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LARGE BUCKLING DEFORMATIONS OF GENERAL LAMINATES

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ABSTRACT

The paper presents the nonlinear buckling analysis of general asymmetric laminated columns subjected to combined compression and bending. Using geometrically nonlinear theory, the buckling response of regular and irregular asymmetric columns is examined. It is shown that, under certain conditions, asymmetric columns tend to exhibit bifurcation due to the effects produced by bending-stretching couplings. The potentials of these effects in terms of improving the performance characteristics of laminated structures are discussed.

INTRODUCTION

Composite laminates with asymmetric material lay-ups have been rarely considered as a viable option in structural design primarily because their behavior has proved to be more complex and less predictable than that of symmetric laminated structures. In fact, the response of asymmetric composites is complicated by the effects of mechanical couplings that cause, for example, bending due to membrane forces, elongation or contraction due to shear flow, or twisting due to bending moments. Typically, the problems introduced by such effects are not readily amenable to conventional engineerings analyses.

In recent years, however, new emerging technologies have increased the demand for materials with novel characteristics, and many modern approaches to design have relied on the use of unique materials specifically tailored for optimum performance in the intended applications. These technological trends have stimulated the resurgence of interest in composite materials, and have modified the traditional perception regarding the functional potentials of asymmetric laminates.

A number of problems of immediate practical interest arise from the buckling response of asymmetric laminated structures. Theoretical and experimental studies [1] - [10] indicate that the buckling characteristics of asymmetric laminates are different than those of their symmetric and isotropic counterparts. It is observed, for example, that asymmetric composite plates with no imperfections produce out-of-plane deflections under in-plane loading conditions. In some cases, however, the laminates can suddenly change their initial buckling direction following an entirely new deformation path, [1], or tend to remain flat when compression is combined unparalleled buckling characteristics of composite structures requires analyses based on nonlinear deformation theory.

This paper presents a continuation of previous work [8] regarding the buckling and postbuckling response of regular asymmetric laminated columns. The term "regular", as introduced in [4], identifies a laminated structure composed of a number of layers of different materials that have the same thickness. In the present study, the analysis concerns the behavior of general asymmetric laminates with an arbitrary number of layers of different thicknesses. The objective is to examine the typical buckling characteristics of such laminates depending on their composition and material properties.

STATEMENT OF THE PROBLEM

Consider a laminated column composed of n elastic layers bonded together. The column has a rectangular cross-section (bxh) and length l. The thickness of the laminate

$$h = \sum_{k=1}^{n} h_k \tag{1}$$

where h_k ($k = 1,2,3,\dots,n$) is the thickness of an individual layer. Each layer has elastic modulus E_k. In general, it is assumed that $h_k \neq h_j$, and $E_k \neq E_j$ $(k \neq j)$. The column is subjected to axial compression and bending by a force P and moment M_o applied at the free end as shown in Fig. 1. The other end of the column is rigidly fixed.

The coupled bending-compression response of the composite column is defined by the equations

$$\begin{bmatrix} P \\ M \end{bmatrix} = \begin{bmatrix} Q_1 & Q_2 \\ Q_2 & Q_3 \end{bmatrix} \begin{bmatrix} \epsilon_o \\ -\kappa \end{bmatrix} \tag{2}$$

where M denotes the bending moment, ϵ_o and κ denote, respectively, the axial strain and curvature, and Q_i ($i = 1,2,3$) are defined as

$$Q_1 = \sum_{k=1}^{n} \left(\int_{A_k} dA\right) E_k;\quad Q_2 = \sum_{k=1}^{n} \left(\int_{A_k} y\,dA\right) E_k;\quad Q_3 = \sum_{k=1}^{n} \left(\int_{A_k} y^2\,dA\right) E_k \tag{3}$$

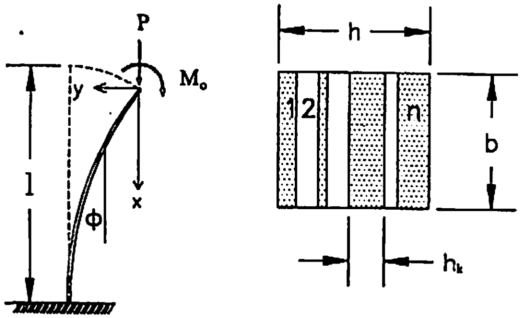

Figure 1. Schematic of composite column

$$\kappa = \frac{d\phi}{ds} \tag{4}$$

where ϕ is defined by

$$\sin\phi = \frac{dy}{ds} \tag{5}$$

Two simultaneous Eqns (2) can be replaced by a single differential equation

$$-\kappa\left[Q_1 Q_3 - (Q_2)^2\right] = Q_1 M - Q_2 P \tag{6}$$

in which

$$M = Py + M_o \tag{7}$$

Substituting Eqns (4) and (7) into Eqn (6), differentiating by s, and introducing Eqn (5) leads to the governing equation of the problem in the form

$$\frac{d^2\phi}{ds^2} + P\frac{Q_1}{Q_1 Q_3 - (Q_2)^2}\sin\phi = 0 \tag{8}$$

where the function $\phi(s)$ must satisfy the boundary conditions

$$\phi|_{s=l} = 0 \tag{9}$$

and

$$\left.\frac{d\phi}{ds}\right|_{s=0} = \frac{Q_2 P - Q_1 M_o}{Q_1 Q_3 - (Q_2)^2} \tag{10}$$

