

# Dependent evidence combination based on decision-making trial and evaluation laboratory method

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## Funding information

National Natural Science Foundation of China, Grant/Award Numbers: 61503237, 61573290

## Abstract

Dempster-Shafer is widely used to address the problems of uncertainty. One assumption mentioned in this theory is that the distribution of information should be independent. In practice, the requirement cannot be fulfilled. One of the efficient methods to deal with dependent evidence is to calculate the correlation discounting. However, existing coefficient can only be applied to show the direct relation between evidence A and B but do not take the indirect relationship into consideration. To address this issue, in this paper, a new method to combine dependent evidence based on decision-making trial and evaluation laboratory is presented, not only considering the relation between evidence A and B and the relation between evidence B and C, but also considering the transitive influence between evidence A and C. Finally, the experiments on some benchmark data sets are illustrated to show the efficiency of the proposed method.

## KEYWORDS

D-S evidence theory, DEMATEL, dependent evidence combination

## 1 | INTRODUCTION

Dempster-Shafer (D-S) evidence theory is one of the most efficient math tools to deal uncertainty.<sup>1,2</sup> It is widely used model uncertainty in real engineering systems, such as risk and reliability analysis<sup>3–8</sup> and decision-making under uncertainty.<sup>9–12</sup> Though the fusion result in some highly conflicting situation is not convinced,<sup>13</sup> evidence theory provides the statistical evidence, which is also a tool to measure uncertainty.<sup>14,15</sup>

One of the most important reasons why D-S evidence theory is used to handle uncertainty and imprecision<sup>16,17</sup> is that the utilization of D-S theory avoids the necessity of assigning prior probabilities (which would be extremely difficult to estimate) and provides more intuitive tools for managing uncertain knowledge.<sup>18–20</sup> Belief structure is efficient to model different kinds of uncertainty.<sup>21–24</sup> Also, the Dempster combination rule which assumes the independence between the evidence was proposed to combine the evidence.<sup>25</sup> However, the assumption of the independence is not convincing, since the associations of the evidence must be taken into consideration.<sup>26–28</sup> To date, how to deal with the associations of the evidence in decision-making is still an open issue.<sup>29</sup> Yager<sup>30</sup> presented a method considering the dependence degree as a discounting factor. Then the weighted evidence combination was proposed.<sup>31</sup> Besides, Su et al<sup>32</sup> proposed a model based on the Pearson correlation coefficient to measure the dependence of the evidence. Some researchers also identified independence in Bayesian networks whereas others's idea is to measure the degree of conflict and define the distance<sup>33</sup> or the dissimilarity between evidence<sup>34</sup> considering combining the evidence to address conflict.<sup>35–38</sup> Recently, a method based on shearman coefficient to combine dependent is presented.<sup>39</sup> However, the indirect relationship among evidence is neglected to some degree.

To address this issue, a new method based on decision-making trial and evaluation laboratory (DEMATEL) is proposed to deal with dependent evidence combination. The proposed method is motivated by two points. One is the application of DEMATEL method.<sup>40,41</sup> DEMATEL method is an efficient way to develop supplier selection criteria<sup>42</sup> and to make management strategies.<sup>43–45</sup> This method is also combined with analytic hierarchy process (AHP) or analytic network process to make selection using the multiple-criteria decision analysis technique,<sup>46–48</sup> to make evaluation<sup>49,50</sup> and to make information security risk control assessment.<sup>51</sup> Besides, this method is widely applied to make decision for service quality. The other is that the dependence of evidence can be measured by correlation coefficient mentioned above.<sup>39,52</sup> Therefore, in this paper, a initial discounting factor is calculated based on the correlation coefficients to measure the relations between evidence A and B. Then, the DEMATEL method is finally used to measure the indirect correlation between evidence apart from direct correlation between evidence, which means that not only the relation between A and B but also B and C is considered, but also the transitive influence between A and C also is taken into consideration.

The paper is organized as follows. Section 2 is the brief introduction of the preliminaries. Section 3 presents the proposed method in detail based on the Pearson correlation coefficient and the DEMATEL method, which are used to calculate the discounting factor to modify the Dempster combination rule. The experiment's data and the analysis of the result are presented in Section 4. The paper ends with a short conclusion in Section 5.

