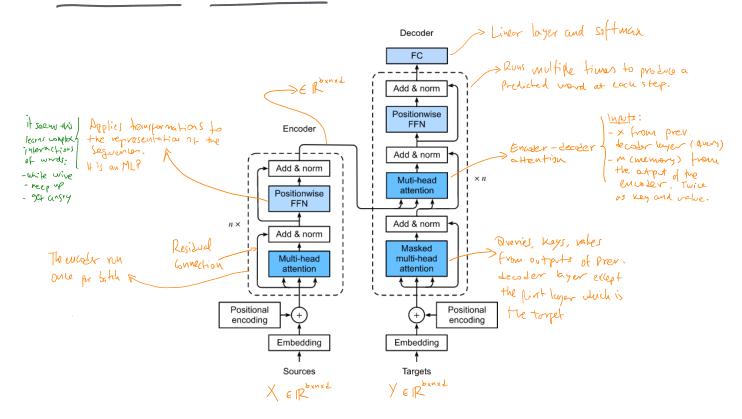
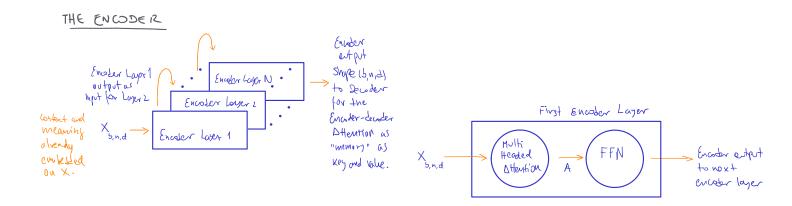
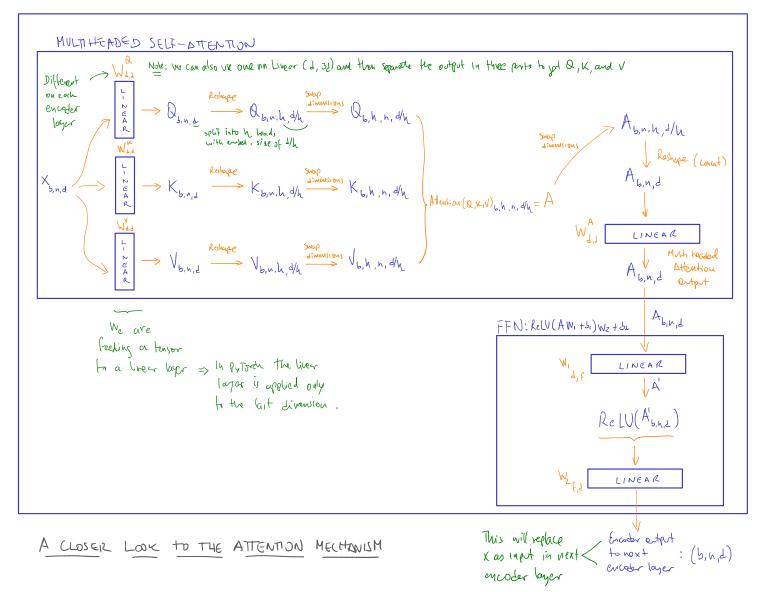
TRANSFORMER REVIEW



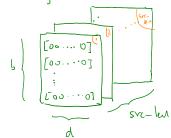






THE INPUTS to ATTENTION:

In RNNs we were interested in processing batches of the i-th word of each sentence at each time stop something like this:



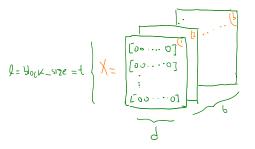
Lukano.

b -> A begin of all i-ty words of all sentences

d -> The ansedding size for each word

sve_len. The low of the largest sentence in the input dataset.

