Computational Modelling of the Solar System

N-Body Problem
ASTRONOMY CLUB

Neel, Prabhat
Deepanshu, Harshit, Karnpriya, Sandra, Shyam

Alen, Hariprasad, Ishaan, Ritu, Shiny, Stephen, Vedanta, Vivek[1]
[1]Undergraduate Students, Department of Physics, St. Stephen's College

Keywords: N-body, Solar System, simulation, stability, asteroids, Triton, Planet 9

5935 (errors:1) Words 16^{th} July, 2020

Abstract
We develop a numerical model of our solar system using the direct method for force calculation and integration using 2nd order leapfrog algorithm. We perform several tests to assess the strengths and weaknesses of our model. With necessary inclusions and modifications we then analyse and verify hypotheses and phenomenon suited to the model's capabilities. Making use of both available research as well as trial and error for parameter values. The 8 planet model is tested stable for $100,000$ years while Asteroid belt having ≈ 30 asteroids for 1000 years. For the unconfirmed planet IX we found eccentricity $= 0.3$, mass $= 8M_{\oplus}$ and semimajor axis $= 800$ AU that gave best estimates of ETNOs orbits. Hypothetical vulcan and the possible theory of triton's capture in retrograde orbit are simulated and analysed for relatively short times. Through successive longer simulations, secular variations of orbital elements of few major planets are studied: $30,000$ years for Earth and Mars and $100,000$ years for Jupiter and Saturn. Changes in eccentricity were comparable: 0.011,0.015 and 0.03 for earth,mars and jupiter while higher: 0.07 for saturn. Inclination changes in jupiter's orbit were half that of saturn's. Influence of Jupiter and Saturn are prominent but small enough over these time periods to cause any instability. Previous research shows that significant long-term variations need atleast Gigayear timeperiods for which our model and computing power is not suited.

Contents

1 Introduction — Interpretation of the question 3
  1.1 The problem . . . 3
2 Definition of Variables 5
3 Assumptions 6
4 Methods and Equations 7
  4.1 N body dynamics . . . 7
    4.1.1 The Numerical Algorithm . . . 8
  4.2 Simulation specific methods . . . 9
  4.3 Asteroid Orbits . . . 9
5 Discussion 12
  5.1 Stability of the Base model . . . 12
  5.2 Calculating Parameters . . . 12
  5.3 Particular Asteroids . . . 13
    5.3.1 (153814) 2001 WN_5 . . . 13
    5.3.2 99942 *Apophis* . . . 14
    5.3.3 2010 RF_{12} . . . 15
  5.4 Comet Bernardinelli-Bernstein . . . 18
  5.5 Triton . . . 20
  5.6 Asteroid belts . . . 22
  5.7 Planet IX . . . 22
  5.8 Long Term Evolution of Orbital Parameters . . . 24
6 Results 26
A References 31

1 Introduction — Interpretation of the question

The n body problem in physics refers to the problem of predicting the motion of a given number of bodies given their instantaneous positions, velocities and the nature of interaction between them. The problem arises naturally in the context of celestial mechanics where Sun, planets and other bodies interact with each other through their mutual gravitational forces. Being of much theoretical and practical interest the problem has a rich history starting from none other than Newton himself. He showed analytically the existence of well-known elliptic Kepler orbits for a Sun-planet system-an n body system with $n = 2$. But the influences from other planets were known to distort the ideal Kepler orbits of planets in the solar system even by Newton. There was a possibility that these influences accumulate over time in a way to make the solar system unstable. And the problem of finding a general solution by taking all the influences into account became the challenge. Even with the simple Newtonian gravitation model, analytic solution can be found only for restricted 3-body problems and in other cases of practical importance scientists resort to numerical methods.

Our solar system clearly constitutes an n body problem of relatively high importance. Through numerous researches it has been found that orbits of most planets are chaotic having Lyapunov time periods of the order of Mega years[1]. This is far shorter than the age of the solar system over which our solar system has evidently been stable. This means that the uncertainty in measured positions and velocities of planets at the present is a large enough range that after Megayears, possible solutions to those conditions diverge far from each other making any deterministic prediction useless. Nonetheless statistical studies of evolution and stability are useful and provides insights into n body dynamics.[2]

For the solar system any significant instabilities in the orbits of planets only arise after hundreds of Mega years. Special purpose integration algorithms and computing power are needed to perform such reliable long-term integrations. This limits us in this project to much shorter time periods and the study of phenomenon therein.

1.1 The problem

1. Set up a numerical model for the solar system with the Sun and 8 planets: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. The model should be able to compute trajectories of all the bodies given the initial conditions and masses.

2. Check the model's stability through simulations encompassing thousands of years. For reliability, the model should not predict any instability like ejection of bodies within these periods.

3. Test the model further by extracting data for the orbital parameters and comparison with accepted values for present times.

4. Implement and explore confirmed theories or phenomenon and other unconfirmed hypotheses related to the solar system and suited for the model

5. Check any long-term variations in eccentricities, semi major axis and other parameters for the planets. Effects of removing major planets like Jupiter and Saturn on these variations can be studied.

2 Definition of Variables

n = total number of bodies
t_i = discrete time
t_f = final time
dt = finite time-step
N = Number of steps in the simulation
m_i = Mass of i^{th} body
$\mathbf{r_i}$ = position vector of i^{th} body having the 3 Cartesian components:
= $[x \quad y \quad z]_i$
$\mathbf{v_i}$ = velocity vector of i^{th} body
= $[v_x \quad v_y \quad v_z]_i$
$\mathbf{F_i}$ = Force on i^{th} body due to all other bodies
= $[F_x \quad F_y \quad F_z]_i$
e = eccentricity (dimensionless)
s = length of semi major axis
i = inclination of orbital plane with respect to ecliptic (degrees)
L = angular momentum
E = energy
KE = kinetic energy
U_{eff} = effective potential energy in the 2 body system
r_0 = distance corresponding to minimum U_{eff}
r_p = perihelion distance
r_a = aphelion distance
M_s = mass of the sun
m_a = mass of the asteroid
d_a = distance of asteroid from the sun
v_t = tangential speed
v_r = radial speed

S.no.	**Quantity**	**Unit**
1.	Distance	Kilometre
2.	Time	Day
3.	Mass	Kilogram

Table 1
Chosen units for base quantities

3 Assumptions

We made a number of reasonable assumptions and simplifications employed in studies of celestial n body problems which were apt for our purposes, as discussed below:

1. **Gravitational force as the only interaction.** This is self-evident since we are analyzing the motion of planetary bodies no other force carries any weight in this domain.

2. **Newtonian mechanics as the model for gravity.** The area where general relativity gives much different results than Newtonian model is very strong gravitational interactions such as near black holes. To analyse the dynamics of bodies such as sun, a medium sized star, and its planets, Newtonian model proves sufficiently accurate.

3. **Isolated solar system.** This means neglecting any gravitational influence of nearby stars and other significant source of gravity on our system. The huge distance between these external sources and the solar system practically makes their influence zero and it can be easily ignored.

4. **Point mass objects.** Taking all the bodies in the simulation: sun, planets, asteroids etc. as point masses. This assumption is valid for a number of reasons. First, any significantly massive bodies in the solar system and the universe in general is a sphere due to its own gravitational pull on itself. This leads to forces exerted by them on other bodies as if their whole mass was concentrated at the centre.

5. **Collision-less system.** For the analysis of current solar system and phenomenon therein, neglecting collisions is reasonable. The planets and sun, in which we are interested does not suffer any significant collisions. They occur mostly in the asteroid belts within and outside the outer solar system and thus have no impact on the dynamics of planets. Further such situations require highly sophisticated algorithms to be properly simulated in n body codes. So we avoid analysing and implementing such situations in our simulation. Thus the distances between bodies in any simulation are always greater than their sizes. This assumption further justifies taking the point-particle approximation.

6. **Fixed mass of all bodies.** This is valid since the timescales of our simulation are very short for any significant change in masses due to physical processes to occur. Such as the predicted loss in solar mass.

The choice of which bodies to include and which can be excluded is dependent on the particular simulation and its purpose. These are crucial assumptions that will be discussed in their respective cases.

4 Methods and Equations

This section is divided into two parts: the first describing our brute-force approach to the n body problem and its implementation on python and the second covering main formulas relevant to specific simulations.

4.1 N body dynamics

In a self-gravitating n body system observed from an inertial reference frame, each body experiences a force at any given time given by the following equation:

$$\mathbf{F_i} = \sum_{j=1, j\neq i}^{n} -Gm_i m_j \frac{(\mathbf{r_i} - \mathbf{r_j})}{|\mathbf{r_i} - \mathbf{r_j}|^3}$$

Thus the acceleration is:

$$\mathbf{a_i} = \sum_{j=1, j\neq i}^{n} -Gm_j \frac{(\mathbf{r_i} - \mathbf{r_j})}{|\mathbf{r_i} - \mathbf{r_j}|^3}$$

This vector equation containing three component equations for the x,y,z directions in the Cartesian coordinate system is the core of our n body simulation.
Thus we have the following system of vector first order ODEs

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \tag{1}$$
$$\frac{d\mathbf{v}}{dt} = \mathbf{a} \tag{2}$$

Where $\mathbf{r} = (\mathbf{r_1}, \mathbf{r_2}, \ldots, \mathbf{r_n})$, $\mathbf{v} = (\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n})$ and $\mathbf{a} = (\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_n})$
This resulting system of coupled differential equations cannot be solved analytically. The only thing left is to have initial conditions $(\mathbf{r_0}, \mathbf{v_0})$ at time $t = 0$ and solve this by a suitable numerical algorithm giving us $(\mathbf{r}, \mathbf{v})$ at the discretized time points.
Total energy is conserved in the system at all times and is given by:

$$E = T + U$$

where T the total Kinetic energy and U the total potential energy are given as:

$$T = \sum_{i=1}^{n} \frac{1}{2} m_i \mathbf{v_i}.\mathbf{v_i}$$

$$U = \sum_{i=1}^{n} \sum_{j>i}^{n} -\frac{Gm_i m_j}{|\mathbf{r_i} - \mathbf{r_j}|}$$

