

FlowPools: Lock-Free Deterministic Concurrent Data-Flow Queues

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Abstract. Implementing correct and deterministic parallel programs is hard (additional motivation sentence after this). We present the design and implementation of a fundamental data structure for deterministic parallel data-flow computation. Additionally, we provide a proof of correctness, showing that the implementation is linearizable, lock-free, and deterministic. Finally, we provide microbenchmarks which compare our *flow pools* against corresponding operations on other popular concurrent data structures, in addition to performance benchmarks on a real *XYZ* application using real data. (Keep abstract between 70 and 150 words).

Keywords: data-flow, concurrent data-structure, determinism

1 Introduction

1.1 Motivation

- we want a deterministic model - we do not block in the programming model (i.e. there are no operations which cause blocking until a value becomes available) - we want a non-blocking data-structure (i.e. the operations on the data-structures should themselves be non-blocking) - programs run indefinitely => we need to GC parts of the data structure we no longer need - we want to reduce heap allocation and inline the datastructure as much as possible => lower memory consumption, better cache behaviour and fewer GC cycles

Obligatory multicore motivation paragraph.

Lock-free is better, and why.

Introduction and motivation for data-flow programming model.

2 Model of Computation

Producer-consumer parallelism. Description and image of queue/stream of values, producer, and multiple consumers.

3 Programming Model

The FlowPool suite supports the following operations:

- `Builder.<<(x: T): Builder`
Inserts an element into the underlying FlowPool.
- `Builder.seal(n: Int): Unit`
Seals the underlying FlowPool at `n` elements. The need for the size argument is explained below. A sealed FlowPool may only contain `n` elements. This allows for callback cleanup and termination.
- `FlowPool.doForAll(f: T => Unit): Future[Int]`
Instructs the FlowPool to execute the closure `f` exactly once for each element inserted into the FlowPool (asynchronously). The returned future contains the number of elements in the FlowPool and completes once `f` has been executed for all elements.
- `FlowPool.mappedFold[U, V <: U](acc: V)(cmb: (U,V) => V)(map: T => U): Future[(Int, V)]` Reduces the FlowPool to a single value of type `V`, by first mapping each element to an internal representation using `map` and then aggregating using `cmb`. No guarantee is given about synchronization or order. Returns a future containing a tuple with number of elements in the FlowPool and the aggregated value.
- `FlowPool.builder`
Returns a builder for this FlowPool.
- `Future.map[U](f: T => U)`
Maps this future to another future executing the function `f` exactly once when the first future completes.
- `future[T](f: () => T): Future[T]`
Asynchronously dispatch execution of `f` and return a future with its result.

Determinism of seal We will show that the final size of the FlowPool is required as an argument to the `seal` method in order to satisfy the determinism property of the FlowPool. Look at the following program:

```
val p = new FlowPool[Int]()
val b = p.builder

future {
  for (i <- 1 to 10) { b << i }
  b.seal
}

future { for (i <- 1 to 10) { b << i } }
```

Depending on which for-loop completes first, this program completes successfully or yields an error. A similar program with `b.seal(20)` will always succeed.

Generators In the following we'll present a couple of generators for FlowPools based on common generators in the Scala standard library.

```
def iterate[T](start: T, len: Int)(f: (T) => T) = {
  val p = new FlowPool[T]
  val b = p.builder
  future {
    var e = start
    for (i <- 1 to len) { b << e; e = f(e) }
    b.seal(len)
  }; p
}
```

```
def tabulate[T](n: Int)(f: (Int) => T) = {
  val p = new FlowPool[T]
  val b = p.builder
  future {
    for (i <- 0 to (n-1)) {
      b << f(i)
    }
    b.seal(n)
  }; p
}
```

```
def fill[T](n: Int)(elem: => T) = {
  val p = new FlowPool[T]
  val b = p.builder
  future {
    for (i <- 1 to n) { b << elem }
    b.seal(n)
  }; p
}
```

