Hazelnut: A Bidirectionally Typed Structure Editor Calculus

(Supplemental Material)

1 Hazelnut

The full collection of rules defining the semantics of Hazelnut are reproduced here in their definitional order for reference.

1.1 H-Types and H-Expressions

Type Compatibility and Incompatibility

$\boxed{\dot{\tau} \sim \dot{\tau}'}$

$$\frac{}{(\!|\,|\!) \sim \dot{\tau}} \tag{1a}$$

$$\frac{}{\dot{\tau} \sim (\!|\,|\!)} \tag{1b}$$

$$\frac{}{\dot{\tau} \sim \dot{\tau}} \tag{1c}$$

$$\frac{\dot{\tau}_1 \sim \dot{\tau}_1' \qquad \dot{\tau}_2 \sim \dot{\tau}_2'}{\dot{\tau}_1 \to \dot{\tau}_2 \sim \dot{\tau}_1' \to \dot{\tau}_2'} \tag{1d}$$

$\boxed{\dot{\tau} \nsim \dot{\tau}'}$

$$\frac{}{\dot{\tau}_1 \to \dot{\tau}_2 \nsim \texttt{num}} \tag{2a}$$

$$\frac{}{\texttt{num} \nsim \dot{\tau}_1 \to \dot{\tau}_2} \tag{2b}$$

$$\frac{\dot{\tau}_1 \nsim \dot{\tau}_1'}{\dot{\tau}_1 \to \dot{\tau}_2 \nsim \dot{\tau}_1' \to \dot{\tau}_2'} \tag{2c}$$

$$\frac{\dot{\tau}_2 \nsim \dot{\tau}_2'}{\dot{\tau}_1 \to \dot{\tau}_2 \nsim \dot{\tau}_1' \to \dot{\tau}_2'} \tag{2d}$$

Synthesis and Analysis The judgements $\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}$ and $\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}$ are defined mutually inductively by Rules (3) and Rules (4), respectively.

$\boxed{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}$

$$\frac{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}{\dot{\Gamma} \vdash \dot{e} : \dot{\tau} \Rightarrow \dot{\tau}} \tag{3a}$$

$$\frac{}{\dot{\Gamma}, x : \dot{\tau} \vdash x \Rightarrow \dot{\tau}} \tag{3b}$$

$$\frac{\dot{\Gamma} \vdash \dot{e}_1 \Rightarrow \dot{\tau}_2 \to \dot{\tau} \qquad \dot{\Gamma} \vdash \dot{e}_2 \Leftarrow \dot{\tau}_2}{\dot{\Gamma} \vdash \dot{e}_1(\dot{e}_2) \Rightarrow \dot{\tau}} \tag{3c}$$

$$\frac{}{\dot{\Gamma} \vdash \underline{n} \Rightarrow \texttt{num}} \tag{3d}$$

$$\frac{\dot{\Gamma} \vdash \dot{e}_1 \Leftarrow \texttt{num} \qquad \dot{\Gamma} \vdash \dot{e}_2 \Leftarrow \texttt{num}}{\dot{\Gamma} \vdash \dot{e}_1 + \dot{e}_2 \Rightarrow \texttt{num}} \tag{3e}$$

$$\frac{}{\dot{\Gamma} \vdash (\!|\,|\!) \Rightarrow (\!|\,|\!)} \tag{3f}$$

$$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}{\dot{\Gamma} \vdash (\!|\dot{e}|\!) \Rightarrow (\!|\,|\!)} \tag{3g}$$

$$\frac{\dot{\Gamma} \vdash \dot{e}_1 \Rightarrow (\!|\,|\!) \qquad \dot{\Gamma} \vdash \dot{e}_2 \Leftarrow (\!|\,|\!)}{\dot{\Gamma} \vdash \dot{e}_1(\dot{e}_2) \Rightarrow (\!|\,|\!)} \tag{3h}$$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$

$$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}' \qquad \dot{\tau} \sim \dot{\tau}'}{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}} \tag{4a}$$

$$\frac{\dot{\Gamma}, x : \dot{\tau}_1 \vdash \dot{e} \Leftarrow \dot{\tau}_2}{\dot{\Gamma} \vdash \lambda x.\dot{e} \Leftarrow \dot{\tau}_1 \to \dot{\tau}_2} \tag{4b}$$

Complete H-Types and H-Expressions By convention, we use the metavariable τ rather than $\dot{\tau}$ for complete H-types, and e rather than $\dot{e}$ for complete H-expressions.

