$$
\begin{array}{llcl}
\text{Types} & T & ::= & \alpha \mid \top \mid \tau_1 \to \tau_2 \mid \forall\alpha.\,\tau \mid \tau_1 \,\&\, \tau_2 \mid \{l{:}\tau\} \\
\text{Expressions} & E & ::= & x \mid \top \mid \lambda(x{:}\tau).\,e \mid e_1\ e_2 \mid \Lambda\alpha.\,e \mid e\ \tau \mid e_1,,e_2 \mid \{l = e\} \mid e.l \mid e \setminus l \\
 & & \mid & \texttt{let}\ x = e\ \texttt{in}\ e \\
 & & \mid & \texttt{sig}\ x\ [\overline{x}]\ \texttt{where}\ \overline{x : \tau}\ \texttt{in}\ e \\
 & & \mid & \texttt{sig}\ x\ [\overline{x}]\ \texttt{extends}\ \overline{x\ [\overline{x}]}\ \texttt{where}\ \overline{x : \tau}\ \texttt{in}\ e \\
\text{Contexts} & \Gamma & ::= & \epsilon \mid \gamma, \alpha \mid \gamma, x{:}\tau \\
\text{Labels} & l & &
\end{array}
$$

Figure 1: Source syntax.

$$
\begin{array}{llcl}
\text{Types} & \tau & ::= & \alpha \mid \top \mid \tau_1 \to \tau_2 \mid \forall\alpha.\,\tau \mid \tau_1 \,\&\, \tau_2 \mid \{l{:}\tau\} \\
\text{Expressions} & e & ::= & x \mid \top \mid \lambda(x{:}\tau).\,e \mid e_1\ e_2 \mid \Lambda\alpha.\,e \mid e\ \tau \mid e_1,,e_2 \mid \{l = e\} \mid e.l \mid e \setminus l \\
\text{Contexts} & \gamma & ::= & \epsilon \mid \gamma, \alpha \mid \gamma, x{:}\tau \\
\text{Labels} & l & &
\end{array}
$$

Figure 2: $F_{\&}$ syntax.

$$
\boxed{\tau \perp \tau} \qquad
\frac{\alpha_1 \neq \alpha_2}{\alpha_1 \perp \alpha_2}\ \textit{\textbf{o}}\textsc{var} \qquad
\frac{\tau_1 \perp \tau_3 \qquad \tau_2 \perp \tau_4}{\tau_1 \to \tau_2 \perp \tau_3 \to \tau_4}\ \textit{\textbf{o}}\textsc{fun}
$$

$$
\frac{\tau_1 \perp [\alpha_1/\alpha_2]\tau_2}{\forall\alpha_1.\,\tau_1 \perp \forall\alpha_2.\,\tau_2}\ \textit{\textbf{o}}\textsc{forall} \qquad
\frac{\tau_1 \perp \tau_3 \qquad \tau_2 \perp \tau_3}{\tau_1 \,\&\, \tau_2 \perp \tau_3}\ \textit{\textbf{o}}\textsc{and-left}
$$

$$
\frac{\tau_1 \perp \tau_2 \qquad \tau_1 \perp \tau_3}{\tau_1 \perp \tau_2 \,\&\, \tau_3}\ \textit{\textbf{o}}\textsc{and-right} \qquad
\frac{l_1 \neq l_2}{\{l_1 : \tau_1\} \perp \{l_2 : \tau_2\}}\ \textit{\textbf{o}}\textsc{rec}
$$

Figure 3: Orthogonality between $F_{\&}$ types.

$$
\boxed{\gamma \vdash \tau} \qquad
\frac{\alpha \in \gamma}{\gamma \vdash \alpha}\ \textit{\textbf{wf}}\textsc{var} \qquad
\frac{}{\gamma \vdash \top}\ \textit{\textbf{wf}}\textsc{top} \qquad
\frac{\gamma \vdash \tau_1 \qquad \Gamma \vdash \tau_2}{\gamma \vdash \tau_1 \to \tau_2}\ \textit{\textbf{wf}}\textsc{fun}
$$

$$
\frac{\gamma, \alpha \vdash \tau}{\gamma \vdash \forall\alpha.\,\tau}\ \textit{\textbf{wf}}\textsc{forall} \qquad
\frac{\gamma \vdash \tau_1 \qquad \Gamma \vdash \tau_2 \qquad \tau_1 \perp \tau_2}{\gamma \vdash \tau_1 \,\&\, \tau_2}\ \textit{\textbf{wf}}\textsc{and}
$$

$$
\frac{\gamma \vdash \tau}{\gamma \vdash \{l{:}\tau\}}\ \textit{\textbf{wf}}\textsc{rec}
$$

Figure 4: Well-formedness of $F_{\&}$ types.