Monadic Operations In the following we'll present some monadic operations on top of the basic FlowPool operations. This will also show some use-cases of the futures as result type of `doForAll`.

```
def map[S](f: T => S) = {
  val fp = new FlowPool[S]
  val b = fp.builder
  doForAll { x =>
    b << f(x)
  } map { b.seal _ }
  fp
}
```

```

def filter(f: T => Boolean) = {
  val fp = new FlowPool[T]
  val b = fp.builder

  mappedFold(0)(_ + _) { x =>
    if (f(x)) { b << x; 1 } else 0
  } map { case (c,fc) => b.seal(fc) }

  fp
}

def flatMap[S](f: T => FlowPool[S]) = {
  val fp = new FlowPool[S]
  val b = fp.builder

  mappedFold(future(0))(_ <+> _) { x =>
    f(x).doForAll(b << _)
  } map { case (c,cfut) => cfut.map(b.seal _) }

  fp
}

```

where `<+>` is the future of the sum of two `Future[Int]`.

4 Implementation

5 Proofs

5.1 Abstract Pool Semantics

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5.2 Linearizability

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5.3 Lock-Freedom

5.4 Determinism

6 Experimental Results

7 Related Work

Things to probably cite: Oz, gpars, Java CLQ, our futures writeup.

Things we should probably have a look at: Microsoft TPL, Dataflow Java, FlumeJava...

Forcing a bib, [1], [2], [3], [4], [5], [6]

8 Conclusion

References

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A Proof of Correctness

Definition 1 (Data types). A **Block** b is an object which contains an array $b.array$, which itself can contain elements, $e \in Elem$, where **Elem** represents the type of e and can be any countable set. A given block b additionally contains an index $b.index$ which represents an index location in $b.array$, a unique index identifying the array $b.blockIndex$, and $b.next$, a reference to a successor block c where $c.blockIndex = b.blockIndex + 1$. A **Terminal** $term$ is a sentinel object, which contains an integer $term.sealed \in -1 \cup \mathbb{N}$, and $term.callbacks$, a set of functions $f \in Elem \Rightarrow Unit$.

Definition 2 (FlowPool). A **FlowPool** $pool$ is an object that has a reference $pool.start$, to the first block b_0 (with $b_0.blockIndex = 0$), as well as a reference $pool.current$ typically pointing to some subsequent block b_n where $b_n = b_0$ or where b_n is reachable from b_0 following $next$ references. Initially, $pool.current = pool.start$. The pool **state** \mathbb{S} is defined as the sequence of blocks reachable from $pool.start$ by following $next$ references within blocks. More formally, the relation $reachable(b, c)$ on two blocks b, c holds iff $b = c \vee b.next = c \vee \exists b' : reachable(b, b') \wedge reachable(b', c)$. A **state changing** instruction is any atomic write or CAS instruction that changes an object that can be accessed from $pool.start$.

Definition 3 (Invariants).

INV1 let $b : Block$ such that $reachable(start, b) \wedge b.next = \perp$. $\exists i$ such that $b.array(i) = Terminal \wedge \forall i < j \leq LASTELEMPOS, b.array(j) = \perp$

Definition 4 (Abstract state). An **abstract state** \mathbb{A} is a tuple $(elems, sealed, callbacks)$ such that $\mathbb{A} \in \{(elems, sealed, callbacks) \mid elems \subset Elem, sealed \in \{-1\} \cup \mathbb{N}, callbacks \subset Elem \Rightarrow Unit\}$ Abstract state operations on some abstract state \mathbb{A} are $append(\mathbb{A}, e) = \mathbb{A}'$ where $\mathbb{A}' = (elems \cup \{e\}, sealed, callbacks)$ if $\mathbb{A} = (elems, sealed, callbacks)$, $seal(\mathbb{A}, sealSize) = \mathbb{A}''$ where $\mathbb{A}'' = (elems, sealSize, callbacks) : sealSize \in \mathbb{N}$ if $\mathbb{A} = (elems, sealed, callbacks)$ and $sealed = -1$, the unsealed state, or

$sealed = sealSize$ already, and $doForAll(\mathbb{A}, fun) = \mathbb{A}'''$ where $\mathbb{A}''' = (elems, sealed, callbacks \cup \{fun\})$.

