COUPLED RIGID-DEFORMABLE BODY FORMULATION FOR ANALYSIS OF JOINT STRESS AND DEFORMATION DURING GAIT

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INTRODUCTION

Analysis of human gait often uses rigid body dynamics to predict joint motions and forces [Andriacchi, *et al.*, 1997]. While these approaches can help identify healthy and pathological human movement, the rigid body assumption does not allow determination of stress and deformation within the joint. This information is desirable not only to help understand the etiology of joint disease, but also for determining how joint replacements will perform once implanted. Finite element analysis, used to determine joint stress and deformation, employs boundary conditions which are intended to simulate physiological conditions [Maxian, *et al*, 1996]. Because these boundary conditions are defined *a priori*, it remains unclear as to how well these applied tractions and displacements match what is truly occurring during human motion.

It would be useful to extend rigid body human gait models to include deformable bodies at areas of special interest, such as at the knee or hip. This extension would then allow the results from gait studies to be bridged to deformable body joint models in a consistent way. The objective of this paper is to compute stress and deformation of the hip joint during a gait simulation. To this end, we propose a unified framework where the rigid and deformable body analyses are coupled, allowing motions of the rigid body as well as stress and deformation of the deformable body to be obtained from a single analysis.

MATERIAL AND METHODS

The variational form of the nonlinear rigid body dynamics equations with conservative loading

$$\delta \boldsymbol{q} \cdot \left(\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}}} - \frac{\partial \mathcal{L}}{\partial \boldsymbol{q}} - \boldsymbol{Q} \right) = 0 \;\; \forall \;\; \delta \boldsymbol{q} \in \mathcal{V}_q \qquad (1)$$

governs the rigid (multi-)body mesh Ω_R (Fig. 1). In Eq. (1), $\boldsymbol{q}$ is a vector of generalized coordinates, with variation $\delta \boldsymbol{q}$, $\dot{\boldsymbol{q}}$ is a vector of generalized speeds, $\mathcal{L}$ is the Lagrangian, and $\boldsymbol{Q}$ is a vector of generalized forces.

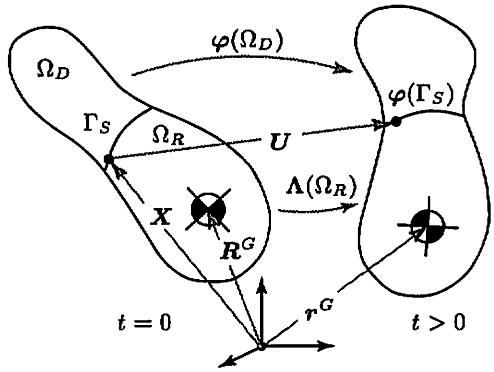

Figure 1: Rigid body Ω_R coupled to deformable body Ω_D at interface Γ_S at reference and current configurations.

The variational form of the nonlinear elastodynamics equations

$$\int_{\Omega} \delta \boldsymbol{U} \cdot \left(\rho_0 \frac{d^2 \boldsymbol{U}}{dt^2} - \mathrm{DIV} \boldsymbol{P} - \boldsymbol{B} \right) d\Omega = 0 \;\; \forall \;\; \delta \boldsymbol{U} \in \mathcal{V}_U \qquad (2)$$

governs the deformable body mesh Ω_D (Fig. 1). In Eq. (2), $\boldsymbol{U}$ is the displacement field, its variation $\delta \boldsymbol{U}$, $\boldsymbol{P}$ is the First Piola-Kirchhoff stress tensor, and $\boldsymbol{B}$ is the body force vector field.

The rigid and deformable bodies are attached along a common boundary Γ_S (Fig. 1). The deformable body at points $\boldsymbol{X}$ along the boundary is required to undergo displacements $\boldsymbol{U}$ compatible with the rigid body through the holonomic constraint equation $\boldsymbol{c}$ as

$$\boldsymbol{c} = \boldsymbol{\varphi} - \boldsymbol{r}^G - \boldsymbol{\Lambda} \left[\boldsymbol{X} - \boldsymbol{R}^G \right] = \boldsymbol{0} \;\; \forall \;\; \boldsymbol{X} \in \Gamma_S. \qquad (3)$$

The functions $\varphi(\boldsymbol{X}, t)$ and $\Lambda(q, t)$ map the deformable body and the rigid body from the reference to the current configurations, respectively.

