Hazelnut: A Bidirectionally Typed Structure Editor Calculus

(Supplemental Material)

1 Hazelnut

The full collection of rules defining the semantics of Hazelnut are reproduced here in their definitional order for reference.

1.1 H-Types and H-Expressions

Type Compatibility and Incompatibility

$\boxed{\dot{\tau} \sim \dot{\tau}'}$

$$\frac{}{(\!|\,|\!) \sim \dot{\tau}} \tag{1a}$$

$$\frac{}{\dot{\tau} \sim (\!|\,|\!)} \tag{1b}$$

$$\frac{}{\dot{\tau} \sim \dot{\tau}} \tag{1c}$$

$$\frac{\dot{\tau}_1 \sim \dot{\tau}_1' \qquad \dot{\tau}_2 \sim \dot{\tau}_2'}{\dot{\tau}_1 \to \dot{\tau}_2 \sim \dot{\tau}_1' \to \dot{\tau}_2'} \tag{1d}$$

$\boxed{\dot{\tau} \nsim \dot{\tau}'}$

$$\frac{}{\dot{\tau}_1 \to \dot{\tau}_2 \nsim \texttt{num}} \tag{2a}$$

$$\frac{}{\texttt{num} \nsim \dot{\tau}_1 \to \dot{\tau}_2} \tag{2b}$$

$$\frac{\dot{\tau}_1 \nsim \dot{\tau}_1'}{\dot{\tau}_1 \to \dot{\tau}_2 \nsim \dot{\tau}_1' \to \dot{\tau}_2'} \tag{2c}$$

$$\frac{\dot{\tau}_2 \nsim \dot{\tau}_2'}{\dot{\tau}_1 \to \dot{\tau}_2 \nsim \dot{\tau}_1' \to \dot{\tau}_2'} \tag{2d}$$

Synthesis and Analysis The judgements $\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}$ and $\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}$ are defined mutually inductively by Rules (3) and Rules (4), respectively.

$\boxed{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}$

$$\frac{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}{\dot{\Gamma} \vdash \dot{e} : \dot{\tau} \Rightarrow \dot{\tau}} \tag{3a}$$

$$\frac{}{\dot{\Gamma}, x : \dot{\tau} \vdash x \Rightarrow \dot{\tau}} \tag{3b}$$

$$\frac{\dot{\Gamma} \vdash \dot{e}_1 \Rightarrow \dot{\tau}_2 \to \dot{\tau} \qquad \dot{\Gamma} \vdash \dot{e}_2 \Leftarrow \dot{\tau}_2}{\dot{\Gamma} \vdash \dot{e}_1(\dot{e}_2) \Rightarrow \dot{\tau}} \tag{3c}$$

$$\frac{}{\dot{\Gamma} \vdash \underline{n} \Rightarrow \mathtt{num}} \tag{3d}$$

$$\frac{\dot{\Gamma} \vdash \dot{e}_1 \Leftarrow \mathtt{num} \qquad \dot{\Gamma} \vdash \dot{e}_2 \Leftarrow \mathtt{num}}{\dot{\Gamma} \vdash \dot{e}_1 + \dot{e}_2 \Rightarrow \mathtt{num}} \tag{3e}$$

$$\frac{}{\dot{\Gamma} \vdash (\!|\,|\!) \Rightarrow (\!|\,|\!)} \tag{3f}$$

$$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}{\dot{\Gamma} \vdash (\!|\dot{e}|\!) \Rightarrow (\!|\,|\!)} \tag{3g}$$

$$\frac{\dot{\Gamma} \vdash \dot{e}_1 \Rightarrow (\!|\,|\!) \qquad \dot{\Gamma} \vdash \dot{e}_2 \Leftarrow (\!|\,|\!)}{\dot{\Gamma} \vdash \dot{e}_1(\dot{e}_2) \Rightarrow (\!|\,|\!)} \tag{3h}$$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$

$$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}' \qquad \dot{\tau} \sim \dot{\tau}'}{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}} \tag{4a}$$

$$\frac{\dot{\Gamma}, x : \dot{\tau}_1 \vdash \dot{e} \Leftarrow \dot{\tau}_2}{\dot{\Gamma} \vdash \lambda x.\dot{e} \Leftarrow \dot{\tau}_1 \to \dot{\tau}_2} \tag{4b}$$

Complete H-Types and H-Expressions By convention, we use the metavariable τ rather than $\dot{\tau}$ for complete H-types, and e rather than $\dot{e}$ for complete H-expressions.