$\boxed{\tau \ \mathsf{complete}}$

$$\frac{\tau_1 \ \mathsf{complete} \qquad \tau_2 \ \mathsf{complete}}{\tau_1 \to \tau_2 \ \mathsf{complete}} \tag{5a}$$

$$\frac{}{\texttt{num} \ \mathsf{complete}} \tag{5b}$$

$\boxed{e \text{ complete}}$

$$\frac{\dot{e}\ \mathsf{complete} \qquad \dot{\tau}\ \mathsf{complete}}{\dot{e} : \dot{\tau}\ \mathsf{complete}} \tag{6a}$$

$$\frac{}{x\ \mathsf{complete}} \tag{6b}$$

$$\frac{\dot{e}\ \mathsf{complete}}{\lambda x.\dot{e}\ \mathsf{complete}} \tag{6c}$$

$$\frac{\dot{e}_1\ \mathsf{complete} \qquad \dot{e}_2\ \mathsf{complete}}{\dot{e}_1(\dot{e}_2)\ \mathsf{complete}} \tag{6d}$$

$$\frac{}{\underline{n}\ \mathsf{complete}} \tag{6e}$$

$$\frac{\dot{e}_1\ \mathsf{complete} \qquad \dot{e}_2\ \mathsf{complete}}{\dot{e}_1 + \dot{e}_2\ \mathsf{complete}} \tag{6f}$$

1.2 Z-Types and Z-Expressions

Type Focus Erasure

$\boxed{\hat{\tau}^{\diamond} = \dot{\tau}}$ is a metafunction defined as follows:

$$(\triangleright\dot{\tau}\triangleleft)^{\diamond} = \dot{\tau} \tag{7a}$$
$$(\hat{\tau} \to \dot{\tau})^{\diamond} = \hat{\tau}^{\diamond} \to \dot{\tau} \tag{7b}$$
$$(\dot{\tau} \to \hat{\tau})^{\diamond} = \dot{\tau} \to \hat{\tau}^{\diamond} \tag{7c}$$

Expression Focus Erasure

$\boxed{\hat{e}^{\diamond} = \dot{e}}$ is a metafunction defined as follows:

$$(\triangleright\dot{e}\triangleleft)^{\diamond} = \dot{e} \tag{8a}$$
$$(\hat{e} : \dot{\tau})^{\diamond} = \hat{e}^{\diamond} : \dot{\tau} \tag{8b}$$
$$(\dot{e} : \hat{\tau})^{\diamond} = \dot{e} : \hat{\tau}^{\diamond} \tag{8c}$$
$$(\lambda x.\hat{e})^{\diamond} = \lambda x.\hat{e}^{\diamond} \tag{8d}$$
$$(\hat{e}(\dot{e}))^{\diamond} = \hat{e}^{\diamond}(\dot{e}) \tag{8e}$$
$$(\dot{e}(\hat{e}))^{\diamond} = \dot{e}(\hat{e}^{\diamond}) \tag{8f}$$
$$(\hat{e} + \dot{e})^{\diamond} = \hat{e}^{\diamond} + \dot{e} \tag{8g}$$
$$(\dot{e} + \hat{e})^{\diamond} = \dot{e} + \hat{e}^{\diamond} \tag{8h}$$
$$(\!|\hat{e}|\!)^{\diamond} = (\!|\hat{e}^{\diamond}|\!) \tag{8i}$$

1.3 Action Model

Type Actions

$\boxed{\hat{\tau} \xrightarrow{\alpha} \hat{\tau}'}$

Type Movement

$$\frac{}{\triangleright\dot{\tau}_1 \to \dot{\tau}_2\triangleleft \xrightarrow{\texttt{move firstChild}} \triangleright\dot{\tau}_1\triangleleft \to \dot{\tau}_2} \tag{9a}$$

$$\frac{}{\triangleright\dot{\tau}_1\triangleleft \to \dot{\tau}_2 \xrightarrow{\texttt{move parent}} \triangleright\dot{\tau}_1 \to \dot{\tau}_2\triangleleft} \tag{9b}$$

$$\frac{}{\dot{\tau}_1 \to \triangleright\dot{\tau}_2\triangleleft \xrightarrow{\texttt{move parent}} \triangleright\dot{\tau}_1 \to \dot{\tau}_2\triangleleft} \tag{9c}$$

$$\frac{}{\triangleright\dot{\tau}_1\triangleleft \to \dot{\tau}_2 \xrightarrow{\texttt{move nextSib}} \dot{\tau}_1 \to \triangleright\dot{\tau}_2\triangleleft} \tag{9d}$$

$$\frac{}{\dot{\tau}_1 \to \triangleright\dot{\tau}_2\triangleleft \xrightarrow{\texttt{move prevSib}} \triangleright\dot{\tau}_1\triangleleft \to \dot{\tau}_2} \tag{9e}$$