The total angular momentum, a vector quantity is also conserved:

$$\mathbf{L} = \sum_{i=1}^{n} m_i \mathbf{r_i} \times \mathbf{v_i}$$

We compute these conserved quantities at every step in the algorithm as the standard way to check the errors in the simulation. We pick the barycentre of the solar system as our reference frame. This acts as a check for the error in the program since evaluating the Centre of Mass of our system at each time step and knowing the drift of the COM also indicates errors in the program. To get the actual initial conditions for the bodies in the solar system we used the HORIZONS ephemeris system developed by NASA-JPL[3]. **01 January 2021** is taken as the starting time t_0 in every simulation

4.1.1 The Numerical Algorithm

We chose to work with leapfrog integrator of the 2nd order for its simplicity, ease of application and properties very useful in integrating Hamiltonian systems such as time-reversibility and energy, angular momentum conservation.
We have

$$N = \frac{t_f - t_0}{dt}$$

Thus $t_1 = t_0 + dt$, $t_2 = t_0 + 2dt$ and so on till $t_f = t_0 + Ndt$. Beginning with $i = 0$ the following set of equations contains the leapfrog method.

$$\mathbf{v}(t_i + dt/2) = \quad \mathbf{v}(t_i) + \frac{dt}{2} \times \mathbf{a}(t_i, \mathbf{r}(t_i)) \tag{3}$$

$$\mathbf{r}(t_i + dt) = \quad \mathbf{r}(t_i) + dt \times \mathbf{v}(t_i + dt/2) \tag{4}$$

$$\mathbf{v}(t_i + dt) = \quad \mathbf{v}(t_i + dt/2) + \frac{dt}{2} \times \mathbf{a}(t_i + dt, \mathbf{r}(t_i + dt)) \tag{5}$$

These sets of equations are solved iteratively from $i = 0$ to $i = N - 1$ The third equation is only useful in getting the velocity at the same instants as the positions. Otherwise the method calculates them in a staggered fashion. Removing the third equation shows the symmetry inherent in the method:

$$\mathbf{v}(t_i + dt/2) = \quad \mathbf{v}(t_i - dt/2) + dt \times \mathbf{a}(t_i, \mathbf{r}(t_i)) \tag{6}$$

$$\mathbf{r}(t_i + dt) = \quad \mathbf{r}(t_i) + dt \times \mathbf{v}(t_i + dt/2) \tag{7}$$

But since initial conditions cannot be given at two instants, the first half-velocity is evaluated using first-order Euler expansion to kick-start the algorithm, as shown in previous set
The python implementation using 3D numpy arrays is given in appendix.

4.2 Simulation specific methods

The perihelion r_p and aphelion r_a distances of each planet or any other body is found by the concept of local maxima and minima. During each orbit, when the body is about to pass through the perihelion position, its distance from the sun decreases, reaches a minima(r_p) and again starts increasing. Similarly on reaching aphelion, distance increases, reaches a maxima and then starts decreasing.
For calculating eccentricity of an orbit:

$$e = 1 - \frac{2}{\frac{r_a}{r_p} + 1}$$

The semi-major axis is simply:

$$s = \frac{r_a + r_p}{2}$$

The orbital inclination angle is found by taking the cross product of two consecutive position vectors in the orbit of the body. This produces a normal vector to the orbit. Then taking its dot product with the $\hat{\mathbf{z}}$ vector gives the inclination with respect to z axis. Inclinations with respect to ecliptic plane can then be found by subtraction.

4.3 Asteroid Orbits

We have given the asteroids initial conditions such that if the sun had been the only other body in the solar system, the asteroid's orbit would have been closed (elliptical) and stable forever. The orbital parameters (eccentricity, semi-major axis and inclination) for these asteroids have been chosen randomly from a range corresponding to the real orbital parameters of the majority of asteroids in the main belt. The longitude of ascending node has been chosen randomly without constraint. The initial distance of the asteroid from the sun (d_a) has been chosen randomly between the perihelion and aphelion distances. The method is detailed below. The equations of the two body problem with one body much more massive than the other have been assumed.

Given the semi-major axis (s) and the eccentricity (e), the perihelion and aphelion distance are respectively:

$$r_p = s(1 - e)$$

$$r_a = s(1 + e)$$

From the theory of the 2 body system we have the angular momentum of the asteroid, its total energy, the PE, and KE to be:

$$L = \sqrt{r_0 M_s m_a^2}$$

$$E = -\frac{GM_s m_a}{2s}$$

$$U = -\frac{GM_s m_a}{d_a}$$

$$KE = E - U$$

The centrifugal term (which is basically the angular kinetic energy) of the effective potential energy,

$$= \frac{La^2}{2m_a d_a^2}$$

The radial KE is,

$$= KE - \frac{La^2}{2m_a d_a^2}$$

The effective potential energy will therefore be,

$$U_{eff} = U + = \frac{La^2}{2m_a d_a^2}$$

And the distance corresponding to minimum U_{eff} (r_0) is:

$$r_0 = s(1 - e^2)$$

We proceed to find initial position and velocity for an asteroid with given eccentricity, semi-major axis, inclination and ϕ (which is the angle between the x axis and the projection of the semi major axis on the xyplane; it is a non standard quantity, which for our purposes plays the role of ascending node longitude of ascending node). We will first find such an orbit in the xy plane and then use matrix transformations to convert the orbit into a 3 dimensional one with the required eccentricity and longitude of ascending node.
The tangential and radial speed will be respectively:

$$v_t = \frac{L}{m_a d_a}$$

$$v_r = \sqrt{\frac{2 \times KE}{m_a}}$$

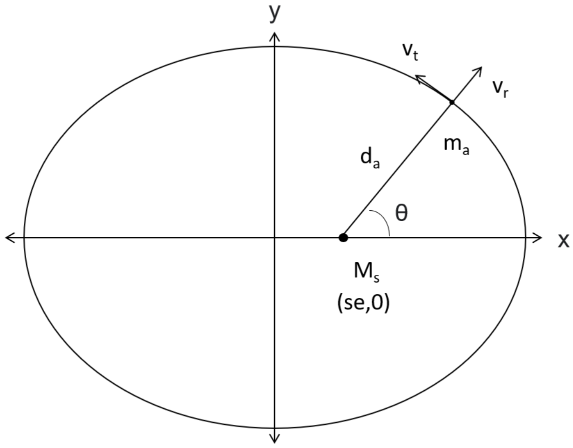


Fig. 1. Figure 1

Satisfying the coordinates in the equation for an ellipse (refer to 1):

$$\frac{(r\cos\theta + ae)^2}{a^2} + \frac{r^2\sin^2\theta}{a^2(1-e^2)} = 1$$

Solving for $\cos\theta$ we get*:

$$\Longrightarrow \cos\theta = \frac{2d_a se(1-e^2) - \sqrt{4d_a^2 s^2 e^2 (1-e^2)^2 + 4e^2 d_a^2\{d_a^2 + s^2e^2(1-e^2) - s^2(1-e^2)\}}}{2e^2 d_a^2}$$

$\sin\theta$ can take one of these 2 values,

$$\pm\sqrt{1-\cos^2\theta}$$

The position coordinates of the asteroid after shifting the origin to the sun's position will be:

$$[d_a\cos\theta \quad d_a\sin\theta \quad 0]$$

The velocity components will be:

$$[v_r\cos\theta - v_t\sin\theta \quad v_r t\sin\theta + v_t\cos\theta \quad 0]$$

Now, in order to obtain orbits with the required inclination and ϕ we pre-multiply the following matrix product to our position and velocity coordinates:

*There will be 2 values of $\cos\theta$ obtained, we reject one of them as it can take values greater than 1 or less than -1

$$\begin{vmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \cos i & 0 & -\sin i \\ 0 & 1 & 0 \\ \sin i & 0 & \cos i \end{vmatrix}$$

To the position and velocity vectors so obtained, we add the position and velocity vector of the sun to convert from the sun's frame to barycentric coordinates.

The 4 most massive asteroids (which contribute 50% of the asteroid belt's mass) were added with their real initial conditions taken from HORIZONS. 25 other asteroids were added with initial conditions as explained above. They contribute the rest 50% of the mass of the asteroid belt in our model.

5 Discussion

We first describe the numerical tests performed to assess the validity and robustness of the model and algorithm, comparing the results with the actual Solar System. Afterwards, we perform and analyse a number of simulations for different purposes by modifying the base model.

5.1 Stability of the Base model

The Sun and the 8 planets of the solar system is taken as the base Solar System model. With the initial conditions taken from the HORIZONS platform with the source body selected as "Barycentre", the centre of mass of this base solar system model was found to be shifted from the origin and having non-zero velocity components This is explained by noting that we have not included all bodies present in the actual solar system. And later we find that on inclusion of other significant mass bodies such as heavy moons, asteroids etc, the position and velocity of the COM converges to 0. To come exactly in the COM of this system, we added a code segment that changed the initial conditions so that $\mathbf{r_{CM}}$, $\mathbf{v_{CM}}$ became zero.
We ran the simulation 50,000 years backward and 50,000 years forward in time from the epoch of 1st January, 2021 with time-step $dt = 1$ day. Covering a total of 100,000 years. No signs of instability were found and each body remained in its orbit. The fractional energy changes were bound throughout because of the symplectic nature of the algorithm and were of the order of 10^{-6}. This order of error is needed to prevent unphysical variations in the orbital parameters of planets.[1] The angular momentum was conserved exactly as is expected from leapfrog integrator.