$\boxed{\tau \text{ complete}}$

$$\frac{\tau_1 \text{ complete} \qquad \tau_2 \text{ complete}}{\tau_1 \to \tau_2 \text{ complete}} \tag{5a}$$

$$\frac{}{\mathtt{num} \text{ complete}} \tag{5b}$$

$\boxed{e \text{ complete}}$

$$\frac{\dot{e} \text{ complete} \qquad \dot{\tau} \text{ complete}}{\dot{e} : \dot{\tau} \text{ complete}} \tag{6a}$$

$$\frac{}{x \text{ complete}} \tag{6b}$$

$$\frac{\dot{e} \text{ complete}}{\lambda x.\dot{e} \text{ complete}} \tag{6c}$$

$$\frac{\dot{e}_1 \text{ complete} \qquad \dot{e}_2 \text{ complete}}{\dot{e}_1(\dot{e}_2) \text{ complete}} \tag{6d}$$

$$\frac{}{\underline{n} \text{ complete}} \tag{6e}$$

$$\frac{\dot{e}_1 \text{ complete} \qquad \dot{e}_2 \text{ complete}}{\dot{e}_1 + \dot{e}_2 \text{ complete}} \tag{6f}$$

1.2 Z-Types and Z-Expressions

Type Focus Erasure

$\boxed{\hat{\tau}^\diamond = \dot{\tau}}$ is a metafunction defined as follows:

$$(\triangleright\dot{\tau}\triangleleft)^\diamond = \dot{\tau} \tag{7a}$$
$$(\hat{\tau} \to \dot{\tau})^\diamond = \hat{\tau}^\diamond \to \dot{\tau} \tag{7b}$$
$$(\dot{\tau} \to \hat{\tau})^\diamond = \dot{\tau} \to \hat{\tau}^\diamond \tag{7c}$$

Expression Focus Erasure

$\boxed{\hat{e}^\diamond = \dot{e}}$ is a metafunction defined as follows:

$$(\triangleright\dot{e}\triangleleft)^\diamond = \dot{e} \tag{8a}$$
$$(\hat{e} : \dot{\tau})^\diamond = \hat{e}^\diamond : \dot{\tau} \tag{8b}$$
$$(\dot{e} : \hat{\tau})^\diamond = \dot{e} : \hat{\tau}^\diamond \tag{8c}$$
$$(\lambda x.\hat{e})^\diamond = \lambda x.\hat{e}^\diamond \tag{8d}$$
$$(\hat{e}(\dot{e}))^\diamond = \hat{e}^\diamond(\dot{e}) \tag{8e}$$
$$(\dot{e}(\hat{e}))^\diamond = \dot{e}(\hat{e}^\diamond) \tag{8f}$$
$$(\hat{e} + \dot{e})^\diamond = \hat{e}^\diamond + \dot{e} \tag{8g}$$
$$(\dot{e} + \hat{e})^\diamond = \dot{e} + \hat{e}^\diamond \tag{8h}$$
$$(\!|\dot{e}|\!)^\diamond = (\!|\hat{e}^\diamond|\!) \tag{8i}$$

1.3 Action Model

Type Actions

$\boxed{\hat{\tau} \xrightarrow{\alpha} \hat{\tau}'}$

Type Movement

$$\frac{}{\triangleright\dot{\tau}_1 \to \dot{\tau}_2\triangleleft \xrightarrow{\texttt{move firstChild}} \triangleright\dot{\tau}_1\triangleleft \to \dot{\tau}_2} \tag{9a}$$

$$\frac{}{\triangleright\dot{\tau}_1\triangleleft \to \dot{\tau}_2 \xrightarrow{\texttt{move parent}} \triangleright\dot{\tau}_1 \to \dot{\tau}_2\triangleleft} \tag{9b}$$

$$\frac{}{\dot{\tau}_1 \to \triangleright\dot{\tau}_2\triangleleft \xrightarrow{\texttt{move parent}} \triangleright\dot{\tau}_1 \to \dot{\tau}_2\triangleleft} \tag{9c}$$

$$\frac{}{\triangleright\dot{\tau}_1\triangleleft \to \dot{\tau}_2 \xrightarrow{\texttt{move nextSib}} \dot{\tau}_1 \to \triangleright\dot{\tau}_2\triangleleft} \tag{9d}$$

$$\frac{}{\dot{\tau}_1 \to \triangleright\dot{\tau}_2\triangleleft \xrightarrow{\texttt{move prevSib}} \triangleright\dot{\tau}_1\triangleleft \to \dot{\tau}_2} \tag{9e}$$

