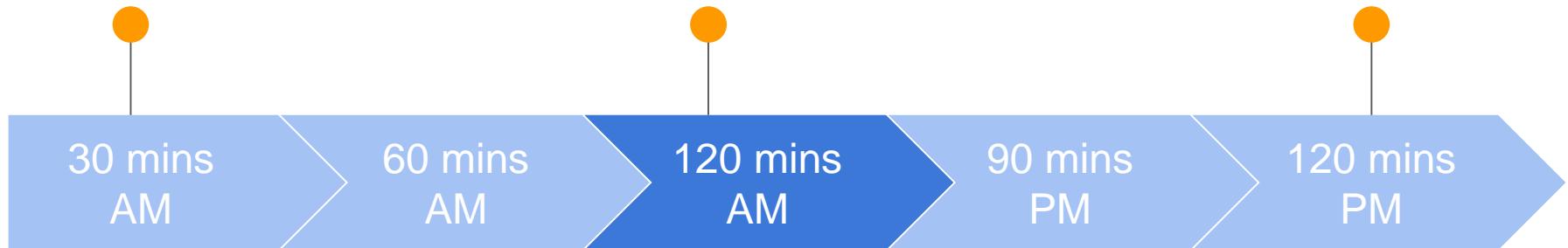


Motivations

## Network Embedding

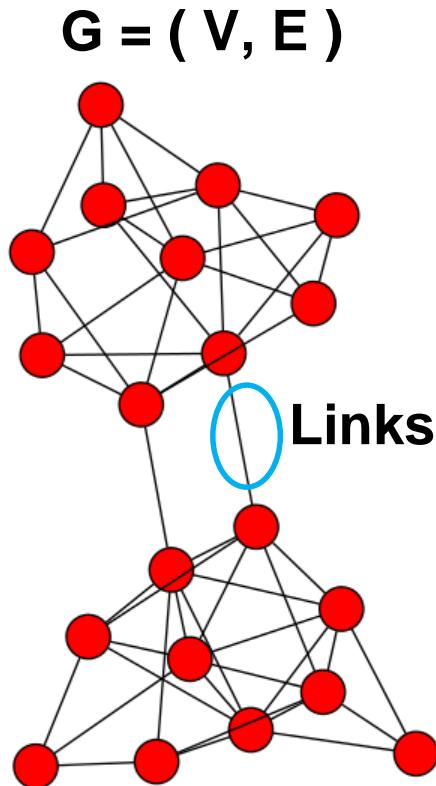
Graph Neural  
Networks



Feature  
Selection

Attributed Network  
Embedding

# The problems of network representation



Iterative &  
Combinatorial

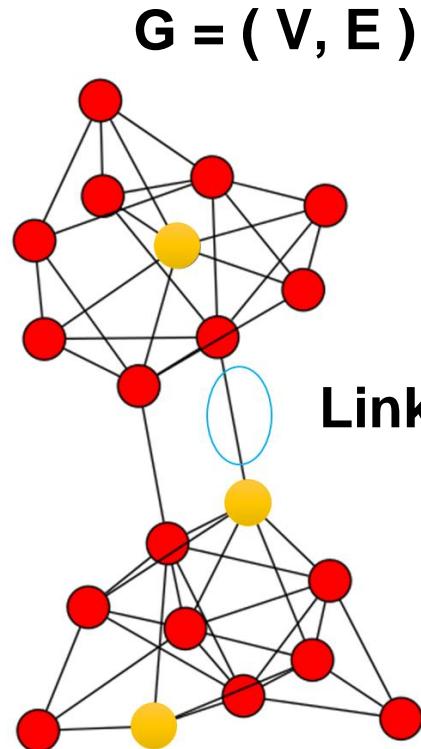
Complexity

Computability

Coupling

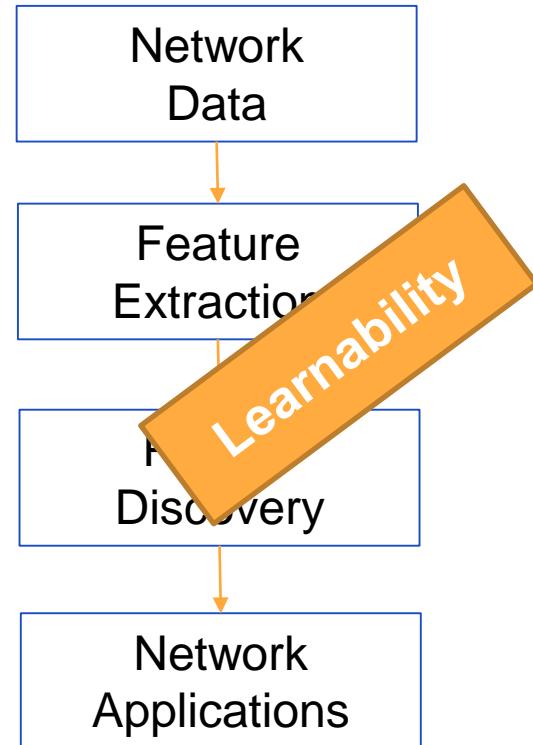
Parallelizability

# The problems of network representation



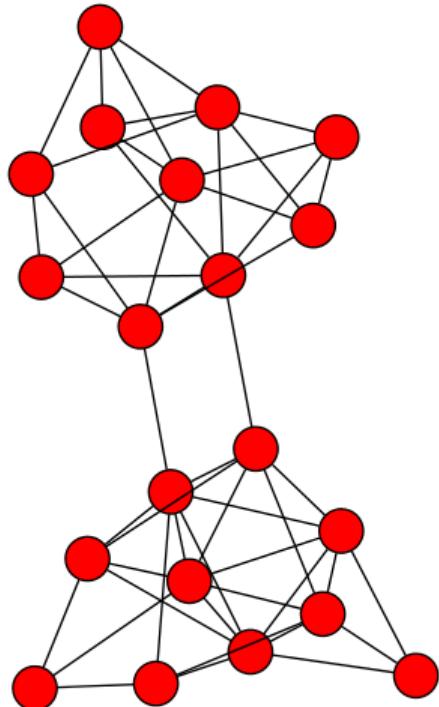
Inapplicability of  
ML methods

Pipeline for network analysis



# Revisit network representation

$$G = (V, E)$$

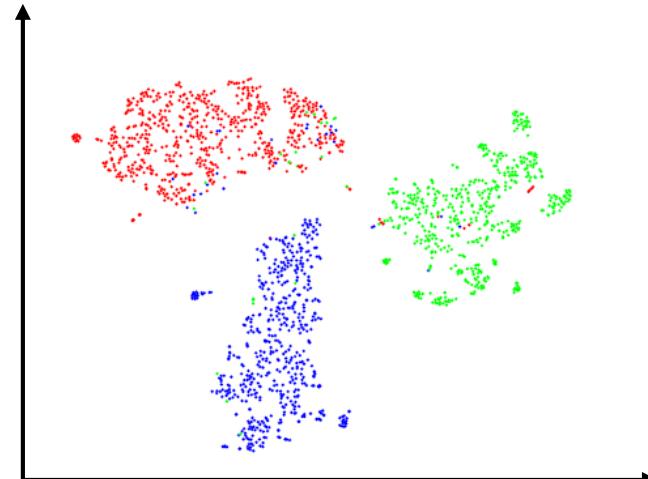


$$G = (V)$$

Vector Space

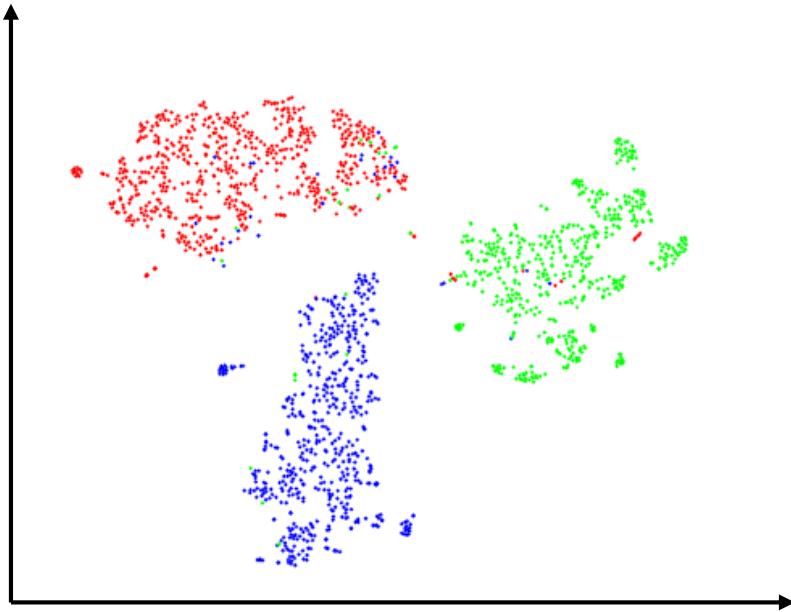
generate

embed



- Easy to parallel
- Can apply classical ML methods

# The ultimate goal

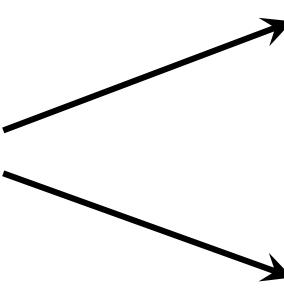
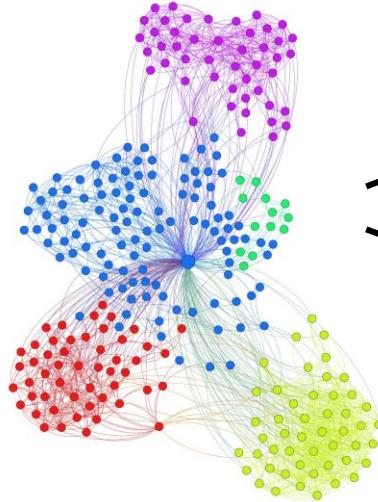
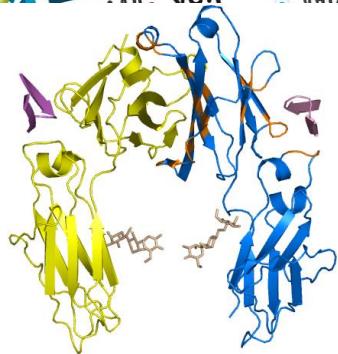


## Network Inference

- Node importance
- Community detection
- Network distance
- Link prediction
- Node classification
- Network evolution
- ...

*in Vector Space*

# The information encoded in networks



Topological  
Information



Semantic  
Information



Nodes Attributes

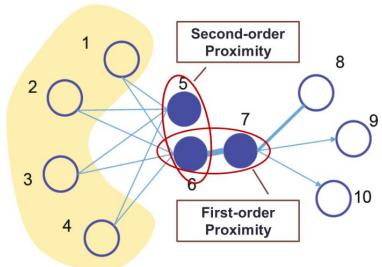
Topology Vectorization is the key problem.

# Network Embedding

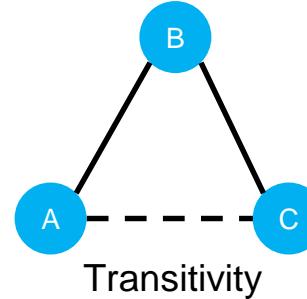
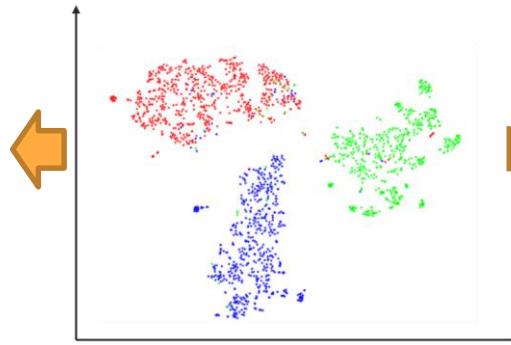
**Goal** Support network inference in vector space



Reflect network structure



Maintain network properties



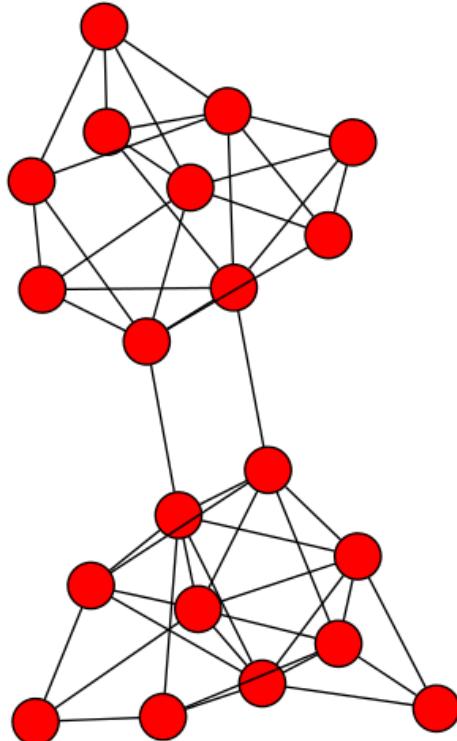
# Outline

- **Structure-preserved network embedding**
- **Property-preserved network embedding**
- **Dynamic network embedding**
- **Robustness, Explainability and Applicability**
- **Network embedding for biomedical applications**

# Outline

- **Structure-preserved network embedding**
- **Property-preserved network embedding**
- **Dynamic network embedding**
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- **Network embedding for biomedical applications**

# Network Structures



**Nodes & Links**



**Pair-wise Proximity**



**Community Structures**



**Hyper Edges**

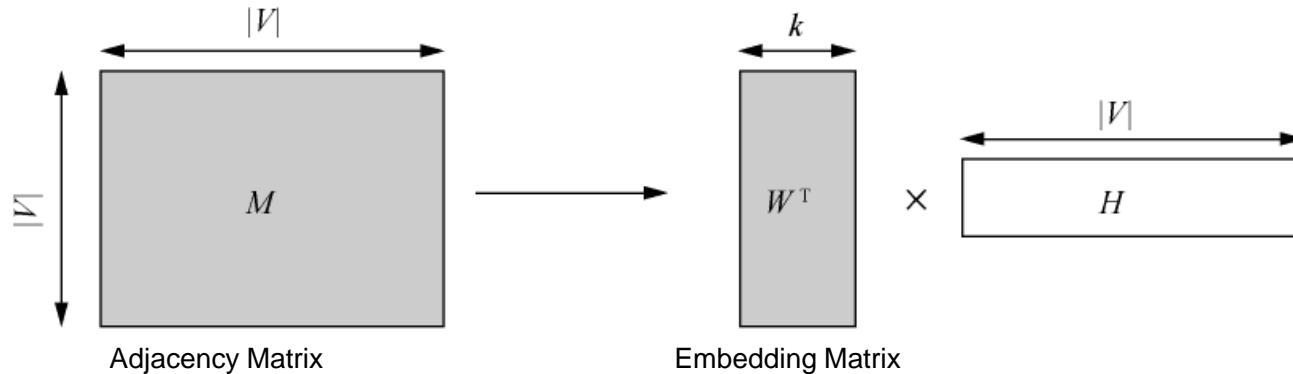


**Global Structure**

# Nodes & Links

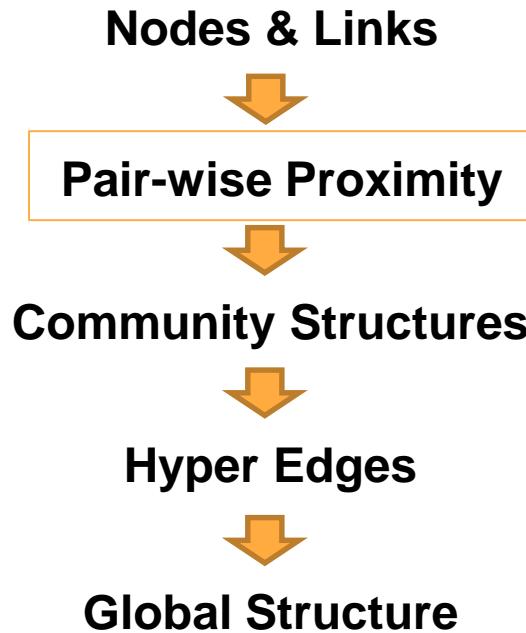
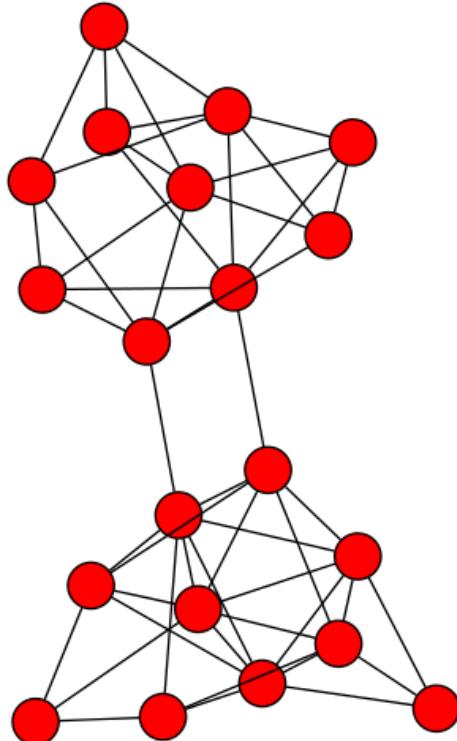
Reconstruct the original network

## Matrix Factorization



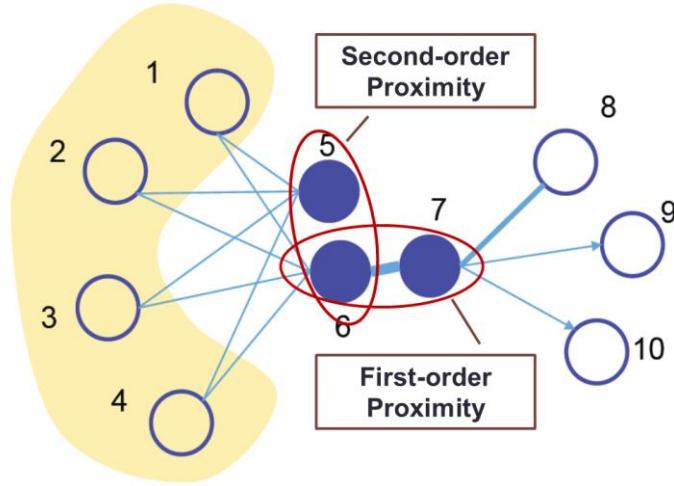
Reconstruct all the links? May cause overfitting.  
The network inference ability is seriously limited.

# Network Structures



# High-Order Proximity

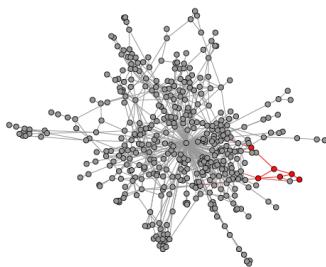
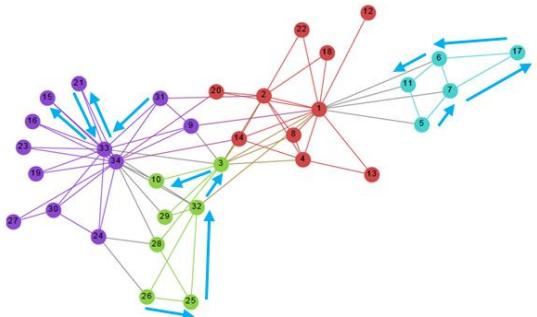
- Capturing the underlying structure of networks



- Advantages:
  - Solve the sparsity problem of network connections
  - Measure indirect relationship between nodes

# Deepwalk

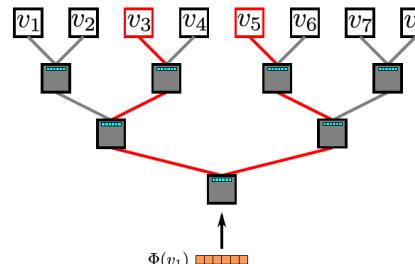
- Exploit truncated random walk to define neighborhood of a node.



(a) Random walk generation.

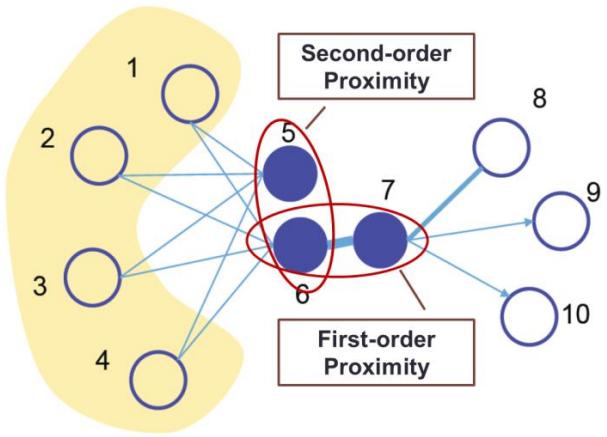
$$\mathcal{W}_{v_4} = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 5 \\ 1 \\ \vdots \end{bmatrix} v_j \rightarrow \Phi^d_j$$

(b) Representation mapping.



(c) Hierarchical Softmax.