With transformers we can pass a botch of entire scatteries so the inputs might book like this:



Where

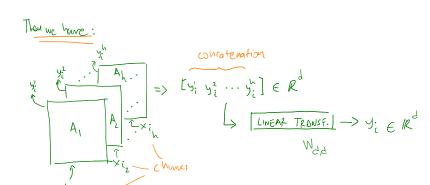
5: The # of sentences in the input dataset

2: The # of wards of sentence i of the dalatet

2: The Linewism of each work unsedding in the fortence

This is a tensor of shape (5, l, d) when applying softmax over this tensor we do it now-vise, meaning, the second Limension

when transpoking this tensors are need to forget the Satches for a moment and just think about the dimensions independently of the batch dimension. It is not a good idea using the Toperand in PyTorch $(5,l,d)^T \rightarrow (d,l,b)$. Better (5,l,d). transpose $(1,2) \Rightarrow (5,d,l)$ so we can do e with forth-smm: ? a single token? (b,l,d) x(b,d,l) > (b,l,l) As we one working with self attention each input vector Xi assures 3 who: - Querier To make things earier we'll define 3 new vectors as linear - Valvos transformations of Xi through 3 dxd matrices: Wx, Wq, Wr Shapes: (2,1) (2,2) (2,1) $q_i = W_q \times i$ $K_i = W_K \times i$ $V_i = W_V \times i$ Ki = Wkxi Attention Layer Attention entput: Y: = \(\times w; \tilde{\tau} \) -> yi of shape (ex) W 24 V4 Attention Listribution: Wij = Softmax (Wij) $W_{21}\sqrt{1}$ $w_3 = W_k x_3$ Attention sorcs: wij = 4 kj $\widetilde{W_q x_2}$... To avoid gradient explosion > we clip the attention scores Xi as WqXi = qi $\omega_{ij} = \frac{q_i \; K_j}{m}$ of shape (dxi) $w'_{23} = q_2^T K_3 = W_q X_2^T W_K X_3$ W23 = SOftmax (WqX2WxX3) As human communication is complex we can't not capture the entire meaning with one attention pass. 12 = W2, V. +W22 V2+W23 V3+W24 V4 We need to stack more layers! => Multi-headed Affention. > MULTI - HEADED SELF ATTENTION We actually split the d-Lim. Upsts in d/h beads We just "stack" several self-Attention medanisms: veil have a set of weight matrice per layer where h is the # of kyers. => $[y_i^1, y_i^2, \dots, y_i^n] \in \mathbb{R}^d$ Attention Xie Rd Heards LINEAR TRONSF.] -> Y.E IR Attenting inpot > This is expensive = It is better to: Wg vg ··· wg => Wge Rdxd Not sure about this representation Input for A1 Input for A2 ... Input for Ah Better WKWF ...WK => WKEREXE INPUT (Hunks -> [00...0] [00...0] -. [00...0] e R Each set contains = Wy Wk W' W' W' W' W' W' Wh Wh cho ERTh xth for multi $W'_{v_{2}} \cdots W'_{v_{n}} \Rightarrow \overline{W}_{v_{n}} \in \mathbb{R}^{1 \times d}$ Attention Concofenation Tust 3 motrices 7# of Attention heads



X:=[X:, X:2 ... Xin]; X: ell

CAUSAL MASK

A vanilla multi-head masked self attention layer with a projection at the end. This can also be achieved with forch m. Multi head Attention ()

Key, query, value ER where d=embedding size.=>x,q,v Projections for all heads: nn. Linear (d,d)

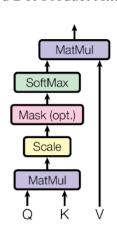
Regularization layers -> Attention Iroport is hn. Droport (attention_droport) > 0.1

L> Residual connection droport: nn. Droport (residual_droport)

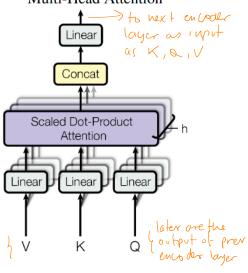
Output projection > hn. Linear (d.d)

Mark to wide "futive" input for attention (right part) > [111 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Scaled Dot-Product Attention



Multi-Head Attention



CSZZAN - ASSIGNMENT 5

1. Attention exploration (21 points)

(a) (2 points) Copying in attention: Recall that attention can be viewed as an operation on a query $q \in \mathbb{R}^d$, a set of value vectors $\{v_1, \dots, v_n\}, v_i \in \mathbb{R}^d$, and a set of key vector $\{k_i, \dots, k_n\}, k_i \in \mathbb{R}^d$,

$$z = \sum_{i=1}^{n} v_i \alpha_i \qquad \text{(vector)}$$

$$\alpha$$
 is attention distribition $\alpha_i = \frac{\exp(k_i^\top q)}{\sum_{j=1}^n \exp(k_j^\top q)}$ at time step (2)

where α_i are frequently called the "attention weights", and the output $c \in \mathbb{R}^d$ is a correspondingly weighted average over the value vectors.