Type Deletion

$$\frac{}{\triangleright\dot{\tau}\triangleleft \xrightarrow{\texttt{del}} \triangleright(\!|\,|\!)\triangleleft} \tag{9f}$$

Type Construction

$$\frac{}{\triangleright\dot{\tau}\triangleleft \xrightarrow{\texttt{construct arrow}} \dot{\tau} \to \triangleright(\!|\,|\!)\triangleleft} \tag{9g}$$

$$\frac{}{\triangleright(\!|\,|\!)\triangleleft \xrightarrow{\texttt{construct num}} \triangleright\texttt{num}\triangleleft} \tag{9h}$$

Zipper Cases

$$\frac{\hat{\tau} \xrightarrow{\alpha} \hat{\tau}'}{\hat{\tau} \to \dot{\tau} \xrightarrow{\alpha} \hat{\tau}' \to \dot{\tau}} \tag{9i}$$

$$\frac{\hat{\tau} \xrightarrow{\alpha} \hat{\tau}'}{\dot{\tau} \to \hat{\tau} \xrightarrow{\alpha} \dot{\tau} \to \hat{\tau}'} \tag{9j}$$

Expression Movement Actions

$\boxed{\hat{e} \xrightarrow{\texttt{move } \delta} \hat{e}'}$

Ascription

$$\frac{}{\triangleright \dot{e} : \dot{\tau} \triangleleft \xrightarrow{\texttt{move firstChild}} \triangleright \dot{e} \triangleleft : \dot{\tau}} \tag{10a}$$

$$\frac{}{\triangleright \dot{e} \triangleleft : \dot{\tau} \xrightarrow{\texttt{move parent}} \triangleright \dot{e} : \dot{\tau} \triangleleft} \tag{10b}$$

$$\frac{}{\dot{e} : \triangleright \dot{\tau} \triangleleft \xrightarrow{\texttt{move parent}} \triangleright \dot{e} : \dot{\tau} \triangleleft} \tag{10c}$$

$$\frac{}{\triangleright \dot{e} \triangleleft : \dot{\tau} \xrightarrow{\texttt{move nextSib}} \dot{e} : \triangleright \dot{\tau} \triangleleft} \tag{10d}$$

$$\frac{}{\dot{e} : \triangleright \dot{\tau} \triangleleft \xrightarrow{\texttt{move prevSib}} \triangleright \dot{e} \triangleleft : \dot{\tau}} \tag{10e}$$

Lambda

$$\frac{}{\triangleright \lambda x.\dot{e} \triangleleft \xrightarrow{\texttt{move firstChild}} \lambda x.\triangleright \dot{e} \triangleleft} \tag{10f}$$

$$\frac{}{\lambda x.\triangleright \dot{e} \triangleleft \xrightarrow{\texttt{move parent}} \triangleright \lambda x.\dot{e} \triangleleft} \tag{10g}$$

Application

$$\frac{}{\triangleright \dot{e}_1(\dot{e}_2) \triangleleft \xrightarrow{\texttt{move firstChild}} \triangleright \dot{e}_1 \triangleleft (\dot{e}_2)} \tag{10h}$$

$$\frac{}{\triangleright \dot{e}_1 \triangleleft (\dot{e}_2) \xrightarrow{\texttt{move parent}} \triangleright \dot{e}_1(\dot{e}_2) \triangleleft} \tag{10i}$$

$$\frac{}{\dot{e}_1(\triangleright \dot{e}_2 \triangleleft) \xrightarrow{\texttt{move parent}} \triangleright \dot{e}_1(\dot{e}_2) \triangleleft} \tag{10j}$$

$$\frac{}{\triangleright \dot{e}_1 \triangleleft (\dot{e}_2) \xrightarrow{\texttt{move nextSib}} \dot{e}_1(\triangleright \dot{e}_2 \triangleleft)} \tag{10k}$$

$$\frac{}{\dot{e}_1(\triangleright \dot{e}_2 \triangleleft) \xrightarrow{\texttt{move prevSib}} \triangleright \dot{e}_1 \triangleleft (\dot{e}_2)} \tag{10l}$$

