

Modelling Sustainable Systems and Semantic Web Systems and Development

**Lecture in the Modul 10-202-2309
for Master Computer Science**

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Modelling Systems

Two problems:

- (1) Build new system
- (2) Rebuild existing system

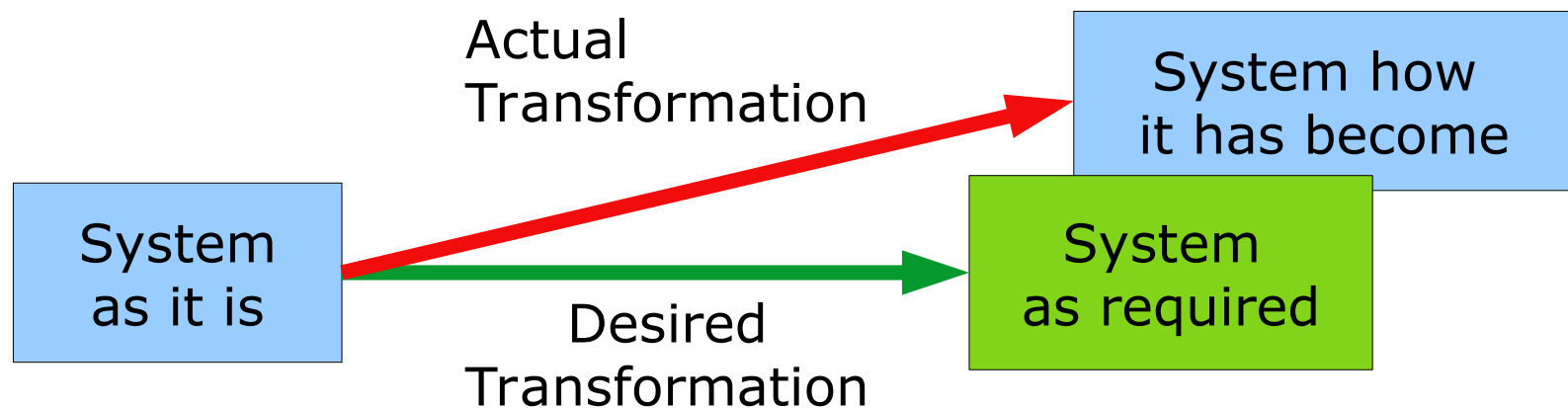
(1) can be considered as a special case of (2), since every need for a new system comes with at least *rough ideas* about that new system, so there is also under (1) an at least *rough description form* of the system to be created.

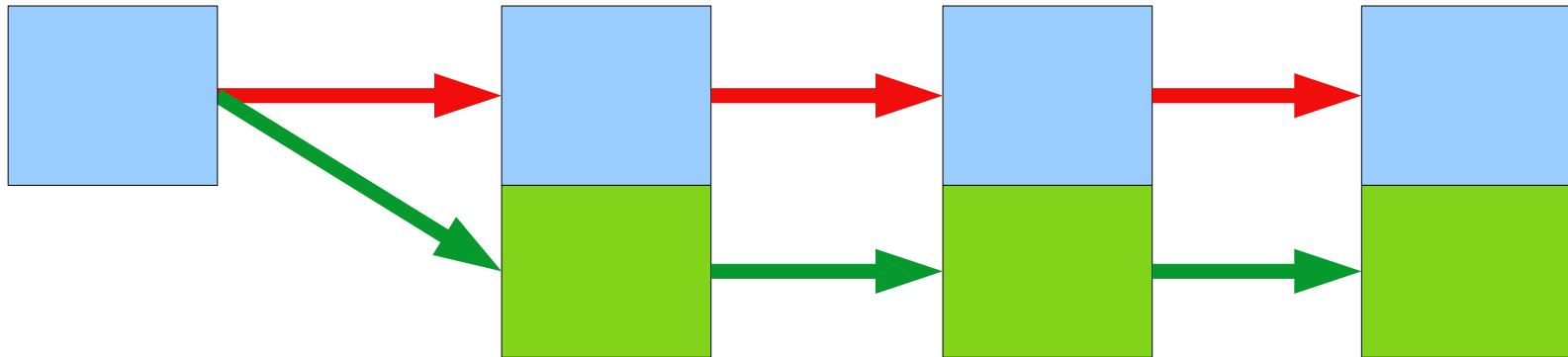


Modelling Systems

This basic scheme fits not only technical systems, but also the modelling of social, socio-ecological and cultural systems, so it is sufficiently universal.

How does such a system evolve over time?