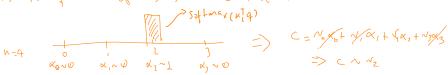
We'll first show that it's particularly simple for attention to "copy" a value vector to the output c. Describe (in one sentence) what properties of the inputs to the attention operation would result in the output c being approximately equal to v_j for some $j \in \{1, ..., n\}$. Specifically, what must be true about the query q, the values $\{v_1, \ldots, v_n\}$ and/or the keys $\{k_1, \ldots, k_n\}$?

FERM, K, ER, V; ER ieh 1.... ny # hidden des in encoder

All colubations are tope at each time step for all i.

$$\frac{\cancel{X}\cancel{V}}{\cancel{C}}$$
 => (\cancel{X}) $\sim \varphi$

Michon kind & bear mis sprokos >> num << 0 >> xi ~ 0



The atention office will be a copy of one of the inputs when the all the keys except one of them oppose to the query. $k_j^* \neq 0$; $j \neq i \rightarrow k_{ej} k_j$ opposes to the query (the of inity between the query and keys) 4ND $k_j^* \neq \infty$ $k_j^* \neq \infty$

(b) (4 points) An average of two: Consider a set of key vectors $\{k_1,\ldots,k_n\}$ where all key vectors are perpendicular, that is $k_i \perp k_j$ for all $i \neq j$. Let $||k_i|| = 1$ for all i. Let $\{v_1, \ldots, v_n\}$ be a set of arbitrary value vectors. Let $v_a, v_b \in \{v_1, \dots, v_n\}$ be two of the value vectors. Give an expression for a query vector q such that the output c is approximately equal to the average of v_a and v_b , that is, $\frac{1}{2}(v_a+v_b)$. Note that you can reference the corresponding key vector of v_a and v_b as k_a and k_b .

KILKZ=DKI.KZ=DK, LK3 KZLKZ



2 v, v, vs & ||K, || = ||K2|| = ||K3|| = 1

K=[1,0,0] KIE [0,1,0]

$$||\vec{p}|| = |p_1|^2 + |p_2|^2 + |p_3|^2$$

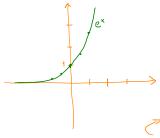
$$||\vec{p}|| = \sqrt{|p_1|^2 + |p_2|^2 + |p_3|^2}$$

To make this happen we need that

\[
\alpha = \preceq \alpha \frac{1}{2}, \frac{\and \alpha; \preceq 0 \tag{and \ki!} \quad \text{between 9 and \ki!} \]

 \Rightarrow softmax $(K_a^T q) = softmax(K_a^T q) \approx \frac{1}{2}$

Hint: while the softmax function will never exactly average the two vectors, you can get close by using a large scalar multiple in the expression



taking the light into account: Multiple of what? I guess the key vectors since q => 9 = 5(Ka+Ks) 14+exacts with them. where s is a sis scalar

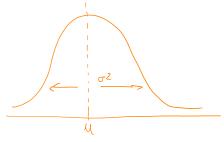
For ie fall $X_{a} = \frac{1}{2e^{s}}$ and X_{a} X_{a}

become the Key rectors are orthogonal then if jetas4

For if Jass & . ~ o if there is no affinity between the Guerry and the Kers to the values for i'm these cases. This means of and it one opposits