Plus

$$\frac{}{\triangleright \dot{e}_1 + \dot{e}_2 \triangleleft \xrightarrow{\texttt{move firstChild}} \triangleright \dot{e}_1 \triangleleft + \dot{e}_2} \tag{10m}$$

$$\frac{}{\triangleright \dot{e}_1 \triangleleft + \dot{e}_2 \xrightarrow{\texttt{move parent}} \triangleright \dot{e}_1 + \dot{e}_2 \triangleleft} \tag{10n}$$

$$\frac{}{\dot{e}_1 + \triangleright \dot{e}_2 \triangleleft \xrightarrow{\texttt{move parent}} \triangleright \dot{e}_1 + \dot{e}_2 \triangleleft} \tag{10o}$$

$$\frac{}{\triangleright \dot{e}_1 \triangleleft + \dot{e}_2 \xrightarrow{\texttt{move nextSib}} \dot{e}_1 + \triangleright \dot{e}_2 \triangleleft} \tag{10p}$$

$$\frac{}{\dot{e}_1 + \triangleright \dot{e}_2 \triangleleft \xrightarrow{\texttt{move prevSib}} \triangleright \dot{e}_1 \triangleleft + \dot{e}_2} \tag{10q}$$

Non-Empty Hole

$$\frac{}{\triangleright(\!|\dot{e}|\!)\triangleleft \xrightarrow{\texttt{move firstChild}} (\!|\triangleright\dot{e}\triangleleft|\!)} \tag{10r}$$

$$\frac{}{(\!|\triangleright\dot{e}\triangleleft|\!) \xrightarrow{\texttt{move parent}} \triangleright(\!|\dot{e}|\!)\triangleleft} \tag{10s}$$

Synthetic and Analytic Expression Actions The synthetic and analytic expression action performance judgements are defined mutually inductively by Rules (11) and Rules (12), respectively.

$\boxed{\dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \dot{\tau}'}$

Movement

$$\frac{\hat{e} \xrightarrow{\texttt{move}\ \delta} \hat{e}'}{\dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\texttt{move}\ \delta} \hat{e}' \Rightarrow \dot{\tau}} \tag{11a}$$

Deletion

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{del}} \triangleright(\!|\,|\!)\triangleleft \Rightarrow (\!|\,|\!)} \tag{11b}$$

Construction

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{construct asc}} \dot{e} : \triangleright\dot{\tau}\triangleleft \Rightarrow \dot{\tau}} \tag{11c}$$

$$\frac{}{\dot{\Gamma}, x : \dot{\tau} \vdash \triangleright(\!|\,|\!)\triangleleft \Rightarrow (\!|\,|\!) \xrightarrow{\texttt{construct var}\ x} \triangleright x\triangleleft \Rightarrow \dot{\tau}} \tag{11d}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright(\!|\,|\!)\triangleleft \Rightarrow (\!|\,|\!) \xrightarrow{\texttt{construct lam}\ x} \lambda x.(\!|\,|\!) : \triangleright(\!|\,|\!)\triangleleft \to (\!|\,|\!) \Rightarrow (\!|\,|\!) \to (\!|\,|\!)} \tag{11e}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau}_1 \to \dot{\tau}_2 \xrightarrow{\texttt{construct ap}} \dot{e}(\triangleright(\!|\,|\!)\triangleleft) \Rightarrow \dot{\tau}_2} \tag{11f}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow (\!|\,|\!) \xrightarrow{\texttt{construct ap}} \dot{e}(\triangleright(\!|\,|\!)\triangleleft) \Rightarrow (\!|\,|\!)} \tag{11g}$$

$$\frac{\dot{\tau} \nsim (\!|\,|\!) \to (\!|\,|\!)}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{construct ap}} (\!|\dot{e}|\!)(\triangleright(\!|\,|\!)\triangleleft) \Rightarrow (\!|\,|\!)} \tag{11h}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau} \xrightarrow{\texttt{construct arg}} \triangleright(\!|\,|\!)\triangleleft(\dot{e}) \Rightarrow (\!|\,|\!)} \tag{11i}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright(\!|\,|\!)\triangleleft \Rightarrow (\!|\,|\!) \xrightarrow{\texttt{construct lit}\ n} \triangleright\underline{n}\triangleleft \Rightarrow \texttt{num}} \tag{11j}$$

$$\frac{\dot\tau \sim \texttt{num}}{\dot\Gamma \vdash \triangleright\dot e\triangleleft \Rightarrow \dot\tau \xrightarrow{\texttt{construct plus}} \dot e + \triangleright(\!|\!|\triangleleft \Rightarrow \texttt{num}} \tag{11k}$$

$$\frac{\dot\tau \nsim \texttt{num}}{\dot\Gamma \vdash \triangleright\dot e\triangleleft \Rightarrow \dot\tau \xrightarrow{\texttt{construct plus}} (\!|\dot e|\!) + \triangleright(\!|\!|\triangleleft \Rightarrow \texttt{num}} \tag{11l}$$