Type Deletion

$$\frac{}{\triangleright\dot{\tau}\triangleleft \xrightarrow{\texttt{del}} \triangleright(\!|\,|\!)\triangleleft} \tag{9f}$$

Type Construction

$$\frac{}{\triangleright\dot{\tau}\triangleleft \xrightarrow{\texttt{construct arrow}} \dot{\tau} \to \triangleright(\!|\,|\!)\triangleleft} \tag{9g}$$

$$\frac{}{\triangleright(\!|\,|\!)\triangleleft \xrightarrow{\texttt{construct num}} \triangleright\texttt{num}\triangleleft} \tag{9h}$$

Zipper Cases

$$\frac{\hat{\tau} \xrightarrow{\alpha} \hat{\tau}'}{\hat{\tau} \to \dot{\tau} \xrightarrow{\alpha} \hat{\tau}' \to \dot{\tau}} \tag{9i}$$

$$\frac{\hat{\tau} \xrightarrow{\alpha} \hat{\tau}'}{\dot{\tau} \to \hat{\tau} \xrightarrow{\alpha} \dot{\tau} \to \hat{\tau}'} \tag{9j}$$

Expression Movement Actions

$\boxed{\hat{e} \xrightarrow{\texttt{move}\ \delta} \hat{e}'}$

Ascription

$$\frac{}{\triangleright \dot{e} : \dot{\tau} \triangleleft \xrightarrow{\texttt{move firstChild}} \triangleright \dot{e} \triangleleft : \dot{\tau}} \tag{10a}$$

$$\frac{}{\triangleright \dot{e} \triangleleft : \dot{\tau} \xrightarrow{\texttt{move parent}} \triangleright \dot{e} : \dot{\tau} \triangleleft} \tag{10b}$$

$$\frac{}{\dot{e} : \triangleright \dot{\tau} \triangleleft \xrightarrow{\texttt{move parent}} \triangleright \dot{e} : \dot{\tau} \triangleleft} \tag{10c}$$

$$\frac{}{\triangleright \dot{e} \triangleleft : \dot{\tau} \xrightarrow{\texttt{move nextSib}} \dot{e} : \triangleright \dot{\tau} \triangleleft} \tag{10d}$$

$$\frac{}{\dot{e} : \triangleright \dot{\tau} \triangleleft \xrightarrow{\texttt{move prevSib}} \triangleright \dot{e} \triangleleft : \dot{\tau}} \tag{10e}$$

Lambda

$$\frac{}{\triangleright \lambda x.\dot{e} \triangleleft \xrightarrow{\texttt{move firstChild}} \lambda x.\triangleright \dot{e} \triangleleft} \tag{10f}$$

$$\frac{}{\lambda x.\triangleright \dot{e} \triangleleft \xrightarrow{\texttt{move parent}} \triangleright \lambda x.\dot{e} \triangleleft} \tag{10g}$$

Application

$$\frac{}{\triangleright \dot{e}_1(\dot{e}_2) \triangleleft \xrightarrow{\texttt{move firstChild}} \triangleright \dot{e}_1 \triangleleft (\dot{e}_2)} \tag{10h}$$

$$\frac{}{\triangleright \dot{e}_1 \triangleleft (\dot{e}_2) \xrightarrow{\texttt{move parent}} \triangleright \dot{e}_1(\dot{e}_2) \triangleleft} \tag{10i}$$

$$\frac{}{\dot{e}_1(\triangleright \dot{e}_2 \triangleleft) \xrightarrow{\texttt{move parent}} \triangleright \dot{e}_1(\dot{e}_2) \triangleleft} \tag{10j}$$

$$\frac{}{\triangleright \dot{e}_1 \triangleleft (\dot{e}_2) \xrightarrow{\texttt{move nextSib}} \dot{e}_1(\triangleright \dot{e}_2 \triangleleft)} \tag{10k}$$

$$\frac{}{\dot{e}_1(\triangleright \dot{e}_2 \triangleleft) \xrightarrow{\texttt{move prevSib}} \triangleright \dot{e}_1 \triangleleft (\dot{e}_2)} \tag{10l}$$

