

# SAR

Tool for estimating precision of smolt-to-adult ratio from PIT-tag data

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## Introduction

Smolt-to-adult ratio (SAR), an estimate of survival rate from the smolt to adult stage of salmon, is used to gauge the viability of salmon runs on the Columbia River and to gauge the effectiveness of the smolt transportation program. The SAR estimate itself is made possible by tagging groups of fish, releasing them, then counting the number of adult recoveries from the release groups. Currently, integrated transponder (PIT) tags are routinely used on juvenile salmonids in the Columbia River basin. Over 15 million salmonids, primarily juvenile spring/summer Chinook and summer steelhead, have been tagged. The tagging has been a fruitful area of applied research, with the PTAGIS web site listing more than 25 peer-reviewed journal articles published pertaining to PIT-tag studies (<http://www.ptagis.org/ptagis>). Examples include estimates of parr-to-smolt survival (Zabel and Achord 2004, Paulsen and Fisher 2005), reach survival from Lower Granite to McNary (Smith et al. 2002), and transport-inriver survival rate comparisons from Fish Passage Center Comparative Survival Studies (<http://www.fpc.org/>).

We develop a simple program that may be used to calculate SAR and its precision as measured by standard error (SE) and coefficient of variation (CV). Such a tool can be used to help design a monitoring program to deliver the desired precision. The parameter under control is the number of juveniles tagged and released. As the number of juveniles tagged and released increases, the number of adult recoveries will increase, and the precision of SAR will also increase. Tagging and releasing too few juveniles could ruin a monitoring program by delivering poor precision of SARs, while tagging too many would be wasteful. The assumptions of the analysis are given in Table 1. The code for

implementing this power analysis (Appendix A) was implemented in R, a system for statistical computation and graphics (Venerables et al. 2010).

Table 1.—Assumptions used in the statistical analysis of SAR.

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A1	The number of smolts (number of trials) is fixed.
A2	The number of adult recoveries is a random variable that follows a binomial distribution.
A3	The probability that a smolt survives and is recovered as adult is a constant for a given release group.
A4	The event that each fish is recovered as an adult is independent of the event that any other individual is recovered as an adult.
A5	The probability that an adult is detected if it is alive (detection rate) is 100%.

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## Methods

To conduct the statistical analysis, a model was formulated and maximum likelihood estimators were derived (Mood et al. 1974). The assumptions detailed in Table 1 imply that the number of adults detected,  $N_d$ , follows a binomial random variable. The maximum likelihood estimator of  $S$  is well-known:

$$\hat{S} = N_d/N_r \quad (1)$$

where  $N_d$  represents the number of adults detected from a total of  $N_r$  smolts released. This estimate is known as the SAR. The variance formula for the estimate of  $S$  is well-known and depends on the unknown value of  $S$ . As an approximation, the  $S$  in the theoretical formula is replaced by its estimate, the SAR. Using this approximation, the variance is a function of the survival rate, the number detected, and the number released (Burnham et al. 1987, p. 115):

$$\text{Var}(\hat{S}) \cong \hat{S}^2(1/N_d - 1/N_r) \quad (2)$$

Note that this is much-simplified from their version because detection efficiency is assumed to be equal to one.

The standard error of the SAR is therefore

$$E(\hat{S}) \cong \hat{S} \sqrt{1/N_d - 1/N_r} \quad (3)$$

and the CV, the standard error divided by the estimate, is equal to

$$CV(\hat{S}) = SE(\hat{S})/\hat{S} \cong \sqrt{1/N_d - 1/N_r}. \quad (4)$$

Notice that the survival rate,  $\hat{S}$ , cancels, leaving the number detected and the number released determining the *sampling* variability of the survival estimate. Since in many cases (e.g., SARs) the survival rate is quite low, the number of fish detected will dominate the sampling variation, and the variation obviously decreases as the number of survivors detected increases.

*Example.* —We take as an example the data of stream-type Chinook salmon from the Columbia River (Muir et al. 2006). Juvenile Chinook salmon were collected at Lower Granite Dam (rkm 695) on the Snake River and either transported downstream of Bonneville Dam (rkm 234) on the Columbia River or released back into the Snake River to continue their migration. We use updated data sets from Muir et al. (2006) in our analysis of SAR, courtesy of Ben Sandford (NOAA, National Marine Fisheries Service, Northwest Fisheries Science Center, Pasco Facility). We used these data to estimate the precision of SARs for fish that originated in a hatchery, and also the SARs for fish that originated in the wild. The data for Snake River stream-type Chinook salmon are given in Table 2. Notice that, for this data set, CV is mainly a function of adult recoveries (Figure 1). This occurs because the adult recoveries are generally very small compared to the number released.

Table 2.—SARs for Snake River stream-type Chinook salmon.