# LINE



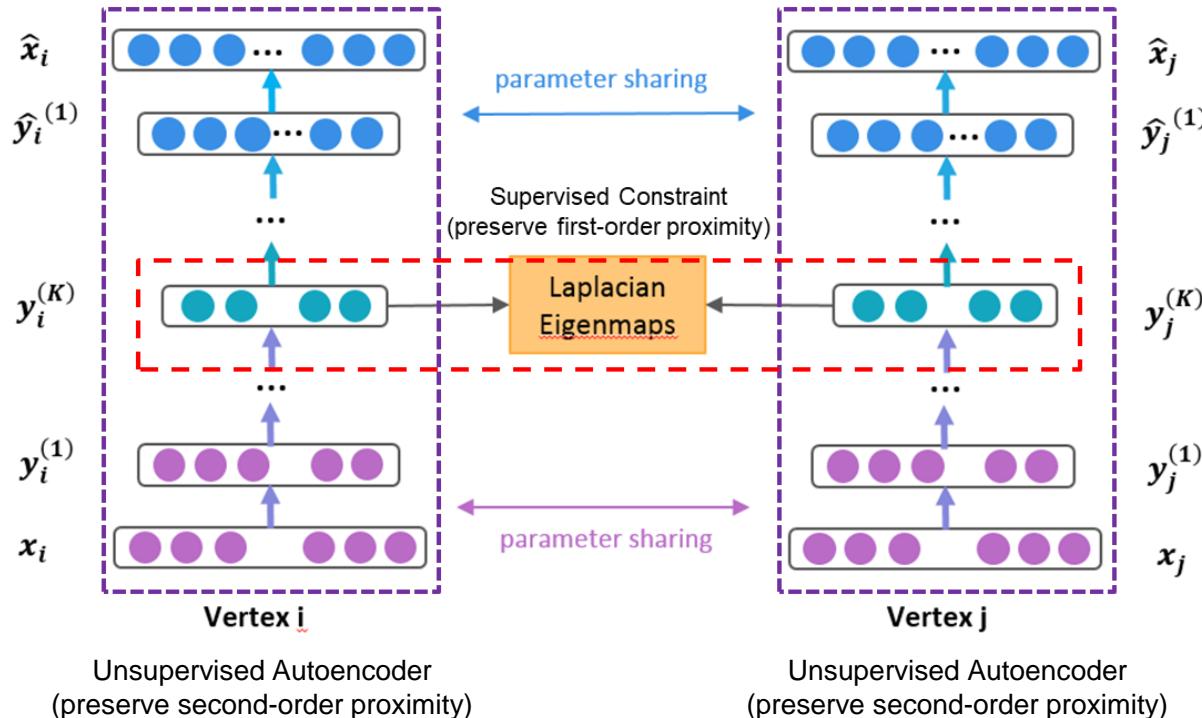
LINE with First-order Proximity:  
local pairwise

$$O_1 = - \sum_{(i,j) \in E} w_{ij} \log p_1(v_i, v_j)$$

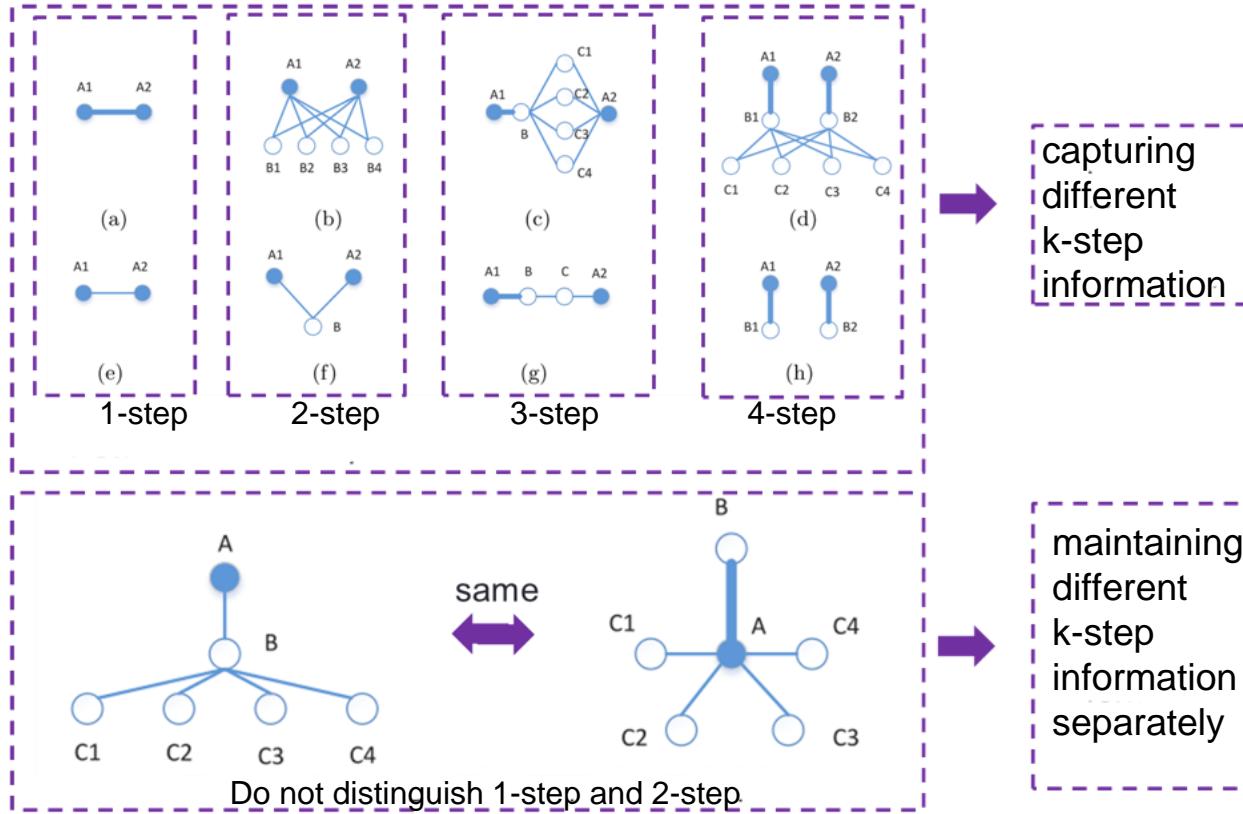
LINE with Second-order Proximity:  
neighborhood structures

$$O_2 = \sum_{i \in V} \lambda_i d(\hat{p}_2(\cdot | v_i), p_2(\cdot | v_i))$$

# SDNE – Structural Deep Network Embedding

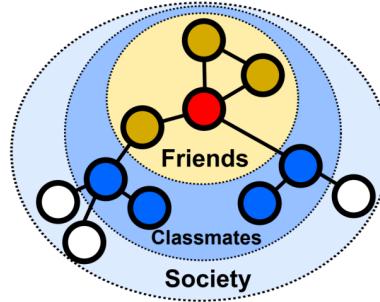


# GraRep



# What is the *right* order?

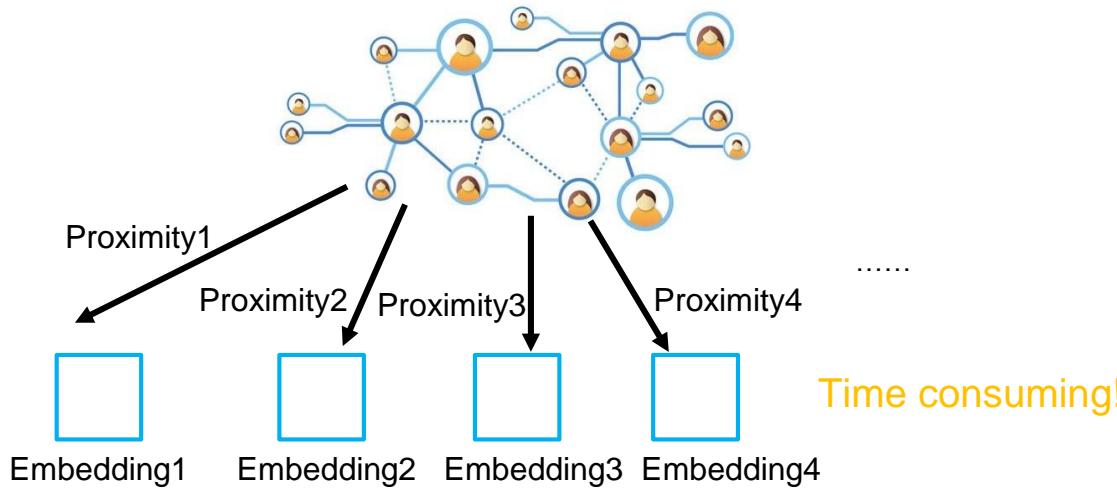
- Different networks/tasks require different high-order proximities
  - E.g., multi-scale classification (Bryan Perozzi, et al, 2017)



- E.g., networks with different scales and sparsity
- Proximities of different orders can also be arbitrarily weighted
  - E.g., equal weights, exponentially decayed weights (Katz)

# What is the *right* order?

- Existing methods can only preserve one fixed high-order proximity
  - Different high-order proximities are calculated separately



- -> How to preserve arbitrary-order proximity while guaranteeing accuracy and efficiency?

# Problem Formulation

- High-order proximity: a polynomial function of the adjacency matrix

$$S = f(A) = w_1 A^1 + w_2 A^2 + \cdots + w_q A^q$$

- $q$ : order;  $w_1 \dots w_q$ : weights, assuming to be non-negative
- $A$ : could be replaced by other variations (such as the Laplacian matrix)

- Objective function: matrix factorization

$$\min_{U^*, V^*} \|S - U^* V^{*T}\|_F^2$$

- $U^*, V^* \in \mathbb{R}^{N \times d}$ : left/right embedding vectors
- $d$ : dimensionality of the space

- Optimal solution: Singular Value Decomposition (SVD)

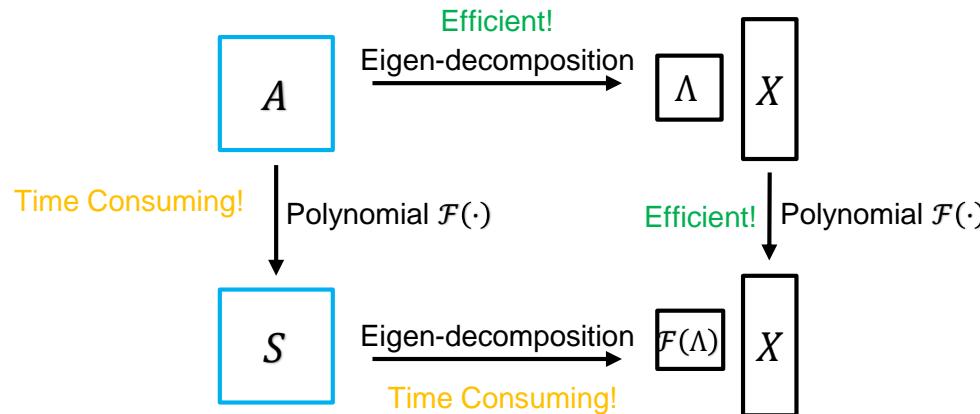
- $[U, \Sigma, V]$ : top-d SVD results

$$U^* = U\sqrt{\Sigma}, \quad V^* = V\sqrt{\Sigma}$$

# Eigen-decomposition Reweighting

- Eigen-decomposition reweighting

THEOREM 4.2 (EIGEN-DECOMPOSITION REWEIGHTING). *If  $[\lambda, \mathbf{x}]$  is an eigen-pair of  $\mathbf{A}$ , then  $[\mathcal{F}(\lambda), \mathbf{x}]$  is an eigen-pair of  $\mathbf{S} = \mathcal{F}(\mathbf{A})$ .*

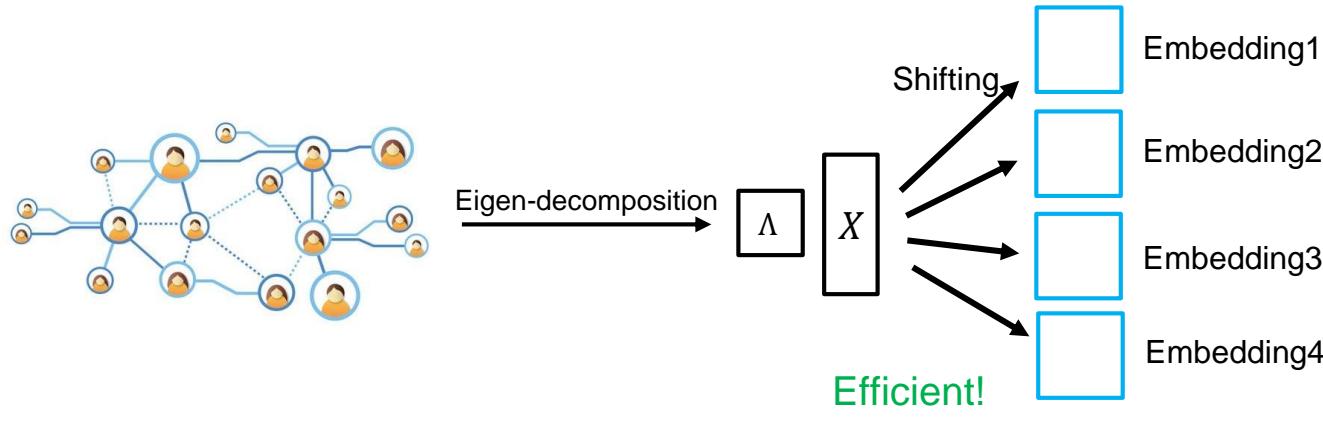


- **Insights:** high-order proximity is simply re-weighting dimensions!

$$U^* = U\sqrt{\Sigma}, V^* = V\sqrt{\Sigma}$$

# Preserving Arbitrary-Order Proximity

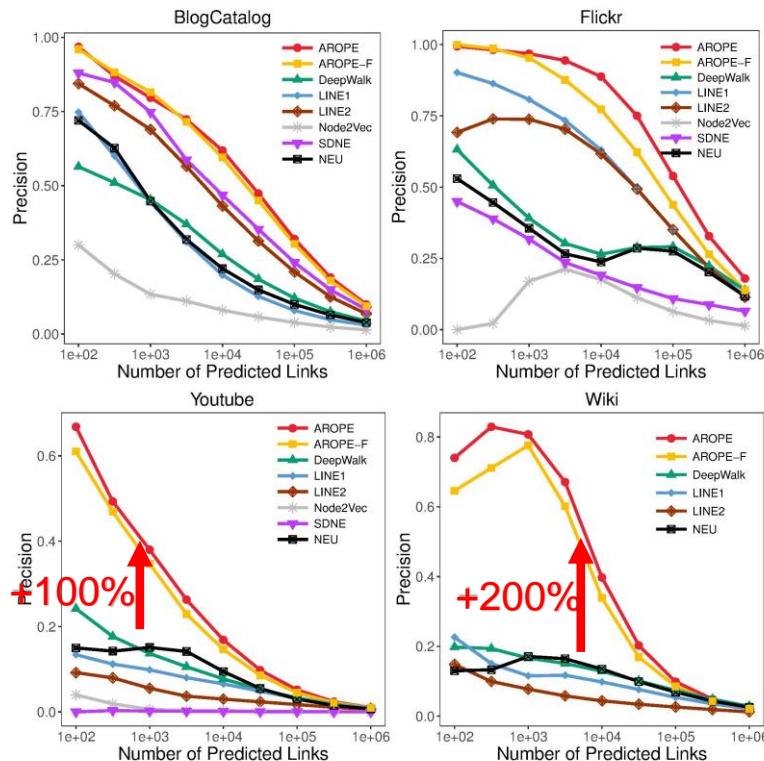
- Shifting across different orders/weights:



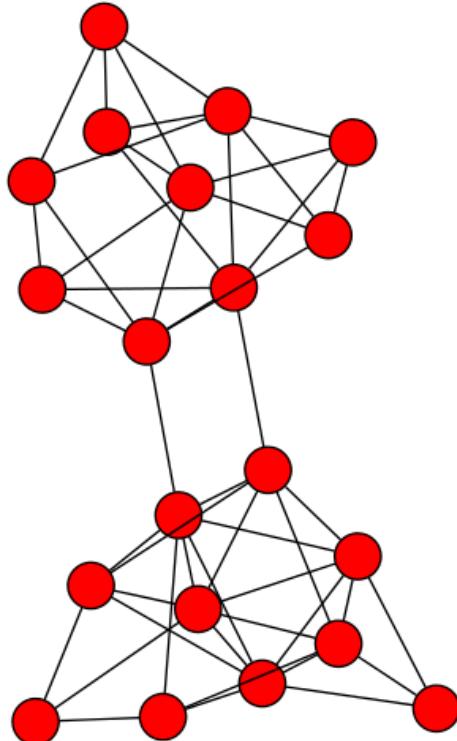
- Preserving arbitrary-order proximity
- Low marginal cost
- Accurate and efficient

# Experimental Results

- Link Prediction



# Network Structures



**Nodes & Links**



**Pair-wise Proximity**



**Community Structures**



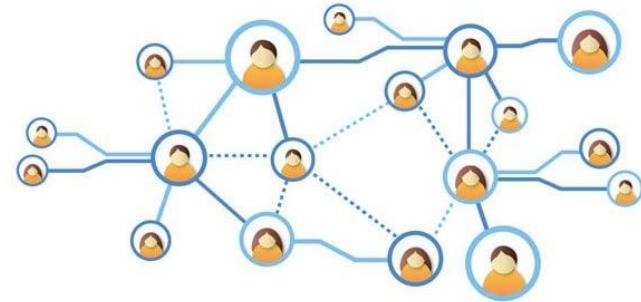
**Hyper Edges**



**Global Structure**

# Motivation

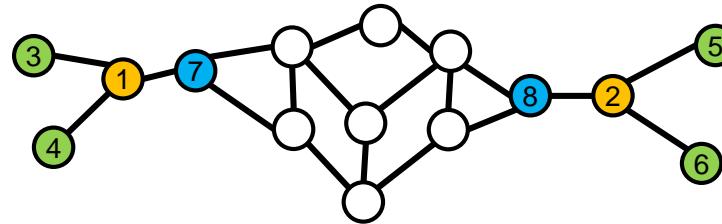
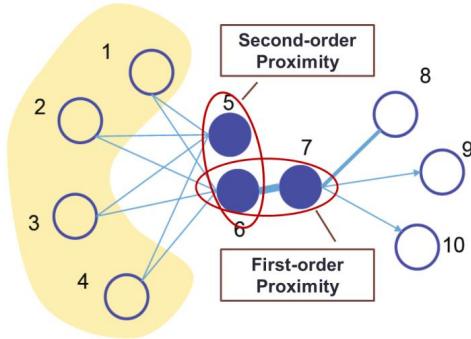
- Vertexes in different parts of the network may have similar roles(global position)
- Example:
  - Managers in the social network of a company
  - Outliers in a network in the task of anomaly detection



Social network with different position

**How to reflect the role or importance of a vertex in embedding space?**

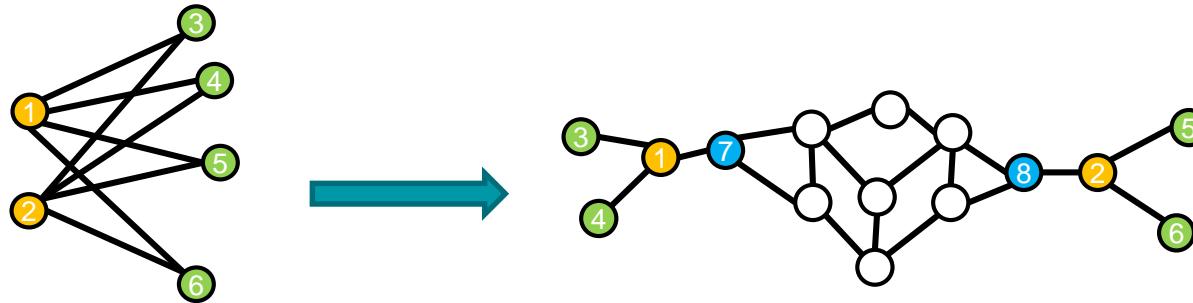
# Existing embedding methods



- They can only preserve local proximity(Structural equivalence), can not reflect the global position
- Embeddings of node 5,6 in left network will be similar but embeddings of node 1, 2 in right network will not be similar.

# Regular Equivalence

Two nodes are regularly equivalent if their network neighbors are themselves similar (i.e. regularly equivalent).



- Structural equivalence  $s$ 
  - $N(u) = N(v)$
  - Direct way
  - Common neighbors
- Regular equivalence  $r$ 
  - $\{r(i) | i \in N(u)\} = \{r(j) | j \in N(v)\}$
  - Recursive way
  - Similar global position

Regular equivalence is largely ignored in network embedding

# Naïve Solutions

- Basis: two regularly equivalent nodes should have similar embeddings
  1. Explicitly calculate the regular equivalence of all vertex pairs
    - infeasible for large-scale networks due to the high complexity of calculating regular equivalence
  2. Replace regular equivalence into simpler graph theoretic metrics
    - centrality measures
    - one centrality can only capture a specific aspect of network role
    - some centrality measures also bear high computational complexity

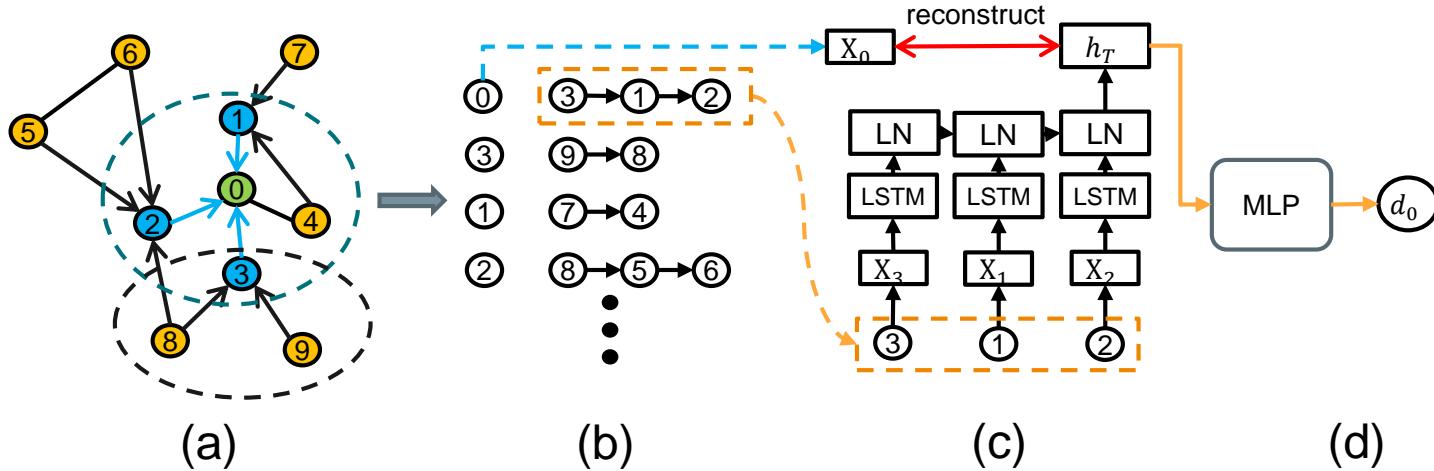
# Deep Recursive Network Embedding

- The definition of regular equivalence is recursive
  - Aggregating neighbors' information in a **recursive** way

$$\mathcal{L}_1 = \sum_{v \in V} \|\mathbf{X}_v - \text{Agg}(\{\mathbf{X}_u | u \in N(v)\})\|_F^2,$$

- How to design the aggregating function
  - Variable length of neighbors
  - Highly nonlinear
  - → Layer-normalized LSTM

# Deep Recursive Network Embedding



- (a) Sampling neighborhoods
- (b) Sorting neighborhoods by their degree
- (c) Aggregate neighbors
- (d) A Weakly guided regularizer

# Theoretical Analysis

THEOREM 3.5. If the centrality  $C(v)$  of node  $v$  satisfies that  $C(v) = \sum_{u \in N(v)} F(u)C(u)$  and  $F(v) = f(\{F(u), u \in N(v)\})$  where  $f$  is any computable function, then  $C(v)$  is one of the optimal solutions of our model.