## 2 | PRELIMINARIES

Many math tools are presented to model uncertainty, such as  $Z$  numbers,<sup>53–55</sup>  $D$  numbers,<sup>56–61</sup> and entropy model.<sup>62–67</sup> Among then, evidence theory is widely used due to its efficiency to deal with uncertainty. In this section, the traditional D-S evidence,<sup>1</sup> the Pearson correlation coefficient,<sup>52</sup> the Spearman correlation coefficient,<sup>68</sup> and the DEMATEL method are introduced.

In the D-S evidence theory,  $\Theta = (A_1, A_2, A_3, \dots, A_N)$  is an identification framework.  $A_i (1 \leq i \leq N)$  represents the identification of a focal element in the framework.  $N$  is to identify the number of elements in the framework.

Basic belief assignment (BBA), a mass function, is one of the most basic and important definitions of D-S evidence theory.<sup>69,70</sup>  $\Theta$  is known to identify the framework. There are  $2^\Theta$  subsets of the framework  $\Theta$ . Each subset's mapping is BBA. BBA has two features:  $m(\emptyset) = 0$  and  $\sum_{A \subseteq \Theta} m(A) = 1$ .

Assuming the identification framework is  $\Theta$ ,  $m_1, m_2, m_3, \dots, m_n$  are  $N$  BBAs which are all independent. According to the Dempster combination rule, the result is presented as follows<sup>71</sup>:

$$m = m_1 \oplus m_2 \oplus m_3 \oplus \dots \oplus m_n, \alpha \in [0, 1], \quad (1)$$

where

$$m(A) = \begin{cases} 0 & \text{if } A = \emptyset, \\ K^{-1} \sum_{\bigcap A_j = A} \prod_{i=1}^n m_i(A_j) & \text{otherwise,} \end{cases} \quad (2)$$

where  $K$  is the normalization factor, defined as follows:

$$K = 1 - \sum_{\bigcap A_j = \emptyset} \prod_{i=1}^n m_i(A_j). \quad (3)$$

Given the discounting factor  $\alpha$  ( $\alpha \in [0, 1]$ ),  $m$  is one of the BBAs on the identification frame  $\Theta$ .  ${}^\alpha m$  is defined as a discounted mass function, shown as follows:

$${}^\alpha m(A) = \begin{cases} \alpha m(A) & \text{if } A \subset \Theta, A \neq \Theta, \\ 1 - \alpha + \alpha m(\Theta) & \text{otherwise.} \end{cases} \quad (4)$$

It should be mentioned that the classical evidence theory has many assumptions, for example, in close world while it may not be coincide with the really.<sup>72</sup> Many correlation coefficients are presented to model the relationship between random variables.<sup>73</sup> The Pearson correlation coefficient is a linear correlation coefficient, which is used to reflect the linear correlation of two normal continuous variables. Assume  $X$  and  $Y$  are two samples: the sample  $X$  contains  $n$  sample observations  $(x_1, x_2, x_3, \dots, x_n)$  and sample  $Y$  contains  $n$  sample observations  $(y_1, y_2, y_3, \dots, y_n)$ . Then the Pearson correlation coefficient is defined as follows<sup>52</sup>:

$$r = \frac{(N \sum x_i y_i - \sum x_i \sum y_i)}{\sqrt{N \sum x_i^2 - (\sum x_i)^2} \sqrt{N \sum y_i^2 - (\sum y_i)^2}}. \quad (5)$$

The Spearman correlation coefficient is also called the rank correlation coefficient. It is a nonparametric parameter and does not rely on the distribution of the samples. Therefore, the rank correlation coefficient can be used to describe the correlation between variables when the sample variables do not strictly follow the normal distribution. Similarly, assume  $X$  and  $Y$  are two samples. The sample  $X$  contains  $n$  sample observations  $(x_1, x_2, x_3, \dots, x_n)$  and the sample  $Y$  contains  $n$  sample observations  $(y_1, y_2, y_3, \dots, y_n)$ . The coordinates of  $X, Y$  are in accordance with the order from large to small (or from small to large).  $x'_i, y'_i$  are used to record the position of

