Lecture 6
The DPLL Algorithm

Introduction to Logic for Computer Science

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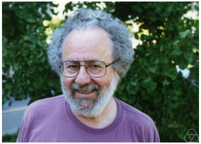
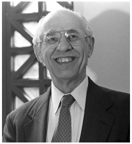
These slides are minor variants of those made by Prof Worrell and Dr Haase for their logic course at Oxford.

Davis–Putnam–Logemann–Loveland

DPLL algorithm:

- Combines search and deduction to decide satisfiability.
- Underlies most modern SAT solvers.
- Over 50 years old.

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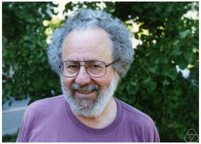
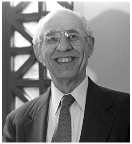
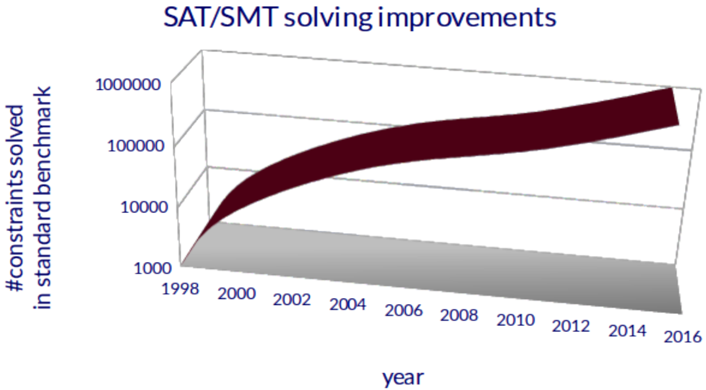



Dramatic progress of DPLL-based SAT solvers since 1990.

Emerged enhancement:

- **Clause learning**.
- **Non-chronological backtracking**.
- Branching heuristics.
- Lazy evaluation.

Performance increase of SAT solvers



DPLL: idea

Depth-first search.

At every unsuccessful leaf of search tree (called **conflict**), use resolution to compute a **conflict clause**.

Add the clause to the formula we're deciding about.

Think of conflict clauses as "caching" previous search results. So we "learn from previous mistakes".

Conflict clauses also determine backtracking.

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   - Sequence of assignments $\langle p_1 \mapsto b_1, \ldots, p_r \mapsto b_k \rangle$ where $p_1, \ldots, p_k$ are distinct prop. variables and $b_1, \ldots, b_k \in \{0, 1\}$.
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   - $p_i \mapsto b_i$ may be annotated with a clause and other info.
3. Specialisation $F|_{\mathcal{A}}$.
   - F simplified under $\mathcal{A}$.
   - Delete any clause containing the true literal under $\mathcal{A}$, and remove from each remaining clause the false literal under $\mathcal{A}$.

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Input: CNF formula F. **Output:** Satisfying valuation $\mathcal{A}$ or UNSAT.

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5. Use a **decision strategy** to determine a new assignment $p \mapsto b$. Add it to $\mathcal{A}$. Go to step 2.

Example run with $\{\{\neg p_1\}, \{p_1, p_2, \neg p_3\}, \{p_3, p_4, p_5\}, \{p_4, \neg p_5\}\}$.

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Unit propagation – deduction step.

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Decision – search step.

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Clause learning – deduction step.

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A **state** of algorithm is a pair of CNF formula F and valuation $\mathcal{A}$.

Successful state when $\mathcal{A} \models F$.

Conflict state when $\mathcal{A} \models \neg F$.

Note: Conflict state if $F|_{\mathcal{A}} \ni \square$. Successful state if $F|_{\mathcal{A}} = \emptyset$.

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Each assignment $p_i \mapsto b_i$ in $\mathcal{A}$ classified as **decision assignment** or **implied assignment**.

$p_i \mapsto b_i$ by a decision strategy (step 5) is a **decision assignment**. p_i called **decision variable**.

$p_i \mapsto b_i$ by a unit propagation on a clause C (step 2) is an **implied assignment**. Denoted by $p_i \overset{C}{\mapsto} b_i$.

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Unit propagation.

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Example: start with set of clauses $F = \{C_1, \ldots, C_5\}$, where

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\begin{aligned}
C_1 &= \{\neg p_1, \neg p_4, p_5\}, \\
C_2 &= \{\neg p_1, p_6, \neg p_5\}, \\
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Let the current valuation is $\mathcal{A} = \langle p_1 \mapsto 1, p_2 \mapsto 0, p_3 \mapsto 0, p_4 \mapsto 1\rangle$.