5.2 Calculating Parameters

Values of Eccentricity, Semi-major axis and orbital inclination were found for the Eight planets in the base model. The inner planets needed a smaller dt for their compact orbits to be

resolved as accurately as the outer planets. This also gets reflected in the energy conservation. We found that on keeping just the outer four planets, $dt = 4$ gave comparable energy error as $dt = 1$ when all 8 planets are included. The results are shown in the tables: 2, 3, 4 with actual data[4]. For Mercury to Earth, $dt = 0.01$ is used and for Jupiter to Neptune, $dt = 1$.
The 2nd order leapfrog integrator lacks in computing accurate positions at any given instant. Thus, particular values of parameters found are bound to be inaccurate to some extent. Though on using a small dt most values converge. The variations in the orbital parameters are more important and the integrator closely resembles the actual system here, compared to other non-symplectic integrators.
For the semi major axes and inclinations, the errors are too small to confidently characterise their cause. Interestingly, the planets having high errors in eccentricity have much less errors in semi-major axis. This can imply that r_a is estimated less and r_p more than actual value, so that e is changed but not the semi major axis. For inclination angle, the errors are small, but the last two columns show that the values are either always lower or higher for most planets. One possible reason could be change in Earth's ecliptic and other planets' inclination between 2000 to 2021.

5.3 Particular Asteroids

In order to check the accuracy of our simulation we measured the close approaches of several asteroids within the next 100 years to Earth. We calculated the date and distance of their closest approach and compared the results to the high accuracy predictions by NASA JPL. In order to make the predictions more accurate we worked with a small time step of $dt = 0.001\ days$. We took initial conditions for the asteroids from the NASA JPL Small Body Database[5]. Since our code begins from 1st January 2021, all asteroid conditions were also taken correspondingly. We took a total of 20 bodies in our system including the 9 planets, Earth's moon(since it can effect asteroid close approach calculations strongly), and a range of asteroids. Additionally asteroid masses are not well defined hence some of them have to be approximated on basis of Mean Asteroid Density and approximate radius of the asteroid. This was performed for a total of 3 asteroids and one comet each of which are discussed below. We find the predictions to agree closely with the NASA high accuracy prediction(specially with date of closest approach) confirming that our model works in accordance with existing models.

5.3.1 $(153814)\ 2001\ WN_5$

(153814) 2001 WN5 is a sub-kilometer asteroid, classified as near-Earth object and potentially hazardous asteroid of the Apollo group. It was discovered by the Lowell Observatory Near-Earth-Object Search at Anderson Mesa Station on 20 November 2001[6] The potentially hazardous asteroid was removed from the Sentry Risk Table on 30 January 2002.
NASA predicts that this asteroid will make its closest approach within 0.65 Lunar Distances on 26^{th} June, 2028, our code predicts that the asteroid will make its closest approach 2732.71 days

from the 1st of January 2021. To the nearest day this takes us exactly to the date predicted by NASA. We however predict the closest approach distance to be 1.09 Lunar Distances. This deviation from the NASA prediction can be justified by the fact that we have taken a second order approximation of time, hence although we calculate the correct day the distance calculation is not as accurate. Figure 2 shows the distance of the asteroid from Earth as a function of time. The point marked in red demonstrates the minimum. This also demonstrates

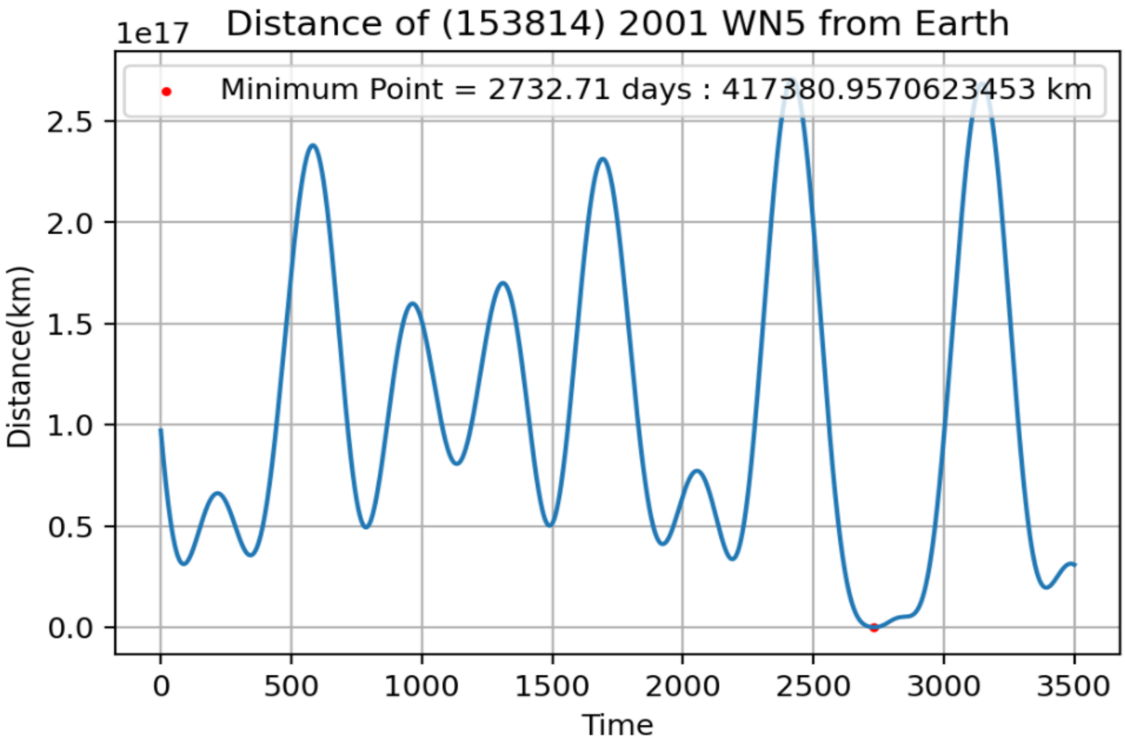


Fig. 2. Earths Distance from (153814) 2001 WN5 over time

that this approach is far closer than the approaches of the asteroid before 2028. Figure 3 shows the orbits of Earth and the asteroid on the same plot.

5.3.2 99942 *Apophis*

99942 Apophis is a near-Earth and potentially hazardous asteroid which was discovered in June 2004. Apophis made a close approach to Earth in December 2004 and caused a great stir in the media when initial predictions suggested Apophis will impact Earth with a probability of upto 2.7%. However the possibility of impact was soon eliminated by 2006. During the short time when it had been of greatest concern, Apophis set the record for highest rating ever on the Torino scale, reaching level 4 on December 27, 2004.[7]. Interestingly Apophis had a very close approach to Earth in 2021 March and recently the chance of collision in 2068 was ruled out by new calculations.

We specifically aimed to study the predicted close approach of Apophis on 13^{th} April 2029 at a distance of 0.1 Lunar Distances. We predict that Apophis will make its closest approach 3024.63 days from 1^{st} January 2021 leading us to 14^{th} April 2029. Interestingly the closest

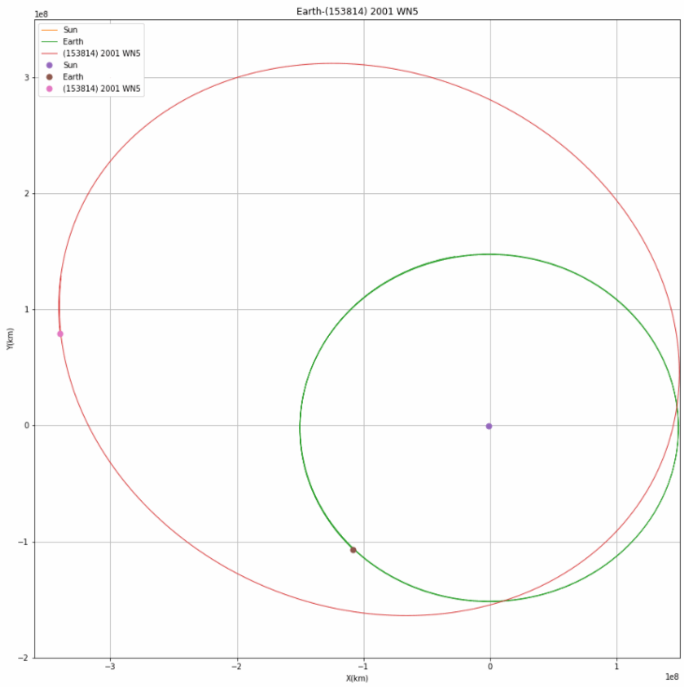


Fig. 3. Earth and 2001WN5 orbit

approach of Apophis is expected at 21:46 UT on 13^{th} April 2029 also giving us closest day as 14^{th} April 2029. We additionally predict closest approach distance to be 0.1079 Lunar Distances corresponding closely with the NASA prediction. Figure 4 shows the distance of Earth from Apophis as a function of time; it also demonstrates the close approach of Apophis in 2021. Figure 5 demonstrates Apophis' orbit with Earth's.

5.3.3 $2010\ RF_{12}$

2010 RF12 is a very small asteroid, classified as near-Earth object of the Apollo group, it was discovered in September 2010 when it passed between the Earth and Moon. It is listed on the Sentry Risk Table as the asteroid with the greatest known probability (5%) of impacting Earth.[8]. NASA predicts a very close approach of this asteroid at a distance of 0.1 Lunar Distances on 6^{th} September 2095. We predict the closest approach 27273.8 days from 1^{st} January 2021 which on the closest day brings us to 4^{th} September 2095. Additionally we find that this closest approach takes place at a distance of 0.1082 Lunar Distances which closely agrees with NASA's prediction. Figure 6 shows the distance of the asteroid from Earth as a function of time. Interestingly the plot demonstrates that this asteroid makes close approaches to Earth regularly(at an approximately constant frequency). Figure 7 shows the orbit of the asteroid with Earth's.