Plus

$$\frac{}{\triangleright \dot{e}_1 + \dot{e}_2 \triangleleft \xrightarrow{\texttt{move firstChild}} \triangleright \dot{e}_1 \triangleleft + \dot{e}_2} \tag{10m}$$

$$\frac{}{\triangleright \dot{e}_1 \triangleleft + \dot{e}_2 \xrightarrow{\texttt{move parent}} \triangleright \dot{e}_1 + \dot{e}_2 \triangleleft} \tag{10n}$$

$$\frac{}{\dot{e}_1 + \triangleright \dot{e}_2 \triangleleft \xrightarrow{\texttt{move parent}} \triangleright \dot{e}_1 + \dot{e}_2 \triangleleft} \tag{10o}$$

$$\frac{}{\triangleright \dot{e}_1 \triangleleft + \dot{e}_2 \xrightarrow{\texttt{move nextSib}} \dot{e}_1 + \triangleright \dot{e}_2 \triangleleft} \tag{10p}$$

$$\frac{}{\dot{e}_1 + \triangleright \dot{e}_2 \triangleleft \xrightarrow{\texttt{move prevSib}} \triangleright \dot{e}_1 \triangleleft + \dot{e}_2} \tag{10q}$$

Non-Empty Hole

$$\frac{}{\triangleright\llparenthesis\dot{e}\rrparenthesis\triangleleft \xrightarrow{\texttt{move firstChild}} \llparenthesis\triangleright\dot{e}\triangleleft\rrparenthesis} \tag{10r}$$

$$\frac{}{\llparenthesis\triangleright\dot{e}\triangleleft\rrparenthesis \xrightarrow{\texttt{move parent}} \triangleright\llparenthesis\dot{e}\rrparenthesis\triangleleft} \tag{10s}$$

Synthetic and Analytic Expression Actions The synthetic and analytic expression action performance judgements are defined mutually inductively by Rules (11) and Rules (12), respectively.

$\boxed{\dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \dot{\tau}'}$

Movement

$$\frac{\hat{e} \xrightarrow{\texttt{move}\ \delta} \hat{e}'}{\dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\texttt{move}\ \delta} \hat{e}' \Rightarrow \dot{\tau}} \tag{11a}$$

Deletion

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{del}} \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \llparenthesis\rrparenthesis} \tag{11b}$$

Construction

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{construct asc}} \dot{e} : \triangleright\dot{\tau}\triangleleft \Rightarrow \dot{\tau}} \tag{11c}$$

$$\frac{}{\dot{\Gamma}, x : \dot{\tau} \vdash \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \llparenthesis\rrparenthesis \xrightarrow{\texttt{construct var}\ x} \triangleright x\triangleleft \Rightarrow \dot{\tau}} \tag{11d}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \llparenthesis\rrparenthesis \xrightarrow{\texttt{construct lam}\ x} \lambda x.\llparenthesis\rrparenthesis : \triangleright\llparenthesis\rrparenthesis\triangleleft \to \llparenthesis\rrparenthesis \Rightarrow \llparenthesis\rrparenthesis \to \llparenthesis\rrparenthesis} \tag{11e}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau}_1 \to \dot{\tau}_2 \xrightarrow{\texttt{construct ap}} \dot{e}(\triangleright\llparenthesis\rrparenthesis\triangleleft) \Rightarrow \dot{\tau}_2} \tag{11f}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \llparenthesis\rrparenthesis \xrightarrow{\texttt{construct ap}} \dot{e}(\triangleright\llparenthesis\rrparenthesis\triangleleft) \Rightarrow \llparenthesis\rrparenthesis} \tag{11g}$$

