Perturbation Methods
Test 1, Spring '16

NOTES.

- Do all three problems.
- Read the statement of each problem carefully prior to attempting a solution. Answers with appropriate supporting work should be written out legibly. In all the problems, asymptotic solution is sought in the limit $\epsilon \to 0+$. Unless otherwise indicated, you may employ the Van Dyke Matching Principle.
- You may consult your class notes and books, but consultation with another individual is not allowed. Please write and sign the following statement on the first page of your answer book:

  I have abided by the ground rules of this test.

PROBLEMS

1. (a) Noting that $\tan x \to \infty$ as $x \to \pi/2-$, obtain three terms of a perturbative solution to
$$\tan x = 1/\epsilon.$$
   (b) Find a 3-term asymptotic expansion of the function $x(\epsilon)$ defined by the equation
$$\epsilon e^x = 1 + \frac{\epsilon}{1+x}.$$
2. Find a 1-term composite expansion to the solution of the initial-value problem
$$\epsilon \frac{d^2u}{dt^2} + 2t\frac{du}{dt} = t^3, \quad u(0) = 0, \quad u'(0) = \frac{1}{\epsilon}.$$
   Hint: Construct a provisional outer solution, then the inner solution, and based upon the behavior of the inner solution, a revised outer solution.
3. Consider the problem
$$(1+\epsilon)x^2y' = \epsilon[(1-\epsilon)xy^2 - (1+\epsilon)x + y^3 + 2\epsilon y^2], \quad y(1) = 1, \quad 0 \le x \le 1.$$
   (a) Compute three terms of an outer expansion.
   (b) Deduce the location and width of the inner region.
   (c) Compute two terms of an inner expansion. (You may find it convenient to compute one term and match before computing the second term.)
   (d) Form a composite expansion.