S8 Assignment # 3

1/

(a) A dielectric material with dielectric constant K changes the electric field intensities between the capacitor plates by establishing an induced charge distribution within the material. The changed amount is given by,

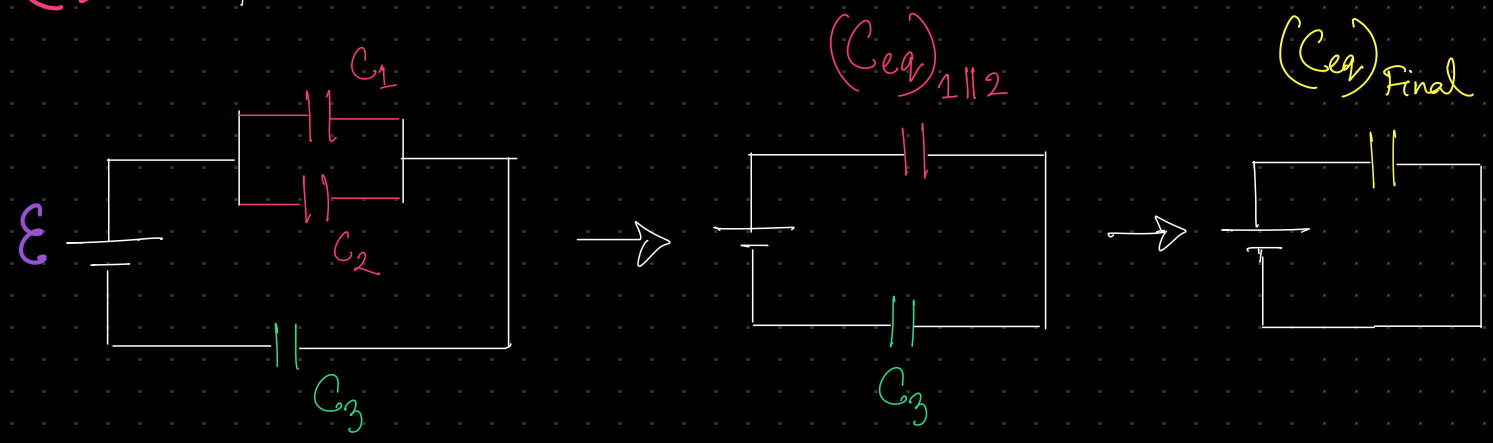
$$E_{new} = E_0 - E_{ind}$$

$$= \frac{\sigma_0}{\epsilon_0} - \frac{\sigma_{ind}}{\epsilon} \quad ; \epsilon = K\epsilon_0$$

since, $\sigma_{ind} < \sigma_0$ and $\epsilon > \epsilon_0$ for any $K > 1$,

$$E_{new} < E_0 .$$

(b) simplify the connections first to find C_{eq}.



$$C_{eq} = \left\{ \frac{1}{(C_1 + C_2)} + \frac{1}{C_3} \right\}^{-1} = \frac{36}{13} \times 10^{-6}\ F$$

$$= 2.769 \times 10^{-6}\ F$$

The battery can supply Q amount of charge for $(C_{eq})_{Final}$. $Q = (C_{eq})_{Final} \times \mathcal{E} = 4.154 \times 10^{-5}\,C$.

This Q will be shared by C_3 and $(C_{eq})_{1\|2}$.

Potential difference across C_3 would be, $(\Delta V_c)_3 = \dfrac{Q}{C_3}$

$$(\Delta V_c)_3 = 10.385\,V.$$

The leftover voltage $\mathcal{E} - (\Delta V_c)_3$ will be dropped through $(C_{eq})_{1\|2}$, which C_1 and C_2 will share

$$\therefore (\Delta V_c)_1 = (\Delta V_c)_2 = \mathcal{E} - (\Delta V_c)_3 = 4.615\,V.$$

Charges across C_1 and C_2 will be,

$$Q_1 = C_1 (\Delta V_c)_1 = 1.3845 \times 10^{-5}\,C.$$

$$Q_2 = C_2 (\Delta V_c)_2 = 2.769 \times 10^{-5}\,C.$$

You may check and verify that $Q = Q_1 + Q_2$

Physics works !

(i) $Q_1 = 1.3845 \times 10^{-5}\,C$.

$Q_2 = 2.769 \times 10^{-5}\,C$.

$Q_3 = 4.154 \times 10^{-5}\,C$.

(ii) $(\Delta V_c)_1 = (\Delta V_c)_2 = 4.615\,V$

$(\Delta V_c)_3 = 10.385\,V$.