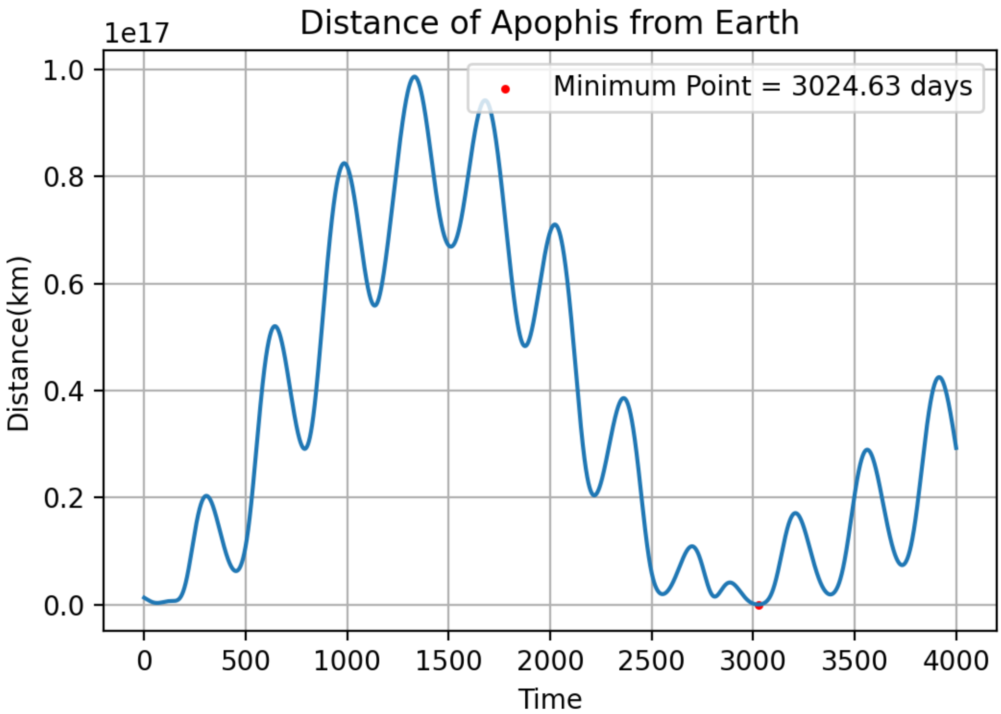


Fig. 4. Distance of Earth from Apophis

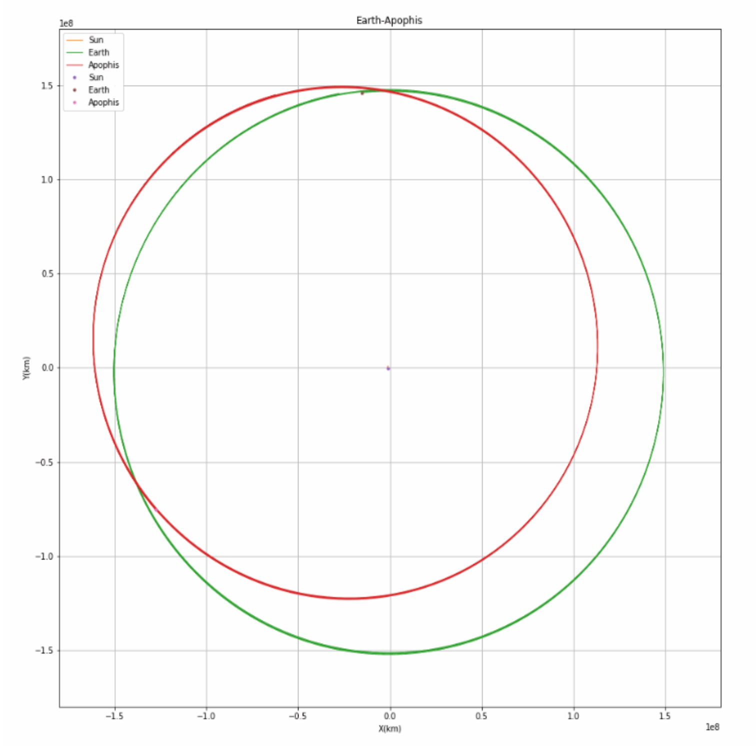


Fig. 5. Orbit of Earth and Apophis

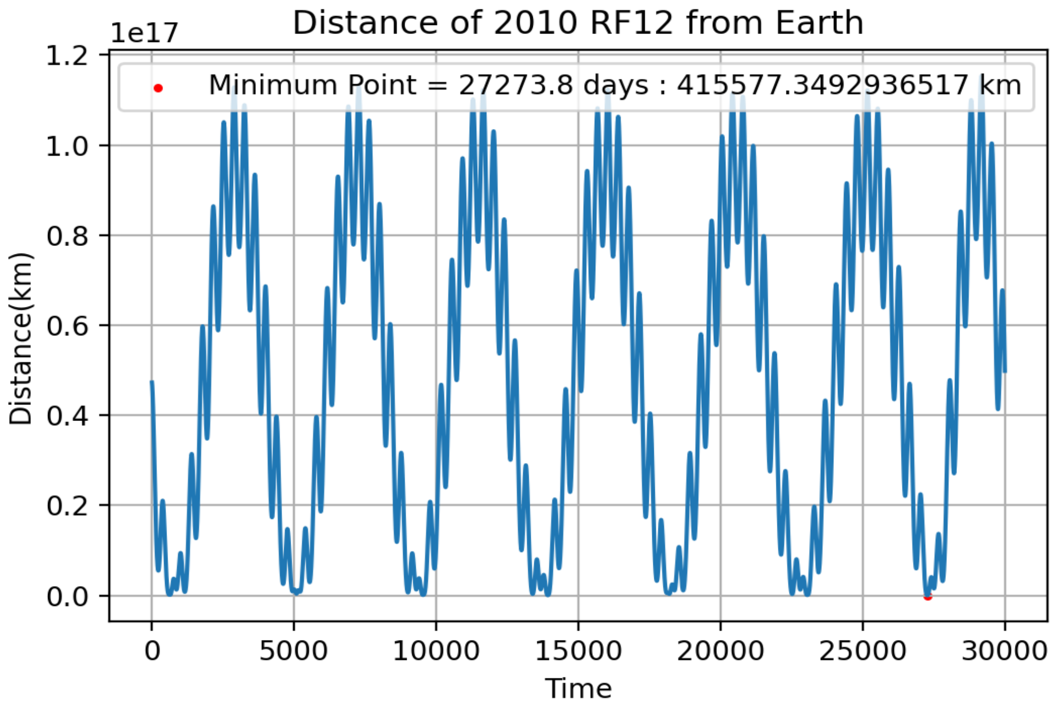


Fig. 6. Earth's Distance from 2010 RF12

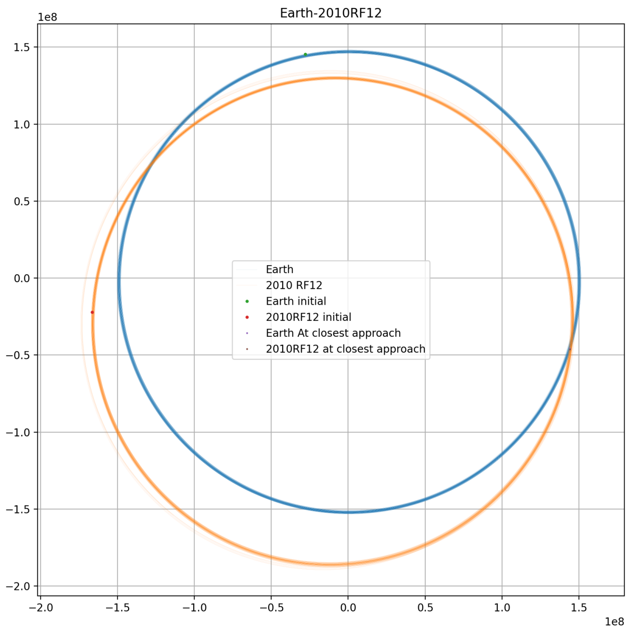


Fig. 7. Orbit of 2010 RF12

5.4 Comet Bernardinelli-Bernstein

Comet is a potentially large Oort cloud comet discovered by astronomers Pedro Bernardinelli and Gary Bernstein in archival images from the Dark Energy Survey.[9]. The object was first discovered in 2014 almost as far as Neptune's orbit and is expected to read perhelion on 23^{rd} January 2021 at a distance of 10.95 AU[10]. Additionally it will make its closest approach to Earth around 5 April 2031 at a distance of $10.11\ AU$[11]. Bernardinelli conservatively estimates that the dusty, icy nucleus is between 62 and 125 miles long, which makes it the largest comet ever seen.[12].

Figure 8 shows the orbit of the comet with respect to the Ecliptic plane and shows that its plane of orbit is approximately perpendicular to the ecliptic. This plot also demonstrates the inclination of Pluto's orbit to the ecpliptic. Figure 9 shows the projection of the comets orbit on the xy(ecliptic) plane and shows that it crosses the plane just beyond the orbit of Saturn.

