MATH-6620 PERTURBATION METHODS SPRING 2016

Homework-4

Assigned Friday March 18, 2016

Due Wednesday March 30, 2016

NOTES

1. Writing solutions in LaTeX is strongly recommended but not required.
2. Show all work. Illegible or undecipherable solutions will be **returned without grading**.
3. Figures, if any, should be neatly drawn (either by hand or by a drawing program), properly labelled and captioned.
4. Please make sure that the pages are stapled together.
5. The assignment can be submitted in the labelled box in Amos Eaton 301, at my office, or in class.

PROBLEMS

1. Consider the signaling problem

$$\epsilon(u_{xx} - u_{tt}) = u_t + 2u_x, \quad 0 < x < \pi,\ t > 0,$$

with auxiliary conditions

$$u(x,0) = u_t(x,0) = 0,\ \ u(0,t) = -\sin t,\ u(\pi,t) = 0.$$

Construct a leading-order solution for $0 < \epsilon \ll 1$, paying due attention to the location of any layers.

2. Consider the elliptic problem

$$\epsilon(u_{xx} + u_{yy}) + u_x + u_y + u = 0, \quad 0 < x < 1,\ 0 < y < 1,$$

with boundary conditions

$$u(x,0) = 0,\ u(x,1) = 1 - x,\ \ u(0,y) = e^{-y},\ u(1,y) = 1 - y.$$

   (a) Construct a leading-order solution for $0 < \epsilon \ll 1$, paying due attention to the location of the layers.

   (b) Repeat the problem if the second boundary condition above is changed to $u(x,1) = 1$.

3. In class we had examined heat transfer on a flat plate in a uniform stream. Now consider heat transfer from a cylinder of radius unity and center at the origin, placed in an otherwise uniform stream. The flow velocity is given by $\boldsymbol{u} = \nabla\phi$, where the potential ϕ is given in polar coordinates as

$$\phi = \left(r + \frac{1}{r}\right)\cos\theta.$$

The temperature T satisfies the PDE

$$\boldsymbol{u} \cdot \nabla T = \epsilon \nabla^2 T, \quad r \geq 1,$$

with boundary conditions

$$T = 1 \text{ on } r = 1, \quad T \to 0 \text{ as } r \to \infty.$$

Find the leading-order solution for $0 < \epsilon \ll 1$. Sketch a graph of the isotherms (lines of constant T). Also, find an expression for $\partial T/\partial r$, the heat flux, from the cylinder surface.

Hint: Look for a similarity solution of the PDE for the inner problem.