$x_i, y_i$  after the arrangement. Set  $d_i = x'_i - y'_i$ , then the Spearman correlation coefficient is defined as follows<sup>68</sup>:

$$r_s = 1 - 6 \sum_{i=1}^n \frac{d_i^2}{(n(n^2 - 1))}. \quad (6)$$

The DEMATEL method<sup>41</sup>: the real world is very complex and needs efficient math tools to address the complexity.<sup>74–77</sup> Many methods such as bioinspired model,<sup>78</sup> complex network,<sup>79–81</sup> average operator,<sup>82,83</sup> and AHP.<sup>84,85</sup> DEMATEL method was originally developed by Battelle Memorial Institute of Geneva Research Center. DEMATEL method can help managers measure the importance and causal relationship of system components through assessing their direct and indirect relations and constructing a map. The procedures of DEMATEL method consist of five steps.<sup>47</sup>

*Step 1.* Establish the measurement scale and extract the direct relation matrix.

Establish the measurement scale for the causal relationship and pairwise comparison among influential criterions.

*Step 2.* Normalize the direct relation matrix.

The normalized direct relations of factors are a mapping from  $d_{i,j}$  to  $[0, 1]$ . For the framework of  $n$  influential characteristics  $\{F_1, F_2, F_3, \dots, F_n\}$ , normalized matrix  $N$  of direct relation matrix  $D = [d]_{i,j} (i, j = 1, 2, 3, \dots)$  is obtained by

$$N = \frac{D}{\max\left(\sum_{j=1}^n d_{i,j}, \sum_{i=1}^n d_{i,j}\right)}. \quad (7)$$

*Step 3.* Calculate total relation matrix.

Due to the characteristics of normalized direct relation matrix  $N$ , total relation matrix which contains direct and indirect relations among factors can be derived from matrix  $T$ . Assume  $N = [n_{i,j}]_{n \times n} (i, j = 1, 2, 3, \dots, n)$  is the normalized direct as

$$T = \lim_{k \rightarrow \infty} (N + N^2 + N^3 + \dots + N^k) = N(I - N)^{-1}, \quad (8)$$

where  $I$  is the identity matrix.

*Step 4.* Calculate the prominence of each criterion.

$\beta$  denotes the prominence of each criterion, showing the impact of  $i$ th influential criterion and its degree of being impacted:

$$\beta = R_i + C_i, \quad (9)$$

where the sum of each row  $R_i$  ( $i = 1, 2, \dots, n$ ) and the sum of each column  $C_i$  ( $i = 1, 2, \dots, n$ ) of the total relation matrix  $T$ .

### 3 | PROPOSED METHOD

To take the indirect relation between the evidence into consideration, a new method, a total relation in DEMATEL method calculated by the Pearson correlation coefficient and the Spearman correlation coefficient is proposed in this section. The total relation matrix not only contains the information of nonparametric correlation between the evidence but also the indirect correlation between the evidence.

Assume the collected evidence  $S_1, S_2, S_3, S_4, \dots, S_N$  are obtained by the sensors. Given two evidence  $S_i, S_j$ , the Pearson correlation coefficient is  $r_{S_i, S_j}^P$  ( $i, j = 1, 2, 3, \dots, N$ ), and the Spearman correlation coefficient is  $r_{S_i, S_j}^S$  ( $i, j = 1, 2, 3, \dots, N$ ). In this paper, we ignore the sign of the correlation coefficient. The main reason is that both positive and negative correlations mean the dependent degree of evidence, to some degree. We use  $d_{S_i, S_j}^P$  and  $d_{S_i, S_j}^S$  to model as the dependent degree of evidence  $S_i, S_j$ .

In this situation, set  $(S_1, S_1), (S_1, S_2), (S_1, S_3), (S_1, S_4), \dots, (S_N, S_N)$  as  $N^2$  pairs of sources of evidence, and then calculate their Pearson correlation coefficient and Spearman correlation coefficient.