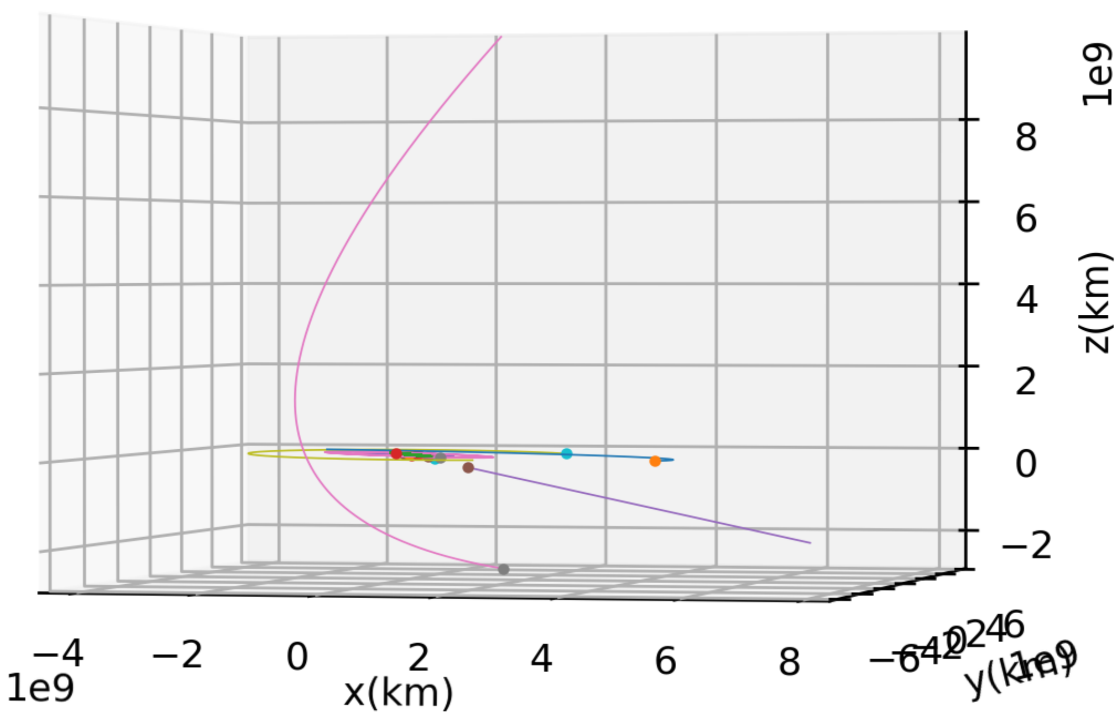


Fig. 8. Comet Orbit with respect to the Ecliptic

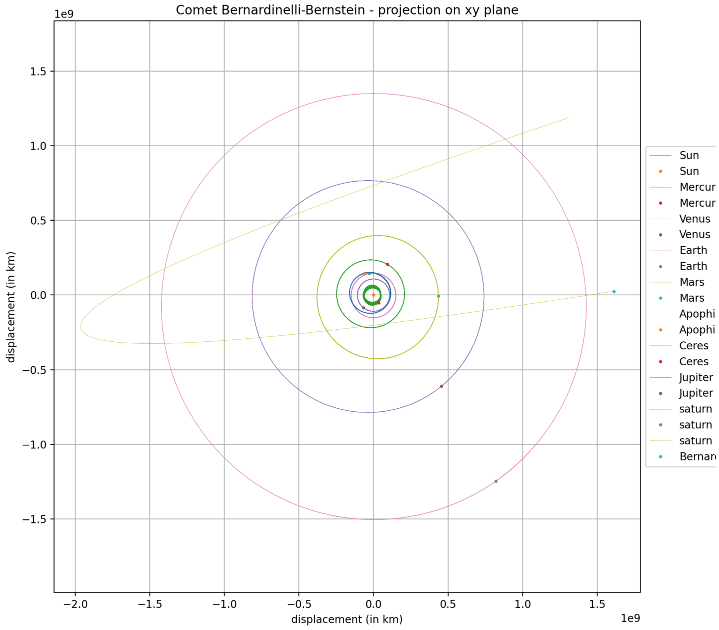
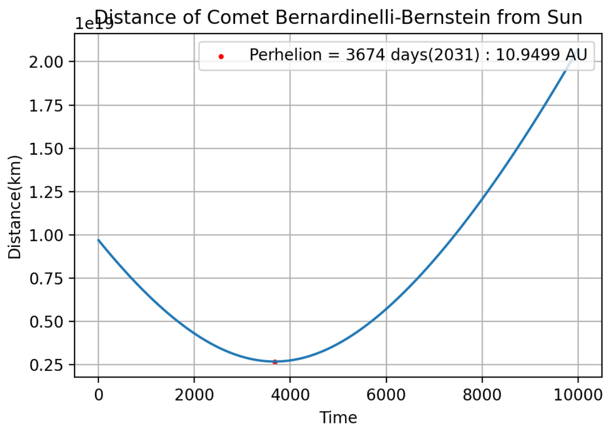
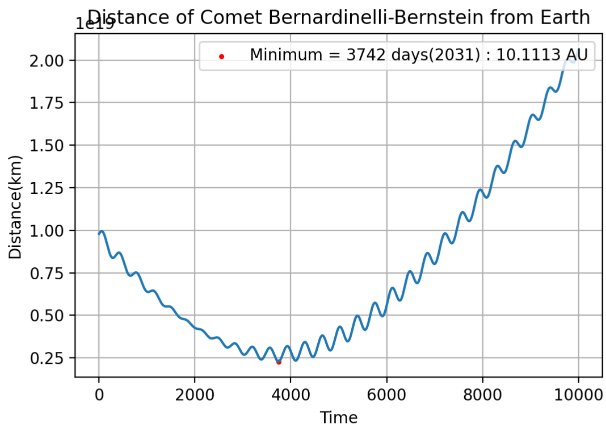


Fig. 9. Comet Orbit Projection

We additionally analyzed the distance of the Comet from the Sun and Earth as functions of time and matched to NASA predictions. Figure 10 shows the distance of the comet from the Sun and Earth as functions of Time(no. of days from 1st January 2021). This helps us predict Perhelion on 23^{rd} January 2031 at a distance of $10.9499\ AU$ closely matching NASA's predictions. We additionally predict the asteroid makes it closest approach to Earth on 1^{st} April 2031 at a distance of $10.1113\ AU$ agreeing closely to NASA's high accuracy prediction.



(a) Distance from Sun



(b) Distance from Earth

Fig. 10. Figure 10

5.5 Triton

Most moons in the solar system orbit around their planets in **Prograde** orbits, i.e. the direction of rotation of the moon around its planet is in the same direction as the rotation of that planet. Triton is unique among all large moons in the Solar System for its retrograde orbit around Neptune and it is almost 12.5 times larger than any other known moon in a retrograde orbit. However all theories of moon formation do not allow for the occurrence of such orbits leading to several theories including that Triton was gravitationally 'captured' by Neptune and that it may have originated in the Kuiper Belt.[13] The theory of Triton's capture can also explain several other features of Neptune's Moons including the high eccentricity of Nereid's orbit and Neptune's scarcity of moons compared to other large planets like Jupiter and Saturn. It is proposed that Triton's initially eccentric orbit could have intersected orbits of irregular moons and disrupted those of smaller regular moons, dispersing them and creating the situation we see today.

There are several proposed mechanisms of the capture of Triton, it is suggested that Triton could have been slowed by collision with an object close to Neptune(probably a moon in orbit) making it lose enough energy to be captured in orbit. We aim to study the feasibility of the more recent hypothesis that, before its capture, Triton was one part of a binary system: when it encountered Neptune, it interacted in such a way that the binary system was broken up with one body expelled and one body(Triton) becoming bound to Neptune. In order to study this we consider the following assumptions:

- The time period in which this interaction occurs is much smaller than the time period of Neptune's orbit around the sun. In this time period we conclude that Neptune is approximately fixed with respect to the sun. Hence we solve with respect to Neptune placing it at the origin initially
- We assume that this interaction is planar
- Since other planets and bodies in the solar system have little effect on interactions so close to Neptune we disregard their influence.

Hence we set up the system as follows: we take 3 bodies into account with Neptune and a binary with both masses equal to that of Triton. In order for a satellite to be caught in a planets orbit, it needs to pass within its Hill Radius with low enough velocity. We use the ideas proposed by Agnor, C., Hamilton (2006)[13] and hit and trial to provide the following initial conditions:

- Neptune:

$$[x, y, z] = [0, 0, 0]$$

$$[v_x, v_y, v_z] = [0, 0, 0]$$

- Mass 1:

$$[x, y, z] = [-20R_N, 10R_N, 0]$$

$$[v_x, v_y, v_z] = \left[2 \times 10^5, 0, 0\right]$$

- Mass 2:

$$[x, y, z] = [-20R_N, -10R_N, 0]$$

$$[v_x, v_y, v_z] = \left[1 \times 10^5, 0, 0\right]$$

Where R_N is the Radius of Neptune in Kilometers

We take a small time step of 0.0001 days and run the code for 365 days. We then find that at first both bodies are held in orbit around Neptune however we find the that eccentricity of one body's orbit increases very fast. This results in the body eventually getting ejected while the other one is held in a stable clockwise elliptical orbit of high eccentricity. Although Triton's orbit today is almost circular, calculations R. A. Jacobson (2009)[14] suggest that this is thought to have happened over time due to gas drag from a prograde debris disc.

Our capture simulation suggests that such a mechanism for the retrograde orbit of Neptune is possible and agrees with the results of R. A. Jacobson (2009)[14].