Centrality	Definition $C(v)$	$F(v)$	$f(\{x_i\})$
Degree	$d_v = \sum_{u \in N(v)} I(d_u)$	$1/d_v$	$1/(\sum I(x_i))$
Eigenvector	$1/\lambda * \sum_{u \in N(v)} C(u)$	$1/\lambda$	mean
PageRank	$\sum_{u \in N(v)} 1/d_u * C(u)$	$1/d_v$	$1/(\sum I(x_i))$

# Experiment --- predict centrality

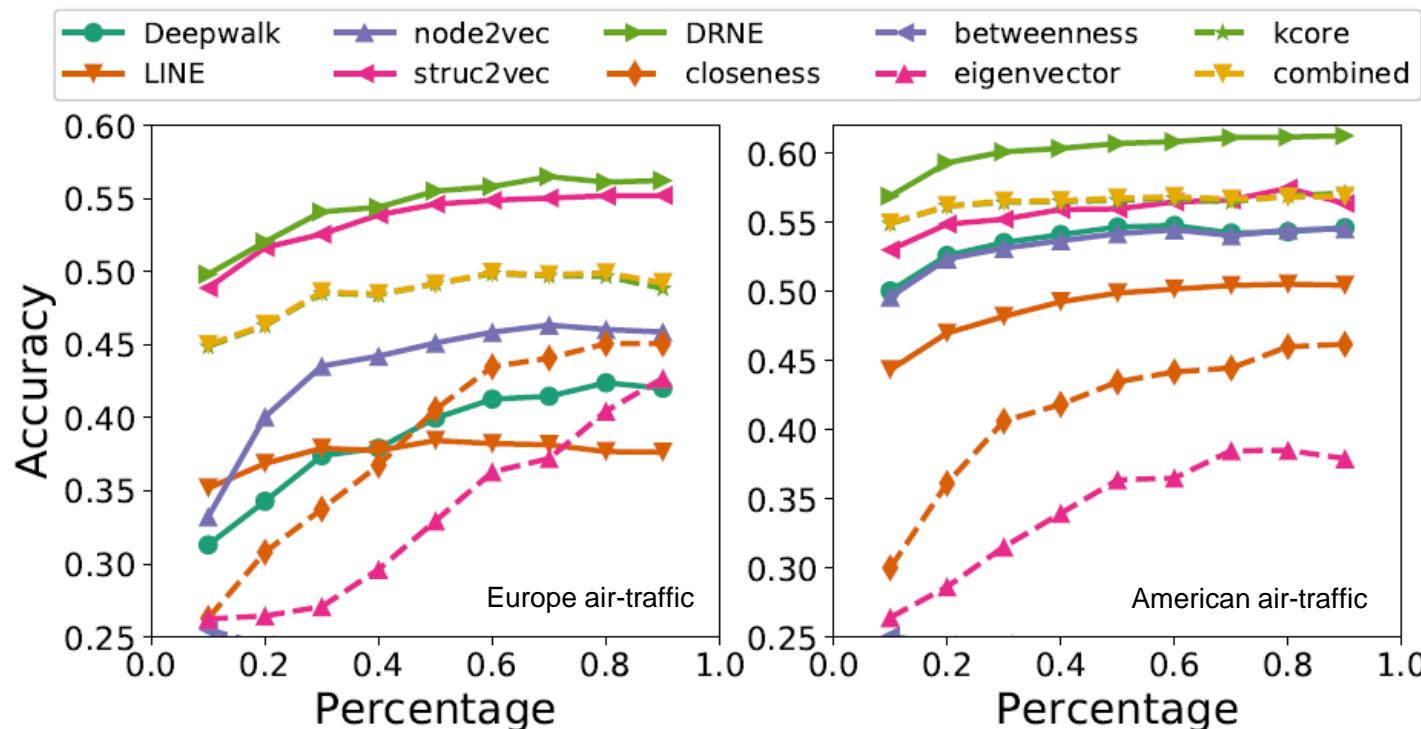
centrality	closeness	betweenness	eignvector	k-core
DeepWalk	0.6016	3.7188	2.1543	13.2755
LINE	0.5153	4.3919	1.5072	15.8179
node2vec	1.0489	3.4065	3.9436	39.2156
struc2vec	0.2365	0.25371	1.0544	9.0858
DRNE	<b>0.1909</b>	<b>0.1261</b>	<b>0.5267</b>	<b>5.5683</b>

The MSE value of predicting centralities on Jazz dataset (\*10–2)

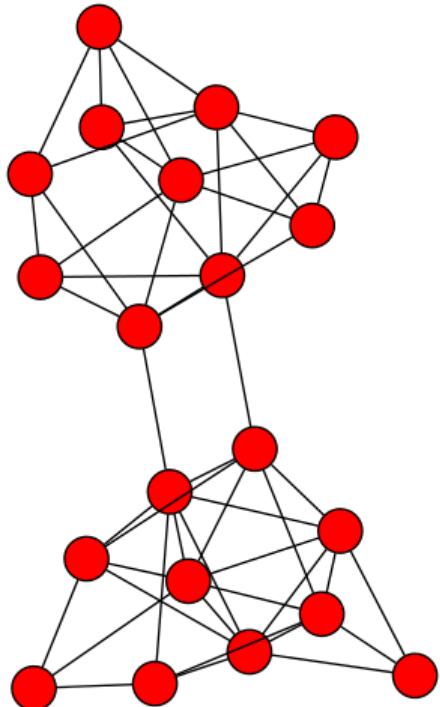
centrality	closeness	betweenness	eignvector	k-core
DeepWalk	0.2982	1.7836	1.1194	19.7016
LINE	0.3979	1.8425	1.5167	34.9079
node2vec	0.3573	1.6958	1.1432	24.1704
struc2vec	0.2947	1.6018	1.0445	25.3047
DRNE	<b>0.1101</b>	<b>0.6676</b>	<b>0.3108</b>	<b>7.7210</b>

The MSE value of predicting centralities on BlogCatalog dataset (\*10–2)

# Experiment - Structural Role Classification



# Section Summary



**Nodes & Links**



**Node Neighborhood**



**Pair-wise Proximity**



**Community Structures**



**Hyper Edges**



**Global Structure**

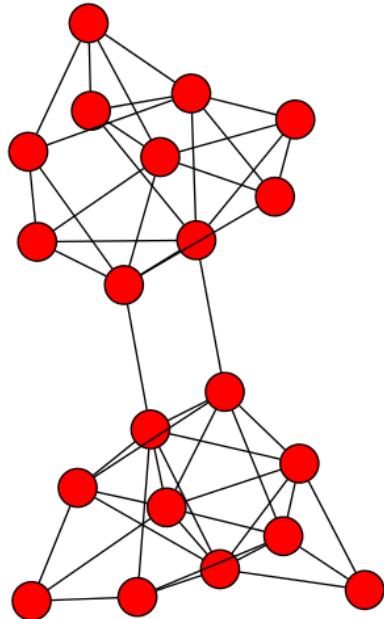
**Network  
Characteristics**

**Application  
Characteristics**

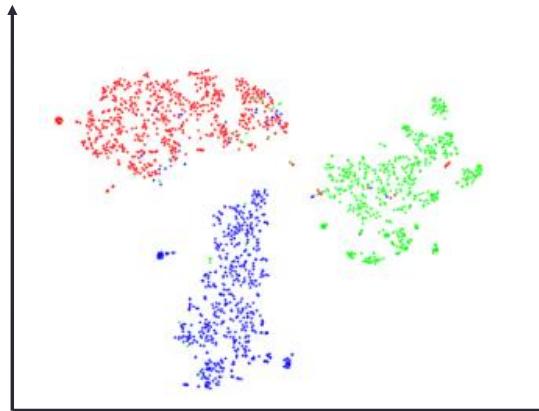
# Outline

- **Structure-preserved network embedding**
- **Property-preserved network embedding**
- **Dynamic network embedding**
- **Robustness, Explainability and Applicability**
- **Network embedding for biomedical applications**

# Why preserve network properties?

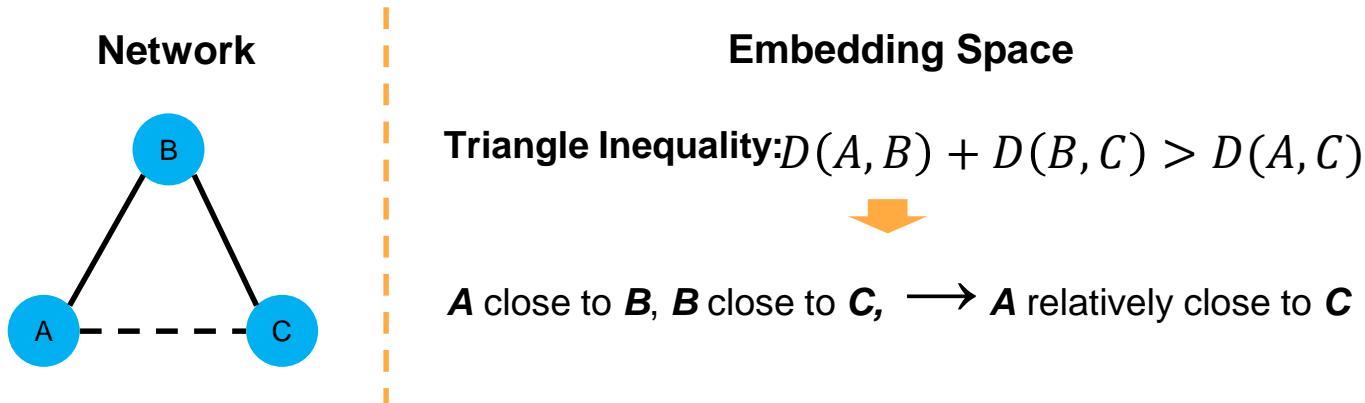


Heterogeneity



# Transitivity

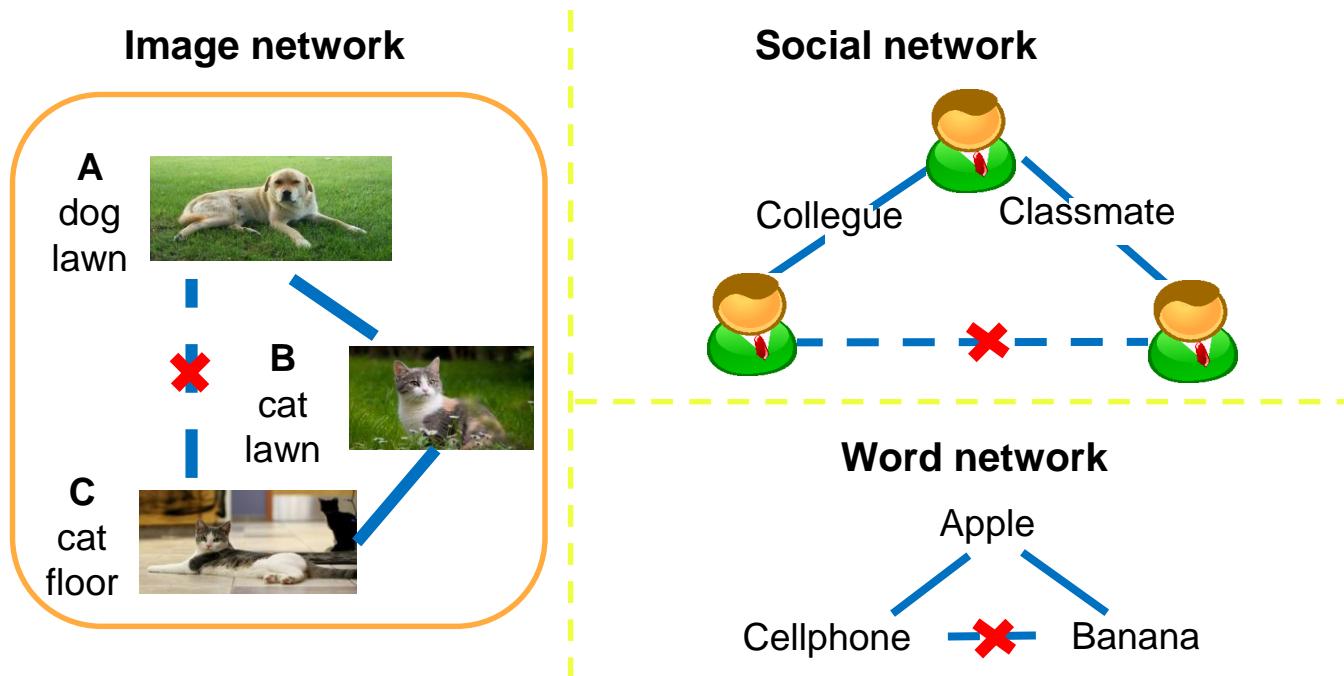
## The Transitivity Phenomenon



However, real network data is complex...

# Non-transitivity

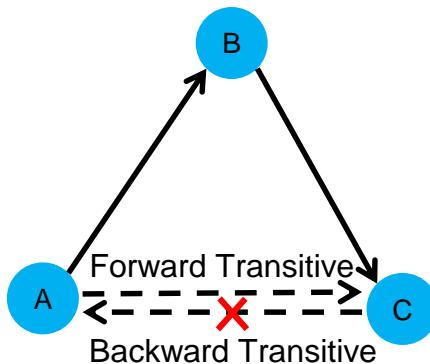
The Co-existence of *Transitivity* and *Non-transitivity*



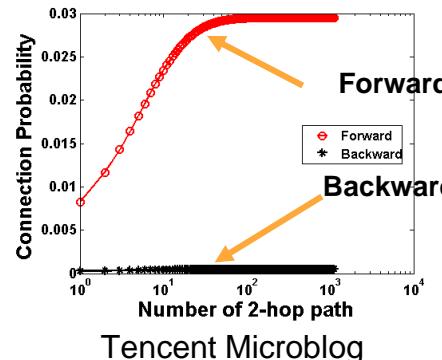
How to incorporate non-transitivity in embedding space?

# Asymmetric Transitivity

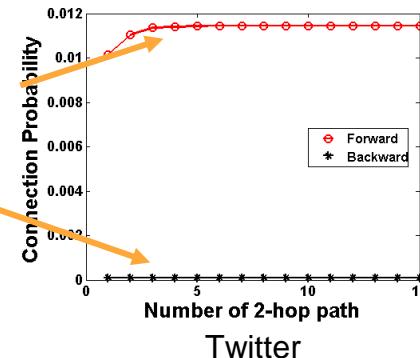
Directed Network



$A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C$ , but not  $C \rightarrow A$



Tencent Microblog



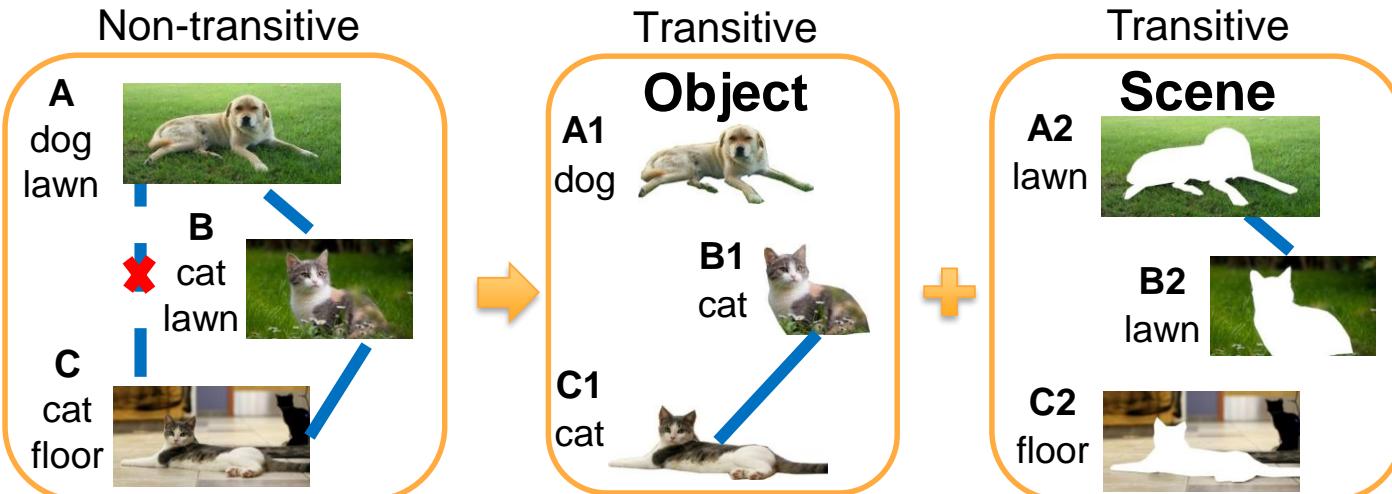
Twitter

Distance metric in embedding space is symmetric.  
How to incorporate *Asymmetric Transitivity*?

# Non-transitivity

The source of non-transitivity:

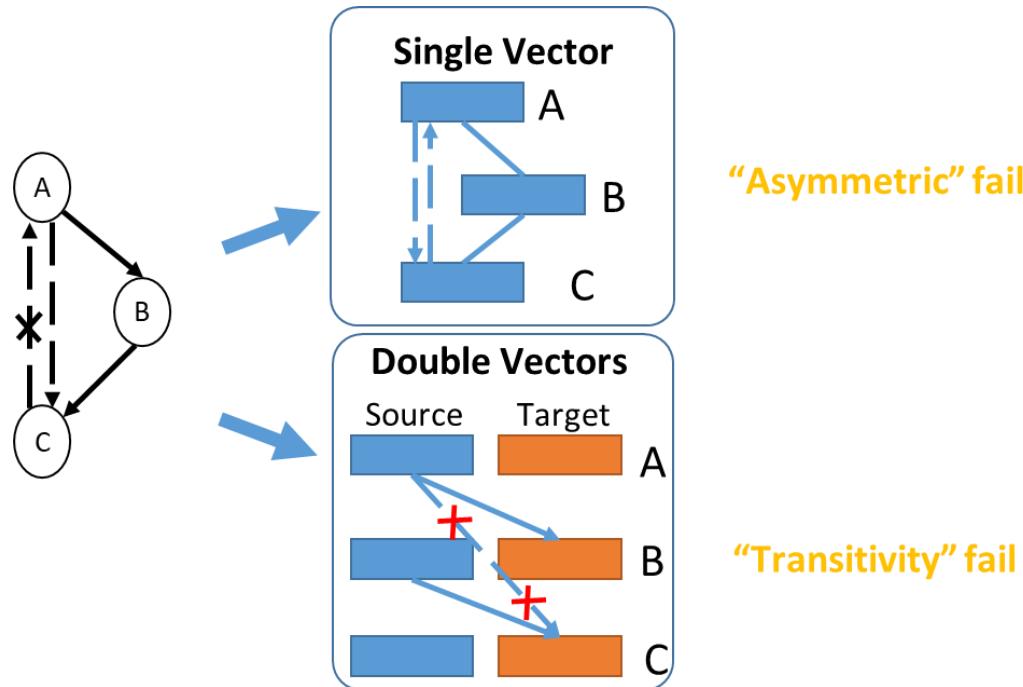
*Each node has multiple similarity components*



**Non-transitive Embedding:** represent non-transitive data with multiple latent similarity components

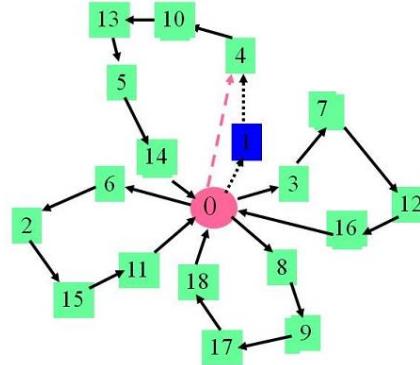
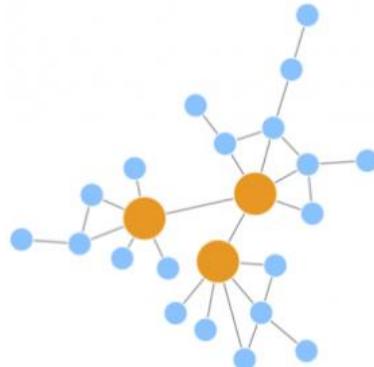
# Asymmetric Transitivity

All existing methods fail..



# Uncertainties in Networks

- The formation and evolution of real-world networks are full of uncertainties
  - E.g., for the nodes with low degree, they contain less information and thus their representations bear more uncertainties than others.
  - E.g., for the nodes across multiple communities, the possible contradiction between their neighboring nodes may also be large and thus cause the uncertainty.



# DVNE for Structure and Uncertainty

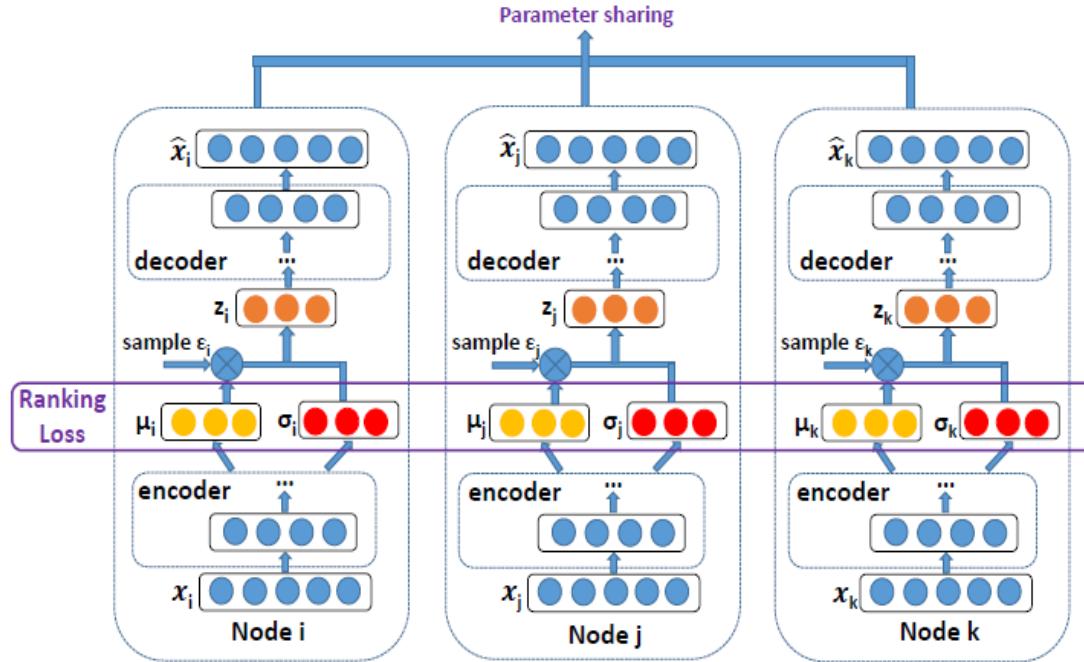


Figure 1: The framework of DVNE.

# Section Summary

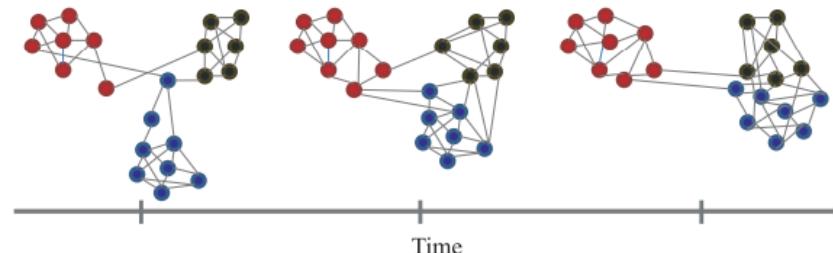
- Compared with network structures, **network properties** have large space to explore in network embedding.
- Transitivity is important for network inference.
- Uncertainty provides evidence in making network inference.
- Many other property issues:
  - The right embedding space: Euclidean space?
  - Power-law distribution
  - ...