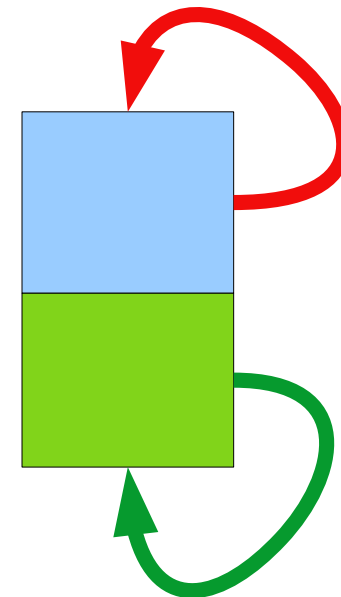




Transitional development as *different versions* of the system over the time.

But can also be understood as development in time of *the same system*.

Transitional management versus adaptive management.

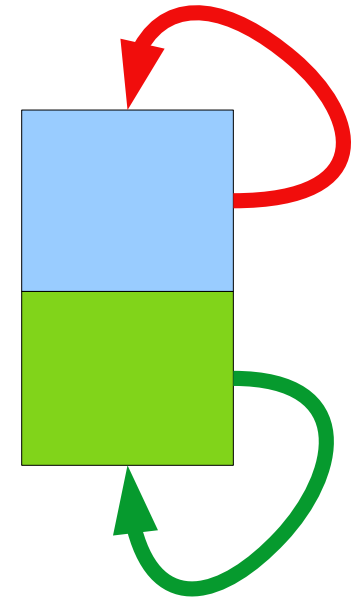


The development of a system can therefore be conceived as a contradiction between an *ideal line of development* and a *real line of development*.

This idea is reflected in the TRIZ concept of the *Ideal Final Result* (IFR).

In the (mathematical) *Theory of Dynamical Systems* (TDS), system development is conceived as a progression of states, which can be described by functions $f(t)$ with values in a phase space.

The *ideal behaviour* is described by mathematical relationships, such as differential equations, whose invariant solutions describe a partial structure of stable states (*trajectories*) in phase space.



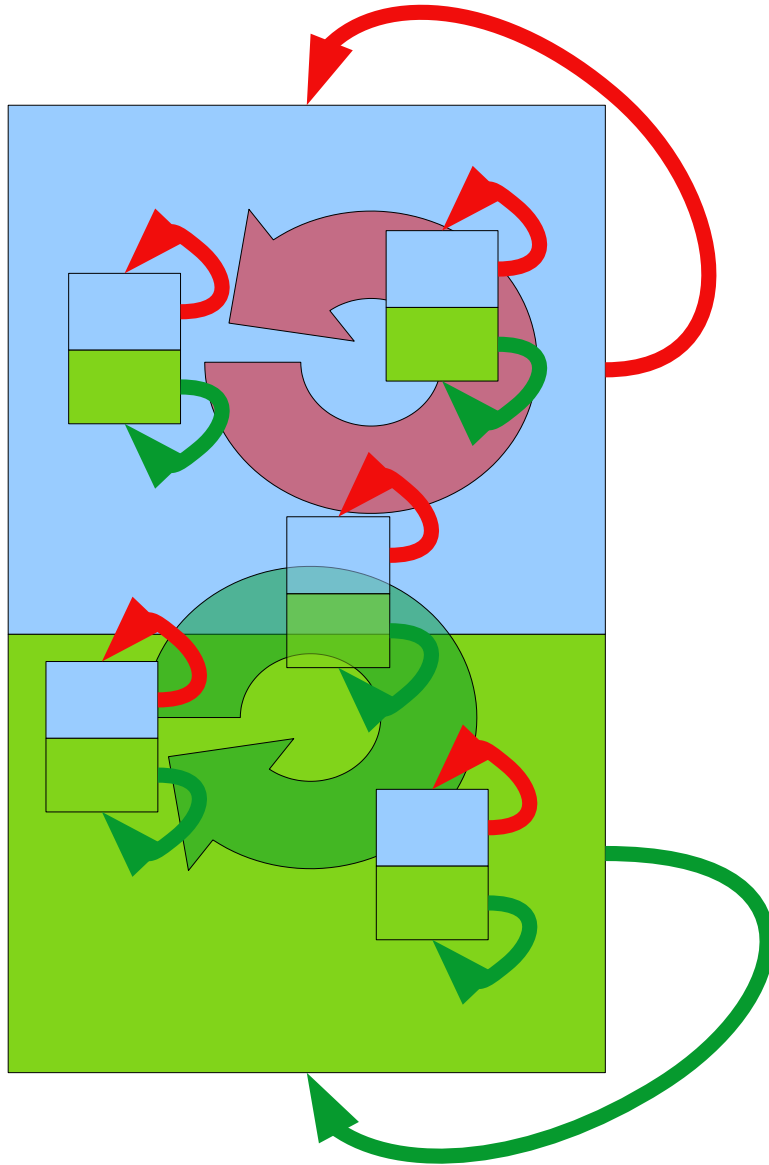
These differential equations and trajectories are part of the *description form of the system* and thus have already been created by *reduction to essentials*.