Given  $d_{S_i, S_j}^P$ , the corresponding Pearson correlation coefficient matrix is defined as follows:

$$\Omega = \begin{bmatrix} d_{S_1, S_1}^P & \cdots & d_{S_1, S_N}^P \\ \vdots & \ddots & \vdots \\ d_{S_N, S_1}^P & \cdots & d_{S_N, S_N}^P \end{bmatrix}. \quad (10)$$

Given  $d_{S_i, S_j}^S$ , the corresponding Spearman correlation coefficient matrix is defined as follows:

$$\Psi = \begin{bmatrix} d_{S_1, S_1}^S & \cdots & d_{S_1, S_N}^S \\ \vdots & \ddots & \vdots \\ d_{S_N, S_1}^S & \cdots & d_{S_N, S_N}^S \end{bmatrix}. \quad (11)$$

In this paper, the proposed method takes the Pearson correlation coefficient and the Spearman correlation coefficient as the measurement scale to generate the total relation matrix in DEMATEL method. In this section, assume the nonparametric dependent degree has the same influence as that of the parametric dependent degree between the evidence, then the direct relation matrix, which will be used to generate the total relation matrix in DEMATEL method, is the arithmetic mean of the Pearson correlation coefficient matrix and the Spearman correlation coefficient matrix.

Given  $\Omega$  and  $\Psi$ , the corresponding direct relation matrix is defined as follows:

$$M = \begin{bmatrix} d_{S_1, S_1}^M & \cdots & d_{S_1, S_N}^M \\ \vdots & \ddots & \vdots \\ d_{S_N, S_1}^M & \cdots & d_{S_N, S_N}^M \end{bmatrix}, \quad (12)$$

where

$$d_{S_i, S_j}^M = \frac{d_{S_i, S_j}^P + d_{S_i, S_j}^S}{2}. \quad (13)$$

The direct relation is calculated as above. According to the procedures of the DEMATEL method, the direct relation matrix  $M$  is normalized in step 2 in Equation (7). And the total relation matrix is calculated in step 3 in Equation (8). They are both defined as follows.

Given the direct relation matrix  $M$ , the corresponding normalized matrix  $M1$  is defined as follows:

$$M1 = \begin{bmatrix} d_{S_1, S_1}^{M1} & \cdots & d_{S_1, S_N}^{M1} \\ \vdots & \ddots & \vdots \\ d_{S_N, S_1}^{M1} & \cdots & d_{S_N, S_N}^{M1} \end{bmatrix}, \quad (14)$$

where

$$d_{S_i, S_j}^{M1} = \frac{d_{S_i, S_j}^M}{\max(\sum_{j=1}^n d_{S_i, S_j}^M, \sum_{i=1}^n d_{S_i, S_j}^M)}. \quad (15)$$

Given the normalized matrix  $M1$ , the corresponding total relation matrix  $M2$  is defined as follows:

$$M2 = \begin{bmatrix} d_{S_1, S_1}^{M2} & \cdots & d_{S_1, S_N}^{M2} \\ \vdots & \ddots & \vdots \\ d_{S_N, S_1}^{M2} & \cdots & d_{S_N, S_N}^{M2} \end{bmatrix}, \quad (16)$$

where

$$\begin{aligned} d_{S_N, S_1}^{M2} &= \lim_{k \rightarrow \infty} (d_{S_i, S_j}^{M1} + \cdots + d_{S_i, S_j}^{M1 k}) \\ &= d_{S_i, S_j}^{M1} (1 - d_{S_i, S_j}^{M1})^{-1}. \end{aligned} \quad (17)$$

As it is mentioned in step 4 of the procedures of the DEMATEL method, the prominence each criterion is calculated based on the total relation matrix in Equation (9):

$$\beta = R_i + C_j, \quad (18)$$

where

$$R_i = \sum_j^n d_{S_i, S_j}^{M2} \quad (i = 1, 2, \dots, n), \quad (19)$$

$$C_j = \sum_i^n d_{S_i, S_j}^{M2} \quad (j = 1, 2, \dots, n). \quad (20)$$

In the proposed method,  $\beta$  in Equation (18) is used to calculate the discounting factor, which is defined as follows:

$$\rho = \frac{1}{\beta}. \quad (21)$$

$\rho$  is taken into consideration as discounting factor in the proposed method,  $\alpha = \rho$  in Equation (4).

