# Modelling Sustainable Systems and Semantic Web

# Immersive and Submersive System Theories

Lecture in the Module 10-202-2309
for Master Computer Science

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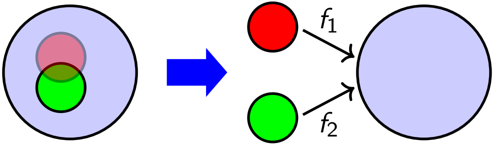
# Relation Between two Systems in a Supersystem

How can the relationship between two systems be conceptualised in their environment?

According to our systems approach, only if we describe *a larger system* (supersystem) and the relation of the two subsystems to the supersystem as *components*.



# Immersive Concept

### Mathematical formulation of the question

We look for functions $f_1 : S_1 \to S,\ f_2 : S_2 \to S$ with certain properties.

For a "generic solution" we ask if for such a constellation exists a **Universal Categorial Object**, i.e. a universal $U$ and universal maps

$$p_1 : S_1 \to U,\ \ p_2 : S_2 \to U,$$

such that for each triple $(f_1, f_2, S)$ the above constellation may be written as

$$f_1 = f \circ p_1 : S_1 \to U \to S,\ \ f_2 = f \circ p_2 : S_2 \to U \to S$$

for a suitable $f = f_1 \oplus f_2 : U \to S$.

# Immersive Concept

$U$ is in some sense the "most general system" that combines the systems $S_1$ and $S_2$ without "further effect".

$U$ in such a case is called a **direct sum** and we write $U = S_1 \coprod S_2$.

$$
\begin{tikzcd}
S_1 \arrow[dr, "f_1"] & \\
& S \\
S_2 \arrow[ur, "f_2"'] &
\end{tikzcd}
\quad \Longrightarrow \quad
\begin{tikzcd}
S_1 \arrow[dr, "p_1"] & & \\
& U \arrow[r, "f"] & S \\
S_2 \arrow[ur, "p_2"'] & &
\end{tikzcd}
$$

# Mathematical Categories

Most mathematical models live in concrete **categories**, for example, the category of sets, vector spaces, fibre bundles, algebraic varieties, and so on.

Each such category is characterised by the fact that the terms **object** and **morphism** have a clear meaning there.

Morphisms between vector spaces, for example, are operationally faithful mappings, i.e. linear mappings that can be described by matrices for finite-dimensional vector spaces.

Such universal objects do not exist in every category.

*Remark:* The construction can easily be extended to finitely many $S_i$ and even to infinitely many $S_i, i \in I$, and so it is defined in mathematics.

# Category of Sets

In this category direct sums $U$ exist for both finite and infinite index sets $I$. This is just the **disjunct union** of the sets $S_i$.

The maps $p_i$ are just the embeddings $p_i : S_i \to U$ of the partial sets in their disjunct union.

The map $f : U \to S$ works as follows: For each $a \in U$ exists exactly one $i$ and one $a' \in S_i$ with $a = p_i(a')$. Put $f(a) = f_i(a')$.

If $|S_1| = a, |S_2| = b$, then $|S_1 \coprod S_2| = a + b$.

The whole is no more than the sum of its parts.

# Submersive Concept

**Invert all Arrows (TRIZ Prinziple 13)**

We look for functions $f_1 : S_1 \leftarrow S$, $f_2 : S_2 \leftarrow S$ with certain properties.

Does for such a constellation exists a **Universal Categorial Object**, i.e. a universal $U$ and universal maps

$$p_1 : S_1 \leftarrow U,\ \ p_2 : S_2 \leftarrow U,$$

such that for each triple $(f_1, f_2, S)$ the above constellation may be written as

$$f_1 = p_1 \circ f : S_1 \leftarrow U \leftarrow S,\ \ f_2 = p_2 \circ f : S_2 \leftarrow U \leftarrow S$$

for a suitable $f = f_1 \otimes f_2 : S \to U$.

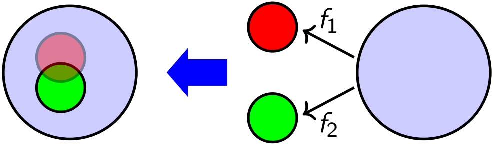
# Submersive Concept

$U$ in such a case is called a **direct product** and we write $U = S_1 \prod S_2$.

$$
\begin{tikzcd}
S_1 & \\
 & S \arrow[ul, "f_1"'] \arrow[dl, "f_2"] \\
S_2 &
\end{tikzcd}
\quad \Longrightarrow \quad
\begin{tikzcd}
S_1 & & \\
 & U \arrow[ul, "p_1"'] \arrow[dl, "p_2"] & S \arrow[l, "f"'] \\
S_2 & &
\end{tikzcd}
$$

# Submersive Concept

How does this change the perspective on the concept of system?



# Category of Sets

In this category direct sums $U$ exist for both finite and infinite index sets $I$. This is just the **cartesian product** of the sets $S_i$.

The maps $p_i$ are just the projections $p_i : U \to S_i$ of the product to the individual components.

The map $f : S \to U$ works as follows: For each $a \in S$ we set $f(a) = (f_i(a)) \in U$.

If $|S_1| = a, |S_2| = b$, then $|S_1 \prod S_2| = a \cdot b$.

**The whole is clearly more than the sum of its parts, most of the "information" is of relational nature.**

# Submersive and Immersive System Theories

System theories rarely make a distinction between between these two approaches.

To distinguish between the approaches, system theories in which the first modelling principle dominates, are called **immersive system theories**. They can be recognised their constructions are essentially based on embeddings (immersions).

System theories that are based on the second modelling principle are called **submersive system theories**. They can be recognised by the fact that their constructions are essentially based on projections (submersions) and thus on processes of staggered complexity reduction.

The Theory of Dynamical Systems is a submersive system theory.