# Outline

- **Structure-preserved network embedding**
- **Property-preserved network embedding**
- **Dynamic network embedding**
- **Robustness, Explainability and Applicability**
- **Network embedding for biomedical applications**

# Dynamic Networks

- **Networks are dynamic in nature**
  - **New (old) nodes are added (deleted)**
    - New users, products, etc.
  - **The edges between nodes evolve over time**
    - Users add or delete friends in social networks, or neurons establish new connections in brain networks.
- **How to efficiently incorporate the dynamic changes when networks evolve?**



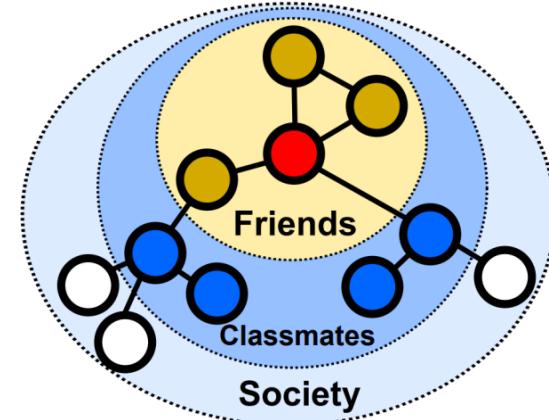
# **Key problems in dynamic network embedding**

- I : Out-of-sample nodes
- II : Incremental edges
- III: Aggregated error
- IV: Scalable optimization

# Challenge: High-order Proximity

- **High-order proximity**

- Critical structural property of networks
- Measure indirect relationship between nodes
- Capture the structure of networks with different scales and sparsity



*Network Embedding V.S. Traditional Graph Embedding*

# Challenge: High-order Proximity

I : Out-of-sample nodes

II : Incremental edges

III: Aggregated error

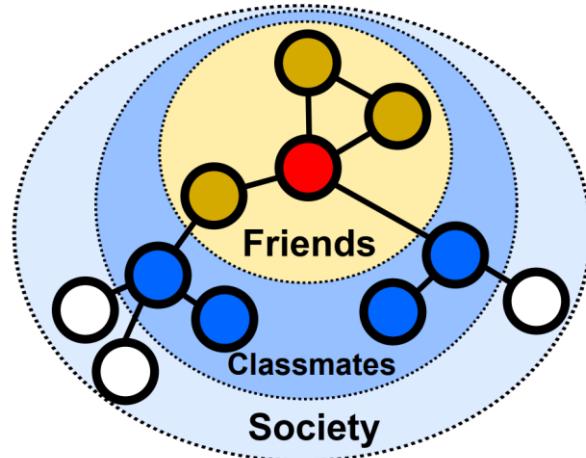
IV: Scalable optimization



*Preserve High-order Proximities*



*Local Change leads to Global Updating*

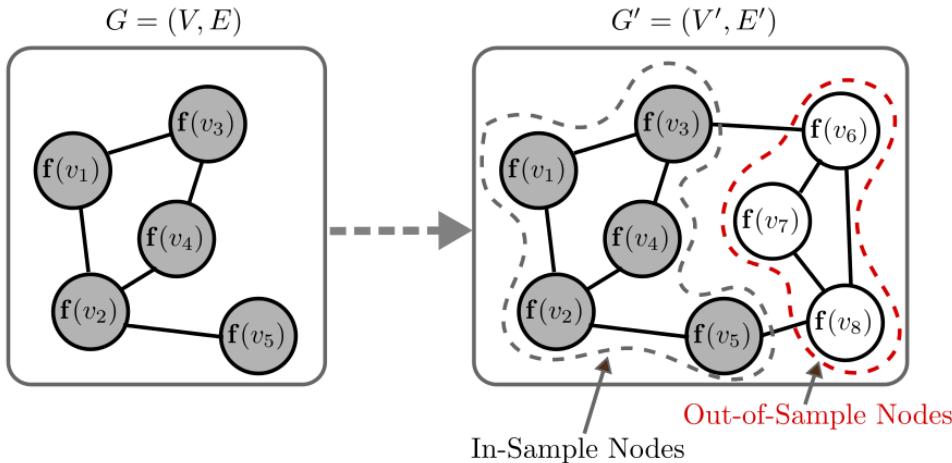


# Key problems in dynamic network embedding

- I : Out-of-sample nodes
- II : Incremental edges
- III: Aggregated error
- IV: Scalable optimization

# Problem

- To infer embeddings for out-of-sample nodes.



- $G=(V, E)$  evolves into  $G'=(V', E')$ , where  $V' = V \cup V^*$ .
- $n$  old nodes:  $V = \{v_1, \dots, v_n\}$ ,  $m$  new nodes:  $V^* = \{v_{n+1}, \dots, v_{n+m}\}$
- Network embedding:  $f: V \rightarrow \mathbb{R}^d$
- We know  $f(v)$  for old nodes, want to infer  $f(v)$  for new nodes.

# Challenges

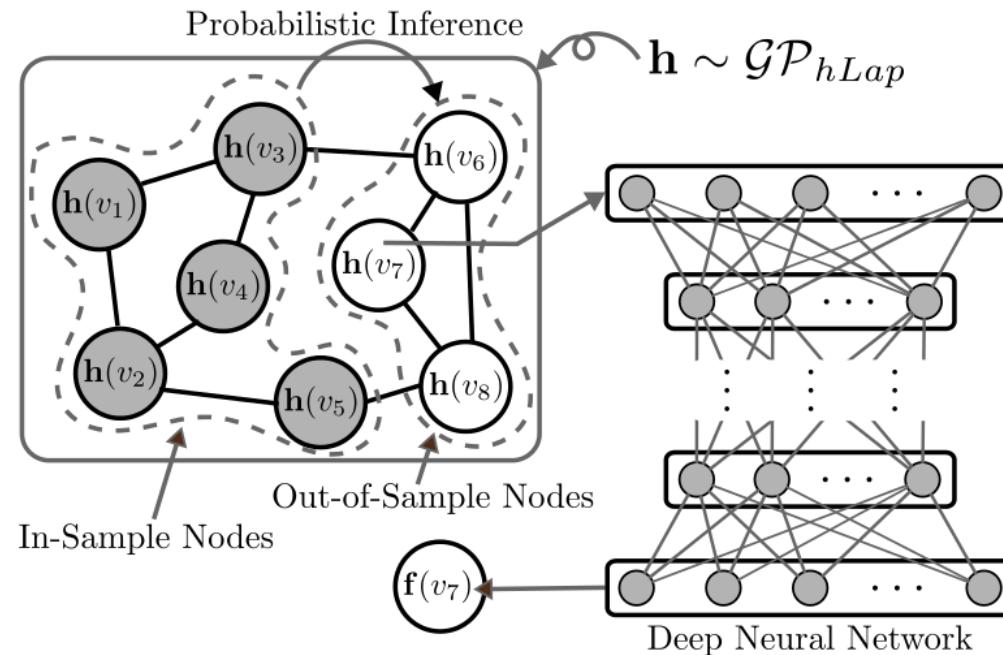
- Preserve network structures
  - e.g. high-order proximity
  - need to incorporate prior knowledge on networks
- Share similar characteristics with in-sample embeddings
  - e.g. magnitude, mean, variance
  - requires a model with great expressive power to fit the data well
- Low computational cost

# Specific vs. General

- Specific
  - A new NE algorithm capable of handling OOS nodes.
- General
  - A solution that helps an arbitrary NE algorithm handle OOS nodes.
- We propose a **general** solution.
  - But it can be easily integrated into an existing NE algorithm (e.g. DeepWalk) to derive a **specific** algorithm (see the paper).

# DepthLGP

- Nonparametric probabilistic modeling + Deep Learning



# DepthLGP

- Design a kernel for the  $k$ th ( $k=1, \dots, s$ ) dimension of  $h(\cdot)$

$$\mathbf{K}_k \triangleq \left[ \mathbf{I} + \eta_k \mathbf{L}(\hat{\mathbf{A}}_k) + \zeta_k \mathbf{L}(\hat{\mathbf{A}}_k \hat{\mathbf{A}}_k) \right]^{-1},$$

First-order Proximity                      Second-order Proximity

$$\hat{\mathbf{A}}_k \triangleq \text{diag}(\boldsymbol{\alpha}_k) \mathbf{A}' \text{diag}(\boldsymbol{\alpha}_k),$$
$$\boldsymbol{\alpha}_k \triangleq [a_{v_1}^{(k)}, a_{v_2}^{(k)}, \dots, a_{v_{n+m}}^{(k)}]^\top,$$

Node Weights  
(to prune uninformative nodes)

<sup>1</sup>The matrix inversion can be bypassed without approximation.

<sup>2</sup> $a_v^{(k)}$  indicates how much attention we pay to a node. It is learned for an in-sample node, but fixed to one for an OOS node, as we are always interested in OOS nodes.

# Task I: Classification

Metric	Embedding	Network	Baselines			This Work		Upper Bound
			LocalAvg	MRG	LabelProp	hLGP	DepthLGP	(rerunning)
Macro-F1(%)	LINE	DBLP	37.89	42.15	40.83	47.33	<b>48.25</b>	(49.07)
		PPI	10.52	10.02	12.42	13.42	<b>13.72</b>	(13.91)
	GraRep	BlogCatalog	13.25	11.30	17.07	17.41	<b>18.03</b>	(18.90)
		DBLP	50.61	55.79	55.02	57.43	<b>58.67</b>	(62.92)
	node2vec	PPI	13.65	13.75	12.38	14.80	<b>14.84</b>	(15.33)
		BlogCatalog	14.76	14.80	14.71	15.94	<b>18.45</b>	(20.15)
		DBLP	53.83	59.34	59.25	60.89	<b>62.63</b>	(64.87)
Micro-F1(%)	LINE	PPI	15.05	13.43	13.78	15.85	<b>16.54</b>	(16.81)
		BlogCatalog	15.10	14.04	19.16	19.77	<b>20.32</b>	(20.82)
	GraRep	DBLP	49.58	50.49	50.88	54.01	<b>54.94</b>	(55.84)
		PPI	18.10	15.71	18.81	20.71	<b>21.42</b>	(21.43)
	node2vec	BlogCatalog	27.40	23.21	30.79	31.36	<b>31.90</b>	(32.20)
		DBLP	60.17	60.62	60.48	61.44	<b>62.29</b>	(65.44)
		PPI	20.23	20.35	20.23	20.79	<b>21.44</b>	(21.88)
	GraRep	BlogCatalog	36.44	30.79	33.90	37.57	<b>38.14</b>	(38.37)
		DBLP	60.54	62.29	62.52	62.83	<b>64.56</b>	(65.63)
	node2vec	PPI	19.70	18.25	18.25	22.63	<b>23.11</b>	(23.41)
		BlogCatalog	34.83	25.82	36.94	37.96	<b>39.64</b>	(40.34)

# Key problems in dynamic network embedding

- I : Out-of-sample nodes
- II : Incremental edges
- III: Aggregated error
- IV: Scalable optimization

# The Static Model

- We aim to preserve high-order proximity in the embedding matrix with the following objective function:

$$\min \|\mathbf{S} - \mathbf{U}\mathbf{U}'^T\|_F^2$$

- where  $\mathbf{S}$  denotes the high-order proximity matrix of the network
  - $\mathbf{U}$  and  $\mathbf{U}'$  is the results of matrix decomposition of  $\mathbf{S}$ .
- 
- For undirected networks,  $\mathbf{U}$  and  $\mathbf{U}'$  are highly correlated.
    - Without loss of generality, we choose  $\mathbf{U}$  as the embedding matrix.

# GSVD

- We choose Katz Index as  $S$  because it is one of the most widely used measures of high-order proximity.

$$S^{Katz} = M_a^{-1} M_b$$

$$M_a = (I - \beta A)$$

$$M_b = \beta A$$

- where  $\beta$  is a decay parameter,  $I$  is the identity matrix and  $A$  is the adjacency matrix
- According to HOPE, the original objective function can be solved by the generalized SVD (GSVD) method

# Generalized Eigen Perturbation

- We propose generalized eigen perturbation to fulfill the task.
  - The goal of generalized eigen perturbation is to update  $X^{(t)}$  to  $X^{(t+1)}$
- Specifically, given the change of adjacency matrix  $\Delta A$  between two consecutive time steps, the change of  $M_a$  and  $M_b$  can be represented as:

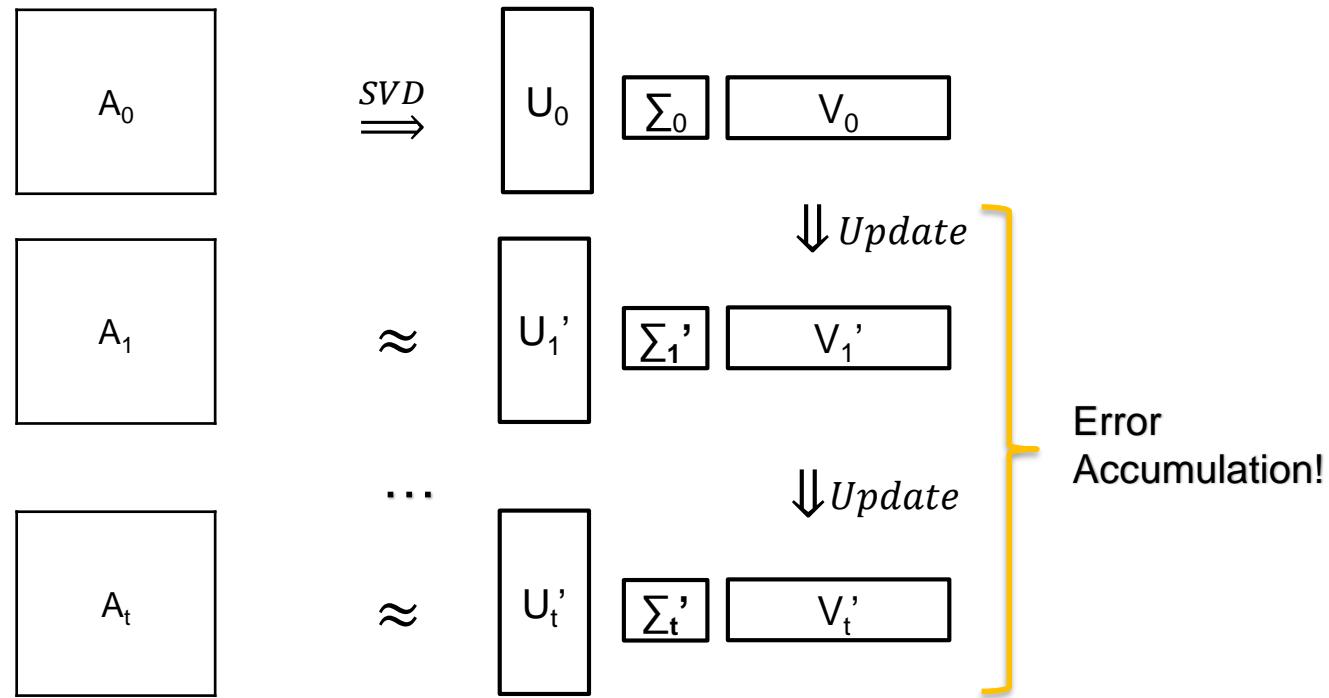
$$\Delta M_a = -\beta \Delta A, \text{ and } \Delta M_b = \beta \Delta A$$

# Key problems in dynamic network embedding

- I : Out-of-sample nodes
- II : Incremental edges
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# Problem: Error Accumulation

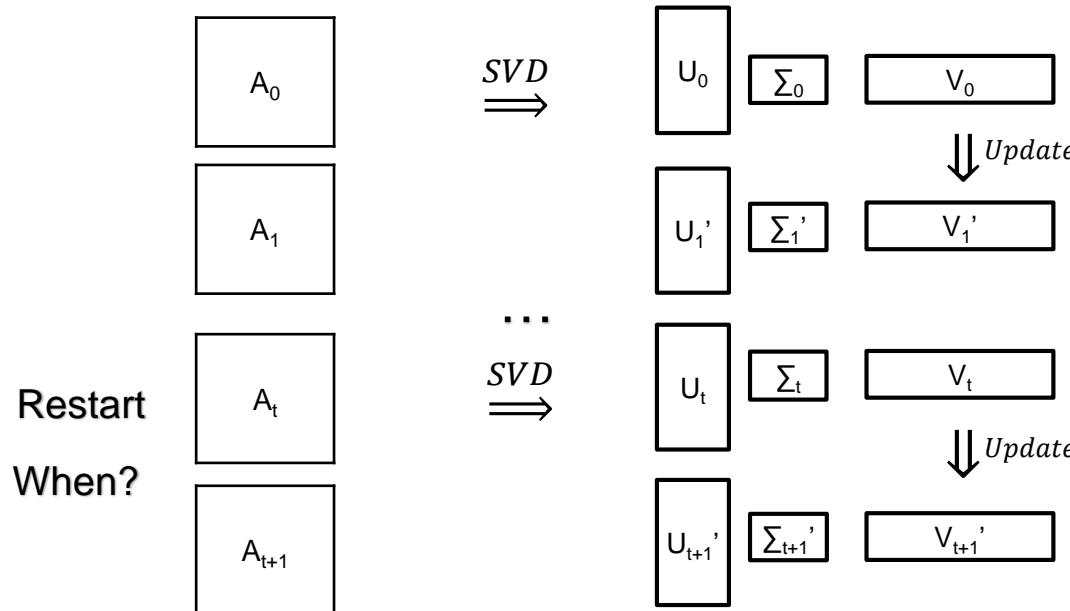
- Eigen perturbation is at the cost of inducing approximation



- Problem: error accumulation is inevitable

# Solution: SVD Restarts

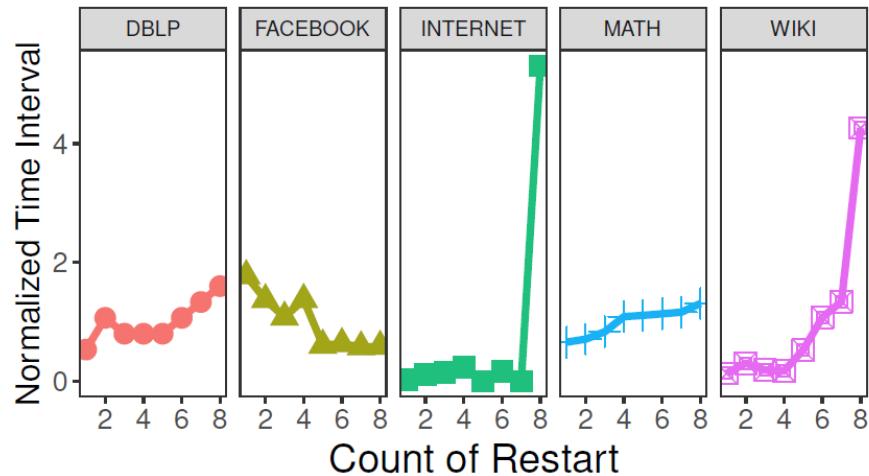
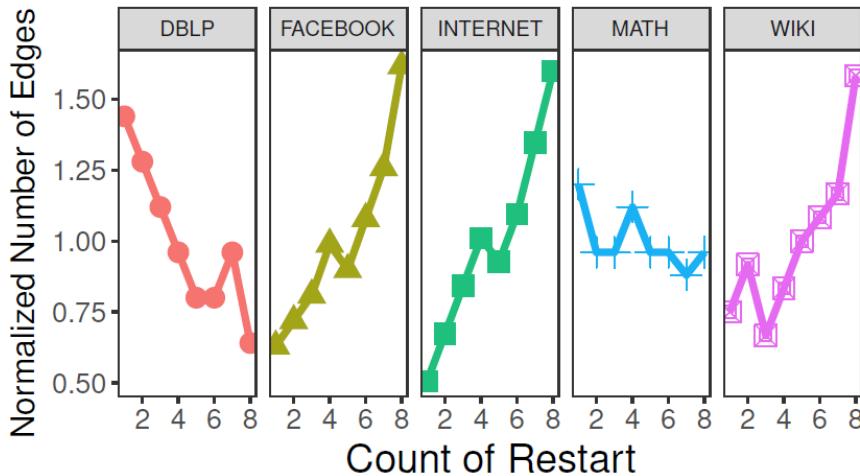
- Solution: restart SVD occasionally



- What are the appropriate time points?
  - Too early restarts: waste of computation resources
  - Too late restarts: serious error accumulation

# Naïve Solution

- Naïve solution: fixed time interval or fixed number of changes
- Difficulty: error accumulation is not uniform



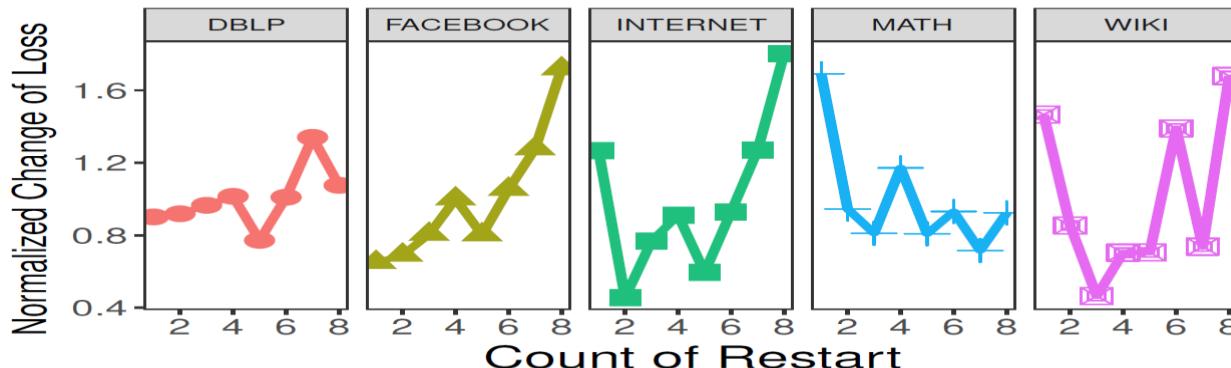
# Existing Method

- Existing method: monitor loss (Chen and Candan, KDD 2014)
- Loss in SVD:

$$\mathcal{J} = \|S - U\Sigma V^T\|_F^2$$

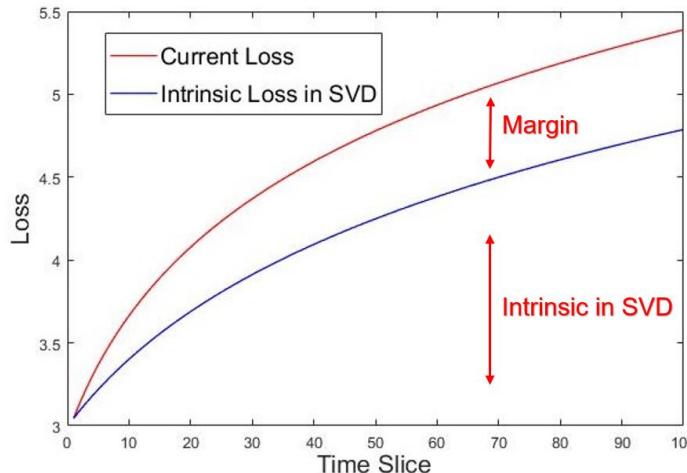
$S$ : target matrix,  $[U, \Sigma, V]$ : results of SVD

- Problem: loss includes approximation error and intrinsic loss in SVD



# Framework: Monitor Margin

- Observation: the margin between the current loss and intrinsic loss in SVD is the actual accumulated error
  - Current loss:  $\mathcal{J} = \|S - U\Sigma V^T\|_F^2$
  - Intrinsic loss:  $\mathcal{L}(S, k) = \min_{U^*, \Sigma^*, V^*} \|S - U^* \Sigma^* V^{*T}\|_F^2, k: \text{dimensionality}$