The algorithm of the proposed method applied to data set is as follows:

*Step 1.* Generate the BBAs.

*Step 2.* Use the BBAs to calculate the Pearson correlation coefficient matrix and the Spearman correlation coefficient matrix.

*Step 3.* Use the Pearson and Spearman correlation coefficient matrices to obtain the direct relation matrix.

*Step 4.* Obtain the normalized matrix based on the direct relation matrix.

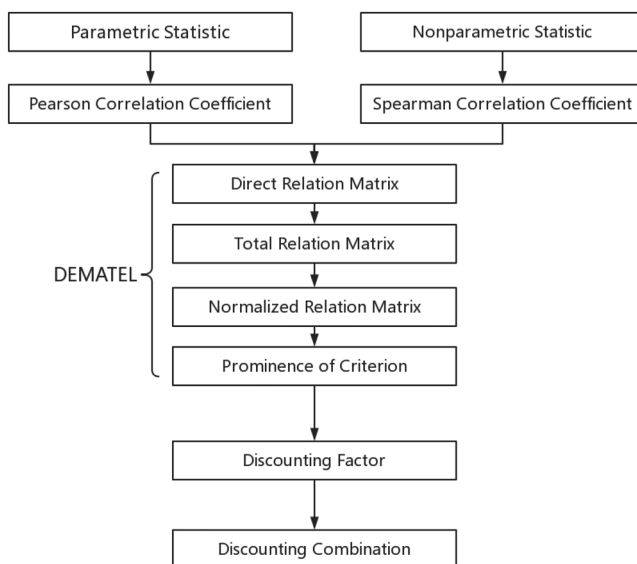
*Step 5.* Obtain the total relation matrix based on the normalized matrix.

*Step 6.* Obtain the prominence of each criterion based on the total relation matrix.

*Step 7.* Use the prominence to generate the discounting factor for the evidence combination.

*Step 8.* Use the pignistic probability transformation formula to convert it into probability.<sup>86</sup>

The flow chart is illustrated in Figure 1.



**FIGURE 1** The flow chart of the proposed method. DEMATEL, decision-making trial and evaluation laboratory

## 4 | EXPERIMENT

### 4.1 | The introduction of data set

*Iris* data set (<https://archive.ics.uci.edu/ml/datasets/Iris>) is used to illustrate the efficiency of the proposed method. The *iris* data set contains three categories: Setosa, Versicolor, and Virginica. Each category has four attributes: SL, SW, PL, and PW. Take these four attributes as the source of evidence to generate BBAs for making decision.

And *wine* data set is used to illustrate the efficiency of the proposed method (<https://archive.ics.uci.edu/ml/datasets/wine>), too. The *wine* data set contains three categories: wine1, wine2, and wine3. And wine1, wine2, and wine3 all have 13 properties, namely, alcohol, malic acid, ash, alkalinity of ash, magnesium, total phenols, flavanoids, nonflavanoid phenols, proanthocyanidins, color intensity, hue, OD280/OD315 of diluted wines, and proline.

*Glass* data set is also selected to illustrate the efficiency of the proposed method (<https://archive.ics.uci.edu/ml/datasets/glass>). Three kinds of glass are chosen, building windows float processed, building windows nonfloat processed, and vehicle windows float processed. Seven features are selected to classify there kinds of glass, naming RI (refractive index), Na, Mg, Al, Si, K, and Ca.

