WEALTH MAXIMIZATION AND THE COST OF CAPITAL: A COMMENT*

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In a recent paper Auerbach [1979] comes to two striking sets of conclusions. The first is that "... in the presence of differential taxation of dividends and capital gains, wealth maximization does not imply maximization of firm market value" [p. 445]; the second is that "the appropriate cost of capital in the presence of personal taxes does not depend directly on ... the tax on dividends. Equity shares have a market value lower than the difference between the reproduction cost of a firm's assets and the market value of its debt obligations" [p. 445]. The purpose of this note is to reconcile the former result with the conventional presumption that (in the absence of uncertainty) optimal policy requires the maximization of the firm's equity value and to establish the relationship between the second set of conclusions and Auerbach's assumption that firms do not issue new equity. The framework used is that of Auerbach's model and the notation follows his.

The firm seeks to maximize the wealth of existing shareholders W_t^0, where

$$W_t^0 = V_t^0 + E_{t-1}. \tag{1}$$

Here E_{t-1} is the net distribution received by shareholders during period $t-1$ (which extends from date $t-1$ to date t) and V_t^0 the ex-dividend value of pre-existing equity at the beginning of t, which when added to new equity sold V_t^N, gives the total ex-dividend equity value V_t. Auerbach uses these definitions together with the identity between the firm's total receipts and total disbursals to express W_t^0 in terms of current and previous period values of equity and debt and current cash flow. He then argues that "since i_{t-1}, V_{t-1}, and B_{t-1} are predetermined at the beginning of period t, W_t^0 is maximized if and only if the firm maximizes

$$W_t^* = (1-c)V_t + (1-\theta)B_t + (1-\theta)x_t - (\theta - c)V_t^N\text{"}$$

[p. 436]. From this Auerbach correctly concludes that wealth maximization does not imply maximization of the sum of current cash flow

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and the market value of the firm's securities, unless the dividend tax rate θ equals the accrual-equivalent rate of capital gains tax c.

It is important, however, to recognize that under standard conditions policies that maximize shareholder wealth can always be characterized as policies that maximize equity value. To establish the result, note that equilibrium in the capital market requires that

$$\rho_t V_t = E_t + (V^0_{t+1} - V_t) \tag{2}$$

[Auerbach's (8)], where ρ_t is the one-period discount rate of equity holders during period t. But from (1) and (2)

$$W^0_t = (1 + \rho_{t-1}) V_{t-1}. \tag{3}$$

With an exogenous discount rate—and recall that this is a world of perfect certainty—maximization of W^0_t is therefore equivalent to maximization of V_{t-1}, the "opening" market value of equity. This is so irrespective of the values of the tax parameters θ and c; all that is needed is equilibrium in the capital market, which ensures that the post-tax return to shareholders over period $t - 1$ is $\rho_{t-1} V_{t-1}$ and is therefore maximized by the policy that maximizes V_{t-1}.

Using the result that wealth maximization is analytically equivalent to the maximization of equity value, we can now show how Auerbach's second set of conclusions must be amended when new equity is the marginal source of finance. In obvious notation, the firm's cash flow at the beginning of period t is equal to post-tax profits less investment:

$$x_t = (1 - \tau)\Pi(K_t,\ t) - I_t,$$

where, for simplicity, the price of capital goods is assumed constant over time and normalized at unity. Assuming, again for simplicity, that the firm is entirely equity-financed and that the equity holder's discount rate is constant over time, we find that

$$V_0 = \sum_{t=0}^{\infty} \left(\frac{1-\theta}{1-c} \{(1-\tau)\Pi(K_{t+1},\ t+1) - I_{t+1} + V^N_{t+1}\} - V^N_{t+1} \right) \beta^{t+1}, \tag{4}$$

where $\beta = 1/(1 + (\rho/(1 - c)))$. The firm's problem is to maximize V_0 subject to

(i) an equation of motion for the capital stock,

$$K_{t+1} = I_t + K_t, \tag{5}$$

which incorporates the simplifying assumption of no depreciation; and

(ii) nonnegativity constraints on dividends D_t and on new equity issues.