# Solution: Lazy Restarts

- Lazy restarts: restart only when the margin exceeds the threshold
- Problem: intrinsic loss is hard to compute
  - Direct calculation has the same time complexity as SVD
- Relaxation: an upper bound on margin
  - A lower bound on intrinsic loss  $\mathcal{L}(S, k)$

$$\mathcal{L}(\mathbf{S}_t, k) \geq B(t) \Rightarrow \frac{\mathcal{J}(t) - \mathcal{L}(\mathbf{S}_t, k)}{\mathcal{L}(\mathbf{S}_t, k)} \leq \frac{\mathcal{J}(t) - B(t)}{B(t)}.$$

$\mathcal{J}(t)$ : current loss;  $\mathcal{L}(S_t, k)$ : intrinsic loss;  $B(t)$ : bound of intrinsic loss

# A Lower Bound of SVD Intrinsic Loss

- Idea: use matrix perturbation

**Theorem 1** (A Lower Bound of SVD Intrinsic Loss). *If  $\mathbf{S}$  and  $\Delta\mathbf{S}$  are symmetric matrices, then:*

$$\mathcal{L}(\mathbf{S} + \Delta\mathbf{S}, k) \geq \mathcal{L}(\mathbf{S}, k) + \Delta\text{tr}^2(\mathbf{S} + \Delta\mathbf{S}, \mathbf{S}) - \sum_{l=1}^k \lambda_l, \quad (9)$$

where  $\lambda_1 \geq \lambda_2 \dots \geq \lambda_k$  are the top- $k$  eigenvalues of  $\nabla_{S^2} = \mathbf{S} \cdot \Delta\mathbf{S} + \Delta\mathbf{S} \cdot \mathbf{S} + \Delta\mathbf{S} \cdot \Delta\mathbf{S}$ , and

$$\Delta\text{tr}^2(\mathbf{S} + \Delta\mathbf{S}, \mathbf{S}) = \text{tr}((\mathbf{S} + \Delta\mathbf{S}) \cdot (\mathbf{S} + \Delta\mathbf{S})) - \text{tr}(\mathbf{S} \cdot \mathbf{S}).$$

- Intuition: treat changes as a perturbation to the original network

# Time Complexity Analysis

**Theorem 2.** *The time complexity of calculating  $B(t)$  in Eqn (13) is  $O(M_S + M_L k + N_L k^2)$ , where  $M_S$  is the number of the non-zero elements in  $\Delta\mathbf{S}$ , and  $N_L, M_L$  are the number of the non-zero rows and elements in  $\nabla_{S^2}$  respectively.*

- If every node has a equal probability of adding new edges, we have:  $M_L \approx 2d_{avg}M_S$ , where  $d_{avg}$  is the average degree of the network .
- For Barabasi Albert model (Barabási and Albert 1999), a typical example of preferential attachment networks, we have:  $M_L \approx \frac{12}{\pi^2} [\log(d_{max}) + \gamma] M_S$ , where  $d_{max}$  is the maximum degree of the network and  $\gamma \approx 0.58$  is a constant.
- Conclusion: the complexity is only linear to the local dynamic changes

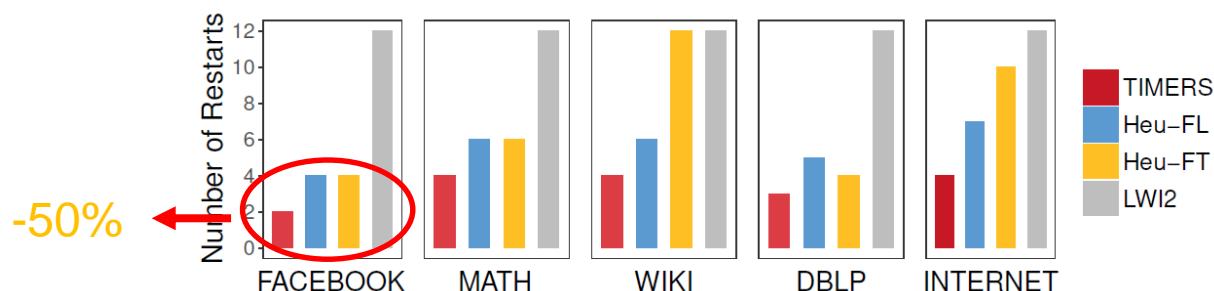
# Experimental Results: Approximation Error

- Fixing number of restarts

Dataset	$avg(r)$				$max(r)$			
	TIMERS	LWI2	Heu-FL	Heu-FT	TIMERS	LWI2	Heu-FL	Heu-FT
FACEBOOK	<b>0.005</b>	0.020	0.009	0.011	<b>0.014</b>	0.038	0.025	0.023
MATH	<b>0.037</b>	0.057	0.044	0.051	<b>0.085</b>	0.226	0.117	0.179
WIKI	<b>0.053</b>	0.086	0.071	0.281	<b>0.139</b>	0.332	0.240	0.825
DBLP	<b>0.042</b>	0.110	0.053	0.064	<b>0.121</b>	0.386	0.198	0.238
INTERNET	<b>0.152</b>	0.218	0.196	0.961	<b>0.385</b>	0.806	0.647	1.897

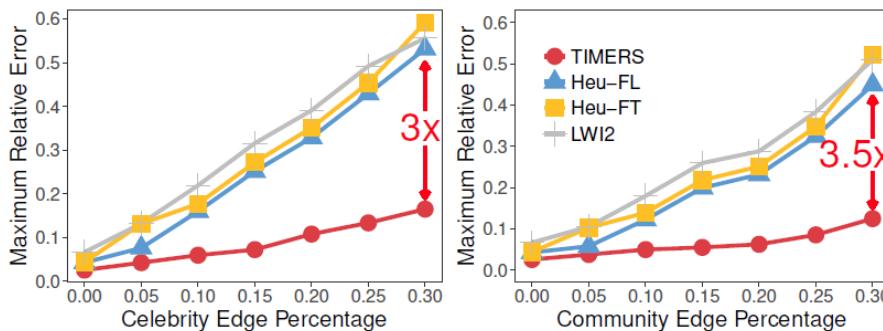
- Fixing maximum error

27%~42% Improvement 

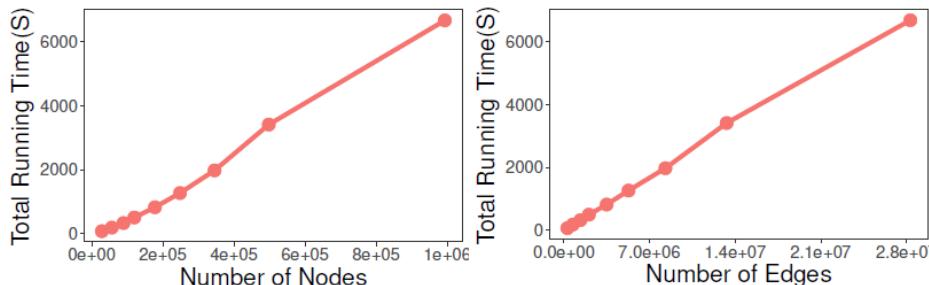


# Experimental Results: Analysis

- Syntactic networks: simulate drastic changes in the network structure



- Robust to sudden changes
- Linear scalability

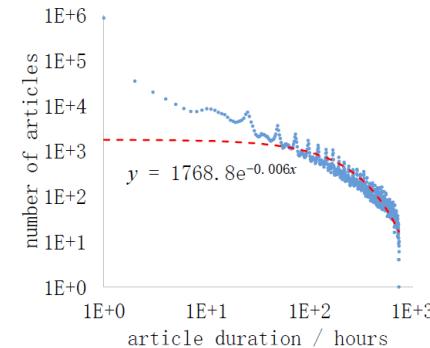
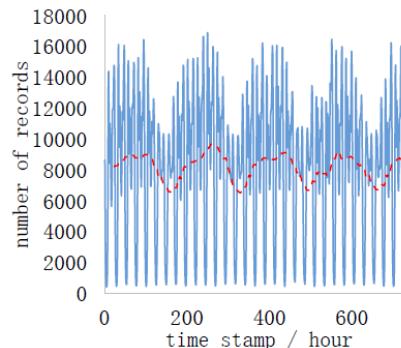
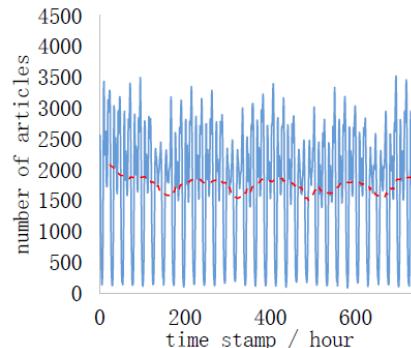


# Key problems in dynamic network embedding

- I : Out-of-sample nodes
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- IV: Scalable optimization

# Highly-dynamic & Recency-sensitive Data

- News recommendation applications: a bipartite graph
- WeChat news recommendation network is highly dynamic
  - 81 articles and 1400 reading records per second
- The network is also recency-sensitive
  - >73% articles died less than 6 hours while no one read again
  - Obvious exponential decay for article duration length.



# Limited resources

- We cannot guarantee convergence in-between every two timestamps.
- Just do it.
- How to do better?
- Non-uniform resource allocation.
- New edges and nodes worth more resources.

# Diffused SGD: Step-wise Weight Diffusion Mechanism

- The Change of a node embedding vector depends on its distance to the changed edge.
- Diffuse across training steps
- For step  $r$ , if edge  $(i, j)$  is chosen by stochastic method

For edge  $(i, j)$ , we have

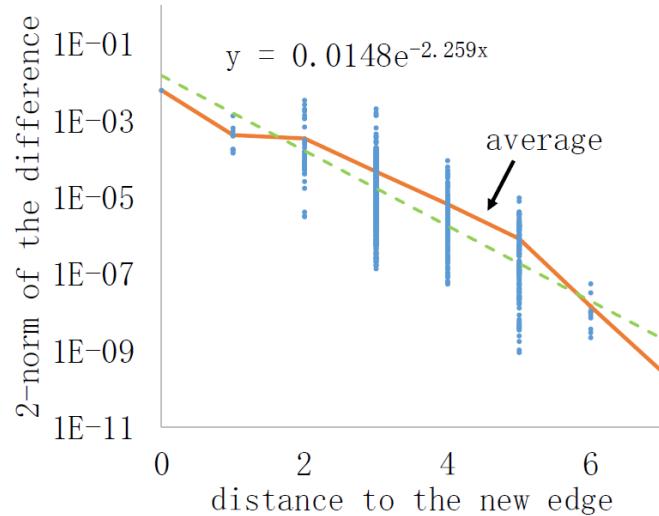
$$p_{i,j}(r) \leftarrow \tau_e(i, p_{i,j}(r-1));$$

for  $(i, k) \in \mathbb{E} \wedge k \neq j$ , we use

$$p_{i,k}(r) \leftarrow p_{i,k}(r-1) + \tau_n(i, p_{i,j}(r-1));$$

and for other edges  $(l, k) \in \mathbb{E} \wedge l \neq i$ ,

$$p_{l,k}(r) \leftarrow p_{l,k}(r-1);$$



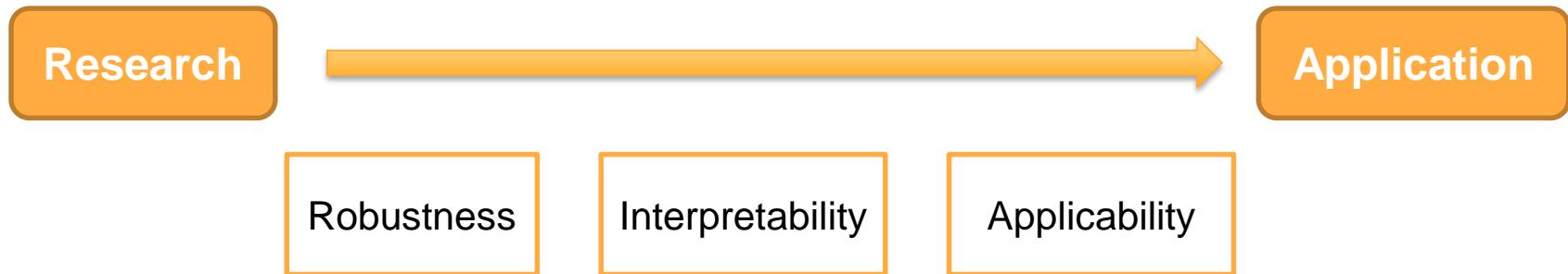
# Section Summary

- **I : Out-of-sample nodes**
  - DepthLGP = Non-parametric GP + DNN
- **II : Incremental edges**
  - DHPE: Generalized Eigen Perturbation
- **III: Aggregated error**
  - TIMERS: A theoretically guaranteed SVD restart strategy
- **IV: Scalable optimization**
  - D-SGD: A iteration-wise weighted SGD for highly dynamic data

# Outline

- **Structure-preserved network embedding**
- **Property-preserved network embedding**
- **Dynamic network embedding**
- **Robustness, Explainability and Applicability**
- **Network embedding for biomedical applications**

# Technical challenges in real applications

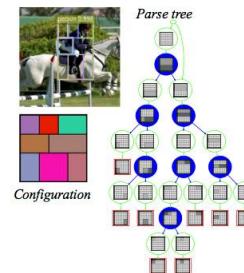


Hot directions in computer vision:

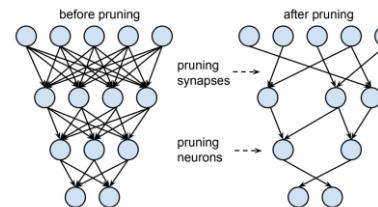
Adversarial



Explainable



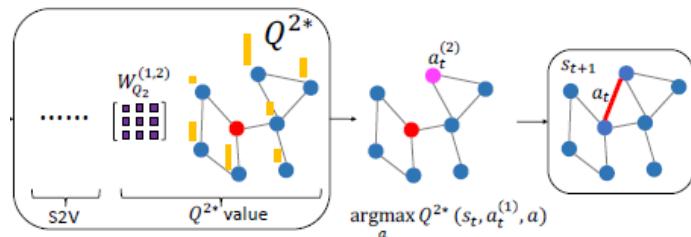
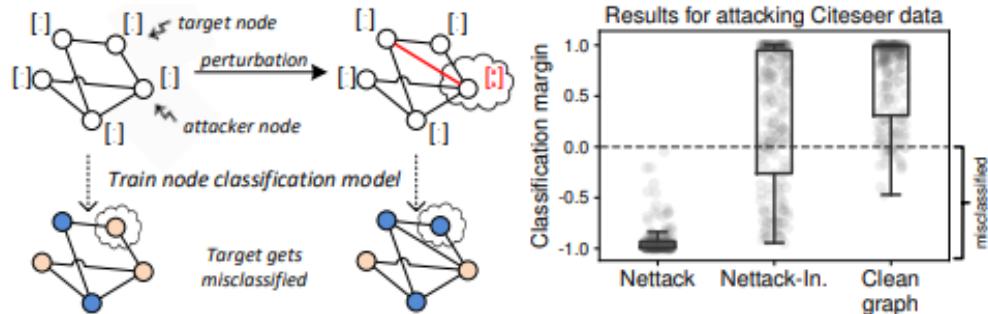
Scalable



# Robustness in network embedding

## □ Adversarial attacks

- small perturbations in graph structures and node attributes
- great challenges for applying GCNs to node classification



# Adversarial Attacks on GCNs

## □ Categories

### □ Targeted VS Non-targeted

- Targeted: the attacker focus on misclassifying some target nodes
- Non-targeted: the attacker aims to reduce the overall model performance

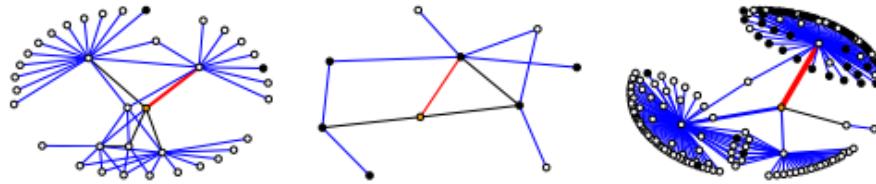
### □ Direct vs Influence

- Direct: the attacker can directly manipulate the edges or features of the target nodes
- Influence: the attacker can only manipulate other nodes except the targets

## □ How to enhance the robustness of GCNs against adversarial attacks?

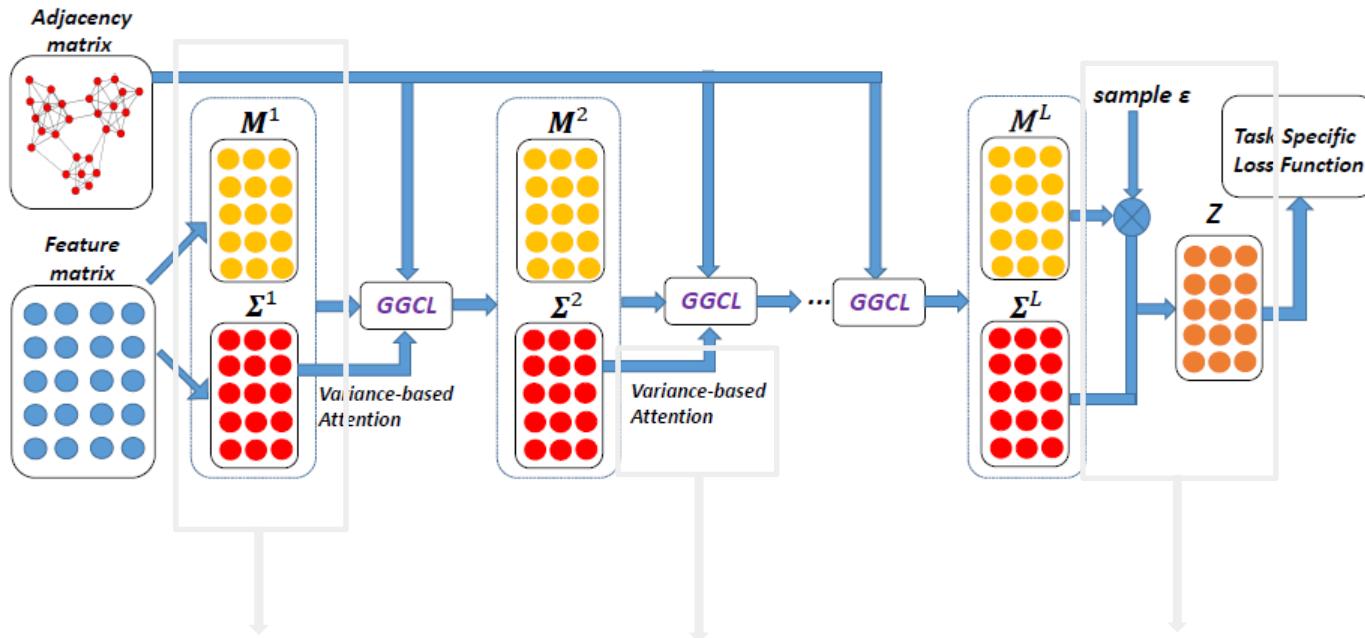
# Robust Graph Convolutional Networks

- Adversarial attacks in node classification
  - Connect nodes from different communities to confuse the classifier



- Distribution V.S. plain vectors
  - Plain vectors cannot adapt to such changes
  - Variances can help to absorb the effects of adversarial changes
  - Gaussian distributions -> Hidden representations of nodes

# The Framework of RGCN



Gaussian Based hidden representations:  
Variance terms absorb the effects of  
adversarial attacks

Attention mechanism:  
Remedy the propagation  
of adversarial attacks

Sampling process:  
Explicitly considers mathematical  
relevance between means and  
variances

# Experimental Results

## □ Node Classification on Clean Datasets

	Cora	Citeseer	Pubmed
GCN	81.5	70.9	79.0
GAT	83.0	72.5	79.0
RGCN	<b>83.1</b>	71.3	79.2

## □ Against Non-targeted Adversarial Attacks

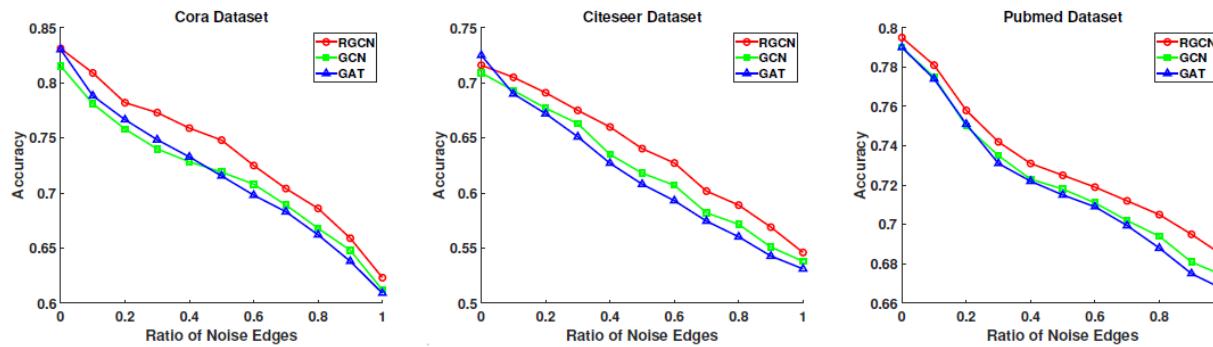
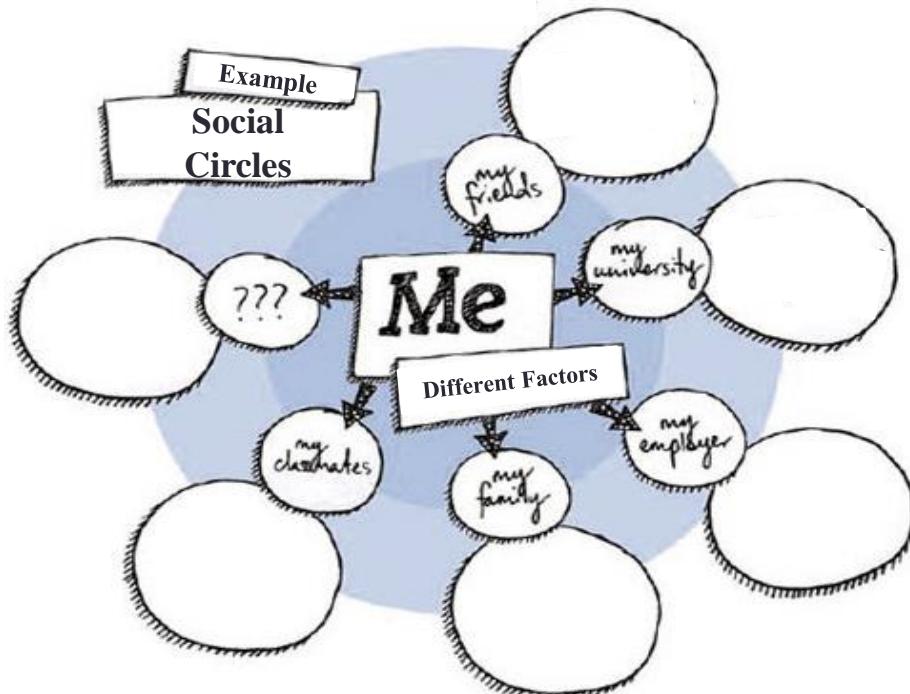


Figure 2: Results of different methods when adopting Random Attack as the attack method.