Finishing

$$\frac{\dot\Gamma \vdash \dot e \Rightarrow \dot\tau'}{\dot\Gamma \vdash \triangleright(\!|\dot e|\!)\triangleleft \Rightarrow (\!|\!|) \xrightarrow{\texttt{finish}} \triangleright\dot e\triangleleft \Rightarrow \dot\tau'} \tag{11m}$$

Zipper Cases

$$\frac{\dot\Gamma \vdash \hat e \xrightarrow{\alpha} \hat e' \Leftarrow \dot\tau}{\dot\Gamma \vdash \hat e : \dot\tau \Rightarrow \dot\tau \xrightarrow{\alpha} \hat e' : \dot\tau \Rightarrow \dot\tau} \tag{11n}$$

$$\frac{\hat\tau \xrightarrow{\alpha} \hat\tau' \qquad \dot\Gamma \vdash \dot e \Leftarrow \hat\tau'^{\diamond}}{\dot\Gamma \vdash \dot e : \hat\tau \Rightarrow \hat\tau^{\diamond} \xrightarrow{\alpha} \dot e : \hat\tau' \Rightarrow \hat\tau'^{\diamond}} \tag{11o}$$

$$\frac{\dot\Gamma \vdash \hat e^{\diamond} \Rightarrow \dot\tau_2 \qquad \dot\Gamma \vdash \hat e \Rightarrow \dot\tau_2 \xrightarrow{\alpha} \hat e' \Rightarrow \dot\tau_3 \to \dot\tau_4 \qquad \dot\Gamma \vdash \dot e \Leftarrow \dot\tau_3}{\dot\Gamma \vdash \hat e(\dot e) \Rightarrow \dot\tau_1 \xrightarrow{\alpha} \hat e'(\dot e) \Rightarrow \dot\tau_4} \tag{11p}$$

$$\frac{\dot\Gamma \vdash \hat e^{\diamond} \Rightarrow \dot\tau_2 \qquad \dot\Gamma \vdash \hat e \Rightarrow \dot\tau_2 \xrightarrow{\alpha} \hat e' \Rightarrow (\!|\!|) \qquad \dot\Gamma \vdash \dot e \Leftarrow (\!|\!|)}{\dot\Gamma \vdash \hat e(\dot e) \Rightarrow \dot\tau_1 \xrightarrow{\alpha} \hat e'(\dot e) \Rightarrow (\!|\!|)} \tag{11q}$$

$$\frac{\dot\Gamma \vdash \dot e \Rightarrow \dot\tau_2 \to \dot\tau \qquad \dot\Gamma \vdash \hat e \xrightarrow{\alpha} \hat e' \Leftarrow \dot\tau_2}{\dot\Gamma \vdash \dot e(\hat e) \Rightarrow \dot\tau \xrightarrow{\alpha} \dot e(\hat e') \Rightarrow \dot\tau} \tag{11r}$$

$$\frac{\dot\Gamma \vdash \dot e \Rightarrow (\!|\!|) \qquad \dot\Gamma \vdash \hat e \xrightarrow{\alpha} \hat e' \Leftarrow (\!|\!|)}{\dot\Gamma \vdash \dot e(\hat e) \Rightarrow (\!|\!|) \xrightarrow{\alpha} \dot e(\hat e') \Rightarrow (\!|\!|)} \tag{11s}$$

$$\frac{\dot\Gamma \vdash \hat e \xrightarrow{\alpha} \hat e' \Leftarrow \texttt{num}}{\dot\Gamma \vdash \hat e + \dot e \Rightarrow \texttt{num} \xrightarrow{\alpha} \hat e' + \dot e \Rightarrow \texttt{num}} \tag{11t}$$

$$\frac{\dot\Gamma \vdash \hat e \xrightarrow{\alpha} \hat e' \Leftarrow \texttt{num}}{\dot\Gamma \vdash \dot e + \hat e \Rightarrow \texttt{num} \xrightarrow{\alpha} \dot e + \hat e' \Rightarrow \texttt{num}} \tag{11u}$$

