

BUBBLE SORT

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program development through refinement, 1971 ACM

ex refinement + transition system } nodes

any problem

algorithm (properties)

- ① deterministic
- ② terminating
- ③ automatic (not interactive)

 $B_1(a^0)$

$$B_1(a^0) = \langle X_B, X_B^0, U_B \xrightarrow{B} Y_B, h_B \rangle$$

given an - array of size n

the state space is the set of all permutations of an

$$X_{B_1}^0 = \{a^0\}$$

$$U_{B_1} = \text{swap}(i, j) \text{ where } 0 \leq i, j < n$$

$$a \xrightarrow[B]{\text{swap}(i, j)} a' \quad \text{if } a' = \text{swap}(a, i, j) \\ \text{where } \text{swap}(a, i, j) \\ = \{i \rightarrow a_j, j \rightarrow a_i\} a$$

ex $[3, 1, 2, 4] \xrightarrow{\text{swap}(0,1)} [1, 3, 2, 4]$
 $\downarrow \text{swap}(1,2)$
 $[1, 2, 3, 4]$

B2

$B2 \langle X_{B2}, X_{B2}^0, U_{B2}, \xrightarrow{B2}, \dots \rangle$

$\text{order}(a_i, i, j) = \text{df} = \text{swap}(a_i, U_{B2}, j)$

$0 \leq i, j < n$

$$X_{B2} = X_{B1}$$

$$X_{B2}^0 = X_{B1}^0$$

$$U_{B2} = \{ \text{order}(i, j) \mid a_i > a_j \}$$

$$a \xrightarrow[B2]{\text{order}(i, j)} a' \quad \text{iff} \quad a_i > a_j$$

B3

$B3 \langle X_{B3}, X_{B3}^0, U_{B3}, \xrightarrow{B3}, \dots \rangle$

$\text{order}(a_i, i, j) = \text{df} = \text{swap}(a_i, U_{B3}, j)$

$$X_{B3} = X_{B1}$$

$$X_{B3}^0 = X_{B1}^0$$

$$U_{B3} = \{ \text{order}(i, j) \mid a_i > a_j \text{ \& } j = i+1 \}$$

$$a \xrightarrow[B3]{\text{order}(i, j)} a' \quad \text{iff} \quad a_i > a_j \text{ \& } j = i+1$$

B₄

$$B_4 = \langle X_{B_4}, X_{B_4}^0, U_{B_4}, \xrightarrow{B_4}, Y_{B_4}, h_{B_4} \rangle$$

$$X_{B_4} = (a, i) \quad \text{where } 0 \leq i < n \\ \text{and } n = |a|$$

$$X_{B_4}^0 = (a^0, 0)$$

$$U_{B_4} = \{ \text{inc}, \text{reset} \}$$

$$(a, i) \xrightarrow[B_4]{\text{inc}} (a', i') \quad \text{iff}$$

$$i' = i + 1 \quad \text{and}$$

$$a \xrightarrow[B_4]{\text{adj}(i)} a' \quad \text{if } a_i > a_j$$

$$a' = a \quad \text{otherwise}$$

- finish B5
- pick iterative algo Coq
- write
- and do a proof.

B5

$$B5 = \langle X_{B5}, X_{B5}^0, U_{B5}, \xrightarrow{B5}, Y_{B5}, h_{B5} \rangle$$

$$X_{B5} = (a, i, j) \quad \text{where } 0 \leq i, j \leq n-1 \text{ and } n = |a|$$

$$X_{B5}^0 = (a^0, 0, n-1)$$

$$U_{B5} = \{ \text{next} \}$$

$$(a, i, j) \rightarrow (a', i', j') \text{ cfb}$$

$$\langle \text{if } 0 \leq j < n \text{ then } (a, i, j) \rightarrow (a, i, j+1) \text{ else } (a, i, j) \rightarrow (a, i, 0) \rangle \text{ term.}$$

$$\text{if } (a, i, j) \rightarrow (a, i, j-1) \text{ cfb, if } (a, i, j) = (a, i, 0) \text{ then } (a, i, j) \rightarrow (a, i, n-1) \text{ reset}$$

$$\text{if } (a, i, j) \rightarrow (a, i, j) \text{ cfb, if } (a, i, j) = (a, i, j) \text{ then } (a, i, j) \rightarrow (a, i, j) \text{ reset}$$

$$\text{if } (a, i, j) \rightarrow (a, i, j) \text{ cfb, if } (a, i, j) = (a, i, j) \text{ then } (a, i, j) \rightarrow (a, i, j) \text{ reset}$$

$$a' = a \text{ otherwise}$$