# Interpretability of network embedding

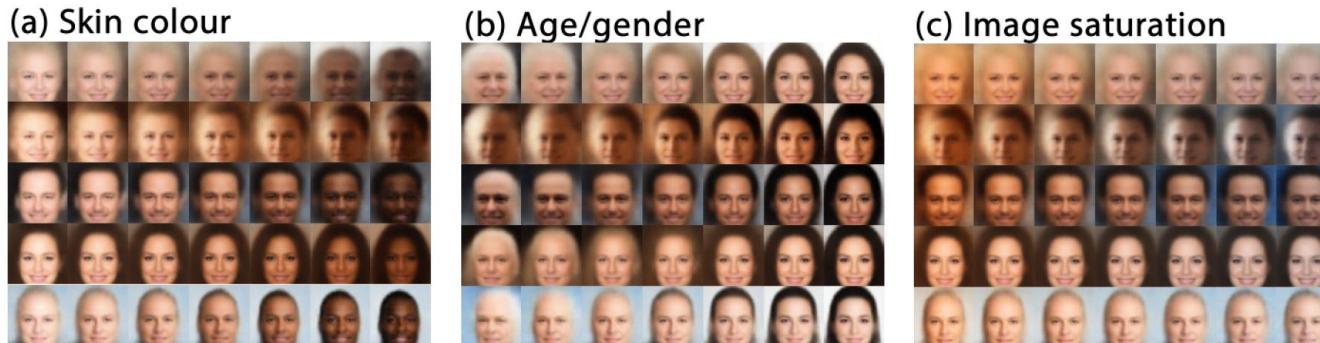
- A real-world graph is typically formed due to *many latent factors*.



- Existing GNNs/GCNs:
  - A holistic approach, that takes in the *whole* neighborhood to produce a *single* node representation.
- We suggest:
  - To disentangle the latent factors.  
(By segmenting the heterogeneous parts, and learning multiple factor-specific representations for a node.)
  - Robustness (e.g., not overreact to an irrelevant factor) & Interpretability.

# Disentangled Representation Learning

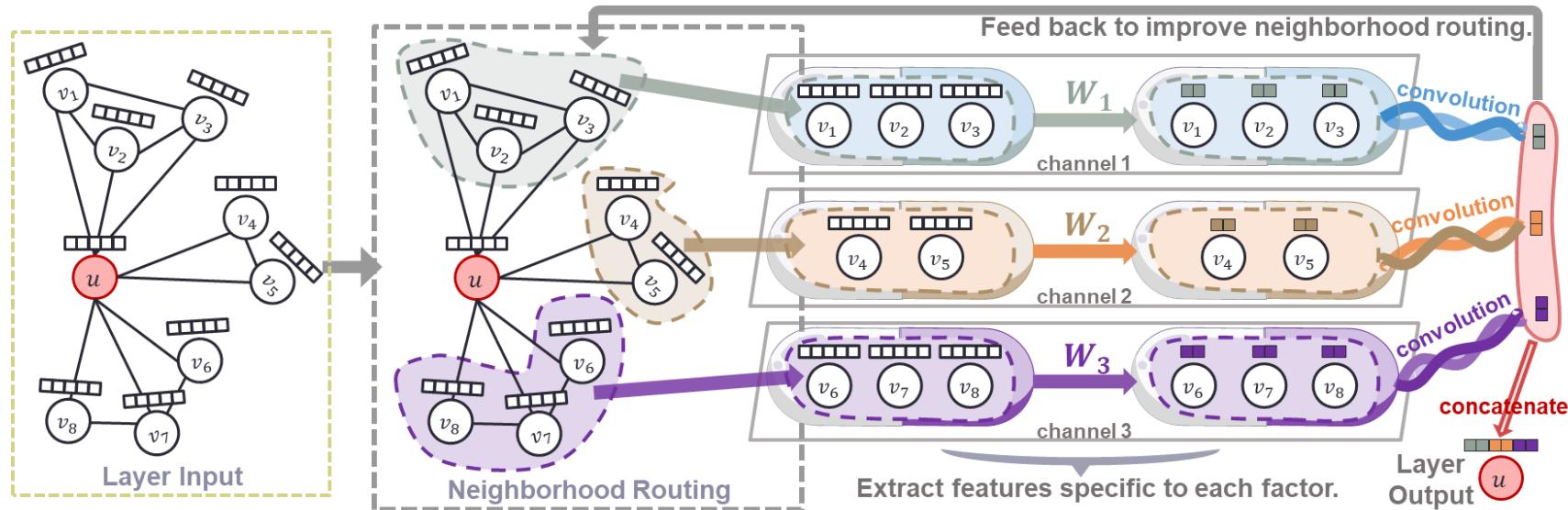
- That is, we aim to learn disentangled node representation,
  - A representation that contains independent components, that describes different aspects (caused by different latent factors) of the observation.
- The topic is well studied in the field of computer vision.
  - But largely unexplored in the literature of GNNs.



Example: Three dimensions that are related skin color, age/gender, and saturation, respectively.

# Method Overview

- We present DisenGCN, the *disentangled* graph convolutional network.
  - DisenConv, a disentangled multichannel convolutional layer (figure below).
  - Each channel convolves features related with a single latent factor.



# Neighborhood Routing: Hypothesis I

- A neighbor is *patched to channel  $k$*  (for further in-channel graph convolution), if the edge between the neighbor and the center node is *caused by factor  $k$* .
- But the actual causes are unknown. *Neighborhood routing* is therefore proposed to infer the latent causes, based on two hypothesis.
- The first is analogous to the second-order proximity.

**Hypothesis 1.** Factor  $k$  is likely to be the reason why node  $u$  connects with a certain subset of its neighbors, if the subset is large and the neighbors in the subset are similar w.r.t. aspect  $k$ , i.e., they form a cluster in the  $k^{\text{th}}$  subspace.

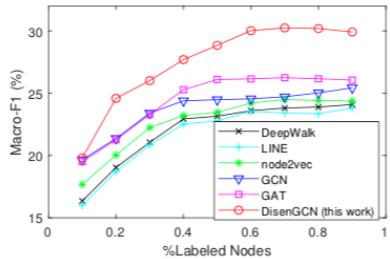
- It inspires us to search for the biggest cluster in each of the  $K$  subspaces.

# Neighborhood Routing: Hypothesis II

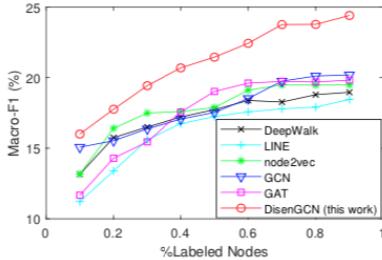
- The second hypothesis is analogous to the first-order proximity.

**Hypothesis 2.** Factor  $k$  is likely to be the reason why node  $u$  and neighbor  $v$  are connected, if the two are similar in terms of aspect  $k$ .
- Hypothesis 2 is not robust if either  $x_u$  or  $x_v$ , misses features about aspect  $k$ , and therefore must be combined with Hypothesis 1. But it can provide a fast guess.

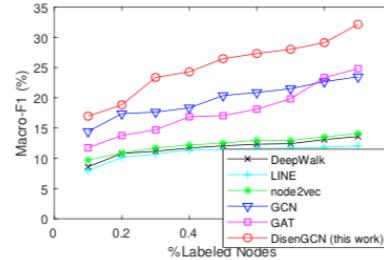
# Results: Multi-label Classification



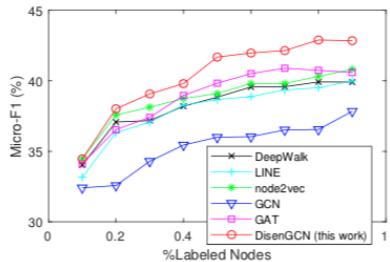
(a) Macro-F1(%), BlogCatalog.



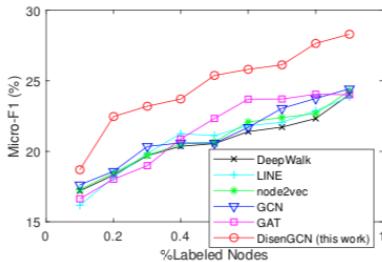
(c) Macro-F1(%), PPI.



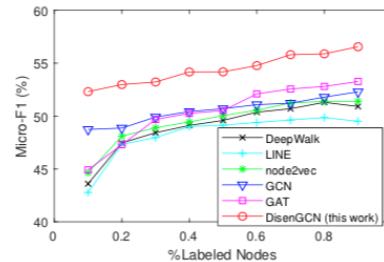
(e) Macro-F1(%), POS.



(b) Micro-F1(%), BlogCatalog.



(d) Micro-F1(%), PPI.



(f) Micro-F1(%), POS.

Figure 2. Macro-F1 and Micro-F1 scores on the multi-label classification tasks. Our approach consistently outperforms the best performing baselines by a large margin, reaching 10% to 20% relative improvement in most cases.

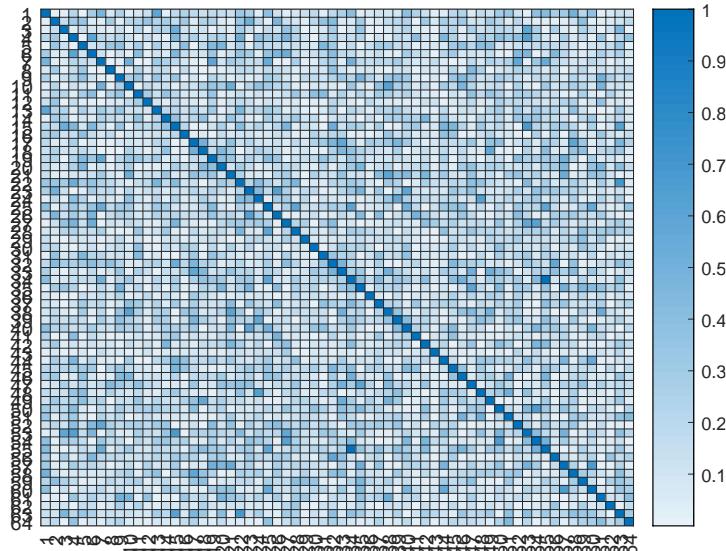
# Results: On Synthetic Graphs

Table 3. Micro-F1 scores on synthetic graphs generated with different numbers of latent factors.

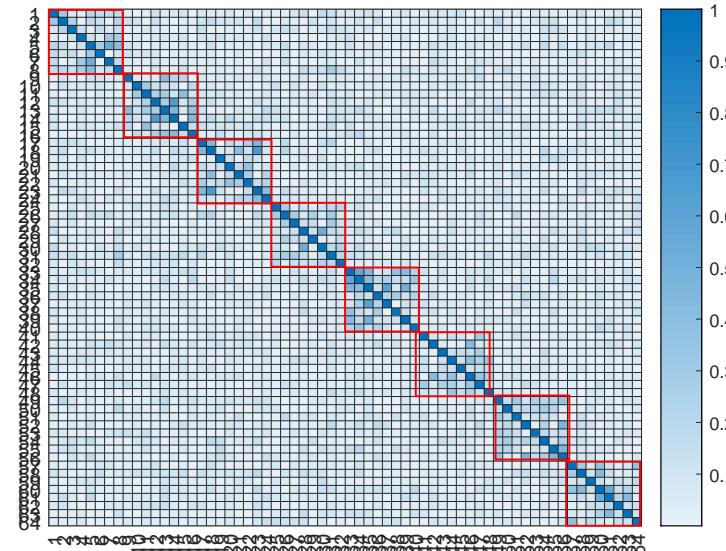
Method	Number of latent factors						
	4	6	8	10	12	14	16
GCN	78.78 ± 1.52	65.73 ± 1.94	46.55 ± 1.55	37.37 ± 1.52	24.49 ± 1.03	18.14 ± 1.50	16.43 ± 0.92
GAT	83.77 ± 2.32	60.89 ± 3.75	45.88 ± 3.79	36.72 ± 3.58	24.77 ± 3.47	20.89 ± 3.57	19.53 ± 3.97
DisenGCN (this work)	<b>93.84</b> ± 1.12	<b>74.68</b> ± 1.92	<b>54.57</b> ± 1.79	<b>43.96</b> ± 1.45	<b>28.17</b> ± 1.22	<b>23.57</b> ± 1.28	<b>21.99</b> ± 1.34
Relative improvement	+12.02%	+13.62%	+17.23%	+17.63%	+13.73%	+12.83%	+12.6%

- Improvement is larger when #factors is relatively large (around 8).
- However, all methods are bad when #factors is extremely large.

# Results: Correlations between the Neurons

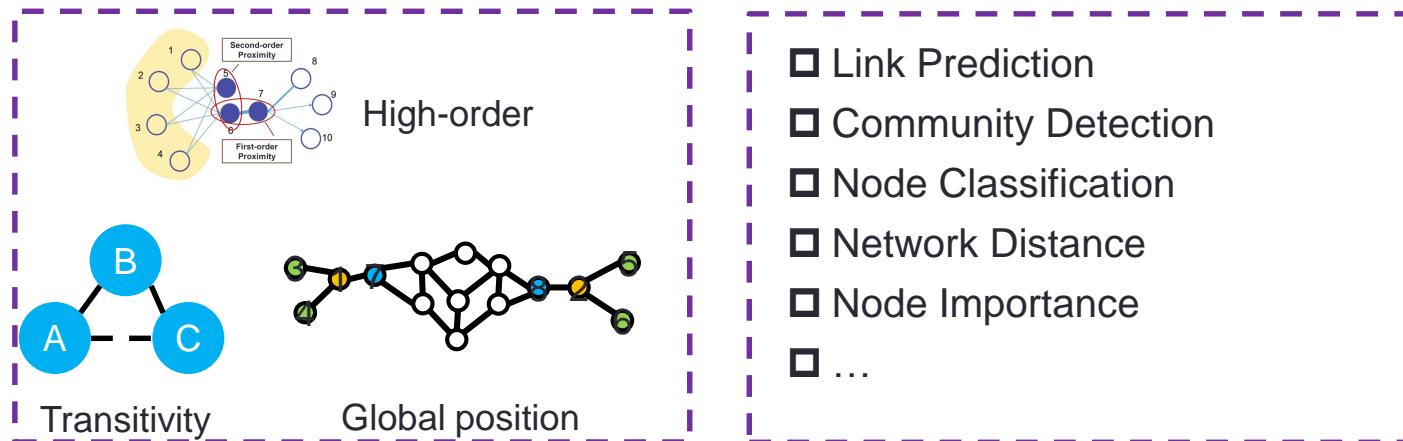


(a) GCN.



(b) DisenGCN (this work).

# Applicability of network embedding



Various network properties

Various applications

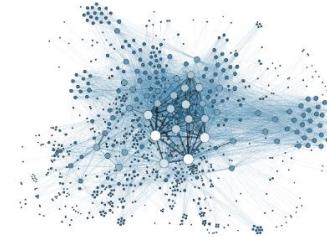


- Leading to **a large number** of hyperparameters
- Must be **carefully tuned**

AutoML

# AutoML

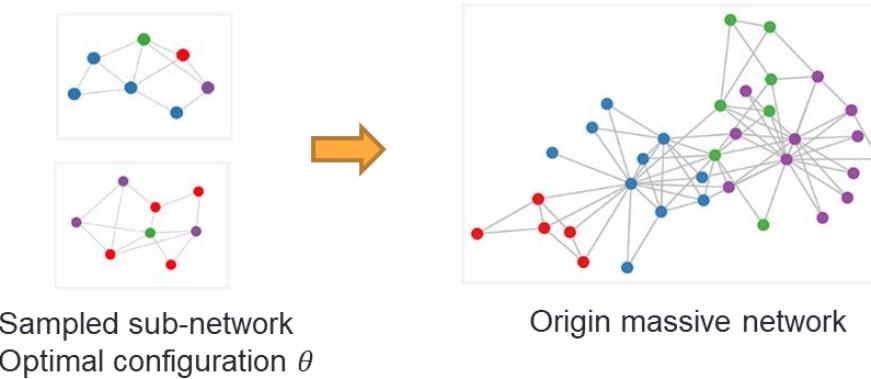
- Ease the adoption of machine learning and reduce the reliance on human experts
  - e.g., hyperparameter optimization
- Largely unexplored on **network** data
- **Large scale issue:**
  - Complexity of Network Embedding is usually at least  $O(E)$ 
    - E is the number of edges (can be 10 billion)
  - Total complexity:  $O(ET)$ , T is the times searching for optimal hyperparameters



How to incorporate AutoML into massive network embedding efficiently?

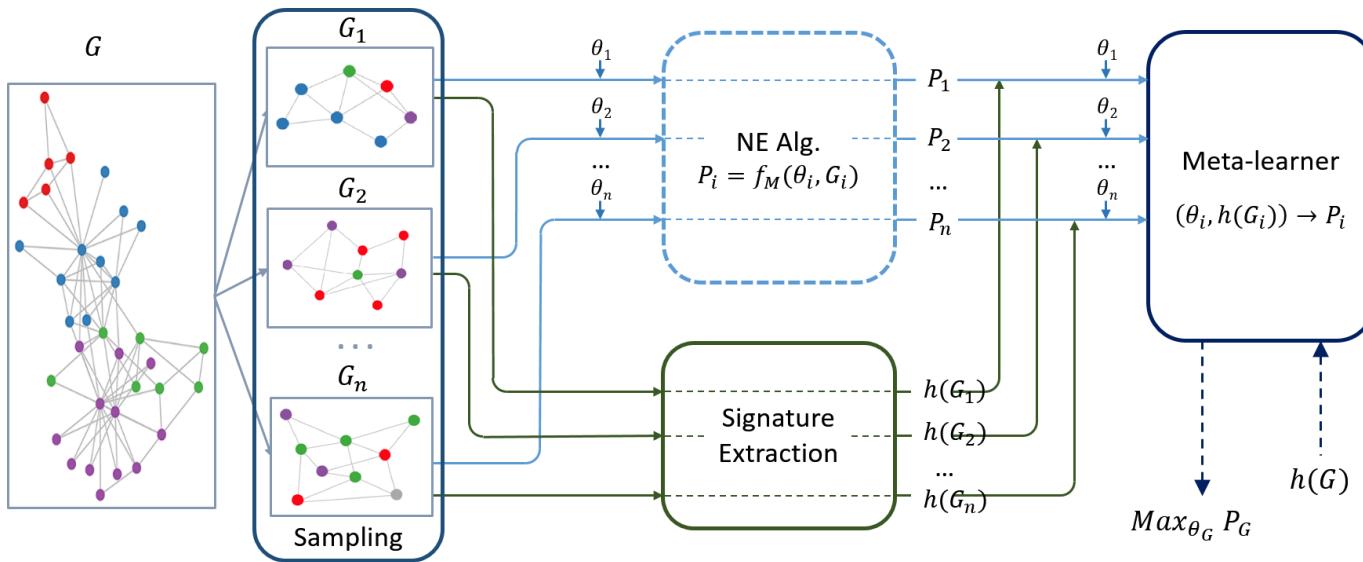
# AutoML for network embedding

- A straightforward way: configuration selection on sampled sub-networks



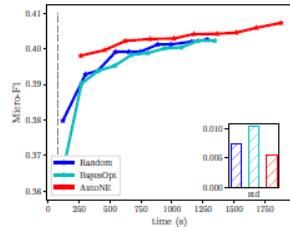
- Transferability
  - $\theta \neq$  optimal configuration on origin network
- Heterogeneity
  - several highly heterogeneous components => carefully designed sampling

# AutoNE

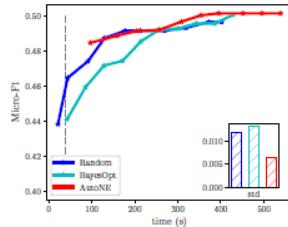


Transfer the knowledge about optimal hyperparameters  
from the sub-networks to the original massive network

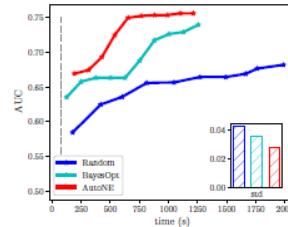
# Experiment --- Sampling-Based NE



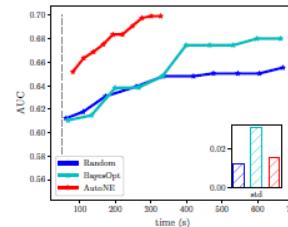
(a) Classification on BlogCatalog



(b) Classification on Wikipedia

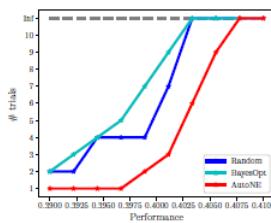


(c) Link prediction on BlogCatalog

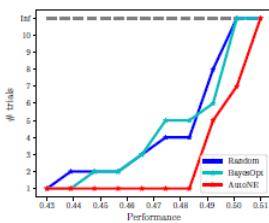


(d) Link prediction on Wikipedia

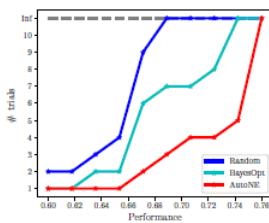
The performance achieved within various time thresholds.



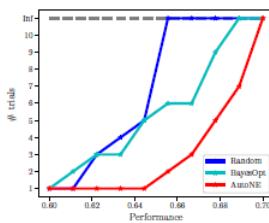
(a) Classification on BlogCatalog



(b) Classification on Wikipedia



(c) Link prediction on BlogCatalog



(d) Link prediction on Wikipedia

The number of trials to reach a certain performance threshold

# A Survey on Network Embedding

IEEE TRANSACTIONS ON  
**KNOWLEDGE AND  
DATA ENGINEERING**

**A Survey on Network Embedding**

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**ABSTRACT**

Network embedding assigns nodes in a network to low-dimensional representations and effectively preserves the network structure. Recently, a significant amount of progresses have been made toward this emerging network analysis paradigm. In this survey, we focus on categorizing and then reviewing the current development on network embedding methods, and point out its future research directions. We first summarize the motivation of network embedding. We discuss the classical graph embedding algorithms and their relationship with network embedding. Afterwards and primarily, we provide a comprehensive overview of a large number of network embedding methods in a systematic manner, covering the structure- and property-preserving network embedding methods, the network embedding methods with side information and the advanced information preserving network embedding methods. Moreover, several evaluation approaches for network embedding and some useful online resources, including the network data sets and softwares, are reviewed, too. Finally, we discuss the framework of exploiting these network embedding methods to build an effective system and point out some potential future directions.

Peng Cui, Xiao Wang, Jian Pei, Wenwu Zhu. **A Survey on Network Embedding.** *IEEE TKDE*, 2018.