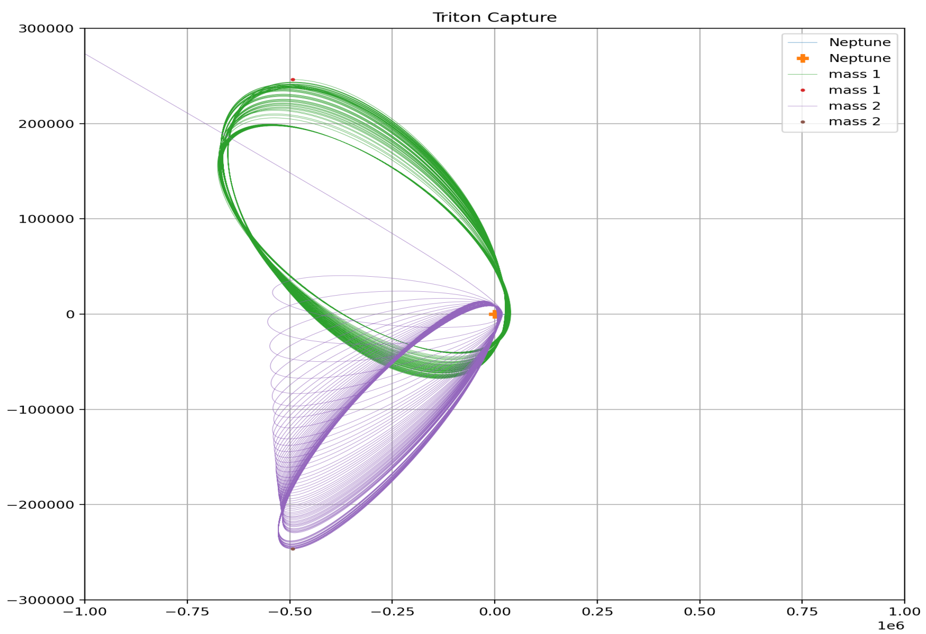


Fig. 11. Possible Triton Capture Mechanism

5.6 Asteroid belts

Since Asteroid belts have a very large amount of discrete asteroid bodies, its very hard to take into account all of them (although one just has to increase number of bodies in simulation but it becomes calculation heavy really early), we tried making our simulation as less calculation heavy as possible and tried to minimize error, we tried many combinations but the following way was what we considered most suitable. We put in the 4 heaviest asteroids (which contribute around 50% of the mass of the asteroid belt) with their real initial conditions from NASA's HORIZONS site. The other asteroids contribute the other 50% of the asteroid belt's mass. Their initial conditions are chosen such that they would have closed (elliptical) orbits in the 2 body problem with one object (the Sun) being much more massive than the other. The semi-major axis, eccentricity and inclination are chosen randomly from a certain range in which the real orbital parameters of the majority of asteroids in the main belt lie. The longitude of the ascending node is chosen randomly without constraint. With these initial conditions, our system was stable for 1000 years. Asteroids with semi-major axes corresponding to the Kirkwood gaps i.e. equal to 2.5 AU (and eccentricity and inclination close to Jupiter's) were also observed to have stable orbits over this duration. We cannot make any confident assertions by plotting the evolution of the semi-major axes of these asteroids during this duration. However, this isn't unusual - the asteroids in the Kirkwood gap take a few million years to leave the asteroid belt and significant changes in their orbits would probably take a lot more time. Since the asteroids' initial conditions correspond to stable orbits in the 2 body case, and since we've observed stable orbits for 1000 years in the presence of the sun and planets, the removal of Jupiter in our model will not significantly affect the orbits. Hence, we skipped this task.We also tried putting asteroid belt close to mars but as usual nothing out of ordinary was seen, except behaviour of pallas which crossed orbit of mars and asteroids while in reality it does not do so, still that is due to program discrepancies but system itself stayed stable so again nothing much of importance. Then we tried putting the asteroid belt between mercury and venus in hopes of some chaos, while the asteroid almost took on orbits of mercury and venus, the system stayed stable so once again no instability was seen. But none of it seemed unusual since instability is seen over a period of at least million years.

5.7 Planet IX

Planet 9 is a hypothetical planet in the outer region of Solar System. Its gravitational effects could explain the clustering of the orbits of a group of extreme trans-Neptunian objects(ETNOs). The discovery of unusual orbit of Sedna led to the speculation that it had encountered a massive body other than the known planets. It has a detached orbit having a semi major axis that is too large to be due to Neptune. It is believed by many authors that Sedna entered this orbit after encountering a massive object on a distant orbit. The discovery of other ETNOs with similar orbits supported the claim that the unusual orbits of these objects was not due to a massive body that passed near the Solar System but in fact because of an unknown Super-Earth in the distant Solar System.[15]

To the basic model of the Solar System we have added the hypothetical planet 9 and four ETNOs which include —90377 Sedna, 2007 TG_{422}, 2004 VN_{112} and 2010 GB_{174}.We have varied the orbital parameters(eccentricity and semi major axis) and mass of planet 9 and observed the orbital parameters of ETNOs for different values. Like other planets the initial positions and velocities of ETNOs have been obtained with respect to the barrycenter from JPL Horizons keeping the initial time as 1st January 2021. The mass of ETNOs was hard to find but a range of mass for 90377 Sedna could be found here, therefore, we have taken a synthetic data for the masses of ETNOs which ranges from 1e21 - 10e21 kgs.

As suggested by Batygin & Brown(2016)[16] we have taken the inclination of planet 9 to be $20°$ and have varied eccentricity form 0.1 - 0.9 with a step size of 0.1. After getting the value of eccentricity we have also varied Planet 9's mass from $5M_{\oplus} - 10M_{\oplus}$ and semi major axis from $400 - 800\ AU$.

For every set of parameters we have ran the programme for 25000 years with a time step of 5 days. This time step might seem a little large but since we are studying Kuiper Belt Objects which have an orbital period of over 10000 years our results were good enough. We were forced to choose a larger step size because of the restricted computational power and the need of running the simulation for a longer duration of time due to large orbital periods.

Our programme gave the best values of parameters for ETNOs for the following set of parameters for planet 9:

- $e = 0.3$
- $M = 8M_{\oplus}$
- $s = 800\ AU$

So we finally ran the programme for $e = 0.35$ with a timestep of 2.5 days which gave us very little deviation from actual values. The error in most of the parameters is less than 1 %. The data for this set of parameters has been given in tables 5, 6 and 7. The actual values of orbital parameters for ETNOs have been obtained from JPL Small Body Database.[17] A 2D plot of all the objects has been given in figure 12.

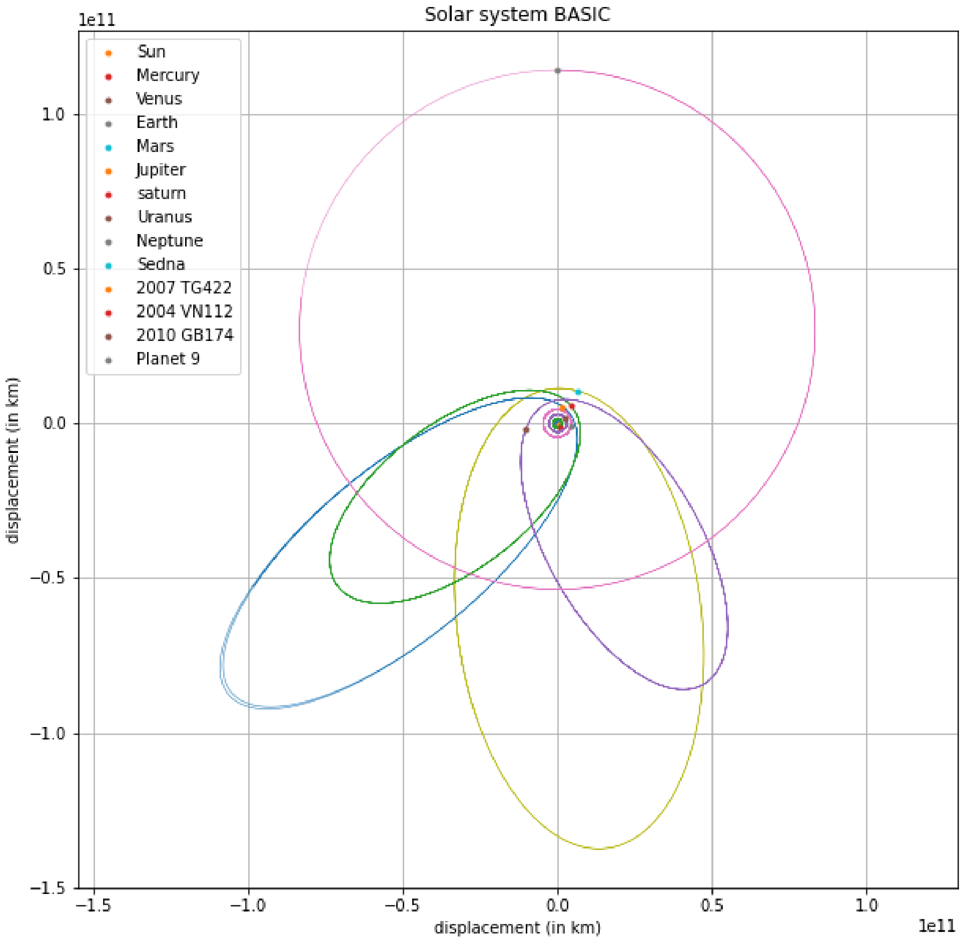


Fig. 12. Orbits of All the bodies including Planet 9 and ETNOs

We also ran our simulation after removing the terrestrial planets with a time step of 2.5 days and we didn't observe any significant change in the orbital parameters of ETNOs. This allows us to conclude that terrestrial planets do not have any significant effect on the orbits of ETNOs.

Also to check whether Planet 9 is actually responsible for the clustering of orbits of ETNOs we ran the programme after removing planet 9 to see any visible changes in their orbits. But there wasn't any change observed. So we conclude that Planet 9 (if it exists) is only responsible for initially elongating the orbits of ETNOs, once they acheive elongated orbits they continue in that orbit without much perturbation by planet 9.

5.8 Long Term Evolution of Orbital Parameters

We ran simulations of different systems and plotted e, i and s of some planets over a course of thousands of years.

First we plotted Earth's eccentricity in a keplerian two body ($n = 2$) system of earth and sun for $40,000$ years , $dt = 1$ to check if any unexpected long-term variations occurs due to errors. No such variation was found. Only minute bounded errors of the order of 10^{-7} were present. The initial conditions having no z component, it remained zero throughout, implying no inclination changes. Thus, we can be confident that the orbital parameter changes found in following simulations are due to perturbations from other planets and are not just numerical errors.

1. We ran the base model with $dt = 1$ and plotted e,i,s vs orbit number for Earth, Mars, Jupiter and Saturn. We did this by running successive simulations of 2000 years each. Mercury, Venus are excluded since dt is not small enough for their orbits. Neptune and Uranus were excluded because of low number of orbits per simulation. Few plots are shown: Secular and periodic variation were found in e and i but not in s. Same simulations were ran again with $dt = 0.2$ and mercury, Venus excluded for better energy conservation $\Delta E/E \approx 10^{-9}$. e and i variations were affected. One immediately sees that e goes to a minimum of 0.05 for Earth whereas earlier it remained bounded with a minimum value around 0.0155. The new results matched with other verified resources.So, we concluded that this bound was caused by low accuracy of the earlier simulation but its difficult to figure out the exact reason it happened.
   The inclination varied slower for both planets but the pattern was unchanged. These inclination cycles represents oscillations of the plane of orbits due to torques from other planets. Finally it can be said that mercury and venus have insignificant contribution to changes in other planet's orbits. Their exclusion in fact led to better energy conservation and less error-induced periodic variations.
   Note: Moon was included but it didn't affected the variations for Earth's elements, just the particular value of eccentricity.