### 4.2 | The application of the proposed method

Select samples from three categories randomly for training, and then the rest for testing. Steps of the experiment are presented as follows:

- Step 1. Generate the BBAs of the *iris* data set.
- Step 2. Use the BBAs to calculate the Pearson correlation coefficient matrix of the *iris* data set shown in Table 1 and the Spearman correlation coefficient matrix of the *iris* data set shown in Table 2.
- Step 3. Use the Pearson and Spearman correlation coefficient matrices to obtain the direct relation matrix of the *iris* data set shown in Table 3.
- Step 4. Obtain the normalized matrix based on the direct relation matrix of the *iris* data set shown in Table 4.
- Step 5. Obtain the total relation matrix based on the normalized matrix of the *iris* data set shown in Table 5.
- Step 6. Obtain the prominence of each criterion based on the total relation matrix of the *iris* data set shown in Tables 6 and 7.

**TABLE 1** The Pearson correlation coefficient of the *iris* data set

	SL	SW	PL	PW
SL	1.0000	0.1473	0.8804	0.8367
SW	0.1473	1.0000	0.2845	0.2760
PL	0.8804	0.2845	1.0000	0.9376
PW	0.8367	0.2760	0.9376	1.0000



**TABLE 2** The Spearman correlation coefficient of the iris data set

	SL	SW	PL	PW
SL	1.0000	0.1473	0.8804	0.8367
SW	0.1473	1.0000	0.2845	0.2760
PL	0.8804	0.2845	1.0000	0.9376
PW	0.8367	0.2760	0.9376	1.0000

**TABLE 3** The direct relation matrix of the iris data set

	SL	SW	PL	PW
SL	1.0000	0.1473	0.8804	0.8367
SW	0.1473	1.0000	0.2845	0.2760
PL	0.8804	0.2845	1.0000	0.9376
PW	0.8367	0.2760	0.9376	1.0000

**TABLE 4** The normalized matrix of the iris data set

	SL	SW	PL	PW
SL	0.3223	0.0475	0.2838	0.2697
SW	0.0475	0.3223	0.0917	0.0890
PL	0.2838	0.0917	0.3223	0.3022
PW	0.2697	0.0890	0.3022	0.3223

**TABLE 5** The total relation matrix of the iris data set

	SL	SW	PL	PW
SL	3.5829	1.3054	3.7265	3.6570
SW	1.3054	0.9486	1.4430	1.4188
PL	3.7265	1.4430	3.9644	3.8863
PW	3.6570	1.4188	3.8863	3.8503

**TABLE 6** The row sum of the prominence of criterion of the iris data set

R	12.2717	5.1158	13.0202	12.8123
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**TABLE 7** The column sum of the prominence of criterion of the iris data set

C
12.2717
5.1158
13.0202
12.8123

**TABLE 8** The DEMATEL discounting factor of the iris data set

SL	SW	PL	PW
0.0407	0.0977	0.0384	0.0390

Abbreviation: DEMATEL, decision-making trial and evaluation laboratory.

*Step 7.* Use the prominence to generate the discounting factor for the evidence combination of the *iris* data set shown in Table 8.

*Step 8.* Use the pignistic probability transformation formula to convert it into probability.<sup>71</sup>

And the procedures of the proposed method are also applied to the *wine* data set, whose results are presented in Tables 10 to 13 and *glass* data set, whose results are presented in Tables 14 to 17.

### 4.3 | The analysis of the experiments

Experiment 1 (iris dataset):

- *Group 1:* The DEMATEL discounting method based on the DEMATEL discounting factor shown in Table 8.
- *Group 2:* The complex discounting method based on the complex discounting factor shown in Table 9.

Experiment 2 (wine dataset):

- *Group 1:* The DEMATEL discounting method based on the DEMATEL discounting factor shown in Table 12.
- *Group 2:* The complex discounting method based on the complex discounting factor shown in Table 13.

Experiment 3 (glass dataset):

- *Group 1:* The DEMATEL discounting method based on the DEMATEL discounting factor shown in Table 16.
- *Group 2:* The complex discounting method based on the complex discounting factor shown in Table 17.