Note that Eqns (8) - (10) describe, in particular, the nonlinear buckling behavior of symmetrically laminated columns for which $Q_2 = 0$. In this case, Eqns (8) and (10) reduce to

$$\frac{d^2\phi}{ds^2} + \frac{P}{Q_3}\sin\phi = 0 \tag{11}$$

and

$$\left.\frac{d\phi}{ds}\right|_{s=0} = -\frac{M_o}{Q_3} \tag{12}$$

In the case of an isotropic column, $Q_3 = EI$, where E and I denote, respectively, the elastic modulus of the material and the inertia moment of the cross-section of the isotropic column.

BIFURCATION

According to Eqns (8) - (10), the buckling response of laminated columns subjected simultaneously to compression and bending is characterized by the development of lateral deflections immediately upon the load application. Under certain conditions, however, such columns exhibit bifurcation, i.e. they tend to remain perfectly straight until, at the critical point, their equilibrium position becomes unstable. The critical bifurcation load and the deflected configuration of the columns in the postbuckling state can be determined by solving the nonlinear eigenvalue problem represented by Eqn (8) and homogeneous boundary conditions.

In the case of asymmetric laminates, the bifurcation condition follows from Eqn (10) in the form

$$\frac{M_o}{P} = \frac{Q_2}{Q_1} \tag{13}$$

The respective lowest value of the critical load P_{cr} is defined as

Note, that both, Eqns (13) and (14), incorporate the effect of bending-stretching coupling. In the case of symmetric laminates, the bifurcation condition requires that $M_o = 0$, i.e. the structure is subjected to a perfectly centered axial compressive force. The respective critical load $P_{cr} = \pi^2 Q_3/4l^2$.

Consider, as a special case, the nonlinear buckling response of laminated columns composed of two layers, h_1 and h_2, with elastic properties defined respectively by E_1 and E_2. Note that $E_1 \neq E_2$ and $h_1 + h_2 = h$. In this case, Eqns (13) and (14) with Q_i (i = 1, 2, 3) determined from Eqns (3) assume, respectively, the form

$$\mu = \frac{\alpha(1-\alpha)(\gamma-1)}{\gamma-\alpha(\gamma-1)} \tag{15}$$

and

$$\lambda = \frac{2}{1+\gamma}\left[\gamma-(\gamma-1)\cdot f(\alpha) - 3\frac{\alpha^2(1-\alpha)^2(1-\gamma)^2}{\gamma-\alpha(\gamma-1)}\right] \tag{16}$$

In Eqns (15) and (16), the normalized parameters μ, λ, α, and γ are defined as

$$\mu = \frac{2M_o}{Ph};\ \lambda = \frac{P_{cr}}{P_o};\ \alpha = \frac{h_1}{h};\ \gamma = \frac{E_2}{E_1} \tag{17}$$

where $P_c = \pi^2 EI/4l^2$ is the critical load of identical isotropic column with the inertia moment $I = bh^3/12$, and elastic modulus $E = (E_1 + E_2)/2$. In Eqn (16), the function $f(\alpha)$ is defined by

$$f(\alpha) = \alpha(4\alpha^2-6\alpha+3) \tag{18}$$

Note that the parameters α and γ characterize the material lay-up of the laminated column, and the parameter μ represents the bending-compression ratio normalized by h/2. By definition, $\alpha < 1$ and $\gamma > 0$. As a result, it follows from Eqn (15) that $\mu < 1$. Further, an assumption that, in Eqn. (15), $\mu > 0$, leads to the requirement that $\gamma > 1$. These conditions imply that in the range

$$0 \le \mu \le 1 \tag{19}$$

bifurcation of the column is possible only if $M_o/P < h/2$, and if the moment M_o acts in the direction of the stronger layer of the column.

ANALYSIS OF RESULTS

The bifurcation condition given by Eqn (15) represents a surface shown in Fig. 2a. For any specified value of γ, the function $\alpha(\mu)$ defines the respective locus of the bifurcation points as shown in Fig. 2b for γ = 1, 2, 5, 10, 30, and 100. These diagrams indicate that, in the case of an isotropic column ($\gamma = 1$), bifurcation is possible only when $\mu = 0$. To satisfy the bifurcation condition in the cases of $\mu > 0$, both parameters, γ and α must be increased. Respectively, as μ increases, the column becomes progressively anisotropic, and the effects produced by bending-stretching couplings are enhanced. The lowest value of γ and the respective α in terms of μ that satisfy the bifurcation condition are obtained in the form