# Deep Learning on Graphs: A Survey

## Deep Learning on Graphs: A Survey

Ziwei Zhang, Peng Cui and Wenwu Zhu

**Abstract**—Deep learning has been shown successful in a number of domains, ranging from acoustics, images to natural language processing. However, applying deep learning to the ubiquitous graph data is non-trivial because of the unique characteristics of graphs. Recently, a significant amount of research efforts have been devoted to this area, greatly advancing graph analyzing techniques. In this survey, we comprehensively review different kinds of deep learning methods applied to graphs. We divide existing methods into three main categories: semi-supervised methods including Graph Neural Networks and Graph Convolutional Networks, unsupervised methods including Graph Autoencoders, and recent advancements including Graph Recurrent Neural Networks and Graph Reinforcement Learning. We then provide a comprehensive overview of these methods in a systematic manner following their history of developments. We also analyze the differences of these methods and how to composite different architectures. Finally, we briefly outline their applications and discuss potential future directions.

**Index Terms**—Graph Data, Deep Learning, Graph Neural Network, Graph Convolutional Network, Graph Autoencoder.

### 1 INTRODUCTION

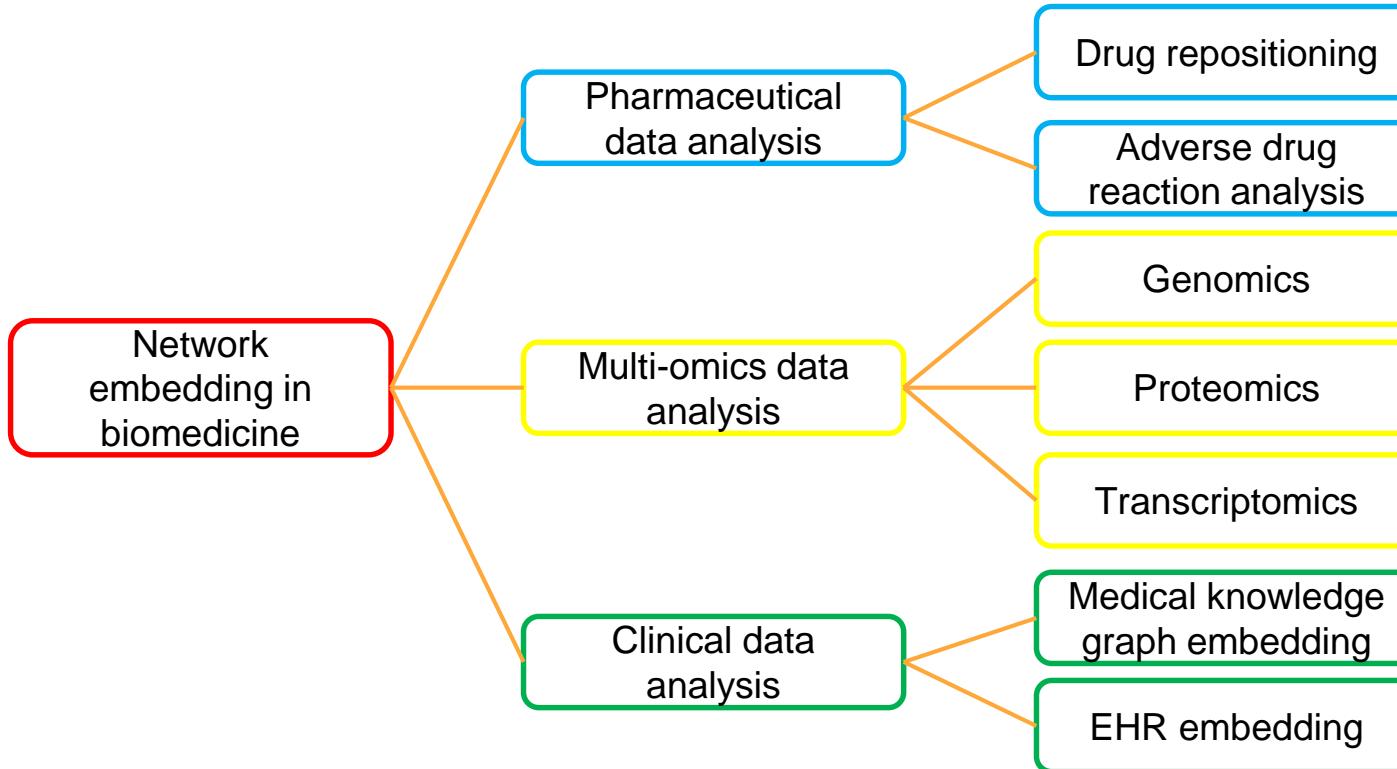
In the last decade, deep learning has been a “crown jewel” in artificial intelligence and machine learning [1], showing superior performance in acoustics [2], images [3] and natural language processing [4]. The expressive power of deep learning to extract complex patterns underlying data has been well recognized. On the other hand, graphs<sup>1</sup> are ubiquitous in the real world, repre-

- **Scalability and parallelization.** In the big-data era, real graphs can easily have millions of nodes and edges, such as social networks or e-commerce networks [8]. As a result, how to design scalable models, preferably with a linear time complexity, becomes a key problem. In addition, since nodes and edges in the graph are interconnected and often need to be modeled as a whole, how to conduct parallel computing is another critical issue.

# Outline

- **Structure-preserved network embedding**
- **Property-preserved network embedding**
- **Dynamic network embedding**
- **Robustness, Explainability and Applicability**
- **Network embedding for biomedical applications**

# Network Embedding for Biomedical Applications



# Pharmaceutical data analysis

## □ Drug repositioning

Exploring new usage for existing drugs.

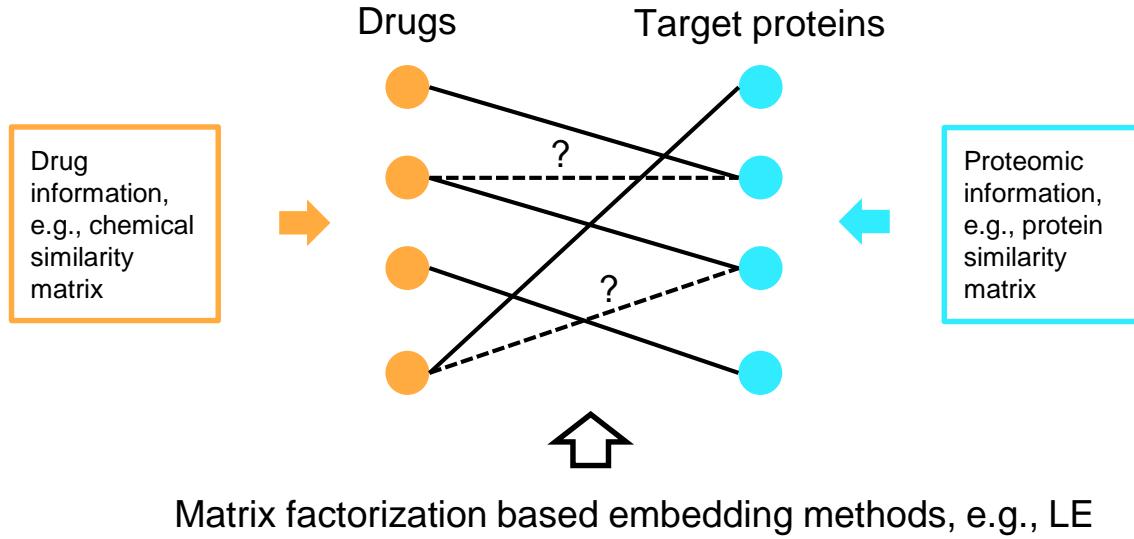
- save drug development cost
- increase productivity

Aiming at predicting:

- unknown drug-target interactions
- unknown drug-disease interactions

# Pharmaceutical data analysis

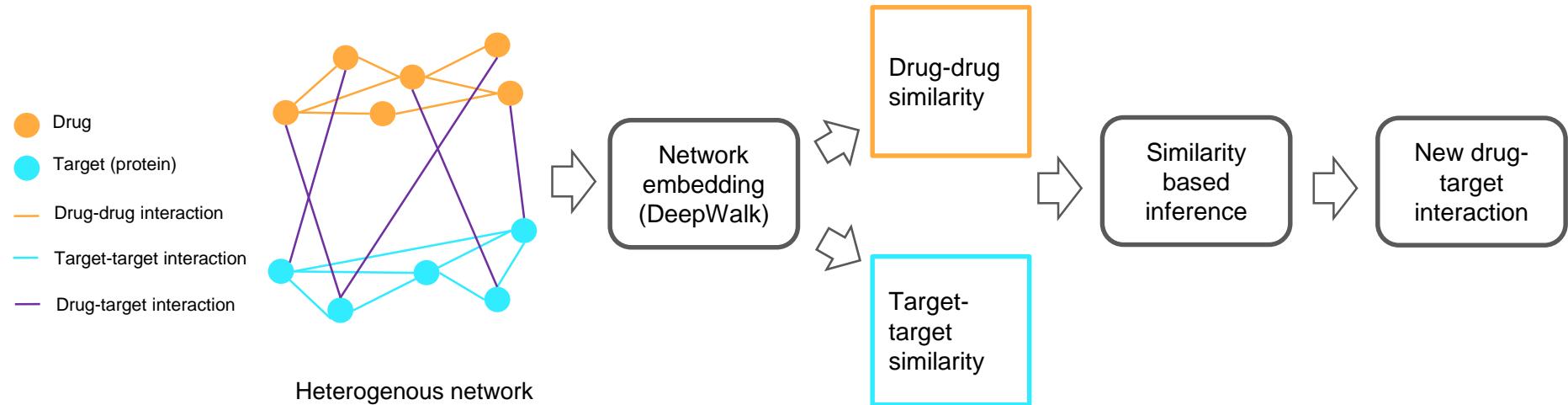
- Drug-target interaction prediction



1. Yamanishi Y, Araki M, Gutteridge A, et al. Prediction of drug–target interaction networks from the integration of chemical and genomic spaces. *Bioinformatics* 2008;24: i232–40.
2. Cobanoglu MC, Liu C, Hu F, et al. Predicting drug–target interactions using probabilistic matrix factorization. *J Chem Inf Model* 2013;53:3399–409.
3. Zheng X, Ding H, Mamitsuka H, Zhu S. Collaborative matrix factorization with multiple similarities for predicting drug–target interactions. *KDD '13*, 2013, pp. 1025–1033. Chicago, Illinois, USA.

# Pharmaceutical data analysis

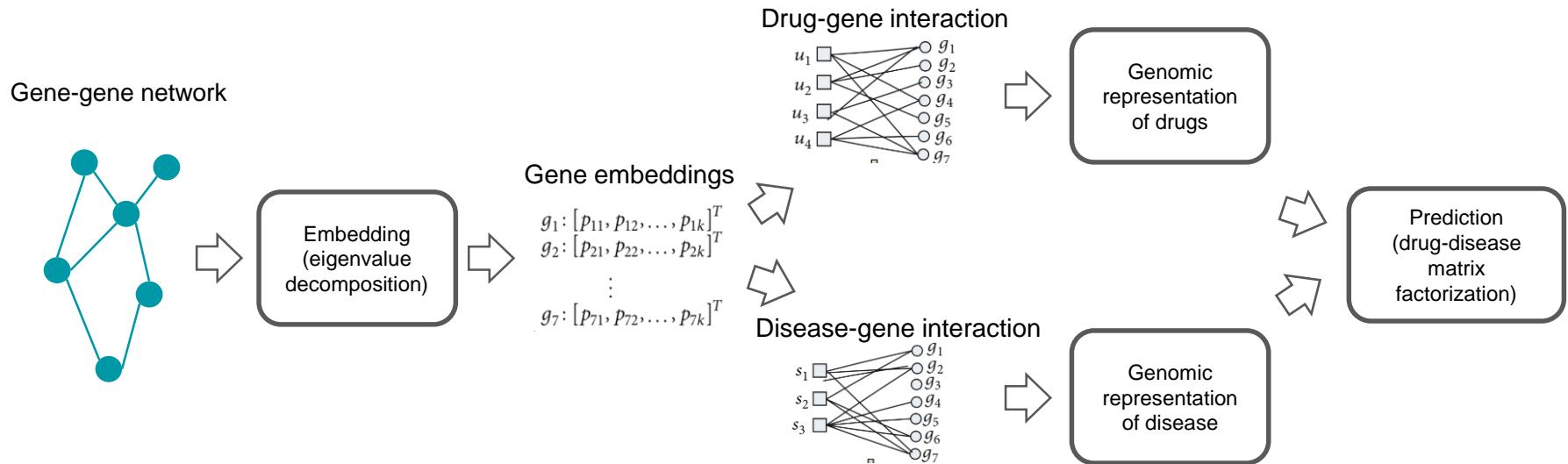
- Drug-target interaction prediction (Zong et al. 2017)



1. Zong N, Kim H, Ngo V, et al. Deep mining heterogeneous networks of biomedical linked data to predict novel drug–target associations. Bioinformatics 2017;33:2337–44.

# Pharmaceutical data analysis

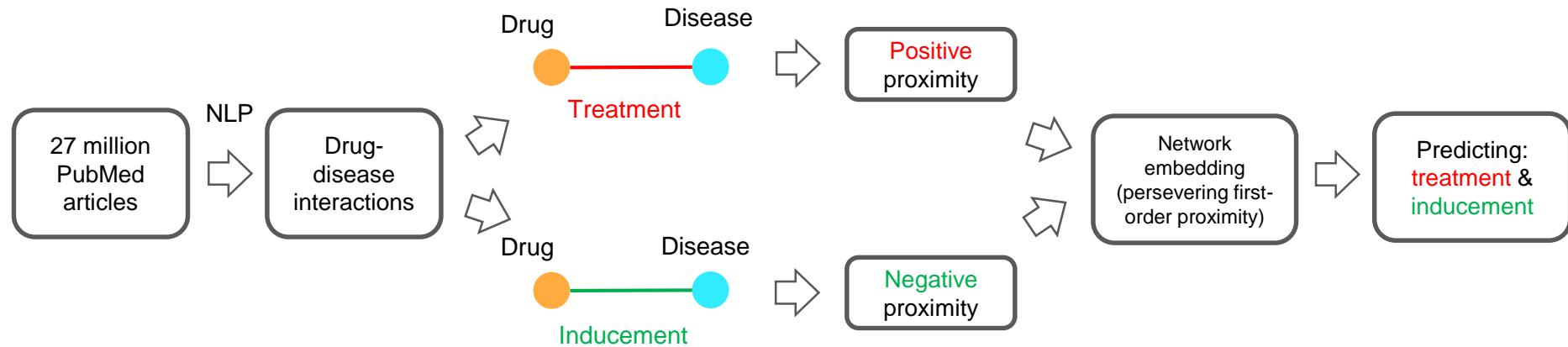
- Drug-disease interaction prediction (Dai et al. 2015)



1. Dai W, Liu X, Gao Y, et al. Matrix factorization-based prediction of novel drug indications by integrating genomic space. Comput Math Methods Med 2015;2015:275045.

# Pharmaceutical data analysis

- Drug-disease interaction prediction (Wang et al. 2017)



1. Wang P, Hao T, Yan J, et al. Large-scale extraction of drug– disease pairs from the medical literature. *J Assoc Inf Sci Technol* 2017;68:2649–61.

# Pharmaceutical data analysis

## □ Adverse drug reaction analysis

An adverse drug reaction (ADR) is defined as any undesirable effect from the medical use of drugs beyond its anticipated therapeutic effects that occurs at a usual dosage.

The ADR study is implemented before a drug is launched on clinical application.

- Adverse drug reaction (ADR) prediction
- Drug-drug interaction (DDI) prediction

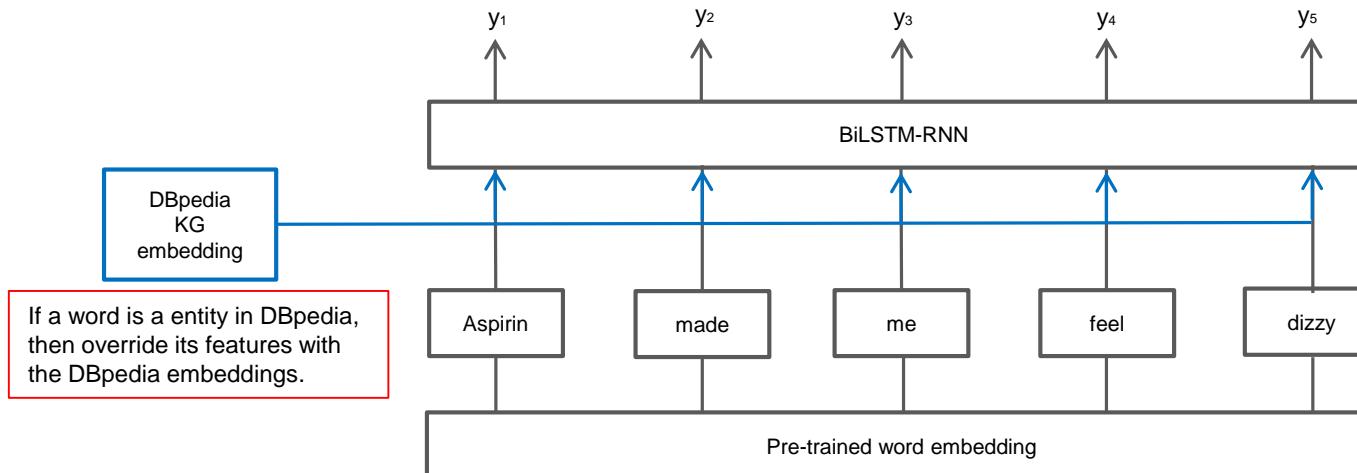
# Pharmaceutical data analysis

- Adverse drug reaction (ADR) prediction (Stanovsky et al. 2017)

Identify ADR from social media posts:

Output labels of each word, B, I, and O:

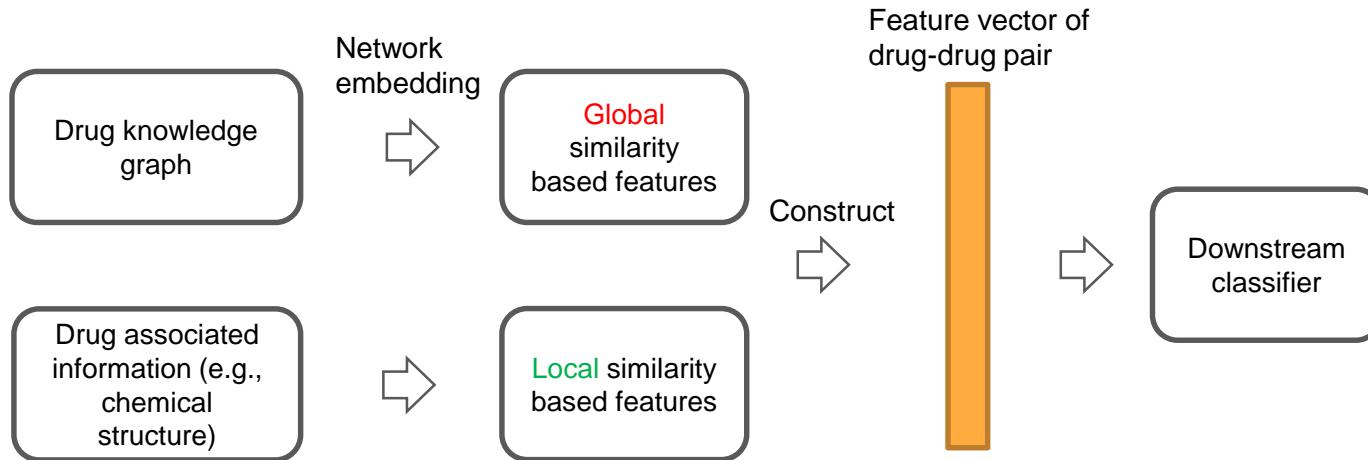
(B) Beginning of an ADR span; (I), Inside an ADR span; (O), Out-side of the span of an ADR



1. Stanovsky G, Gruhl D, Mendes P. Recognizing mentions of adverse drug reaction in social media using knowledge- infused recurrent models. In: Proceedings of the 15th Conference of the European Chapter of the Association for Computational Linguistics, 2017, pp. 142–51. Valencia, Spain.

# Pharmaceutical data analysis

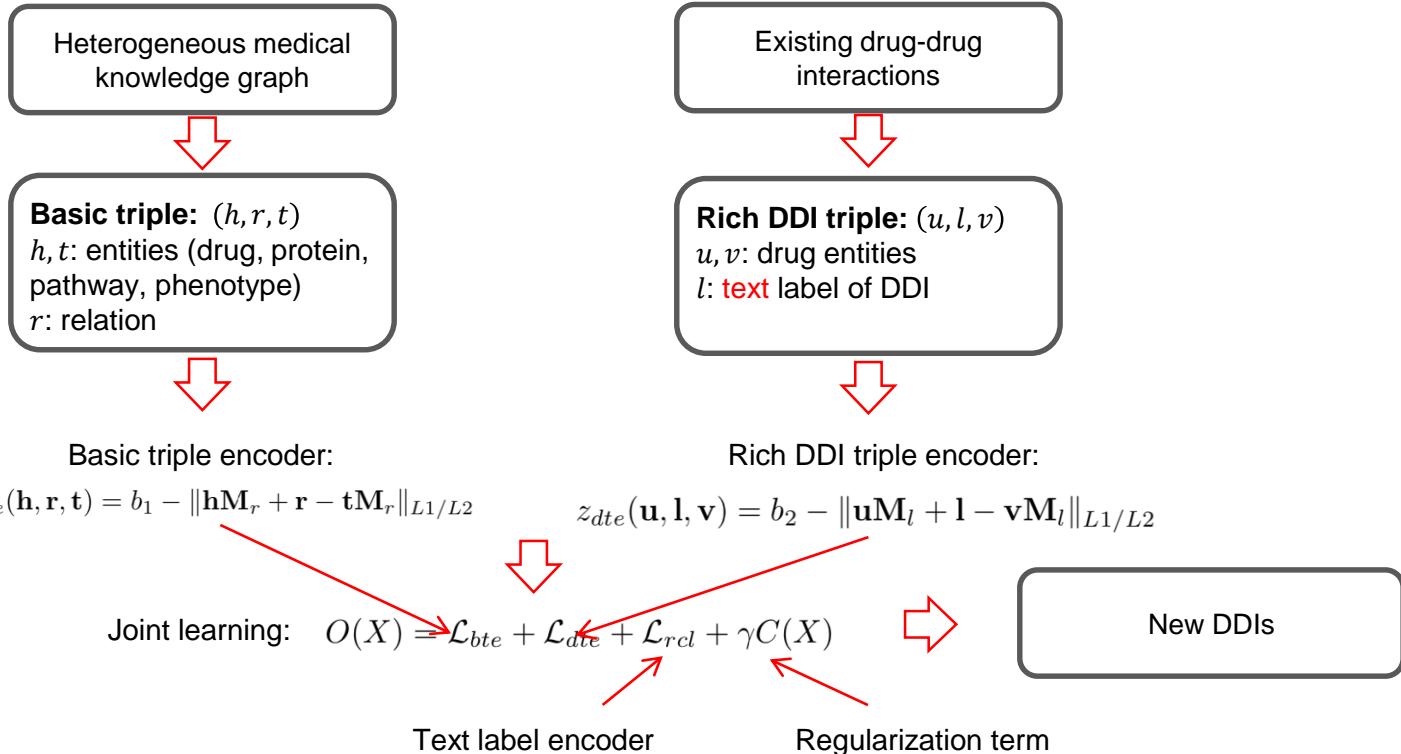
- Drug-drug interaction (DDI) prediction (Abdelaziz et al. 2017)



1. Abdelaziz I, Fokoue A, Hassanzadeh O, et al. Large-scale structural and textual similarity-based mining of knowledge graph to predict drug-drug interactions. *Web Semant* 2017;44:104–17.