Definition 5 (Consistency). A FlowPool state \mathbb{S} of $pool$ with starting block $pool.start$ is consistent with an abstract state $\mathbb{A} = (elems, sealed)$ iff some element $e \in elems \Leftrightarrow \exists b, i : reachable(pool.start, b) \wedge b.array(i) = e$, and $\exists c, j : c.array(j) \in Terminal \wedge c.array(j).sealed = sealed \wedge reachable(pool.start, c)$. A FlowPool operation op is **consistent** with the corresponding abstract state operation op' iff $\mathbb{S}' = op(\mathbb{S})$ is consistent with an abstract state $\mathbb{A}' = op'(\mathbb{A})$. A **consistency change** is a change from state \mathbb{S} to state \mathbb{S}' such that \mathbb{S} is consistent with an abstract state \mathbb{A} and \mathbb{S}' is consistent with an abstract set \mathbb{A}' , where $\mathbb{A} \neq \mathbb{A}'$.

Definition 6 (Lock-freedom). In a scenario where some finite number of threads are executing a concurrent operation, that concurrent operation is *lock-free* if and only if that concurrent operation is completed after a finite number of steps by some thread.

Theorem 1 (Lock-freedom). *FlowPool operations append, seal, and doForAll are lock-free.*

We begin by first proving that there are a finite number of execution steps before a consistency change occurs.

CASes that fail by being completed by other CASes signify progress.

Lemma 1. *After invoking an operation op , if non-consistency changing CAS operations CAS1, CAS3, or CAS4, in the pseudocode fail, they must have already been successfully completed by another thread since op began.*

Proof. Trivial inspection of the pseudocode reveals that since CAS1 makes up a check that precedes CAS2, and since CAS2 is the only operation besides CAS1 which can change the expected value of CAS1, in the case of a failure of CAS1, CAS2 (and thus CAS1) must have already successfully completed or CAS1 must have already successfully completed by a different thread since op began executing.

Likewise, by trivial inspection CAS3 is the only CAS which can update the $b.next$ reference, therefore in the case of a failure, some other thread must have already successfully completed CAS3 since the beginning of op .

Like above, CAS4 is the only CAS which can change the *current* reference, therefore in the case of a failure, some other thread must have already successfully completed CAS4 since op began. \square

Lemma 2. *Invoking the expand operation will execute a non-consistency changing instruction after a finite number of steps. Furthermore, given a total number of blocks $numBlocks$ reachable in a FlowPool pool before invoking expand through append, the number of blocks $numBlocks'$ after some finite number of steps is guaranteed to satisfy $numBlocks' > numBlocks$.*

Proof. From inspection of the pseudocode, it is clear that the only point at which $expand(b)$ can be invoked is under the condition that for some block b , $b.index > LASTELEMPOS$, where $LASTELEMPOS$ is the maximum size set aside for elements of type *Elem* in any block. Given this, we will proceed by

showing that a new block will be created with all related references $b.next$ and $current$ correctly set.

There are two conditions under which a non-consistency changing CAS instruction will be carried out.

- **Case 1:** if $b.next = \perp$, a new block nb will be created and $CAS3$ will be executed. From Lemma 1, we know that $CAS3$ must complete successfully on some thread. Afterwards recursively calling *expand* on the original block b .
- **Case 2:** if $b.next \neq \perp$, $CAS4$ will be executed. Lemma 1 guarantees that $CAS4$ will update $current$ to refer to $b.next$, which we will show can only be a new block. Likewise, Lemma 1 has shown that $CAS3$ is the only state changing instruction that can initiate a state change at location $b.next$, therefore, since $CAS3$ takes place within Case 1, Case 2 can only be reachable after Case 1 has been executed successfully. Given that Case 1 always creates a new block, therefore, $b.next$ in this case, must always refer to a new block.

Therefore, since from Lemma 1 we know that both $CAS3$ and $CAS4$ can only fail if already completed guaranteeing their finite completion, and since $CAS3$ and $CAS4$ are the only state changing operations invoked through *expand*, the *expand* operation must complete in a finite number of steps.

Finally, since we saw in Case 2 that a new block is always created and related references are always correctly set, that is both $b.next$ and $current$ are correctly updated to refer to the new block, it follows that *numBlocks* strictly increases after some finite number of steps. \square

Lemma 3. *After invoking $append(elem)$, if $CAS2$ fails, then some thread has successfully completed $CAS2$ or $CAS5$ (or likewise, $CAS6$) after some finite number of steps.*

Proof. First, we show that a thread attempting to complete $CAS2$ can't fail due to a different thread completing $CAS1$ so long as *seal* has not been invoked after completing the read of *currobj*. We address this exception later on.