Forming a Lagrangean, associating multipliers μ_t with the constraint (5) and λ_t^D and λ_t^N with the nonnegativity constraints, we see that the relevant necessary conditions are then

$$\left(\frac{1-\theta}{1-c} + \lambda_t^D\right)(1-\tau)\,\Pi_K(K_t, t) + \mu_t - \mu_{t-1}\beta^{-1} = 0 \tag{6}$$

$$\mu_t - \left(\frac{1-\theta}{1-c} + \lambda_t^D\right) = 0 \tag{7}$$

$$\frac{1-\theta}{1-c} + \lambda_t^D + \lambda_t^N - 1 = 0, \tag{8}$$

and the usual complementary slackness conditions. We assume that $\theta > c$, as is commonly the case. It is then easily shown that the firm will never simultaneously pay dividends and issue new equity.

The multiplier μ (evaluated at the optimum) gives the effect on the maximized value of equity of a gift of capital, and therefore represents the marginal value of Tobin's q. When the firm pays dividends in t, $\lambda_t^D = 0$ and so, from (7),

$$\mu_t = (1-\theta)/(1-c), \tag{9}$$

while if new equity is issued in t, $\lambda_t^N = 0$ and hence from (7) and (8)

$$\mu_t = 1. \tag{10}$$

Equation (9) shows that marginal q is less than unity when retained profits are the marginal source of investment finance, and corresponds to Auerbach's result on average q. But it is clear from (10) that this result does not extend to the case in which new equity is the marginal source: marginal q is then always unity. This difference between the values of marginal q under the two sources of funds is simply explained. If a firm retains an additional \$1 of after-tax profits, shareholders forgo dividends of $\$(1-\theta)$ and will therefore be indifferent as long as the firm's equity value rises by $\$(1-\theta)/(1-c)$, which, after payment of capital gains tax, increases their wealth by $\$(1-\theta)$. However if the firm issues new equity, shareholders are giving up \$1 of disposable income, so to maintain their wealth constant, this \$1 must produce a corresponding increase in the value of the firm's equity. The point is that retained profit is a source of funds from within the firm and hence already subject to the "trap" of the tax on distributions, but this is not the case for new equity.

We can also use (9) and (10) in (6) to obtain expressions for the net cost of capital (equal, at the optimum, to $(1-\tau)\,\Pi_K$) conditional

TABLE I

		Marginal source of funds at t	
		Retained profits	New equity
Marginal source of funds at $t-1$	Retained profits	$\rho/(1-c)$	$\left(\frac{1-\theta}{1-c}\right)\frac{\rho}{1-c}-\left(\frac{\theta-c}{1-c}\right)$
	New equity	$(\rho+(\theta-c))/(1-\theta)$	$\rho/(1-c)$

upon the marginal sources of funds at dates $t-1$ and t, as shown in Table I.

Whenever the marginal source of funds is the same in two adjacent periods, the cost of capital is $\rho/(1-c)$; Auerbach's result that the cost of capital does not depend on the dividend tax rate therefore extends to the case where new equity finance is used at $t-1$ to finance a marginal investment, the return from which will be used to reduce the amount of new equity issued at t. But if firms are to have positive value, it must be the case that a period of new equity issue is eventually followed by one in which dividends are paid; the cost of capital is then $\{\rho+(\theta-c)\}/(1-\theta)$, and thus there is at least one period in which the cost of capital for new equity finance is raised above $\rho/(1-c)$ by dividend taxes. Conversely, a firm that plans to issue new equity at t will reinvest profits at $t-1$ beyond the point where the cost of capital is $\rho/(1-c)$ in order to reduce the amount of expensive new equity it has to issue; in this case the effect of the dividend tax is to reduce the cost of capital. We conclude that, in this standard model, the dividend tax affects the cost of capital when, and only when, the marginal source of finance is about to change.

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