$$\frac{\dot{\tau} \nsim \llparenthesis\rrparenthesis \to \llparenthesis\rrparenthesis}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{construct ap}} \llparenthesis\dot{e}\rrparenthesis(\triangleright\llparenthesis\rrparenthesis\triangleleft) \Rightarrow \llparenthesis\rrparenthesis} \tag{11h}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{construct arg}} \triangleright\llparenthesis\rrparenthesis\triangleleft(\dot{e}) \Rightarrow \llparenthesis\rrparenthesis} \tag{11i}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \llparenthesis\rrparenthesis \xrightarrow{\texttt{construct numlit}\ n} \triangleright\underline{n}\triangleleft \Rightarrow \texttt{num}} \tag{11j}$$

$$\frac{\dot{\tau} \sim \texttt{num}}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{construct plus}} \dot{e} + \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \texttt{num}} \tag{11k}$$

$$\frac{\dot{\tau} \nsim \texttt{num}}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{construct plus}} \llparenthesis\dot{e}\rrparenthesis + \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \texttt{num}} \tag{11l}$$

Finishing

$$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}'}{\dot{\Gamma} \vdash \triangleright\llparenthesis\dot{e}\rrparenthesis\triangleleft \Rightarrow \llparenthesis\rrparenthesis \xrightarrow{\texttt{finish}} \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau}'} \tag{11m}$$

Zipper Cases

$$\frac{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}}{\dot{\Gamma} \vdash \hat{e} : \dot{\tau} \Rightarrow \dot{\tau} \xrightarrow{\alpha} \hat{e}' : \dot{\tau} \Rightarrow \dot{\tau}} \tag{11n}$$

$$\frac{\hat{\tau} \xrightarrow{\alpha} \hat{\tau}' \qquad \dot{\Gamma} \vdash \dot{e} \Leftarrow \hat{\tau}'^{\diamond}}{\dot{\Gamma} \vdash \dot{e} : \hat{\tau} \Rightarrow \hat{\tau}^{\diamond} \xrightarrow{\alpha} \dot{e} : \hat{\tau}' \Rightarrow \hat{\tau}'^{\diamond}} \tag{11o}$$

$$\frac{\dot{\Gamma} \vdash \hat{e}^{\diamond} \Rightarrow \dot{\tau}_2 \qquad \dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau}_2 \xrightarrow{\alpha} \hat{e}' \Rightarrow \dot{\tau}_3 \to \dot{\tau}_4 \qquad \dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}_3}{\dot{\Gamma} \vdash \hat{e}(\dot{e}) \Rightarrow \dot{\tau}_1 \xrightarrow{\alpha} \hat{e}'(\dot{e}) \Rightarrow \dot{\tau}_4} \tag{11p}$$

$$\frac{\dot{\Gamma} \vdash \hat{e}^{\diamond} \Rightarrow \dot{\tau}_2 \qquad \dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau}_2 \xrightarrow{\alpha} \hat{e}' \Rightarrow \llparenthesis\rrparenthesis \qquad \dot{\Gamma} \vdash \dot{e} \Leftarrow \llparenthesis\rrparenthesis}{\dot{\Gamma} \vdash \hat{e}(\dot{e}) \Rightarrow \dot{\tau}_1 \xrightarrow{\alpha} \hat{e}'(\dot{e}) \Rightarrow \llparenthesis\rrparenthesis} \tag{11q}$$

$$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}_2 \to \dot{\tau} \qquad \dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}_2}{\dot{\Gamma} \vdash \dot{e}(\hat{e}) \Rightarrow \dot{\tau} \xrightarrow{\alpha} \dot{e}(\hat{e}') \Rightarrow \dot{\tau}} \tag{11r}$$

$$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \llparenthesis\rrparenthesis \qquad \dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \llparenthesis\rrparenthesis}{\dot{\Gamma} \vdash \dot{e}(\hat{e}) \Rightarrow \llparenthesis\rrparenthesis \xrightarrow{\alpha} \dot{e}(\hat{e}') \Rightarrow \llparenthesis\rrparenthesis} \tag{11s}$$

$$\frac{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \texttt{num}}{\dot{\Gamma} \vdash \hat{e} + \dot{e} \Rightarrow \texttt{num} \xrightarrow{\alpha} \hat{e}' + \dot{e} \Rightarrow \texttt{num}} \tag{11t}$$

$$\frac{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \texttt{num}}{\dot{\Gamma} \vdash \dot{e} + \hat{e} \Rightarrow \texttt{num} \xrightarrow{\alpha} \dot{e} + \hat{e}' \Rightarrow \texttt{num}} \tag{11u}$$