In the modelling it is assumed that everything essential is taken into account, i.e. that the *real temporal development* $r(t)$ of the system differs from the *ideal temporal development* $f(t)$ only by a small difference $d(t)=r(t)-f(t)$, which *is insignificant for the selected essential*.

While $f(t)$ enables a *quantitative prediction* of the development of the system, the statement that $d(t)$ is "small" or "damped" is a *qualitative statement* of the descriptive form.

Often one also restricts oneself with $f(t)$ to a *qualitative statement* about the exact position of the trajectories as invariants in the solution space and thus to the statement that $r(t)$ oscillates around these trajectories in a damped manner. These trajectories seem to "magically" attract the real states and are therefore also called *attractors*.

For example, the Earth moves on an elliptical orbit around the Sun in the sense that real deviations from this orbit are always compensated for.



Development

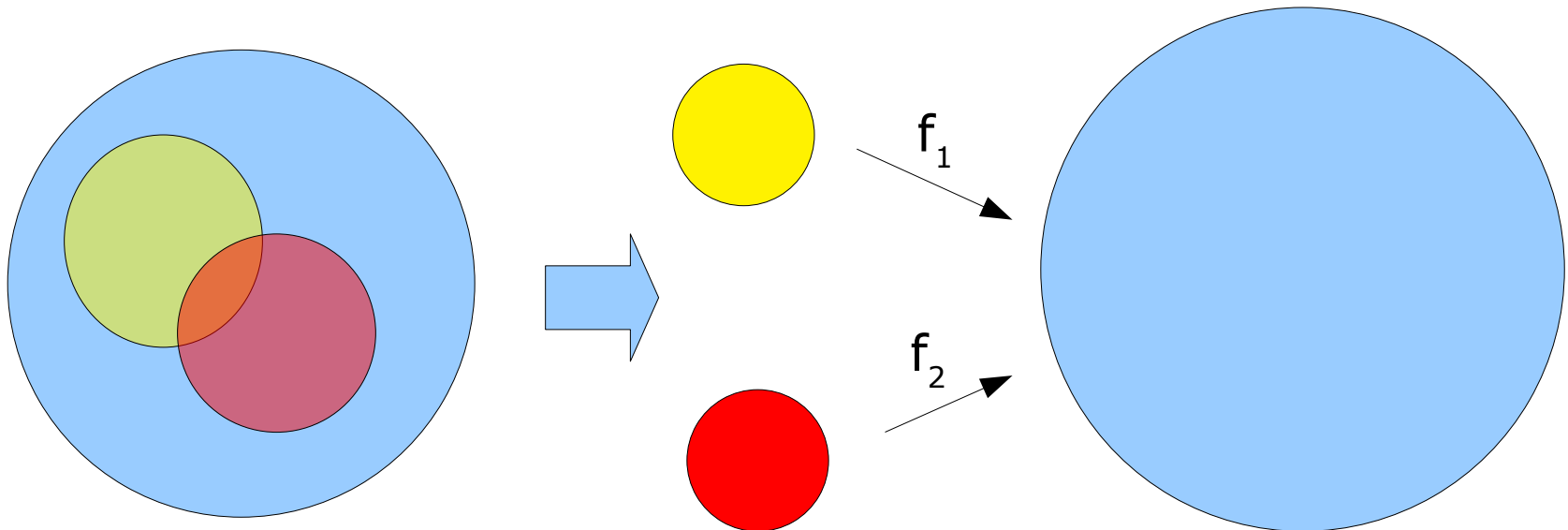
- of the system itself
- the components in the system and
- the relationships in the system

However, let us first take a closer look at how complicated trajectories can be.

See TDS.md

Immersive and submersive System Theories

In which structures one can investigate more complex relations of systems within supersystems?