$$\frac{\dot\Gamma \vdash \hat e^{\diamond} \Rightarrow \dot\tau \qquad \dot\Gamma \vdash \hat e \Rightarrow \dot\tau \xrightarrow{\alpha} \hat e' \Rightarrow \dot\tau' \qquad \hat e' \neq \triangleright(\!|\!|\triangleleft}{\dot\Gamma \vdash (\!|\hat e|\!) \Rightarrow (\!|\!|) \xrightarrow{\alpha} (\!|\hat e'|\!) \Rightarrow (\!|\!|)} \tag{11v}$$

$$\frac{\dot\Gamma \vdash \hat e^{\diamond} \Rightarrow \dot\tau \qquad \dot\Gamma \vdash \hat e \Rightarrow \dot\tau \xrightarrow{\alpha} \triangleright(\!|\!|\triangleleft \Rightarrow (\!|\!|)}{\dot\Gamma \vdash (\!|\hat e|\!) \Rightarrow (\!|\!|) \xrightarrow{\alpha} \triangleright(\!|\!|\triangleleft \Rightarrow (\!|\!|)} \tag{11w}$$

$\boxed{\dot\Gamma \vdash \hat e \xrightarrow{\alpha} \hat e' \Leftarrow \dot\tau}$

Subsumption

$$\frac{\dot{\Gamma} \vdash \hat{e}^{\diamond} \Rightarrow \dot{\tau}' \qquad \dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau}' \xrightarrow{\alpha} \hat{e}' \Rightarrow \dot{\tau}'' \qquad \dot{\tau} \sim \dot{\tau}'' \qquad \alpha \neq \texttt{construct asc} \qquad \alpha \neq \texttt{construct lam } x}{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}} \tag{12a}$$

Movement

$$\frac{\hat{e} \xrightarrow{\texttt{move } \delta} \hat{e}'}{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\texttt{move } \delta} \hat{e}' \Leftarrow \dot{\tau}} \tag{12b}$$

Deletion

$$\frac{}{\dot{\Gamma} \vdash \triangleright \dot{e} \triangleleft \xrightarrow{\texttt{del}} \triangleright (\!|\,|\!) \triangleleft \Leftarrow \dot{\tau}} \tag{12c}$$

Construction

$$\frac{}{\dot{\Gamma} \vdash \triangleright \dot{e} \triangleleft \xrightarrow{\texttt{construct asc}} \dot{e} : \triangleright \dot{\tau} \triangleleft \Leftarrow \dot{\tau}} \tag{12d}$$

$$\frac{\dot{\tau} \nsim \dot{\tau}'}{\dot{\Gamma}, x : \dot{\tau}' \vdash \triangleright (\!|\,|\!) \triangleleft \xrightarrow{\texttt{construct var } x} (\!|\triangleright x \triangleleft|\!) \Leftarrow \dot{\tau}} \tag{12e}$$

$$\frac{}{\dot{\Gamma} \vdash \triangleright (\!|\,|\!) \triangleleft \xrightarrow{\texttt{construct lam } x} \lambda x. \triangleright (\!|\,|\!) \triangleleft \Leftarrow \dot{\tau}_1 \to \dot{\tau}_2} \tag{12f}$$

$$\frac{\dot{\tau} \nsim (\!|\,|\!) \to (\!|\,|\!)}{\dot{\Gamma} \vdash \triangleright (\!|\,|\!) \triangleleft \xrightarrow{\texttt{construct lam } x} (\!|\lambda x.(\!|\,|\!) : \triangleright (\!|\,|\!) \triangleleft \to (\!|\,|\!)|\!) \Leftarrow \dot{\tau}} \tag{12g}$$

$$\frac{\dot{\tau} \nsim \texttt{num}}{\dot{\Gamma} \vdash \triangleright (\!|\,|\!) \triangleleft \xrightarrow{\texttt{construct lit } n} (\!|\triangleright \underline{n} \triangleleft|\!) \Leftarrow \dot{\tau}} \tag{12h}$$

Finishing

$$\frac{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}{\dot{\Gamma} \vdash \triangleright (\!|\dot{e}|\!) \triangleleft \xrightarrow{\texttt{finish}} \triangleright \dot{e} \triangleleft \Leftarrow \dot{\tau}} \tag{12i}$$

Zipper Cases

$$\frac{\dot{\Gamma}, x : \dot{\tau}_1 \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}_2}{\dot{\Gamma} \vdash \lambda x. \hat{e} \xrightarrow{\alpha} \lambda x. \hat{e}' \Leftarrow \dot{\tau}_1 \to \dot{\tau}_2} \tag{12j}$$