**TABLE 9** The complex discounting factor of the iris data set

SL	SW	PL	PW
0.4002	0.8386	0.3650	0.3788

**TABLE 10** The row sum of the prominence of criterion of the wine data set

	<i>R</i>		<i>R</i>
Alcohol	2.7953	Nonflavanoid phenols	2.8656
Malic acid	2.9450	Proanthocyanidins	3.5009
Ash	2.0522	Color intensity	2.3533
Alkalinity of ash	2.9788	Hue	3.4721
Magnesium	2.0898	OD280/OD315	4.0798
Total phenols	4.2521	Proline	3.2218
Flavanoids	4.5584		

**TABLE 11** The column sum of the prominence of criterion of the wine data set

	<i>C</i>		<i>C</i>
Alcohol	2.7953	Nonflavanoid phenols	2.8656
Malic acid	2.9450	Proanthocyanidins	3.5009
Ash	2.0522	Color intensity	2.3533
Alkalinity of ash	2.9788	Hue	3.4721
Magnesium	2.0898	OD280/OD315	4.0798
Total phenols	4.2521	Proline	3.2218
Flavanoids	4.5584		

**TABLE 12** The DEMATEL discounting factor of the wine data set

Alcohol	0.1789	Nonflavanoid phenols	0.1745
Malic acid	0.1698	Proanthocyanidins	0.1428
Ash	0.2436	Color intensity	0.2125
Alkalinity of ash	0.1679	Hue	0.1440
Magnesium	0.2393	OD280/OD315	0.1226
Total phenols	0.1176	Proline	0.1552
Flavanoids	0.1097		

Abbreviation: DEMATEL, decision-making trial and evaluation laboratory.

**TABLE 13** The complex discounting factor of the wine data set

Alcohol	0.4553	Nonflavanoid phenols	0.4777
Malic acid	0.4367	Proanthocyanidins	0.3516
Ash	0.6235	Color intensity	0.4864
Alkalinity of ash	0.4409	Hue	0.3202
Magnesium	0.6155	OD280/OD315	0.2789
Total phenols	0.2568	Proline	0.3986
Flavanoids	0.2287		

**TABLE 14** The row sum of the prominence of criterion of the glass data set

	<b>R</b>
RI (refractive index)	7.8282
Na	5.0578
Mg	4.5434
Al	6.2056
Si	6.12882
K	7.3523
Ca	7.4299

**TABLE 15** The column sum of the prominence of criterion of the glass data set

	<b>R</b>
RI (refractive index)	7.8282
Na	5.0578
Mg	4.5434
Al	6.2056
Si	6.12882
K	7.3523
Ca	7.4299

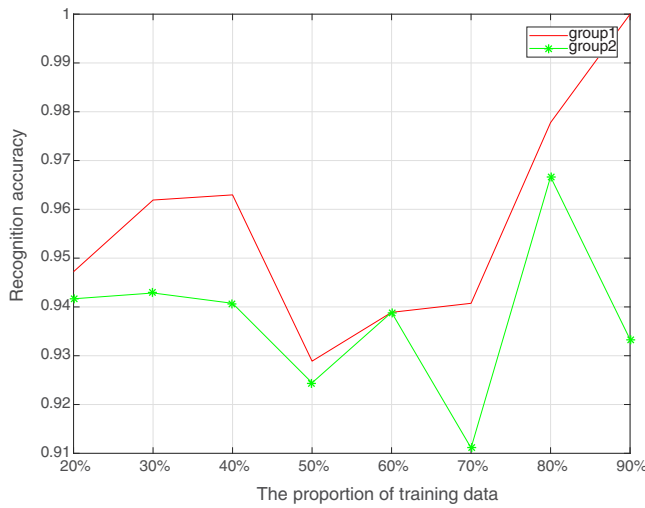
**TABLE 16** The DEMATEL discounting factor of the glass data set

RI (refractive index)	0.0638
Na	0.0988
Mg	0.1100
Al	0.0805
Si	0.0815
K	0.0680
Ca	0.0672

Abbreviation: DEMATEL, decision-making trial and evaluation laboratory.