# Pharmaceutical data analysis

- Drug-drug interaction (DDI) prediction (Wang et al. 2017)



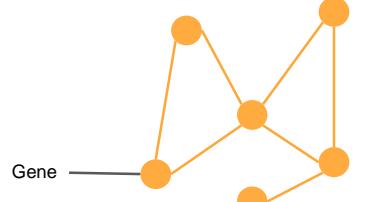
1. Wang M, Chen Y, Qian B, et al. Predicting rich drug–drug interactions via biomedical knowledge graphs and text jointly embedding, 2017, arXiv preprint arXiv:171208875.

# Multi-omics data analysis

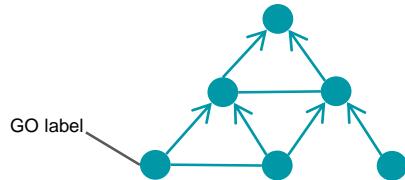
## □ Genomics data analysis

- Gene function prediction (Wang et al. 2015)

Molecular network

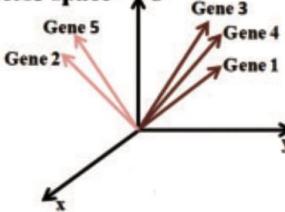


Gene ontology (GO)



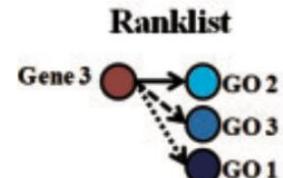
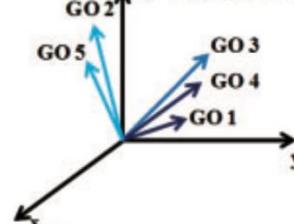
Network embedding  
(random walk based)

Gene vector space

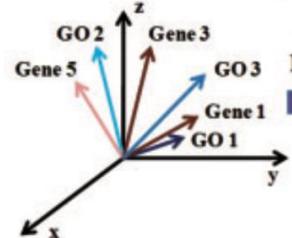


Project  $y^* = wx$

GO label vector space

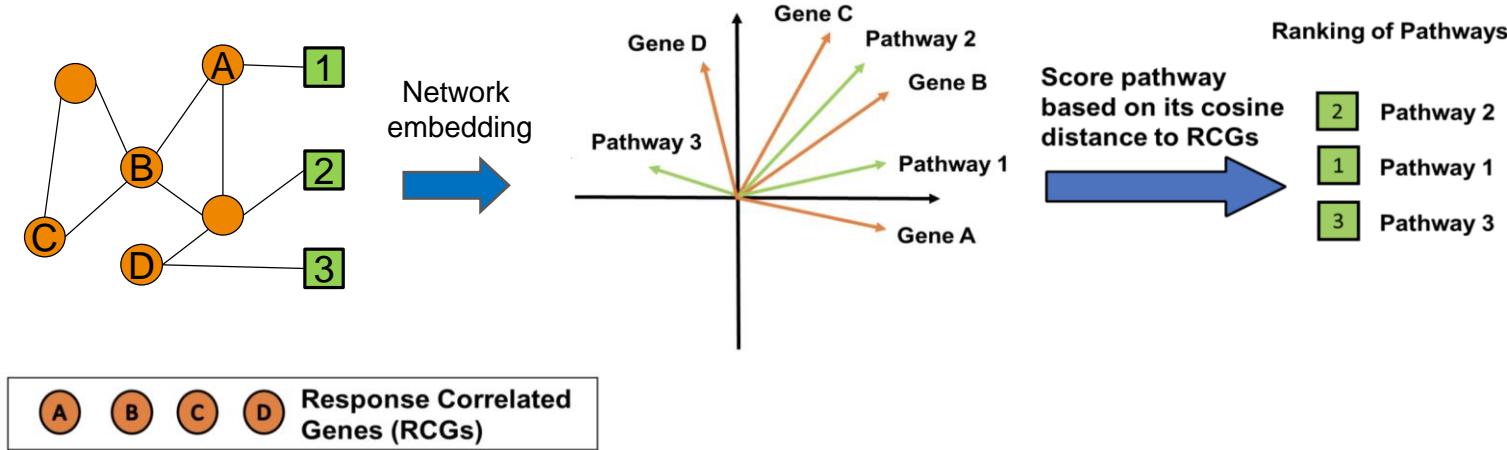


Predict



# Multi-omics data analysis

- Identification of Pathways Associated with Chemosensitivity (Wang et al. 2017)



1. Wang S, Huang E, Cairns J, et al. Identification of pathways associated with chemosensitivity through net-work embedding. 2017, bioRxiv preprint bioRxiv: 168450 doi:10.1101/168450

# Multi-omics data analysis

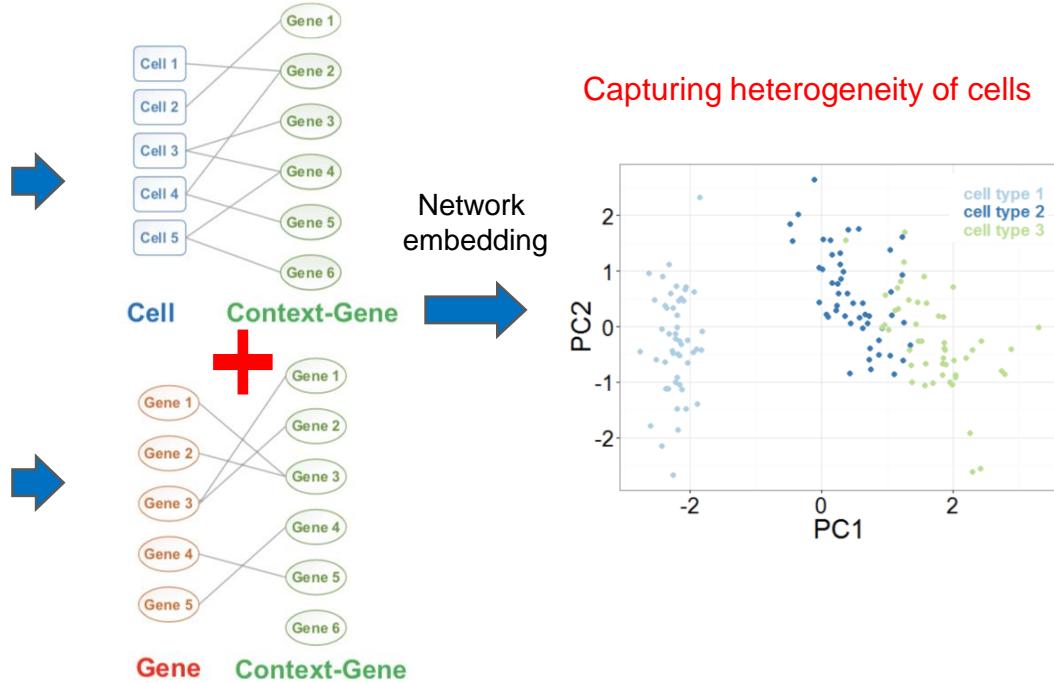
- Cell and gene representation (Li et al. 2017)

scRNA-seq data

	Gene1	Gene2	Gene3	Gene4	Gene5	Gene6
Cell1	0	5	0	0	0	0
Cell2	3	0	0	0	0	0
Cell3	0	0	100	50	0	0
Cell4	0	56	0	0	10	0
Cell5	0	0	0	99	0	10
	—					

Pathway data

	Gene1	Gene2	Gene3	Gene4	Gene5
Gene1	0	0	1	0	0
Gene2	0	0	1	0	0
Gene3	1	1	0	0	0
Gene4	0	0	0	0	1
Gene5	0	0	0	1	0
	—				



1. Wang S, Huang E, Cairns J, et al. Identification of pathways associated with chemosensitivity through net-work embedding. 2017, bioRxiv preprint bioRxiv: 168450 doi:10.1101/168450

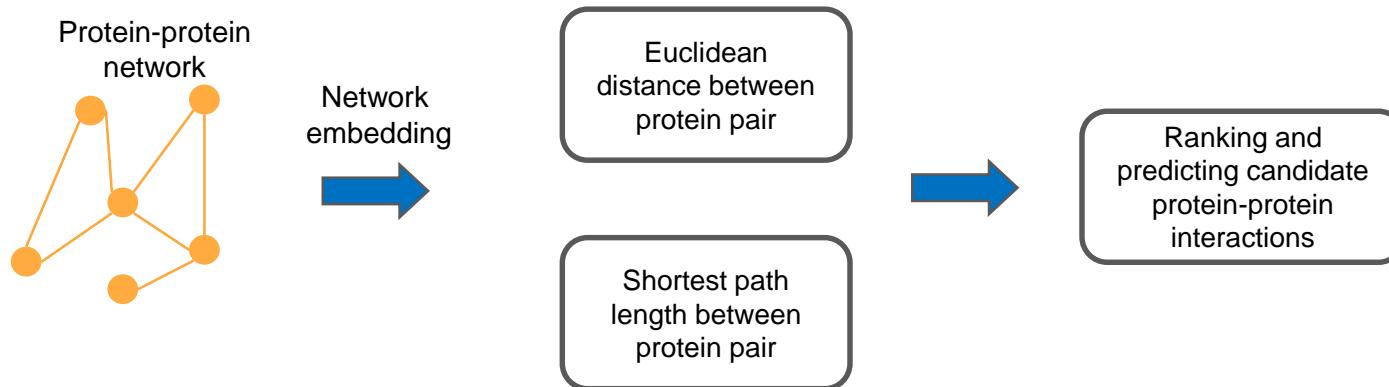
# Multi-omics data analysis

## □ Proteomics data analysis

- Protein-protein interaction (PPI) prediction (Cannistraci et al. 2013)

Experimental identification of PPIs is time-consuming.

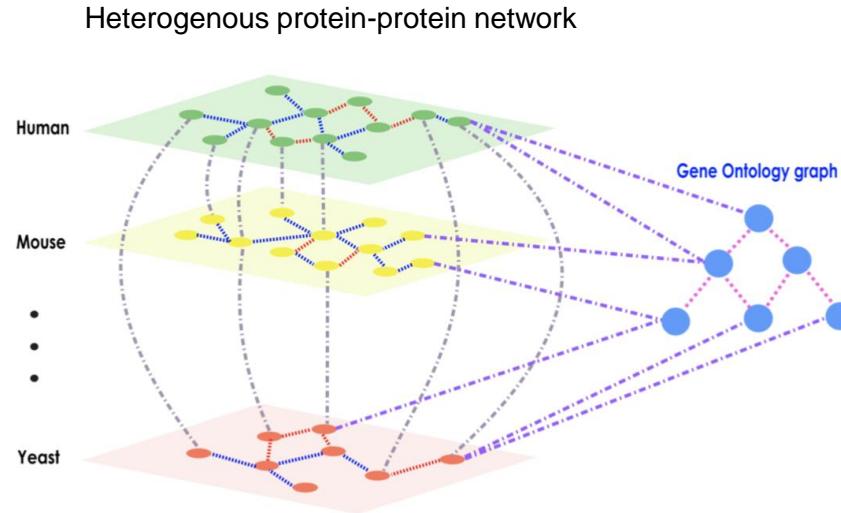
Computational method was proposed to predict candidate PPIs based on existing interactions.



1. Cannistraci CV, Alanis-Lobato G, Ravasi T. Minimum curvilinearity to enhance topological prediction of protein interactions by network embedding. Bioinformatics 2013;29: i199–209.

# Multi-omics data analysis

- Protein function prediction (Wang et al. 2017)



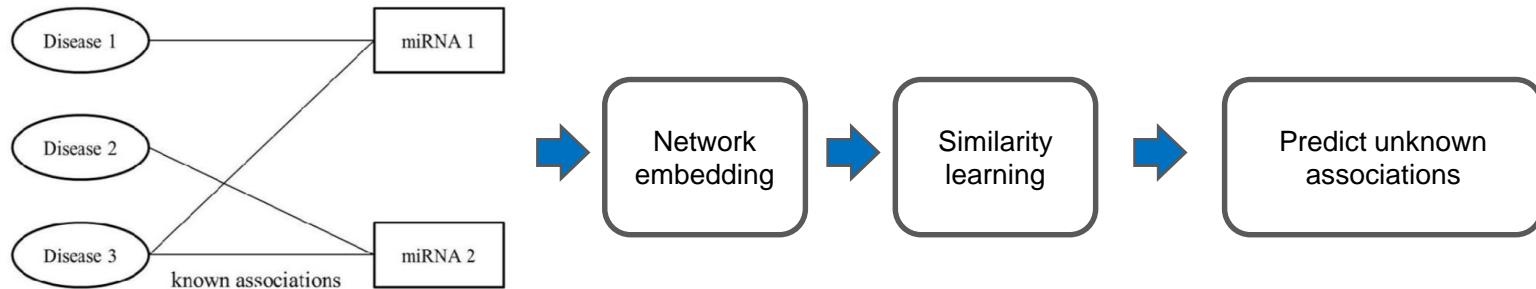
1. Construct protein-protein network;
2. Learn low-dimensional representations for proteins;
3. Calculate an intra-species affinity score and an inter-species affinity score by transferring Gene annotations
4. Rank the score and pick the function(s) with the highest score(s) for queried protein (s)

1. Wang S, Qu M, Peng J. Prosnet: integrating homology with molecular networks for protein function prediction. Pac Symp Biocomput 2017;22:27-38.

# Multi-omics data analysis

## □ Transcriptomics data analysis

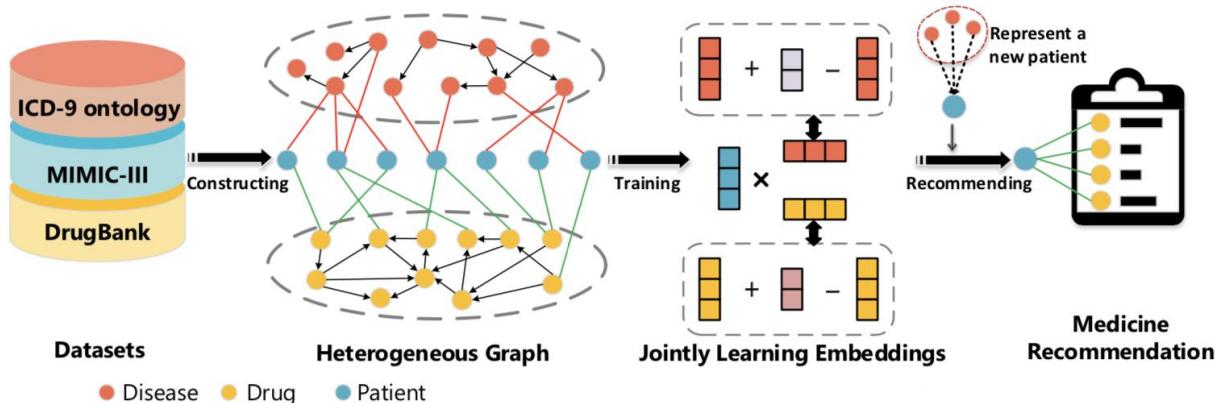
- MicroRNA-disease association prediction (Li et al. 2017)
  - microRNAs (miRNAs) are connected with several complex human diseases;
  - Identifying human disease-related miRNAs will be useful in uncovering novel prognostic markers for cancer.



1. Li G, Luo J, Xiao Q, et al. Predicting microRNA-disease associations using network topological similarity based on DeepWalk. IEEE Access 2017;5:24032–9.

# Medical data analysis

- Medical knowledge graph embedding
  - Safe medication recommendation (Wang et al. 2017) – predicting patient-medicine association



$$\mathcal{L}(X) = \mathcal{L}_{G_m} + \mathcal{L}_{G_d} + \mathcal{L}_{G_{pm}} + \mathcal{L}_{G_{pd}} + \gamma C(X)$$

Regularization term

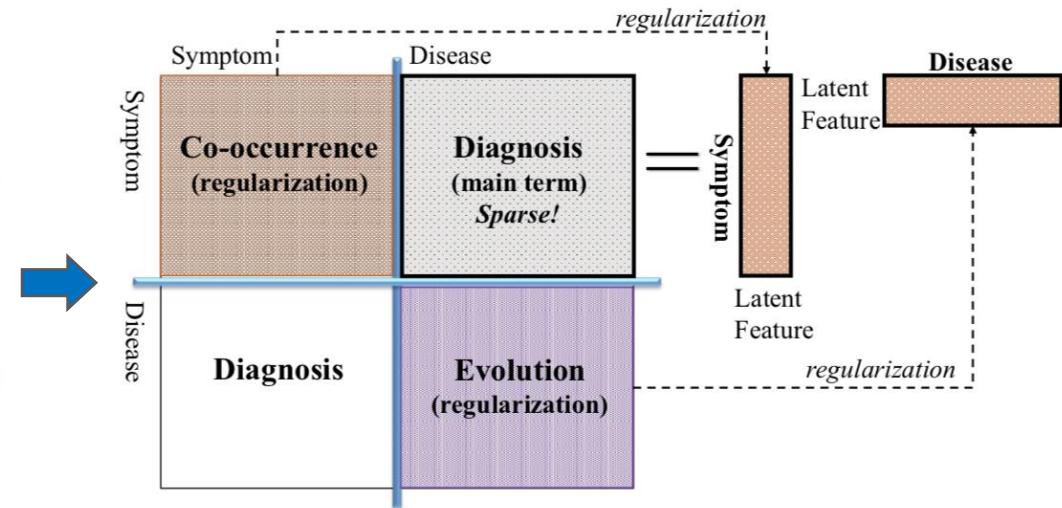
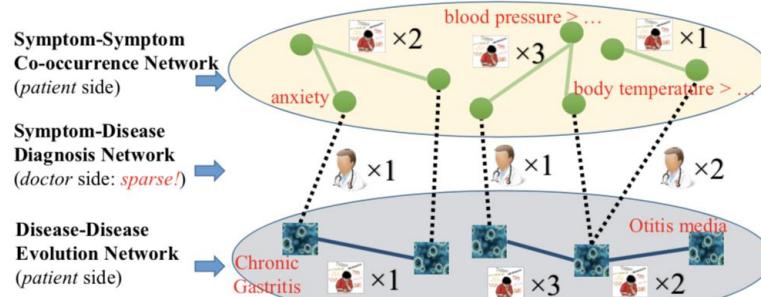
Medicine network embedding      Disease network embedding      Patient-medicine network embedding      Patient-disease network embedding

1. Wang M, Liu M, Liu J, et al. Safe medicine recommendation via medical knowledge graph embedding, 2017, arXiv preprint arXiv:171005980

# Medical data analysis

- Representation learning on medical forum data (Zhao et al. 2017)

Learning low-dimensional representations of diseases and symptoms.

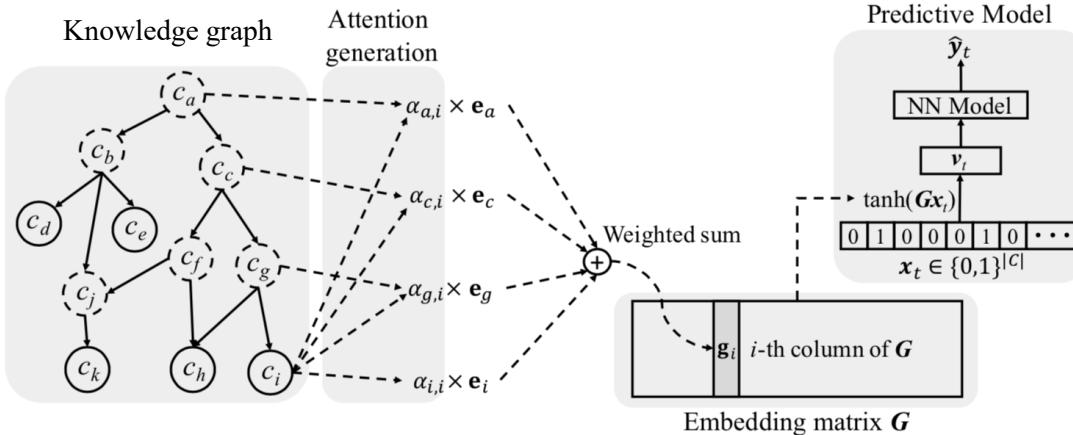


1. Zhao S, Jiang M, Yuan Q, et al. ContextCare: incorporating contextual information networks to representation learning on medical forum data. In: Proceedings of the 26th International Joint Conference on Artificial Intelligence, 2017, pp. 3497–503. AAAI Press, Melbourne, Australia.

# Medical data analysis

## □ EHR embedding

- Healthcare representation learning (Choi et al. 2017)
  - EHR representation with hierarchical information inherent to medical ontologies;
  - The model represents a medical concept as a combination of its ancestors in the ontology via an attention mechanism.



Node: medical concept

Attention is assigned to the node (medical concept) embedding:

$$g_i = \sum_{j \in \mathcal{A}(i)} \alpha_{ij} e_j, \quad \sum_{j \in \mathcal{A}(i)} \alpha_{ij} = 1, \quad \alpha_{ij} \geq 0$$

$$\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_t = \tanh(\mathbf{G}[\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_t]),$$

$$\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_t = \text{RNN}(\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_t, \theta_r),$$

$$\hat{\mathbf{y}}_t = \hat{\mathbf{x}}_{t+1} = \text{Softmax}(\mathbf{W}\mathbf{h}_t + \mathbf{b}),$$

1. Choi E, Bahadori MT, Song L, et al. GRAM: graph-based attention model for healthcare representation learning. In: Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2017, pp. 787–95. ACM, Halifax, Nova Scotia, Canada.

# Network Embedding for Biomedical Applications

## □ Challenges

- Data quality. Networks constructed from the biomedical data are usually noisy and incomplete. For example, the PPI data produced by high-throughput techniques, suffer from high false negative rates up to 70% and high false positive rates up to 64%.
- Local and global. Network embedding and its downstream tasks rely on the type of structural property to preserve. Designing embedding method by properly considering local and global structure properties require the development of novel solutions.
- Network evolution. Networks are always not static, especially in the biomedical domain. Yet most existing embedding models focus on static network.
- Domain complexity. Network structure is highly associated with domain knowledge.

# Network Embedding for Biomedical Applications

## ❑ Opportunities

- Local and global trade-off embedding.
- Dynamic network embedding.
- Text associated embedding. Medical knowledge bases always contain rich text information such as descriptions of entities and relations, which would have high potential to address network incompleteness and improve understanding of topological properties.
- Domain-knowledge-associated embedding. Incorporating external domain knowledge into network embedding.

# Summary and Conclusion

- **Structure-preserved network embedding**
  - **Guarantee information equivalence**
- **Property-preserved network embedding**
  - **Enabling network inference in embedding space**
- **Dynamic network embedding**
  - **Incorporating dynamic changes**
- **Robustness, Explainability and Applicability**
  - **Promote network embedding in real applications**
- **Network embedding for biomedical applications**
  - **Prove the effectiveness of NE in real applications**