Notice that $F|_{\mathcal{A}}$ contains the unit clause $\{p_5\}$.

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Answer: Unit propagation generates $p_5 \overset{C_1}{\mapsto} 1, p_6 \overset{C_2}{\mapsto} 1, p_7 \overset{C_3}{\mapsto} 1$. This leads to conflict, with C_4 being made false.

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Clause learning via conflict analysis.

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Ex: How can we find such C from a conflicted state $(F, \mathcal{A})$?

Clause learning algorithm

Suppose $\mathcal{A} = \langle p_1 \mapsto b_1, \ldots, p_k \mapsto b_k \rangle$ leads to conflict.

Find associated clauses $D_1, \ldots, D_{k+1}$ by backward induction:

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Ex1: Run this algorithm on the example in the unit prop. slide.

Ex2: Prove the proposition.

Clause learning: example

In conflict of above example, learning generates clauses

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\begin{aligned}
D_8 &:= \{\neg p_1, \neg p_7, \neg p_5\} && \text{(clause } C_4\text{)} \\
D_7 &:= \{\neg p_1, \neg p_5, \neg p_6\} && \text{(resolve } D_8,\ C_3\text{)} \\
D_6 &:= \{\neg p_1, \neg p_5\} && \text{(resolve } D_7,\ C_2\text{)} \\
D_5 &:= \{\neg p_1, \neg p_4\} && \text{(resolve } D_6,\ C_1\text{)} \\
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The learned clause D_1 is a conflict clause with only decision variables, including the top-level one p_4.

D_1 records that conflict arose from decision to make p_1, p_4 true.

Adding D_1 makes assignments validating p_1, p_4 unreachable.

Backtracking to the highest level where D_1 is a unit clause ($p_1 \mapsto 1$) and doing unit propagation lead to $p_4 \mapsto 0$.

Example: 4 queens

Problem: place 4 non-attacking queens on a 4x4 chess board.

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Variable p_{ij} models that there is a queen in the square (i, j).

- ≥ 1 in each row: $\bigwedge_{i=1}^{4} \bigvee_{j=1}^{4} p_{ij}$.
- ≤ 1 in each row: $\bigwedge_{i=1}^{4} \bigwedge_{j \neq j'=1}^{4} (\neg p_{ij} \vee \neg p_{ij'})$.
- ≤ 1 in each column: $\bigwedge_{j=1}^{4} \bigwedge_{i \neq i'=1}^{4} (\neg p_{ij} \vee \neg p_{i'j})$.
- ≤ 1 on each diagonal:
  $\bigwedge_{i,j=1}^{4} \bigwedge_{k} (\neg p_{i,j} \vee \neg p_{i+k,j+k}) \wedge \bigwedge_{i,j=1}^{4} \bigwedge_{l} (\neg p_{i,j} \vee \neg p_{i-l,j+l})$

Total number of clauses: 4 + 24 + 24 + 28 = 80.

DPLL: 4 queens

Running the DPLL algorithm:

- Start with $p_{11} \mapsto 1$. Then,
  1) delete $\{p_{11}, p_{12}, p_{13}, p_{14}\}$;
  2) delete $\neg p_{11}$ and generate 9 new unit clauses;
  3) delete 65 clauses subsequently!

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- Next, set $p_{23} \mapsto 1$. Then,
  1) generate 4 new unit clauses: $\{\neg p_{24}\}, \{\neg p_{43}\}, \{\neg p_{32}\}, \{\neg p_{34}\}$;
  2) through the unit propagation of $\{\neg p_{34}\}$, reach UNSAT.

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- Fixing only two literals collapsed 80 clauses to 1 and ruled out 2^{14} of 2^{16} possible assignments!

- Backtrack and set $\langle p_{11} \mapsto 0, p_{12} \mapsto 1\rangle$. Then,
  1) Delete $\{\neg p_{12}\}$ and generate 9 new unit clauses;
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- Answer: $p_{12}, p_{24}, p_{31}, p_{43} \mapsto 1$.

Summary

Resolution:

- Very simple sound and complete proof calculus.

Davis–Putnam algorithm:

- Uses resolution to decide SAT via variable elimination.
- But may generate huge intermediate formulas.

DPLL algorithm:

- Improves resolution with clause learning and backtracking.
- Efficient.
- Basis for modern SAT solvers.