2. In the next system, jupiter and saturn were excluded from the 8 planets.e now changed 7-10 times slower. Rate of change in i increased for Earth, and decreased for Mars. This can be reasonable as Mars and other planets now applies torque in the direction in which Earth's orbit keeps inclining without jupiter and saturn's counter torque slowing it. While for Mars, jupiter and saturn's torque were aligned to its inclination changes. Without them, the inclination change slows down significantly.

3. Finally we took just the 4 outer planets, $dt = 4$ and were able to run for $100,000$ years: 10,000 years per simulation with $\Delta E/E \approx 10^{-6}$. Jupiter and Saturn's variations matched the first system's results.s still remain unchanged throughout. e and i varied in step with each other. This shows how these two affect not just the other minor planets but also themselves and if small perturbations from other planets are removed, their variations tend to get in perfect step with each other. The e variations have

interesting patterns. There exist short periodic cycles ≈ 100 orbits for jupiter and ≈ 50 orbits of Saturn superimposed on the longer-term cycles: $\approx 51,600$ years for jupiter and $\approx 49,300$ years for saturn. These were also observed when the same system was simulated with $dt = 0.2$ for verification.

Due to low number of years per simulation, the variations due to errors are evident in the thickness of the curves. But they are minute and don't overshadow the actual and sufficiently greater secular variations.

6 Results

Table 2

Eccentricities

PLANET	Eccentricity				First 10 orbits	
	Actual data	First orbit value	Round off value	**Percentage Error**	**Min**	**Max**
Mercury	0.2056	0.205658522704313	0.2057	0.03%	0.2056	0.2057
Venus	0.0067	0.00679662186725449	0.0068	1.44%	0.0067	0.0068
Earth	0.0167	0.0158760041879939	0.0159	-4.93%	0.0158	0.0159
Mars	0.0935	0.0934070016449676	0.0934	-0.10%	0.0933	0.0935
Jupiter	0.0489	0.0482994692680768	0.0483	-1.23%	0.0479	0.0489
Saturn	0.0565	0.0539817805180483	0.0540	-4.46%	0.0540	0.0554
Uranus	0.0457	0.047373869736339	0.0474	3.66%	0.0474	0.0485
Neptune	0.0113	0.00891562010393165	0.0089	-21.10%	0.0089	0.0100

Table 3

Semi-major axis

PLANET	Semimajor axis(kms)				First 10 orbits	
	Actual data	First orbit value	**Round off value**	**Percentage Error**	**Min**	**Max**
Mercury	57,909,000.00	57,906,940.14	57,907,000	-0.00%	57,907,000	57907000
Venus	108,209,000.00	108,206,429.00	108,206,000.00	-0.00%	108203000	108206000
Earth	149,596,000.00	149,486,249.00	149,486,000.00	-0.07%	149464000	149469000
Mars	227,923,000.00	227,921,000.00	228,923,000.00	0.44%	227919000	227939000
Jupiter	778,570,000.00	778,034,334.86	778,034,000.00	-0.07%	778016000	778299000
Saturn	1,433,530,000.00	1,426,003,203.14	1,426,000,000.00	-0.53%	1426000000	1429140000
Uranus	2,872,460,000.00	2,870,650,133.69	2,870,650,000.00	-0.06%	2868800000	2871480000
Neptune	4,495,060,000.00	4,498,613,841.14	4,498,610,000.00	0.08%	4459170000	4499300000

Table 4
Orbit Inclination with respect to ecliptic plane

PLANET	Orbit Inclination angle (degrees)				First 10 orbits	
	Actual data	First orbit value	**Round off**	**Percentage Error**	**Min**	**Max**
Mercury	7.005	7.13538504039581	7.135	1.86%	7.078	7.135
Venus	3.395	3.3793867211544	3.379	-0.46%	3.373	3.395
Earth	0	0	0.000	0.00%	0.00	0.00
Mars	1.851	1.84386774418914	1.844	-0.39%	1.840	1.847
Jupiter	1.305	1.3003135726774	1.300	-0.36%	1.294	1.301
Saturn	2.485	2.48802132955896	2.488	0.12%	2.484	2.499
Uranus	0.772	0.768354059858815	0.768	-0.47%	0.759	0.768
Neptune	1.769	1.7617251306493	1.762	-0.41%	1.757	1.766

Table 5
Eccentricity of ETNOs with Planet 9

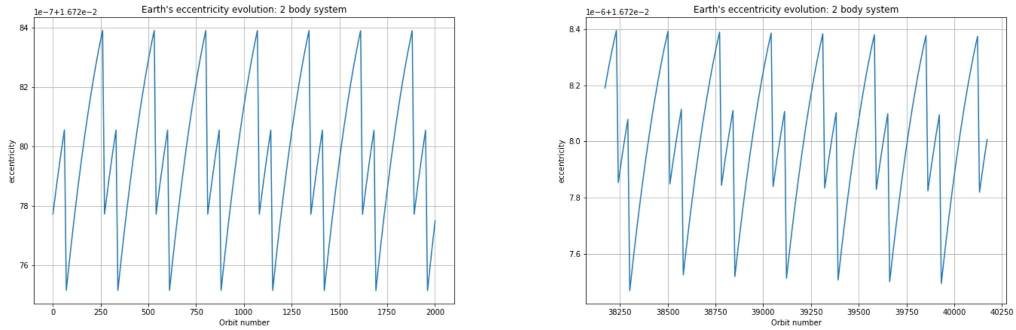
PLANET	Eccentricity			
	Actual Data	Calculated Data	Round off	Percentage Error
90377 Sedna	0.8496	0.849468522	0.8495	-0.01177 %
2007 TG_{422}	0.9248	0.929145455	0.9291	0.4650 %
2004 VN_{112}	0.8579	0.855612428	0.8556	-0.2681 %
2010 GB_{174}	0.869	0.860695481	0.861	-0.920 %

Table 6
Semi Major Axis of ETNOs with Planet 9

PLANET	Semi Major Axis (AU)			
	Actual Data	Calculated Data	Round off	Percentage Error
90377 Sedna	506	506.108542	506	0.00 %
2007 TG_{422}	472.77	497.6163227	497.62	5.2562 %
2004 VN_{112}	332.8	327.3590432	327.4	-1.622 %
2010 GB_{174}	350.7	348.4217574	348.4	-0.6558 %

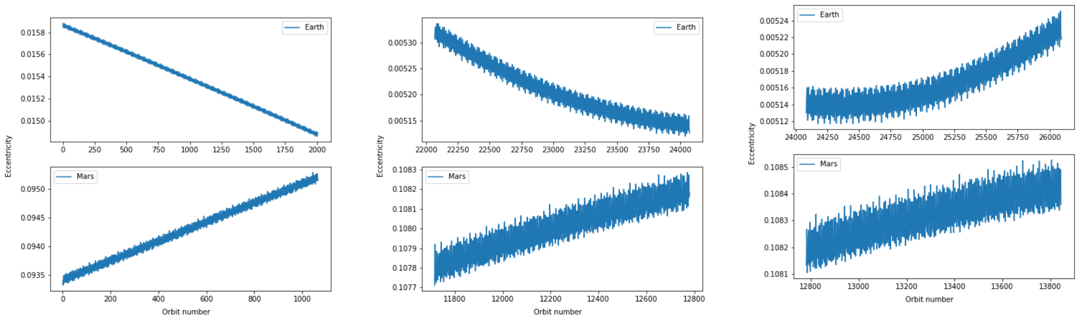
Table 7
Inclination of ETNOs with Planet 9

PLANET	Inclination (degrees)			
	Actual Data	Calculated Data	Round off	Percentage Error
90377 Sedna	11.9307	11.92524997	11.9252	-0.0460996 %
2007 TG_{422}	18.62	18.5954992	18.60	-0.1074 %
2004 VN_{112}	25.572	25.55032477	25.550	-0.086032 %
2010 GB_{174}	21.54	21.56387392	21.56	0.09285 %

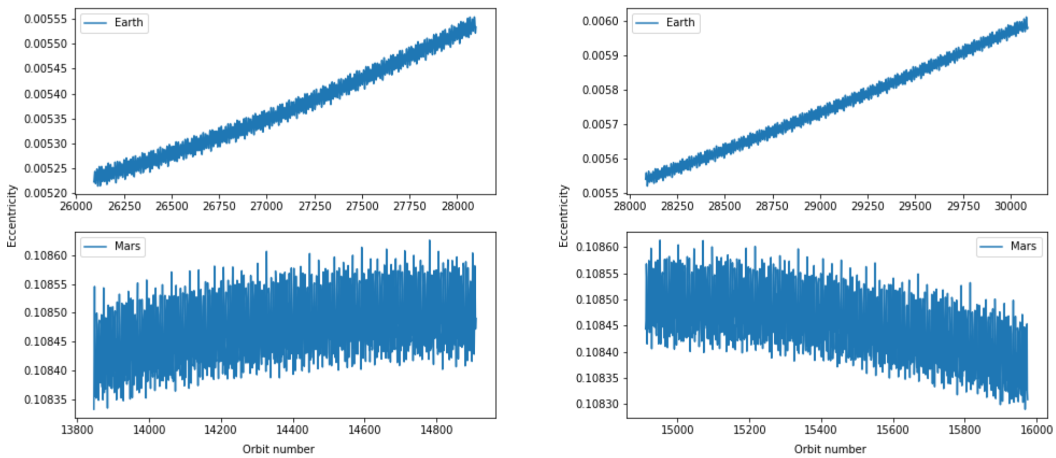


(a) Year 0 to 2000 **(b)** Year 38000 to 40000

Fig. 13. Earth's eccentricity in Keplerian Earth-Sun system

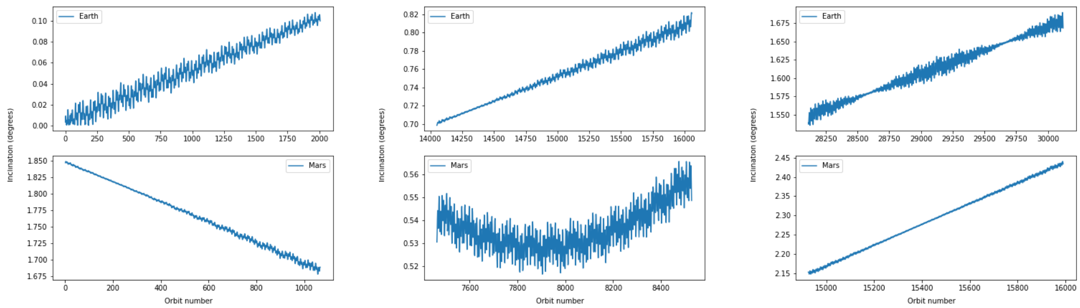


(a) First 2000 years **(b)** Both variations slow down **(c)** Earth's e reaches local minima, starts increasing