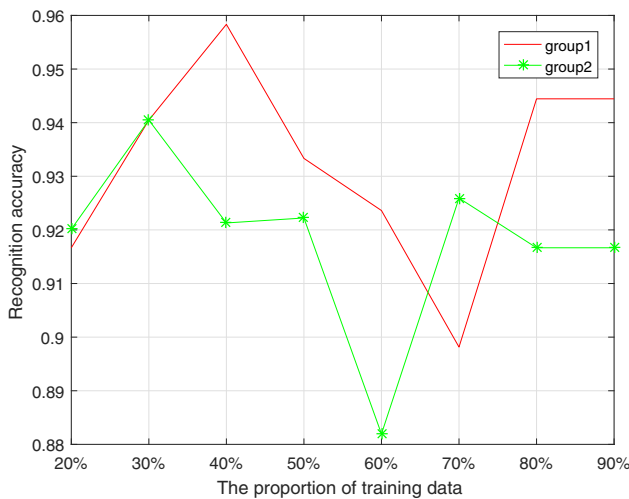
**TABLE 17** The complex discounting factor of the glass data set

RI (refractive index)	0.2454
Na	0.3395
Mg	0.3757
Al	0.3024
Si	0.3011
K	0.2570
Ca	0.2559

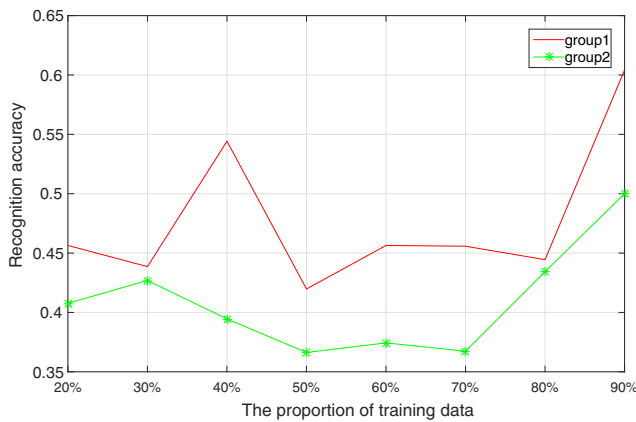


**FIGURE 2** The comparison between the complex discounting method and the proposed method of the iris data set [Color figure can be viewed at wileyonlinelibrary.com]

The result of comparison of the *iris* data set is shown in Figure 2. The average of accuracy of the group 1 is above 0.95, but the average of accuracy of the group 2 is below 0.94. And the result of comparison of the *wine* data set is shown in Figure 3. The average of accuracy of the group 1 is above 0.94, but the average of accuracy of the group 2 is below 0.92. Obviously, the correct rate of recognition in group 1 is always higher than that in group 2. Besides, the result of comparison of the *glass* data set also show the efficiency of the proposed method in Figure 4. The group 2 almost is lower than which means it cannot be used in the classification work in real world. But the group 2 is always above 0.5, which means it can be used in the application of classification. The main reason is that the DEMATEL method not only considers the relation between A and B and the relation between B and C, but also the transitive influence



**FIGURE 3** The comparison between the complex discounting method and the proposed method of the wine data set [Color figure can be viewed at wileyonlinelibrary.com]



**FIGURE 4** The comparison between the complex discounting method and the proposed method of the glass data set [Color figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

between A and C is taken into account. Hence, more dependence among these evidence is discounted to increase the correct rate of recognition.

## 5 | CONCLUSION

Although the existing method based on statistical correlation coefficient is proven efficient in the process of dependent evidence combination, it is not feasible to deal with transitive influence among dependent evidence. In this paper, to address this issue, a new method based on the DEMATEL method is proposed to generate the discounting factor. The result shows that the new proposed method has higher accuracy of recognition. The merit of this proposed method is more dependence among these evidence is discounted to increase the correct rate of recognition. It not only deals with the relation between evidence A and B and the relation between evidence B and C, but also handles with the transitive influence between A and C.

## ACKNOWLEDGMENT

The work is partially supported by National Natural Science Foundation of China (grant nos. 61573290 and 61503237).

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**How to cite this article:** Xu H, Deng Y. Dependent evidence combination based on decision-making trial and evaluation laboratory method. *Int J Intell Syst*. 2019;34: 1555-1571. <https://doi.org/10.1002/int.22107>