Skikj = 0 For iega,si

- (c) (5 points) Drawbacks of single-headed attention: In the previous part, we saw how it was possible for a single-headed attention to focus equally on two values. The same concept could easily be extended to any subset of values. In this question we'll see why it's not a practical solution. Consider a set of key vectors $\{k_1, \ldots, k_n\}$ that are now randomly sampled, $k_i \sim \mathcal{N}(\mu_i, \Sigma_i)$, where the means μ_i are known to you, but the covariances Σ_i are unknown. Further, assume that the means μ_i are all perpendicular; $\mu_i^{\top} \mu_j = 0$ if $i \neq j$, and unit norm, $\|\mu_i\| = 1$.
 - i. (2 points) Assume that the covariance matrices are $\Sigma_i = \alpha I$, for vanishingly small α . Design a query q in terms of the μ_i such that as before, $c \approx \frac{1}{2}(v_a + v_b)$, and provide a brief argument as to why it works.



$$\sum_{i=1}^{\infty} \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \infty \end{bmatrix}$$

 $\Sigma_i = \begin{bmatrix} \alpha & 0 & 0 & \cdots & 0 \\ 0 & \alpha & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}$ vous hing small $\alpha = \gamma$ The variance $\alpha \otimes \beta \times K_i \approx \mathcal{U}_i$ or $g = S(\mathcal{U}_a + \mathcal{U}_b)$ where S is a big solar.

ii. (3 points) Though single-headed attention is resistant to small perturbations in the keys, some types of larger perturbations may pose a bigger issue. Specifically, in some cases, one key vector k_a may be larger or smaller in norm than the others, while still pointing in the same direction as π_a . As an example, let us consider a covariance for item a as $\Sigma_a = \alpha I + \frac{1}{2}(\mu_a \mu_a^{\mathsf{T}})$ for vanishingly small α (as shown in figure 1). Further, let $\Sigma_i = \alpha I$ for all $i \neq a$.

Ma Ma = 1

When you sample $\{k_1, \ldots, k_n\}$ multiple times, and use the q vector that you defined in part i., what qualitatively do you expect the vector c will look like for different samples?

Not see about this but I'l son) Sampling with q as defined in the part i will \rightarrow comparing with the cases where $i \neq a$. Why? because the variance \sum_{i} is greater than \sum_{i} . Then the c vector will replicate the large participations observed in the keys.

(d) (3 points) Benefits of multi-headed attention: Now we'll see some of the power of multi-headed attention. We'll consider a simple version of multi-headed attention which is identical to singleheaded self-attention as we've presented it in this homework, except two query vectors $(q_1 \text{ and } q_2)$ are defined, which leads to a pair of vectors $(c_1 \text{ and } c_2)$, each the output of single-headed attention given its respective query vector. The final output of the multi-headed attention is their average, $\frac{1}{2}(c_1+c_2)$. As in question 1(c), consider a set of key vectors $\{k_1,\ldots,k_n\}$ that are randomly sampled,

 $k_i \sim \mathcal{N}(\mu_i, \Sigma_i)$, where the means μ_i are known to you, but the covariances Σ_i are unknown. Also as before, assume that the means μ_i are mutually orthogonal; $\mu_i^{\top}\mu_j=0$ if $i\neq j$, and unit norm,

i. (1 point) Assume that the covariance matrices are $\Sigma_i = \alpha I$, for vanishingly small α . Design q_1 and q_2 such that c is approximately equal to $\frac{1}{2}(v_a + v_b)$.

For this to happen $c_1 = \sqrt{a}$ and $c_2 = \sqrt{b}$ and as per question a) that will be twe if there is equal affinity between the green and the keys to and its; and no affinity with the for i \$40,59.

Now taking quotion b) into a court we can sup that $q_1 \approx SK_a$ and $q_2 \approx SK_b$. Finally, as $\Sigma_{t}=\alpha I$ for a vowiling small α we know that the variance $t_1^2 \approx 0$:

$$q_1 = SM_a$$
, where S is a sis scalar.
 $q_2 = SM_b$

ii. (2 points) Assume that the covariance matrices are $\Sigma_a = \alpha I + \frac{1}{2}(\mu_a \mu_a^{\mathsf{T}})$ for vanishingly small α , and $\Sigma_i = \alpha I$ for all $i \neq a$. Take the query vectors q_1 and q_2 that you designed in part i. What, qualitatively, do you expect the output c to look like across different samples of the key vectors? Please briefly explain why. You can ignore cases in which $q_i^{\top} k_a < 0$.