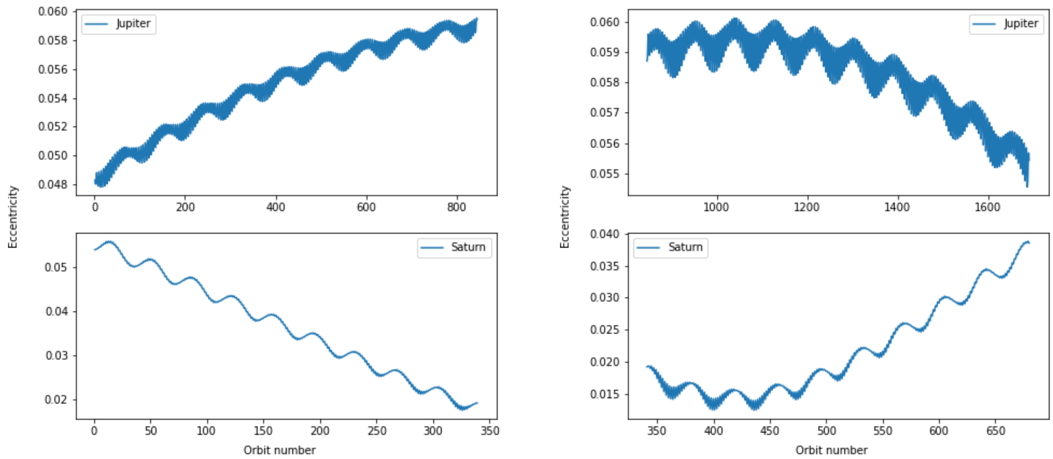
(d) Mars e reaches local maxima **(e)** Mars e now decreasing

Fig. 14. Earth and Mars' eccentricity variations. $dt = 0.2$



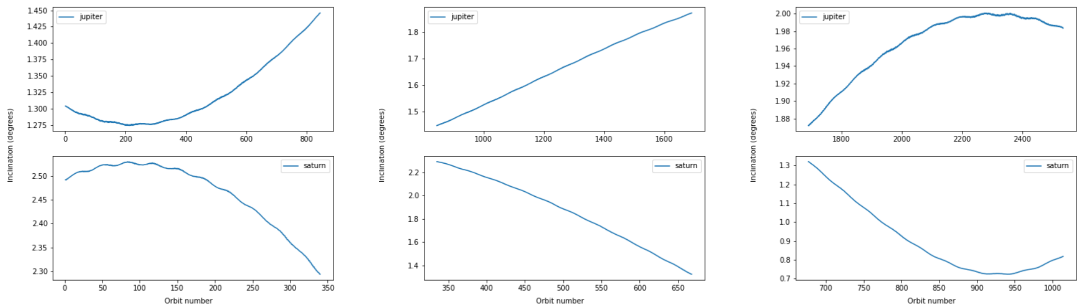
(a) First 2000 years **(b)** Mars' curve reaches local minima **(c)** Last 2000 years

Fig. 15. Earth and Mars' inclination variations. $dt = 0.2$



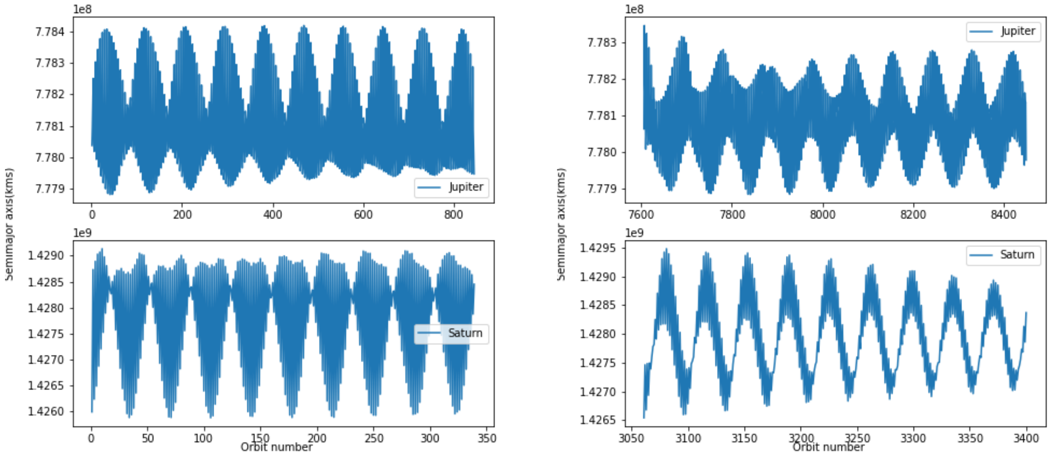
(a) Year 0 to 10,000 **(b)** Year 10,000 to 20,000

Fig. 16. Eccentricity variations of jupiter and saturn : $dt = 4$



(a) **(b)** **(c)**

Fig. 17. Inclination variations of jupiter and saturn : $dt = 4$



(a) Year 0 to 10,000

(b) Year 90,000 to 100,000

Fig. 18. Semi-major axis remaining constant for jupiter and saturn : $dt = 4$

List of Figures

1 Figure 1 . . . 11
2 Earths Distance from (153814) 2001 WN5 over time . . . 14
3 Earth and 2001WN5 orbit . . . 15
4 Distance of Earth from Apophis . . . 16
5 Orbit of Earth and Apophis . . . 16
6 Earth's Distance from 2010 RF12 . . . 17
7 Orbit of 2010 RF12 . . . 17
8 Comet Orbit with respect to the Ecliptic . . . 18
9 Comet Orbit Projection . . . 19
10 Figure 10 . . . 19
(a) Distance from Sun . . . 19
(b) Distance from Earth . . . 19
11 Possible Triton Capture Mechanism . . . 21
12 Orbits of All the bodies including Planet 9 and ETNOs . . . 24
13 Earth's eccentricity in Keplerian Earth-Sun system . . . 28
14 Earth and Mars' eccentricity variations. $dt = 0.2$ . . . 28
15 Earth and Mars' inclination variations. $dt = 0.2$ . . . 29
16 Eccentricity variations of jupiter and saturn : $dt = 4$ . . . 29
17 Inclination variations of jupiter and saturn : $dt = 4$ . . . 29
18 Semi-major axis remaining constant for jupiter and saturn : $dt = 4$ . . . 30

List of Tables

1 Chosen units for base quantities . . . 5
2 Eccentricities . . . 26
3 Semi-major axis . . . 26
4 Orbit Inclination with respect to ecliptic plane . . . 27
5 Eccentricity of ETNOs with Planet 9 . . . 27
6 Semi Major Axis of ETNOs with Planet 9 . . . 27
7 Inclination of ETNOs with Planet 9 . . . 27

A References

1. Batygin, K. & Laughlin, G. On the Dynamical Stability of the Solar System. *The Astrophysical Journal* **683** (2008).
2. *Is the solar sytem stable?* https://www.ias.edu/ideas/2011/tremaine-solar-system.
3. *HORIZONS Web-Interface* https://ssd.jpl.nasa.gov/horizons.cgi.
4. *Planetary factsheets* https://nssdc.gsfc.nasa.gov/planetary/factsheet/.

5. *Jet Propulsion Laborotary* Small Body Database Browser.
6. *Minor Planet Center* (153814) 2001 WN5.
7. *Yeomans, D.; Chesley, S.; Chodas, P. (23 December 2004)* "Near-Earth Asteroid 2004 MN4 Reaches Highest Score To Date On Hazard Scale".
8. *Sentry Risk Table* NASA/JPL Near-Earth Object Program Office.
9. *Kocz, Amanda (25 June 2021).* Giant Comet Found in Outer Solar System by Dark Energy Survey.
10. *"Perihelion in January 2031"* JPL Horizons.
11. *"Closest Approach to Earth 2031"* JPL Horizons.
12. *"The Largest Comet Ever Found Is Making Its Move Into a Sky Near You"* The New York Times.
13. Agnor, C. B. & Hamilton, D. P. Neptune's capture of its moon Triton in a binary–planet gravitational encounter. *Nature* **441,** 192–194 (May 2006).
14. Jacobson, R. A. THE ORBITS OF THE NEPTUNIAN SATELLITES AND THE ORIENTATION OF THE POLE OF NEPTUNE. *The Astronomical Journal* **137,** 4322–4329 (Apr. 2009).
15. *Planet 9* Wikipedia.
16. Brown, M. E. & Batygin, K. OBSERVATIONAL CONSTRAINTS ON THE ORBIT AND LOCATION OF PLANET NINE IN THE OUTER SOLAR SYSTEM. *The Astrophysical Journal* **824,** L23 (June 2016).
17. *JPL Small Body Database Browser* https://ssd.jpl.nasa.gov/sbdb.cgi.