$$\frac{\dot{\Gamma} \vdash \hat{e}^{\diamond} \Rightarrow \dot{\tau} \qquad \dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \dot{\tau}' \qquad \hat{e}' \neq \triangleright\llparenthesis\rrparenthesis\triangleleft}{\dot{\Gamma} \vdash \llparenthesis\hat{e}\rrparenthesis \Rightarrow \llparenthesis\rrparenthesis \xrightarrow{\alpha} \llparenthesis\hat{e}'\rrparenthesis \Rightarrow \llparenthesis\rrparenthesis} \tag{11v}$$

$$\frac{\dot{\Gamma} \vdash \hat{e}^{\diamond} \Rightarrow \dot{\tau} \qquad \dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\alpha} \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \llparenthesis\rrparenthesis}{\dot{\Gamma} \vdash \llparenthesis\hat{e}\rrparenthesis \Rightarrow \llparenthesis\rrparenthesis \xrightarrow{\alpha} \triangleright\llparenthesis\rrparenthesis\triangleleft \Rightarrow \llparenthesis\rrparenthesis} \tag{11w}$$

$\boxed{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}}$

Subsumption

$$\frac{\dot{\Gamma} \vdash \hat{e}^{\diamond} \Rightarrow \dot{\tau}' \qquad \dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau}' \xrightarrow{\alpha} \hat{e}' \Rightarrow \dot{\tau}'' \qquad \dot{\tau} \sim \dot{\tau}'' \quad \alpha \neq \texttt{construct asc} \qquad \alpha \neq \texttt{construct lam } x}{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}} \tag{12a}$$

Movement

$$\frac{\hat{e} \xrightarrow{\texttt{move } \delta} \hat{e}'}{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\texttt{move } \delta} \hat{e}' \Leftarrow \dot{\tau}} \tag{12b}$$

Deletion

$$\frac{}{\dot{\Gamma} \vdash \triangleright \dot{e} \triangleleft \xrightarrow{\texttt{del}} \triangleright (\!|\,|\!) \triangleleft \Leftarrow \dot{\tau}} \tag{12c}$$

Construction

$$\frac{}{\dot{\Gamma} \vdash \triangleright \dot{e} \triangleleft \xrightarrow{\texttt{construct asc}} \dot{e} : \triangleright \dot{\tau} \triangleleft \Leftarrow \dot{\tau}} \tag{12d}$$

$$\frac{\dot{\tau} \nsim \dot{\tau}'}{\dot{\Gamma}, x : \dot{\tau}' \vdash \triangleright (\!|\,|\!) \triangleleft \xrightarrow{\texttt{construct var } x} (\!| \triangleright x \triangleleft |\!) \Leftarrow \dot{\tau}} \tag{12e}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright (\!|\,|\!) \triangleleft \xrightarrow{\texttt{construct lam } x} \lambda x. \triangleright (\!|\,|\!) \triangleleft \Leftarrow \dot{\tau}_1 \rightarrow \dot{\tau}_2} \tag{12f}$$

$$\frac{\dot{\tau} \nsim (\!|\,|\!) \rightarrow (\!|\,|\!)}{\dot{\Gamma} \vdash \triangleright (\!|\,|\!) \triangleleft \xrightarrow{\texttt{construct lam } x} (\!| \lambda x.(\!|\,|\!) : \triangleright (\!|\,|\!) \triangleleft \rightarrow (\!|\,|\!) |\!) \Leftarrow \dot{\tau}} \tag{12g}$$

$$\frac{\dot{\tau} \nsim \texttt{num}}{\dot{\Gamma} \vdash \triangleright (\!|\,|\!) \triangleleft \xrightarrow{\texttt{construct numlit } n} (\!| \triangleright \underline{n} \triangleleft |\!) \Leftarrow \dot{\tau}} \tag{12h}$$

Finishing

$$\frac{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}{\dot{\Gamma} \vdash \triangleright (\!| \dot{e} |\!) \triangleleft \xrightarrow{\texttt{finish}} \triangleright \dot{e} \triangleleft \Leftarrow \dot{\tau}} \tag{12i}$$

Zipper Cases

$$\frac{\dot{\Gamma}, x : \dot{\tau}_1 \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}_2}{\dot{\Gamma} \vdash \lambda x.\hat{e} \xrightarrow{\alpha} \lambda x.\hat{e}' \Leftarrow \dot{\tau}_1 \rightarrow \dot{\tau}_2} \tag{12j}$$