 $C = \frac{c_1 + c_2}{7}$; $\sigma_{\alpha}^2 > \sigma_{i \neq \alpha}^2$ If we have the same q_1 and q_2 then C should be $\approx \frac{(V_{\alpha} + V_{\alpha})}{2}$ then when sampling keys there would be a knowing of having bisser norm for the c vector would wa, but as we are averaging co and cz the

(7 points) **Key-Query-Value self-attention in neural networks:** So far, we've discussed attention as a function on a set of key vectors, a set of value vectors, and a query vector. In Transformers, we perform *self-attention*, which roughly means that we draw the keys, values, and queries from the same data. More precisely, let $\{x_1, \ldots, x_n\}$ be a sequence of vectors in \mathbb{R}^d . Think of each x_i where $x_{ij} = x_{ij} + x_{ij$ (e) (7 points) Key-Query-Value self-attention in neural networks: So far, we've discussed atten-

$$v_i = V x_i \ i \in \{1, \dots, n\}$$
 (3)

$$k_i = Kx_i \quad i \in \{1, \dots, n\} \tag{4}$$

$$q_i = Qx_i \quad i \in \{1, \dots, n\}$$

Then we get a context vector for each input i; we have $c_i = \sum_{j=1}^n \alpha_{ij} v_j$, where α_{ij} is defined as without after i, v. Y.1. q_z $\alpha_{ij} = \frac{\exp(k_j^\top q_i)}{\sum_{i=1}^n \exp(k_i^\top q_i)}$. Note that this is single-headed self-attention.

In this question, we'll show how key-value-query attention like this allows the network to use different aspects of the input vectors x_i in how it defines keys, queries, and values. Intuitively, this allows networks to choose different aspects of x_i to be the "content" (value vector) versus what it uses to determine "where to look" for content (keys and queries.)

particular, let u_a, u_b, u_c, u_d be mutually orthogonal vectors in \mathbb{R}^d , each with equal norm $||u_a|| =$ $||u_b|| = ||u_c|| = ||u_d|| = \beta$, where β is very large. Now, let our x_i be:

$$x_1 = u_d + u_b \tag{6}$$

$$x_2 = u_a \tag{7}$$

$$x_3 = u_c + u_b \tag{8}$$

If we perform self-attention with these vectors, what vector does c_2 approximate? Would it be possible for c_2 to approximate u_b by adding either u_d or u_c to x_2 ? Explain why or why not (either math or English is fine).

$$C_2 = \sum_{j=1}^{n} x_{2j} x_{j}$$
, where $x_{2j} = Softmax(K_j^T q_2)$
Ly the keys showing more affinity with q_2

(4) A vector highlighting
(5) the characteristics of the most values with

networks to choose different aspects of
$$x_i$$
 to be the "content" (value vector) versus what it uses to letermine "where to look" for content (keys and queries.)

i. (3 points) First, consider if we didn't have key-query-value attention. For keys, queries, and values we'll just use x_i ; that is, $v_i = q_i = k_i = x_i$. We'll consider a specific set of x_i . In particular, let u_a, u_b, u_c, u_d be mutually orthogonal vectors in \mathbb{R}^d , each with equal norm $||u_a|| = 1$

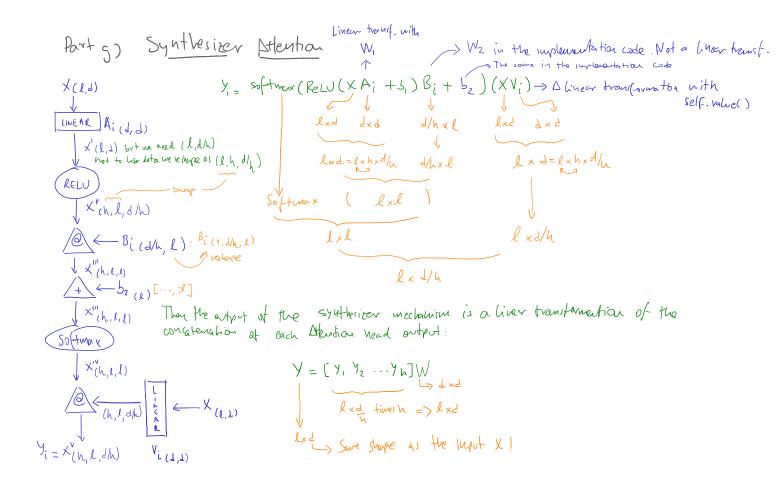
(6)
$$C_2 = X_1 \text{ So f-lmax}(X_1^{\top}X_2) + X_2 \text{ Sof-lmax}(X_2^{\top}X_2)$$
(7) $+ X_3 \text{ Sof-lmax}(X_3^{\top}X_2)$