You may check and verify that

$$\mathcal{E} = (\Delta V_c)_1 + (\Delta V_c)_2 + (\Delta V_c)_3$$

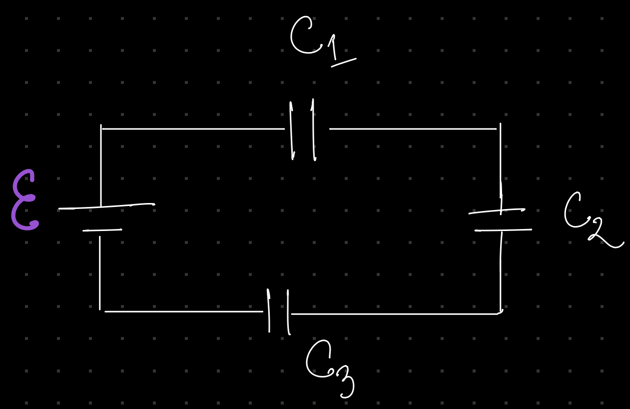
Physics works !

(iii) $(U_c)_1 = \dfrac{Q_1^2}{2C_1} = 3.195 \times 10^{-5}\,J$.

$(U_c)_2 = \dfrac{Q_2^2}{2C_2} = 6.389 \times 10^{-5}\,J$.

$(U_c)_3 = \dfrac{Q_3^2}{2C_3} = 2.157 \times 10^{-4}\,J$.

(c) $K = \dfrac{\epsilon}{\epsilon_0} = 9.04$.



$$(C_1)_{new} = K C_1 = 2.71 \times 10^{-5}\ F$$

$$(C_2)_{new} = K C_2 = 5.424 \times 10^{-5}\ F$$

$$(C_3)_{new} = K C_3 = 3.616 \times 10^{-5}\ F$$

$$(C_{eq})_{new} = 11.75 \times 10^{-5}\ F$$

$$Q = (C_{eq})_{new}\,\mathcal{E} = 1.763 \times 10^{-4}\ C.$$

New values,

$$(U_C)_1 = \frac{Q^2}{2(C_1)_{new}} = 5.73 \times 10^{-6}\ J.$$

$$(U_C)_2 = \frac{Q^2}{2(C_2)_{new}} = 2.87 \times 10^{-4}\ J.$$

$$(U_C)_3 = \frac{Q^2}{2(C_3)_{new}} = 4.298 \times 10^{-4}\ J.$$

Due to the series connection, the energy stored by each capacitors have changed.

Note to ST: Students who measured either the combined energy stored by the series setup or measured the combined energies in the (b) setup gets mark too. As long as their approach and implimentation of the principles are correct!

2) (a)

Use Ohm's law in 3D.

$$J = \sigma E = \frac{I}{A}.$$

Since both wires share I and A to be the same, their J is the same.

$$\therefore \quad E \propto \frac{1}{\sigma}. \qquad \frac{E_2}{E_1} = \frac{\sigma_1}{\sigma_2} = 2.$$

(b) Given, the maximum current $I = 1A$.

Current density of the wire,

$$J = \frac{I}{A} = \frac{I}{\frac{\pi d^2}{4}}.$$

$$\Rightarrow d^2 = \frac{I}{\frac{\pi J}{4}}$$

$$\Rightarrow d = \sqrt{\frac{4I}{\pi J}} = 5.0475 \text{ m}.$$

E and J are related by,

$$E = \rho J$$

$$= (0.025 \times 500 \times 10^{-4})\ Vm^{-1}$$

$$= 0.00125\ Vm^{-1}.$$

(b) (i) $I = \dfrac{\Delta Q}{\Delta t}$; charge carriers per unit second through a cross-section.

$\Delta Q = nq$

$$q = \frac{I}{q} = \frac{0.40\,A}{1.602 \times 10^{-19}} = 2.497 \times 10^{18} \text{ electrons}$$

(ii)

current density in the wire,

$$J = \frac{I}{A} = \frac{0.40\,A}{1.55 \times 10^{-6}\,m^2}$$

$$= 258.065 \times 10^{3}\ Am^{-2}$$

(iii) drift speed,

$$V_d = \frac{J}{nq} = 1.895 \times 10^{-5}\ ms^{-1}$$

(iv) only the current density J and drift speed v_d will be affected by a change in cross-sectional area.

Since $J \propto \frac{1}{A}$ and $v_d \propto \frac{1}{A}$.

Charges passing through per unit second will not change as it is independent of the wire size.