Modelling Sustainable Systems and Semantic Web

Immersive and Submersive System Theories

Lecture in the Module 10-202-2312
for Master Computer Science

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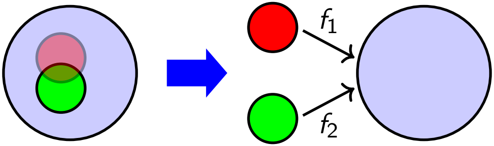
Build up Components to a Larger System

"Components are for Composition". How can one build up a larger system from components?

According to our systems approach, we have to define the relations of the existing components Red and Green to the (yet hypothetical) larger system Blue.



Red and Green are somehow *embedded* in Blue.

Immersive Concept

Mathematical formulation of the question

We look for an appropriate object S and functions $f_1 : S_1 \to S$, $f_2 : S_2 \to S$ with certain properties.

For a "generic solution" we ask if for such a constellation exists a **Universal Categorial Object**, i.e. a universal U and universal maps

$$p_1 : S_1 \to U,\ p_2 : S_2 \to U,$$

such that for each triple (f_1, f_2, S) the above constellation may be written as

$$f_1 = f \circ p_1 : S_1 \to U \to S,\ f_2 = f \circ p_2 : S_2 \to U \to S$$

for a suitable $f = f_1 \oplus f_2 : U \to S$.

Immersive Concept

U is in some sense the "most general larger system" that combines the systems S_1 and S_2 without "further effect".

U in such a case is called a **direct sum** and we write $U = S_1 \coprod S_2$.

$$\begin{tikzcd} S_1 \arrow[dr, "f_1"] & \\ & S \\ S_2 \arrow[ur, "f_2"'] & \end{tikzcd} \quad \Longrightarrow \quad \begin{tikzcd} S_1 \arrow[dr, "p_1"] & & \\ & U \arrow[r, "f"] & S \\ S_2 \arrow[ur, "p_2"'] & & \end{tikzcd}$$

Mathematical Categories

Most mathematical models live in concrete **categories**, for example, the category of sets, vector spaces, fibre bundles, algebraic varieties, and so on.

Each such category is characterised by the fact that the terms **object** and **morphism** have a clear meaning there.

Morphisms between vector spaces, for example, are operationally faithful mappings, i.e. linear mappings that can be described by matrices for finite-dimensional vector spaces.

Such universal objects do not exist in every category.

Remark: The construction can easily be extended to finitely many S_i and even to infinitely many $S_i, i \in I$, and so it is defined in mathematics.

Category of Sets

In this category direct sums U exist for both finite and infinite index sets I. This is just the **disjunct union** of the sets S_i.

The maps p_i are just the embeddings $p_i : S_i \to U$ of the partial sets in their disjunct union.

The map $f : U \to S$ works as follows: For each $a \in U$ exists exactly one i and one $a' \in S_i$ with $a = p_i(a')$. Put $f(a) = f_i(a')$.

If $|S_1| = a, |S_2| = b$, then $|S_1 \coprod S_2| = a + b$.

The whole is no more than the sum of its parts.

Submersive Concept

Invert all Arrows (TRIZ Principle 13)

We look for an appropriate object S and functions $f_1 : S_1 \leftarrow S$, $f_2 : S_2 \leftarrow S$ with certain properties.

Does for such a constellation exists a **Universal Categorial Object**, i.e. a universal U and universal maps

$$p_1 : S_1 \leftarrow U,\ p_2 : S_2 \leftarrow U,$$

such that for each triple (f_1, f_2, S) the above constellation may be written as

$$f_1 = p_1 \circ f : S_1 \leftarrow U \leftarrow S,\ f_2 = p_2 \circ f : S_2 \leftarrow U \leftarrow S$$

for a suitable $f = f_1 \otimes f_2 : S \to U$.

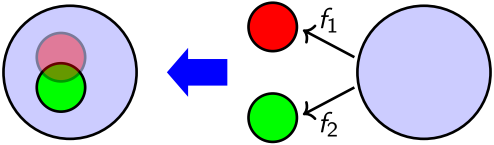
Submersive Concept

U in such a case is called a **direct product** and we write $U = S_1 \prod S_2$.

$$\begin{tikzcd} S_1 & \\ & S \arrow[ul, "f_1"'] \arrow[dl, "f_2"] \\ S_2 & \end{tikzcd} \quad \Rightarrow \quad \begin{tikzcd} S_1 & & \\ & U \arrow[ul, "p_1"'] \arrow[dl, "p_2"] & S \arrow[l, "f"'] \\ S_2 & & \end{tikzcd}$$

Submersive Concept

How does this change the perspective on the concept of system?



Red and Green are somehow *projections* of Blue.

Category of Sets

In this category direct products U exist for both finite and infinite index sets I. This is just the **cartesian product** of the sets S_i.

The maps p_i are just the projections $p_i : U \to S_i$ of the product to the individual components.

The map $f : S \to U$ works as follows: For each $a \in S$ we set $f(a) = (f_i(a)) \in U$.

If $|S_1| = a, |S_2| = b$, then $|S_1 \prod S_2| = a \cdot b$.

The whole is clearly more than the sum of its parts, most of the "information" is of relational nature.

Submersive and Immersive System Theories

System theories rarely make a distinction between between these two approaches.

To distinguish between the approaches, system theories in which the first modelling principle dominates, are called **immersive system theories**. They can be recognised their constructions are essentially based on embeddings (immersions).

System theories that are based on the second modelling principle are called **submersive system theories**. They can be recognised by the fact that their constructions are essentially based on projections (submersions) and thus on processes of staggered complexity reduction.

The Theory of Dynamical Systems is a submersive system theory.