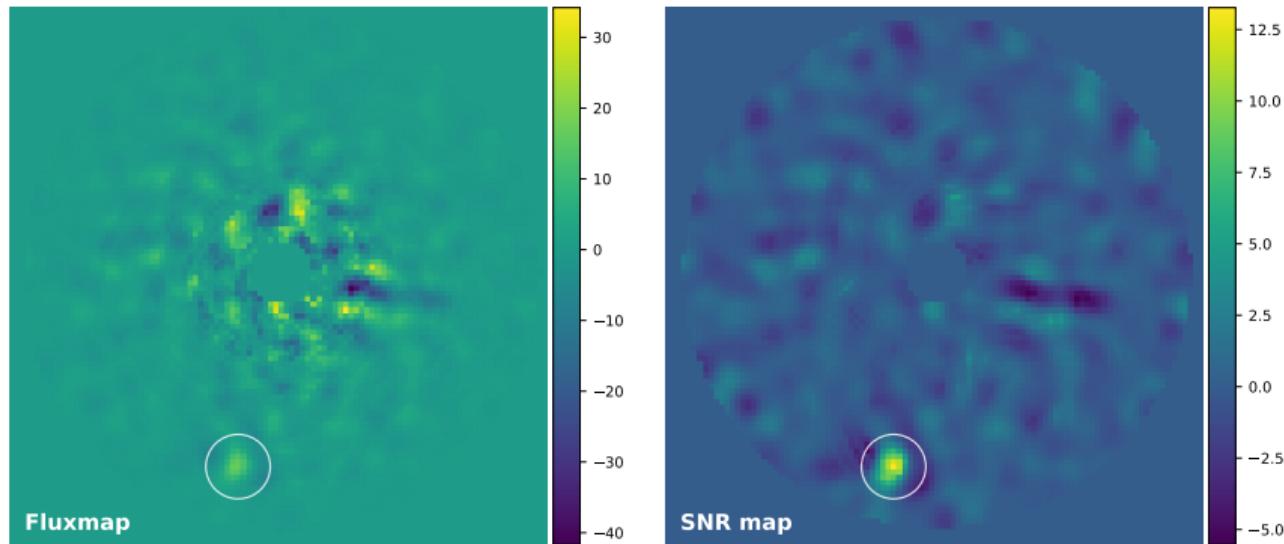


Thank you for your attention!  
Any questions?

[hazan.daglayan@uclouvain.be](mailto:hazan.daglayan@uclouvain.be)  
GitHub: [hazandaglayan/AMAT](https://github.com/hazandaglayan)



# From Fluxmap to SNR



# Contrast Curve

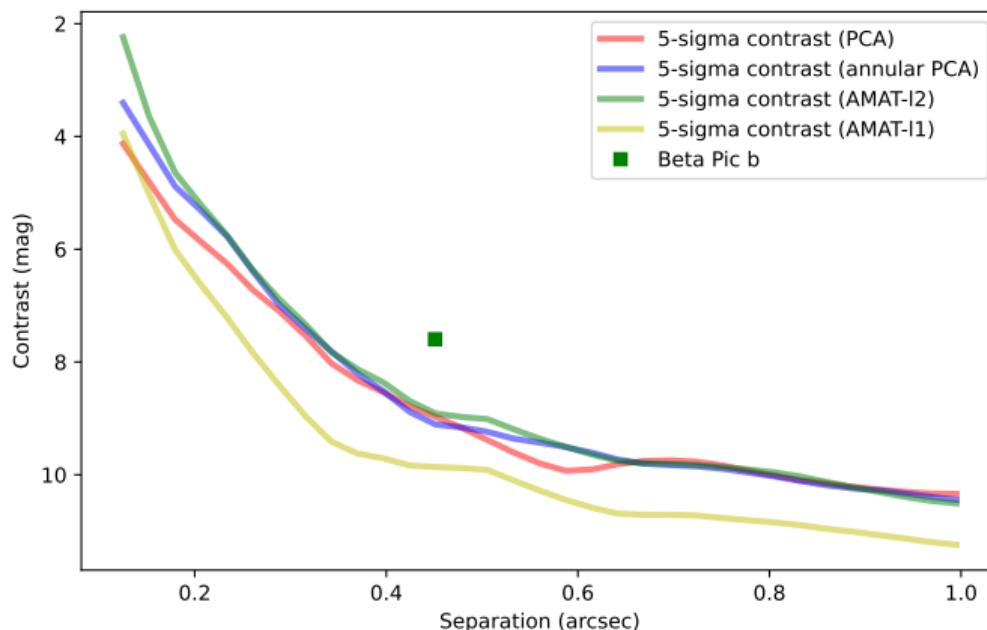


Figure:  $5\sigma$  contrast curve

# Median frame vs Fluxmap