Let's take a look to xix, xix= (ustus) ua. Since the dot product is distributive with addition:

half + halls As wa by and he are orthogonal among from the dot product is O. Some hoppens with XTX,

Then we have:
$$c_1 \approx \chi_2$$
 softwax $(\chi_1^{\dagger} \chi_2) \approx \frac{\chi_2}{2} \frac{e^{\chi_2^{\dagger} \chi_2}}{e^{\chi_1^{\dagger} \chi_2}} \approx \chi_2 = \frac{e^{\chi_2^{\dagger} \chi_2}}{2 + e^{\chi_2^{\dagger} \chi_2}} \approx \chi_2 \approx \chi_2 \approx \chi_2$

Notice: softwax $(\chi_1^{\dagger} \chi_2) = \frac{e^{\varphi}}{2 + e^{\chi_1^{\dagger} \chi_2}} = \frac{1}{2 + e^{\varphi}} \approx \mathcal{O}$
 $e^{\chi_1^{\dagger} \chi_2} + e^{\chi_1^{\dagger} \chi_2} = e^{\chi_1^{\dagger} \chi_2} \approx \chi_2 \approx \chi_2 \approx \chi_2$



ii. (2 points) Why might the *synthesizer* self-attention not be able to do, in a single layer, what the key-query-value self-attention can do?

I's say it is because the synthesizer is not inspecting the potential relationships between each possible countination in the copie. It is burially transforming two countries another representation and convening it with the value representation. It backs the power of the θ , K, V system.

3. Considerations in pretrained knowledge (5 points)

(a) (1 point) Succinctly explain why the pretrained (vanilla) model was able to achieve an accuracy of above 10%, whereas the non-pretrained model was not.

Becase the Vanilla model Wo pretaining was trained as a language model without an "understanding" of the complex relationships in the language (person "" pace). The pretaining activil was able to capture some of that language dependencies boosting the final experive thanks to transfer learning. Also the pretaining tose has better (honos to learn helpful dependencies for the Lownstream toses because its detaset was bigger (68%) than the hiretoning dataset

(b) (2 points) Take a look at some of the correct predictions of the pretrain+finetuned vanilla model, as well as some of the errors. We think you'll find that it's impossible to tell, just looking at the output, whether the model retrieved the correct birth place, or made up an incorrect birth place. Consider the implications of this for user-facing systems that involve pretrained NLP components. Come up with two reasons why this indeterminacy of model behavior may cause concern for such applications.

I think the model has berned to generate a place name after a question like "where was xxx born?" What it gives a convect answer is probably because a telephonship between that person name and a place was corphired and embedded buring pretraining. Otherwise outputs any location name. The implications are big, pows on twist and confidence of the predictions.

Law confidence —> can't be applied to critical processes

Poor prediction -> Low usability rate. Undervable contest ence when acting stindy based on crossers model output

(c) (2 points) If your model didn't see a person's name at pretraining time, and that person was not seen at fine-tuning time either, it is not possible for it to have "learned" where they lived. Yet, your model will produce *something* as a predicted birth place for that person's name if asked. Concisely describe a strategy your model might take for predicting a birth place for that person's name, and one reason why this should cause concern for the use of such applications.

If the prediction size is below a threshold the model can givery a database to retrieve the red answer. This of course defeats the purpose of the model and might wrongly bias the user about the real power of AI. Perhaps it is letter to take advantage of this situation to interact with the user asking for the missing into and/or creating a new data pair from the database for later Minetoning.