Origin	Year	Transported				In-river migrants			
		Smolts	Adults	SAR	CV	Smolts	Adults	SAR	CV
<b>Hatchery</b>	1997	24965	225	0.0090	0.0664	6496	49	0.0075	0.1423
	1998	46560	810	0.0174	0.0348	20811	152	0.0073	0.0808
	1999	24381	681	0.0279	0.0378	11594	175	0.0151	0.0750
	2000	33157	1025	0.0309	0.0307	16734	264	0.0158	0.0611
	2001	54101	590	0.0109	0.0409	57273	25	0.0004	0.2000
	2002	17291	250	0.0145	0.0628	30471	233	0.0076	0.0653
	2003	56937	376	0.0066	0.0514	21919	80	0.0036	0.1116
	2004	52778	190	0.0036	0.0724	46067	74	0.0016	0.1162
	2005	55950	230	0.0041	0.0658	46807	62	0.0013	0.1269
	2006	29111	281	0.0097	0.0594	36912	167	0.0045	0.0772
<b>Wild</b>	1998	5465	34	0.0062	0.1710	8280	53	0.0064	0.1369
	1999	7956	172	0.0216	0.0754	10451	136	0.0130	0.0852
	2002	3804	51	0.0134	0.1391	14134	106	0.0075	0.0968
	2003	5981	18	0.0030	0.2353	22695	26	0.0011	0.1960
	2004	10178	39	0.0038	0.1598	8328	8	0.0010	0.3534
	2005	11135	29	0.0026	0.1855	5498	10	0.0018	0.3159
	2006	19557	127	0.0065	0.0884	9816	64	0.0065	0.1246

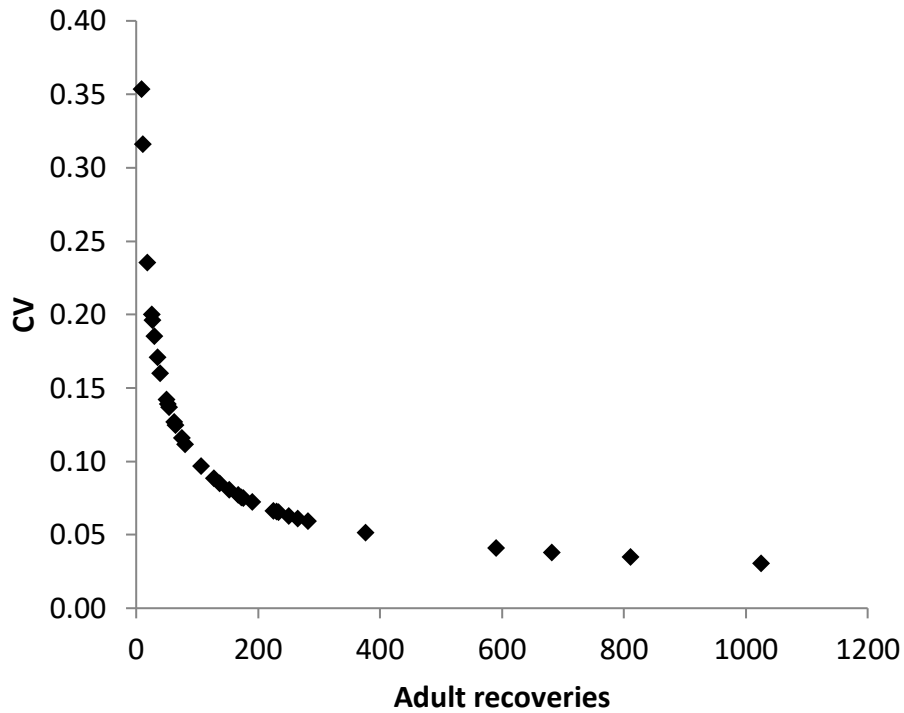


Figure 1.—The CV of SAR is strongly related to the number of adult recoveries in the Snake River stream-type Chinook data set. Depicted here are the CVs from the 34 SAR estimates in Table 2.

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## References

- Burnham, K. P., Anderson, D.R., White, G. C., Brownie, C., Pollock, K. H. 1987. Design and analysis of fish survival experiments based on release-recapture. American Fisheries Society, Bethesda, MD.
- Mood, A.M, Graybill, F.A., and D.C. Boes. 1974. Introduction to the theory of statistics, Third Edition. McGraw-Hill, New York, New York.
- Muir, W.D., D.M. Marsh, B.P. Sandford, S.G. Smith, and J.G. Williams. 2006. Post-hydropower system delayed mortality of transported Snake River stream-type chinook salmon: unraveling the mystery. Transactions of the American Fisheries Society 135:1523-1534.
- Paulsen, C.M. and Fisher, T.R. 2005. Do habitat actions increase juvenile survival? An information-theoretic approach applied to endangered Snake River chinook salmon. Transactions of the American Fisheries Society 143: 68-85
- Smith, S.G., Muir, W.D., Williams, J.G., and J.R. Skalski. 2002. Factors associated with travel time and survival of migrant yearling Chinook salmon and steelhead in lower Snake river. North American Journal of Fisheries Management 22: 385-405.
- Venerables, W.N., Smith, D.M., and R Development Core Team. 2010. An Introduction to R. Notes on R: A Programming Environment for Data Analysis and Graphics Version 2.11.1 (2010-05-31). <http://www.r-project.org/>
- Zabel, R.W. and S. Achord. 2004. Relating size of juveniles to survival within and among populations of Chinook salmon. Ecology 85: 795-806.

## Appendix A. R code used to calculate precision of the SAR estimate

```
# Program to estimate standard error and CV
# of a smolt-to-adult ratio (SAR). Adult recoveries
# are assumed to be binomially distributed
#CODE: Richard A. Hinrichsen, Ph.D.
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#input parameters
#Nr number of juveniles released
#Nd number of adults detected
SAR<-function(Nr,Nd){
  if(Nd>Nr){
    warning("The number of adults detected (Nd) must be no greater than the number of
juveniles released (Nr)")
    return(NULL)}
  sar<-Nd/Nr
  se<-sar*sqrt(1/Nd-1/Nr)
  cv<-sqrt(1/Nd-1/Nr)
  return(list(Nr=Nr,Nd=Nd,sar=sar,se=se,cv=cv))
}
#outputs
#sar survival rate estimate (SAR)
#se standard error
#cv coefficient of variation
```