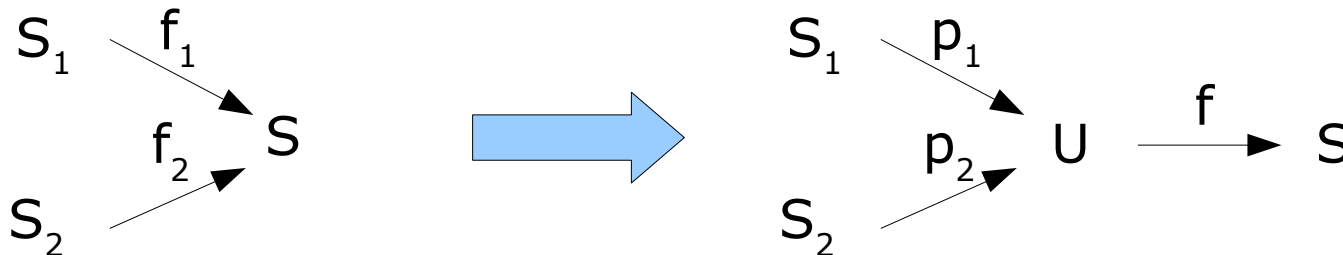
Mathematical formulation of the question

We look for functions $f_1 : S_1 \rightarrow S$, $f_2 : S_2 \rightarrow S$.

Does for such a constellation exist a **universal categorial Object**, i.e. a universal U and universal maps $p_1 : S_1 \rightarrow U$, $p_2 : S_2 \rightarrow U$, such that *for each triple* (f_1, f_2, S) the above constellation may be written as

$$f_1 = f \circ p_1 : S_1 \rightarrow U \rightarrow S, f_2 = f \circ p_2 : S_2 \rightarrow U \rightarrow S$$

for a suitable $f = f_1 \oplus f_2 : U \rightarrow S$. U in such a case is called a **direct sum** and we write $U = S_1 \amalg S_2$.



Mathematical Categories

Most mathematical models live in concrete **categories**, for example, the category of sets, vector spaces, fibre bundles, algebraic varieties, and so on.

Each such category is characterised by the fact that the terms **object** and **morphism** have a clear meaning there.

Morphisms between vector spaces, for example, are operationally faithful mappings, i.e. linear mappings that can be described by matrices for finite-dimensional vector spaces.

Such universal objects do not exist in every category.

Remark: The construction can easily be extended to finitely many S_i and also to infinitely many S_i , $i \in I$, and that is how it is meant in mathematics.

Category of Sets

In this category direct sums U exist for both finite and infinite index sets I . This is just the **disjunctive union** of the sets S_i .

The maps p_i are just the embeddings $p_i : S_i \rightarrow U$ of the partial sets in their disjunct union.

The map $f : U \rightarrow S$ works as follows: For each $a \in U$ exists exactly one i and one $a' \in S_i$ with $a = p_i(a')$. Put $f(a) = f_i(a')$.

If $|S_1| = a$, $|S_2| = b$, then $|S_1 \amalg S_2| = a + b$.

The whole is no more than the sum of its parts.

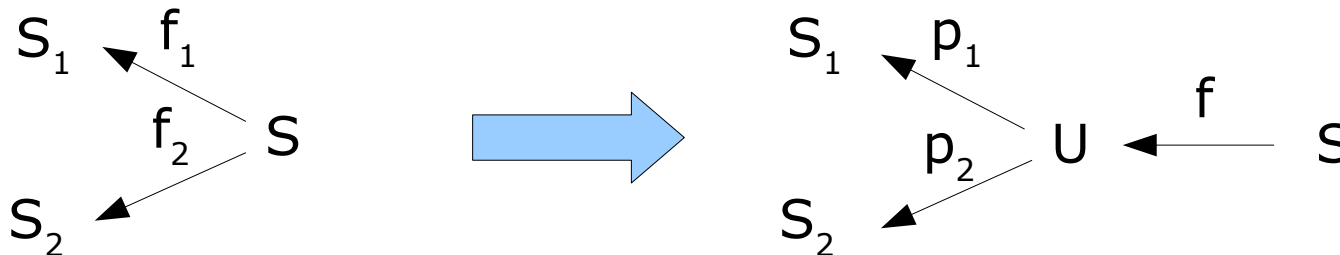
Invert all Arrows (TRIZ Prinziple 13)

We look for functions $f_1 : S_1 \leftarrow S$, $f_2 : S_2 \leftarrow S$.