Since after *check*, the only condition under which $CAS1$, and by extension, $CAS2$ can be executed is the situation where the current object *currobj* with index location *idx* is the *Terminal* object, it follows that $CAS1$ can only ever serve to duplicate this *Terminal* object at location $idx + 1$, leaving at most two *Terminals* in block referred to by *current* momentarily until $CAS2$ can be executed. By Lemma 1, since $CAS1$ is a non-consistency changing instruction, it follows that any thread holding any element $elem'$ can execute this instruction without changing the expected value of *currobj* in $CAS2$, as no new object is ever created and placed in location *idx*. Therefore, $CAS2$ cannot fail due to $CAS1$, so long as *seal* has not been invoked by some other thread after the read of *currobj*.

This leaves only two scenarios in which consistency changing $CAS2$ can fail:

- **Case 1:** Another thread has already completed $CAS2$ with a different element $elem'$.

- **Case 2:** Another thread completes an invocation to the *seal* operation after the current thread completes the read of *currobj*. In this case, *CAS2* can fail because *CAS5* (or, likewise *CAS6*) might have completed before, in which case, it inserts a new *Terminal* object *term* into location *idx* (in the case of a *seal* invocation, *term.sealed* $\in \mathbb{N}$, or in the case of a *doForAll* invocation, $\{term.callbacks \in \{\}\}$).

We omit the proof and detailed discussion of *CAS6* because it can be proven using the same steps as were taken for *CAS5*. \square

Lemma 4. *All operations with the exception of *append*, *seal*, and *doForAll* execute only a finite number of steps between each state changing instruction*

Proof. The **advance**, **check**, **totalElems**, **invokeCallbacks**, and **tryWriteSeal** operations have a finite number of execution steps, as they contain no recursive calls, loops, or other possibility to restart.

While the **expand** operation contains a recursive call following a CAS instruction, it was shown in Lemma 2 that an invocation of *expand* is guaranteed to execute a state changing instruction after a finite number of steps. \square

Lemma 5. *After invoking *append(elem)*, a consistency changing instruction will be completed after a finite number of steps.*

Proof. The **append** operation can be restarted in three cases. We show that in each case, it's guaranteed to either complete in a finite number of steps, or leads to a state changing instruction:

- **Case 1:** The call to *check*, a finite operation by Lemma 4, returns *false*, causing a call to *advance*, also a finite operation by Lemma 4, followed by a recursive call to *append* with the same element *elem* which in turn once again calls *check*.

We show that after a finite number of steps, the *check* will evaluate to *true*, or some other thread will have completed a consistency changing operation since the initial invocation of *append*. In the case where *check* evaluates to *true*, Lemma 3 applies, as it guarantees that a consistency changing CAS is completed after a finite number of steps.

When the call to the finite operation *check* returns *false*, if the subsequent *advance* finds that a *Terminal* object is at the current block index *idx*, then the next invocation of *append* will evaluate *check* to *true*. Otherwise, it must be the case that another thread has moved the *Terminal* to a subsequent index since the initial invocation of *append*, which is only possible using a consistency changing instruction.

Finally, if *advance* finds that the element at *idx* is an *Elem*, it will increment *idx*. By INV1, this can only happen a finite number of times until a *Terminal* is found. In the case that *expand* is meanwhile invoked through *advance*, by Lemma 2 it's guaranteed to complete state changing instructions *CAS3* or *CAS4* in a finite number of steps. Otherwise, some other thread has moved the *Terminal* to a subsequent index. However, this latter case is only possible by successfully completing *CAS2*, a consistency changing instruction, after the initial invocation of *append*.

- **Case 2:** *CAS1* fails, which we know from Lemma 1 means that it must've already been completed by another thread, guaranteeing that *CAS2* will be attempted. If *CAS2* fails, by Lemma 3, after a finite number of steps, a consistency changing instruction will be completed. If *CAS2* succeeds, as a consistency changing instruction, consistency will have clearly been changed.
- **Case 3:** *CAS2* fails, which, by Lemma 3, indicates that either some other thread has already completed *CAS2* with another element, or another consistency changing instruction, *CAS5* or *CAS6* has successfully completed.