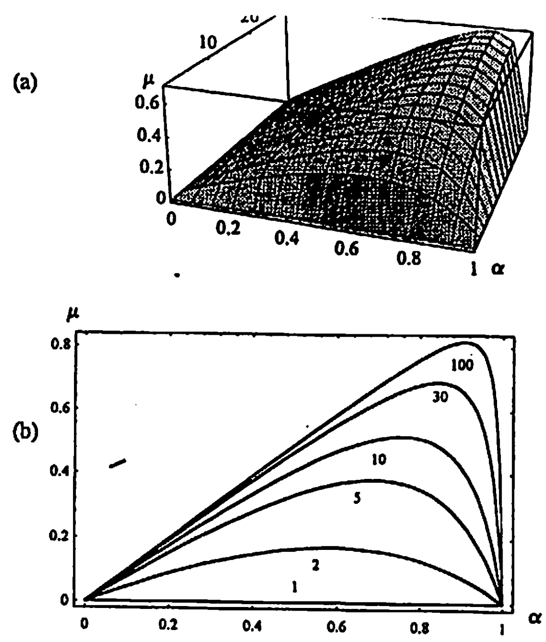

Figure 2. Bifurcation condition

$$\gamma = \frac{(\mu+1)^2}{(\mu-1)^2}\ \text{and}\ \alpha = \frac{1}{2}(\mu+1) \tag{20}$$

As these relations are introduced into Eqn (16), one obtains the critical value of λ as a function of μ presented by the curve 2 in Fig. 3.

Within the scope of this analysis, it is of interest to consider, the buckling response of regular asymmetric composite columns studied in [8]. In the latter case, $\alpha = 2$, and the bifurcation condition, as well as the respective critical value of λ, are obtained from Eqns (15) and (16) in the form

$$\mu = \frac{1}{2}\frac{\gamma-1}{\gamma+1}\ \text{and}\ \lambda = 1-\frac{3}{4}\frac{(\gamma-1)^2}{(\gamma+1)^2} \tag{21}$$

It follows that, in this case

$$\lambda = 1-3\mu^2 \tag{22}$$

This relation is depicted by the curve 1 in Fig. 3.

A comparison of the diagrams in Fig. 3 indicates that, in the range of $\mu \le 0.4$, the critical buckling loads of regular and irregular asymmetric laminates are practically of the same value, however, as μ tends to increase, the response of irregular laminates is characterized by a greater range of μ for which the bifurcation condition is satisfied.

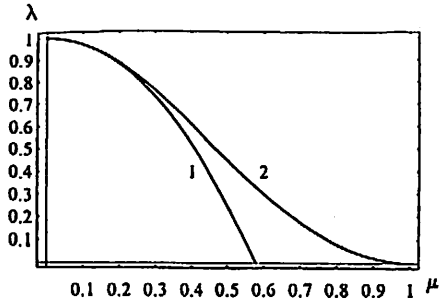

Figure 3. Critical load $\lambda(\mu)$ of regular and irregular asymmetric laminates

›s an illustration, the buckling and postbuckling behavior or asymmetric laminated columns ; depicted in Fig. 4 for $\alpha = 2$ and different values of the parameters γ and μ. These iagrams are obtained by solving the nonlinear eigenvalue problem defined by Eqns (8) - (10) nd (13) using software for bifurcation analysis, AUTO, [11]. The buckling characteristics hown in Fig 4, as well as the diagrams in Fig. 3, indicate that the critical bifurcation load of symmetric laminated composites tends to decrease substantially for increasing values ›f μ and γ.

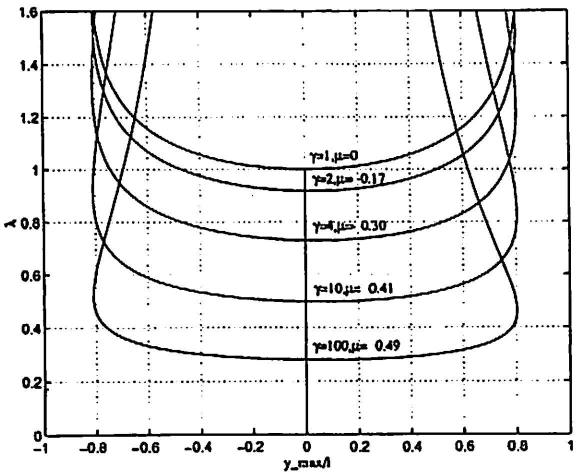

Figure 4. Bifurcation response of asymmetric laminated columns

CONCLUDING REMARKS

In this paper, the buckling and postbuckling behavior of asymmetric laminated columns subjected to compression and bending is examined. The analysis is based on geometrically nonlinear deformation theory. Respectively, the problem under consideration is defined by a nonlinear differential equation and two boundary conditions that incorporate the effects of

It is shown that, in general, the buckling behavior of asymmetric composite columns is characterized by the development of lateral deflections due to the applied force-moment system. However, under certain conditions, the structures exhibit bifurcation. It appears, that the latter property of asymmetric laminates can be utilized to control the lateral buckling of structural members subjected to compression and bending. However, as shown in the study, this approach can be applied only in a narrow range of material parameters, and its practical effectiveness is limited.

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