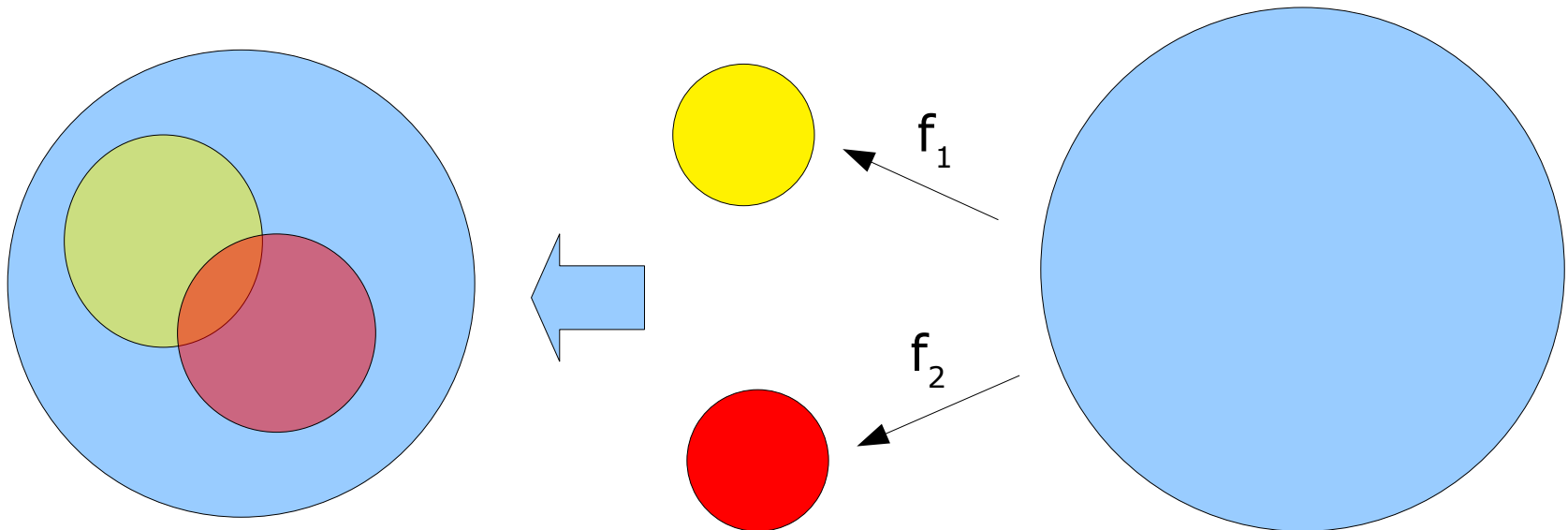
Does for such a constellation exist a **universal categorial Object**, i.e. a universal U and universal maps $p_1 : S_1 \leftarrow U$, $p_2 : S_2 \leftarrow U$, such that *for each triple* (f_1, f_2, S) the above constellation may be written as

$$f_1 = p_1 \circ f : S_1 \leftarrow U \leftarrow S, \quad f_2 = p_2 \circ f : S_2 \leftarrow U \leftarrow S$$

for a suitable $f = f_1 \otimes f_2 : S \rightarrow U$. U in such a case is called a **direct product** and we write $U = S_1 \amalg S_2$.



How does this change the perspective on the concept of system?



Kategorie der Mengen

In this category direct sums U exist for both finite and infinite index sets I . This is just the **cartesian product** of the sets S_i .

The maps p_i are just the projections $p_i : U \rightarrow S_i$ of the product to the individual components.

The map $f : S \rightarrow U$ works as follows: For each $a \in S$ we set $f(a) = (f_i(a)) \in U$.

If $|S_1| = a$, $|S_2| = b$, then $|S_1 \amalg S_2| = a \cdot b$.

The whole is clearly more than the sum of its parts, most of the "information" is of relational nature.

Submersive and Immersive System Theories

Systemtheorien machen selten einen Unterschied zwischen diesen beiden Zugängen.

Zur Unterscheidung der Zugänge bezeichnet man Systemtheorien, in denen das erste Modellierungsprinzip dominiert, als **immersive Systemtheorien**. Man erkennt sie daran, dass ihre Konstruktionen wesentlich auf Einbettungen (Immersionen) aufbauen.

Systemtheorien, die auf dem zweiten Modellierungsprinzip aufbauen, bezeichnet man als **submersive Systemtheorien**. Man erkennt sie daran, dass ihre Konstruktionen wesentlich auf Projektionen (Submersionen) aufbauen und damit auf Prozessen gestaffelter Komplexitätsreduktion.

Die Theorie dynamischer Systeme ist eine submersive Systemtheorie.

Submersive and Immersive System Theories

Systems theories rarely make a distinction between these two approaches.

To distinguish between the approaches, system theories in which the first modelling principle dominates are called **immersive system theories**. They can be recognised by the fact that their constructions are essentially based on embeddings (immersions).

System theories that are based on the second modelling principle are called **submersive system theories**. They can be recognised by the fact that their constructions are essentially based on projections (submersions) and thus on processes of staggered complexity reduction.

The Theory of Dynamical Systems is a submersive system theory.