Therefore, *append* itself as well as all other operations reachable via an invocation of *append* are guaranteed to have a finite number of steps between consistency changing instructions.

Lemma 6. *After invoking $\text{seal}(\text{size})$, if *CAS5* fails, then some thread has successfully completed *CAS5* or *CAS2* after some finite number of steps.*

Proof. Since *CAS1* only duplicates an existing *Terminal*, it can not be the cause for a failing *CAS5*. This leaves only two cases in which *CAS5* can fail:

- **Case 1:** Another thread has already completed *CAS5*.
- **Case 2:** Another thread completes an invocation to the *append(elem)* operation after the current thread completes the read of *currobj*. In this case, *CAS5* can fail because *CAS2* might have completed before, in which case, it inserts a new *Elem* object *elem* into location *idx*.

Lemma 7. *After invoking $\text{seal}(\text{size})$, a consistency changing instruction will be completed after a finite number of steps, or the initial invocation of $\text{seal}(\text{size})$ completes.*

Proof. The *seal* operation can be restarted in two scenarios.

- **Case 1:** The object at the current index location *idx* is of type *Elem*, not *Terminal*, causing a recursive call to *seal* with the same size *size*.
- **Case 2:** The check $\text{idx} \leq \text{LASTELEMPOS}$ fails, indicating that we must move on to the next block, causing first a call to *expand* followed by a recursive call to *seal* with the same size *size*.

Lemma 8. *After invoking $\text{doForAll}(\text{fun})$, a consistency changing instruction will be completed after a finite number of steps.*

We omit the proof for *doForAll* since it proceeds in the exactly the same way as does the proof for *seal* in Lemma 7.

Lemma 9. *Assume a concurrent operation *op* is started. After a finite number of state changing instructions, a consistency changing CAS instruction is guaranteed to be successfully completed.*

Proof.

Lemma 10. *Assume a concurrent operation is started. If a consistency changing CAS instruction completes, some concurrent operation is guaranteed to be completed.*

Proof.

A.1 Scratchpad

Lock-freedom.

By Lemma 2, we know that when we call *expand*, it's guaranteed that there will be a finite number of steps executed followed by a state changing operation.

After invoking *append*, if *check* is false, a consistency changing CAS (CAS2, CAS5, or CAS6) is completed after a finite number of steps.

We have to show that we'll update *current* after a finite number of steps.

The *expand* operation contains two CAS instructions; *CAS3* and *CAS4*.

Lemma 1. *In each operation there is a finite number of execution steps between consecutive CAS instructions..*

Proof. The *append* operation is restarted in three cases. Case 1: iff *check* returns *true* \wedge *CAS1* fails. Case 2: . Case 3:

expand

tryWriteSeal

asyncDoForAll

Then, the operation corresponding to the consistency-changing CAS instruction is guaranteed to be eventually completed

If there's a CAS failing, then some other thread completes the CAS. Consistency change means progress.

OLD: *Proof.* Case 1: The failing *CAS4* happens after a successful *CAS3*. From lemma 1, we know that *CAS3*, which is a check that precedes *CAS4*, is guaranteed to be successfully completed by some thread, so we focus on the implications of failure of *CAS4*. Case 2: *CAS5* takes place if *nb* is *null*. Therefore, in both cases, *CAS4* and *CAS5* successfully complete.

Lemma 3. *consistency-changing CAS operations ... will successfully complete.*

Proof. Uses \mathbb{A} .

Lemma 4. *Assume that the FlowPool is consistent with some abstract state \mathbb{A} . If one of the operations *advance* or *expand* succeeds, the FlowPool will remain consistent with the abstract state \mathbb{A} following the operation.*

Proof. The CAS operations, denoted *CAS3*, and *CAS4* in the pseudo-code, within the *expand* operation neither affect *elems* nor *sealed*, thus by Definition 4, causes no consistency change. Likewise, the *advance* operation either calls *expand* once, or it invokes *CAS1* it may update the index of the current block, neither of which cause a consistency change.

Lemma 5 (). *If a consistency changing CAS completes, then the operation is guaranteed to successfully complete.*

Lemma 6 (). *append operation is lock-free.*

Lemma 7 (). *seal operation is lock-free.*

Lemma 8 (). *doForAll operation is lock-free.*