$$\boxed{\tau <: \tau} \qquad \frac{}{\alpha <: \alpha}\ \textit{\textbf{sub}}\text{VAR} \qquad \frac{}{\tau <: \top}\ \textit{\textbf{sub}}\text{TOP} \qquad \frac{\tau_3 <: \tau_1 \qquad \tau_2 <: \tau_4}{\tau_1 \to \tau_2 <: \tau_3 \to \tau_4}\ \textit{\textbf{sub}}\text{FUN}$$

$$\frac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2}{\forall \alpha_1.\, \tau_1 <: \forall \alpha_2.\, \tau_2}\ \textit{\textbf{sub}}\text{FORALL} \qquad \frac{\tau_1 <: \tau_2 \qquad \tau_1 <: \tau_3}{\tau_1 <: \tau_2 \,\&\, \tau_3}\ \textit{\textbf{sub}}\text{AND}$$

$$\frac{\tau_1 <: \tau_3}{\tau_1 \,\&\, \tau_2 <: \tau_3}\ \textit{\textbf{sub}}\text{AND}_1 \qquad \frac{\tau_2 <: \tau_3}{\tau_1 \,\&\, \tau_2 <: \tau_3}\ \textit{\textbf{sub}}\text{AND}_2 \qquad \frac{\tau_1 <: \tau_2}{\{l : \tau_1\} <: \{l : \tau_2\}}\ \textit{\textbf{sub}}\text{REC}$$

Figure 5: Subtyping between $\mathsf{F}_{\&}$ types.

$$\boxed{\gamma \vdash e : \tau} \qquad \frac{(x,\tau) \in \gamma}{\gamma \vdash x : \tau}\ \textit{\textbf{ty}}\textsc{var} \qquad \frac{}{\gamma \vdash \top : \top}\ \textit{\textbf{ty}}\textsc{top}$$

$$\frac{\gamma, x{:}\tau \vdash e : \tau_1 \qquad \gamma \vdash \tau}{\gamma \vdash \lambda(x{:}\tau).\, e : \tau \to \tau_1}\ \textit{\textbf{ty}}\textsc{lam}$$

$$\frac{\gamma \vdash e_1 : \tau_1 \to \tau_2 \qquad \gamma \vdash e_2 : \tau_3 \qquad \tau_3 <: \tau_1}{\gamma \vdash e_1\ e_2 : \tau_2}\ \textit{\textbf{ty}}\textsc{app}$$

$$\frac{\gamma, \alpha \vdash e : \tau}{\gamma \vdash \Lambda\alpha.\, e : \forall\alpha.\, \tau}\ \textit{\textbf{ty}}\textsc{blam} \qquad \frac{\gamma \vdash e : \forall\alpha.\, \tau_1 \qquad \gamma \vdash \tau}{\gamma \vdash e\ \tau : [\tau/\alpha]\tau_1}\ \textit{\textbf{ty}}\textsc{tapp}$$

$$\frac{\gamma \vdash e_1 : \tau_1 \qquad \gamma \vdash e_2 : \tau_2}{\gamma \vdash e_1 ,, e_2 : \tau_1 \,\&\, \tau_2}\ \textit{\textbf{ty}}\textsc{merge}$$

$$\frac{\gamma \vdash e : \tau}{\gamma \vdash \{l = e\} : \{l{:}\tau\}}\ \textit{\textbf{ty}}\textsc{rec-construct}$$

$$\frac{\gamma \vdash e : \tau \qquad \tau \bullet l = \tau_1}{\gamma \vdash e.l : \tau_1}\ \textit{\textbf{ty}}\textsc{rec-select}$$

$$\frac{\gamma \vdash e : \tau \qquad \tau \setminus l = \tau_1}{\gamma \vdash e \setminus l : \tau_1}\ \textit{\textbf{ty}}\textsc{rec-restrict}$$

$$\boxed{\tau_1 \bullet l = \tau_2} \qquad \frac{}{\{l{:}\tau\} \bullet l = \tau}\ \textit{\textbf{select}} \qquad \frac{\tau_1 \bullet l = \tau}{\tau_1 \,\&\, \tau_2 \bullet l = \tau}\ \textit{\textbf{select}}_1$$

$$\frac{\tau_2 \bullet l = \tau}{\tau_1 \,\&\, \tau_2 \bullet l = \tau}\ \textit{\textbf{select}}_2$$

$$\boxed{\tau_1 \setminus l = \tau_2} \qquad \frac{}{\{l{:}\tau\} \setminus l = \top}\ \textit{\textbf{restrict}} \qquad \frac{\tau_1 \setminus l = \tau}{\tau_1 \,\&\, \tau_2 \setminus l = \tau \,\&\, \tau_2}\ \textit{\textbf{restrict}}_1$$

$$\frac{\tau_2 \setminus l = \tau}{\tau_1 \,\&\, \tau_2 \setminus l = \tau_1 \,\&\, \tau}\ \textit{\textbf{restrict}}_2$$

Figure 6: Typing of $F_{\&}$.