The equations for the rigid body dynamics, elastodyamics, and constraint coupling may be combined into a single variational system, δs, now a function of unknowns $\boldsymbol{d}$ and $\boldsymbol{\lambda}$

$$\left.\begin{array}{r}\delta \boldsymbol{d} \cdot \left[\boldsymbol{R} + \left[\dfrac{\partial c}{\partial \boldsymbol{d}}\right]^T \boldsymbol{\lambda}\right] = 0, \\ \boldsymbol{c} = \boldsymbol{0}\end{array}\right\} = \delta s(\boldsymbol{d}, \boldsymbol{\lambda}) \qquad (4)$$

where $\boldsymbol{d} = \{\boldsymbol{q}, \boldsymbol{U}\}^T$ and $\boldsymbol{R} = \{\boldsymbol{R}_q, \boldsymbol{R}_U\}^T$ is the residual for both the rigid and deformable equations of motion. Newton's method is used to linearize the motion and constraint equations, leading to the standard matrix form, implemented in a predictor-corrector Newmark time integration scheme.

We implement a particular form of Eq. (3) which couples a rigid inverted physical pendulum, representing the stance leg, to a deformable two-dimensional finite element mesh, representing the femoral head (Fig. 2).

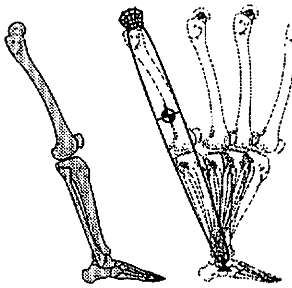

Figure 2: Rigid-deformable human ballistic gait model.

The stance leg uses anthropometric data for geometry and mass. The femoral head uses a nonlinear Kirchhoff-St.Venant model for cancellous bone ($E = 300$ MPa, $\nu = 0.25$). The two domains are constrained to move in unison at the seven nodes on the interface Γ_S. A dead load representing the head, arms, and trunk (HAT) is applied to the proximal end of the finite element mesh.

Initial conditions are set on the rigid leg so that the ballistic gait model produces a configuration at the end of the simulation symmetric to that at the beginning. The nonlinear dynamics equations of motion for the coupled system are integrated using the trapezoidal rule with consistent linearization providing quadratic convergence.

RESULTS

Initially, high compressive stresses exist on the posterior side of the hip joint (Fig. 3). Areas of localized hydrostatic pressure may be observed in the bone directly under the area of dead load application. As the leg moves through the stance phase, the stress profiles appear symmetric (mid-stance) and then fully reverse at the end of the simulation so that high compressive stress appears on the anterior side of the joint. The compressive hydrostatic pressure is shown with plotting threshholds set between 3.0 MPa (white) and 8.0 MPa (black).

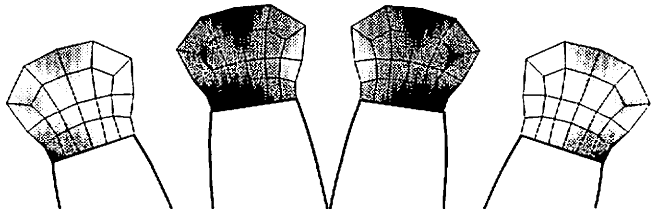

Figure 3: Hydrostatic pressure in the deformable hip for various configurations in the ballistic gait cycle.

DISCUSSION

The ballistic gait model illustrates that the proposed framework allows stress and deformation within a joint to be computed in a musculoskeletal dynamics simulation.

A fundamental feature of this framework is the ability to capture the interplay between displacements generated from deformation of the joint and displacements generated from rigid body movement of limb segments.

It is well-established that the assumptions placed on a rigid-body gait model have significant effect on the resulting motions [Audu and Davy, 1985]. Rigid body gait models assume that bone and soft tissue displacements are negligible compared to the overall motion of the rigid bodies. The proposed coupled system could help prove or disprove this assumption.

Similarly, the assumptions given to a deformable joint model have equally strong influence on the stress and deformation fields [Mow, *et al.*, 1993]. Deformable body models of human joints or implants with prescribed boundary conditions do not directly incorporate how such boundary conditions change with alterations in gait patterns. The coupled system presented here eliminates this restriction and thus could be used to explore how human subjects may modify their gait patterns to reduce stress in a joint.

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