classmate

Date 20.1.2020

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FACTORIAL

def fac(n):

fac-iter (n, 1)

$$\text{fac-iter}(i, a) = a \quad \text{if } i = 0$$
$$= \text{fac-iter}(i-1, a*i) \quad \text{otherwise}$$

$! : \mathbb{N} \to \mathbb{N}$

RTP: fac(n) $\underset{①}{\underline{\text{returns}}}$ with $\underset{②}{\underline{!n}}$

BASE CASE

fac(0)

= fac-iter (0, 1)

= 1

= 0! ✓

INDUCTION STEP

assume fac(k) = !k for k > 0, then

show fac(k+1) = !(k+1)

fac(k+1) = fac-iter (k+1, 1)

= fac-iter (k, k+1)

fac(k) = fac-iter (k, 1)

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TRANSITION SYSTEM

$(i, a) \rightarrow (i-1, a*i)$ if $i>0$

$F: \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N} \times \mathbb{N}$

$(i, a) \rightarrow (i', a')$

ex $(3,7) \rightarrow (2,21) \rightarrow (1,42) \rightarrow (0,42) \not\rightarrow$

$(0,5) \not\rightarrow$

theorm:
S is terminating

given any state $m' x'$ every run is finite.
$m' : x' < m : x$

if $(i, a) \rightarrow (i', a')$ then
① $i > 0$
② $i = i-1$

pendulum amplitude reducing
- well ordered
- minimum

2 parts — invariant generation.

① termination
② correctness (partial)

ex $(3, 1) \to (2, 3) \to (1, 6) \to (0, 6) \not\to$

(markov chains).

INVARIANT

$(n, 1) \to \dots \to (0, n!)$

P is an invariant if

① $P(x_0)$ is true where $x_0 \in X^0$

② if $x \to x'$ and $P(x)$, then $P(x')$
~~P(x')~~

$(i, a) \to (i', a') \to \dots (i'_n, a'_n)$

$P(i, a) = df = i! * a \quad (= n!)$

prove that P_n is an invariant for a when stating $(n, 1)$

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check or verify that
$P_n(x_0)$ is true?

(1) $P_n(i_0, a_0) = P_n(n, 1)$ $\left\{ \begin{array}{l} \text{gcd} \\ \text{fibonacci} \end{array} \right.$

$P_n(n, 1) = n! * 1 = n!$ trivially true

(2) assume $P_n(i, a)$

$(i, a) \to (i', a')$

show $P_n(i', a')$

$$i'! * a = (i-1)! * a * i$$
$$= i! * a \qquad P_n(i, a)$$
$$= n!$$

$i_n = 0$ (term)
$\Rightarrow a = n! \longrightarrow$ done