Introduction to Logic for Computer Science Spring 2023

Assignment 1 (Deadline: 6:00pm on 31 March 2023)

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Submit your solutions in KLMS. (Reminder: We adopt a very strict policy for handling dishonest behaviours. If a student is found to copy answers from fellow students or other sources in his or her homework submission, she or he will get F.)

Notation: symbols F, G, H denote propositional formulas, and p, q denote propositional variables.

1. Let F, G and H be formulas and let $\mathcal{S}$ be a set of formulas. Let $p_1, \ldots, p_n$ be propositional variables. Which of the following statements are true? Justify your answer. **(10 points)**

   (a) If F is unsatisfiable, then $\neg F$ is valid.

   (b) If $F \to G$ is satisfiable and F is satisfiable, then G is satisfiable.

   (c) $p_1 \to (p_2 \to (p_3 \to \ldots (p_n \to p_1) \ldots))$ is valid.

   (d) $\mathcal{S} \models F$ and $\mathcal{S} \models \neg F$ cannot both hold.

2. John and Sue are hosting a dinner party for a few friends. They receive the following response to their invitations:

   (a) If Carol attends, then she brings Ben also.

   (b) At least one of the twins Paul and Anne attend.

   (c) Either Ben or James attend, but not both.

   (d) Either James and Anne both come or they both do not.

   (e) If Paul attends, then so do Anne and Carol.

   Formalise the above statements in propositional logic and use this to conclude the possible combinations of guests. **(20 points)**

3. Suppose that F and G are formulas such that $F \models G$.

   (a) Show that if F and G have no variable in common, then either F is unsatisfiable or G is valid. **(10 points)**

   (b) Now let F and G be arbitrary formulas. Show that there is a formula H, mentioning only propositional variables common to F and G, such that $F \models H$ and $H \models G$. **(20 points)**

4. A **perfect matching** in an undirected graph $G = (V, E)$ is a subset of the edges $M \subseteq E$ such that every vertex $v \in V$ is an endpoint of exactly one edge in M. Given a finite graph G, describe how to obtain a propositional formula F_G such that F_G is satisfiable if and only if G has a perfect matching. The formula F_G should be computable from G in time polynomial in $|V|$. **(10 points)**

5.

(a) Write down a **DNF**-formula equivalent to $(p_1 \vee q_1) \wedge (p_2 \vee q_2) \wedge \cdots \wedge (p_n \vee q_n)$. Here the p_i and q_i are propositional variables. **(10 points)**

(b) Prove that any **DNF**-formula equivalent to the above formula must have at least 2^n clauses. **(20 points)**