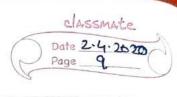


	normal form	description			
	VAL	values			
*	DW/o	Stuck at DIV/o			
	ATM	stuck due to arg type mismatch			
	, ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,				
	e val =>	e A			
	e 011/0 =>	e +>			
	e atm =>	e / >			
	SPANNING LEMMA				
	is e →,	De VAL, or			
		2) e DIV/0, or			
		De ATM			
_					
	+				



PROOF

by induction on e

base case

Oe=n > e VAL

X e DIV/O

De= 5 => e VAL

3 e = e, + e2

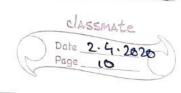
clearly e, >

otherwise e is reducible

by 1H, BI VAL or

el DIV/o or

e1 ATM



(3.1) E1 = n,

cleanly ez >>

otherwise e is reducible

by IH, ez VAL, on

e2 DIV/O, or

ez ATM

3.1.1) C2 VAL (2 cases)

e2 = n2

this is not possible, as it is reduci

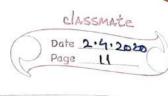
 $\vec{n}_1 + \vec{n}_2 \hookrightarrow \vec{n}_1 + \vec{n}_2$

e₂ = b₂

then we build a derivation in ATM.

NUM-BOOL

proving e= ni + b2 ATM

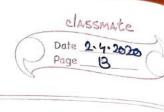


(3.1.2) e2 DIV/0 then we have a deduction in DIV/o en VAL @ DIVO RIGHT eifer Divlo e DIV/O 8.13) e2 ATM then we have of & ATM ei VAL ez ATM (RIGHT) eI (F) e2 ATM (3.1) (ii) e, = 51 dearly en +> otherwise e is reducible. by IH ez VAL, or ez DIV/O on

e2 ATM



(2.1.1) ez VAL (2 cases)	
0		
	$e_2 = \overline{n}_2$	
	then we have a derivation	ATM
		ey) Alla,
	B00L-N	1204
	5, F N2 ATM	014
	0 12 111	
	<u> </u>	
	MTA S	
		N .
	ez = b2	
	then	
	B001-B	00L
	5 € b2 ATM	
	Q AIM	
	Q.	
		<u>.</u>
1 1		
3.1.2		
(0.1.2	2) 62 011/0	in DIVIA
	then we have a deduction	(r) DLV20
		. #
	e2 DIV/0 RIG	1 th')
	\$ 5, (+) e2 DIV/0	
	e	



(3.13) e2 ATM then we have a derivation 51 VAL EZ ATM RIGHT 5, 1 e2 ATM e, DIV/O then we have a derivation in DIV/o CI DIVO LEFT e, Dez DIV/o (8.3) e, ATM then we have a derivation in ATM

CI ATM LEFT

er (F) es ATM

Ø e = e, Ø ez,

this is similar to (3) with.

(4.1.1) B1 = n1

 $c_{2} = \frac{1}{100}$

0= cn (i)

in this case we have

n 1 7 n2 DIV/0

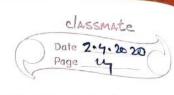
(ii) n2 +0

in this case we have

 $\vec{n}_1 (\vec{n}_2 \rightarrow \vec{n}_1 + \vec{n}_2)$

6

contradicting ex



5 e = (if) e, ez ez

clearly e, +>
otherwise e is reducible.

(5.1) e, VAL

(i) e1 = n1 then we have a deduction in ATM

(if) n, e2 e3 ATM

(ii) e1 = b1

er = true

then we have a derivation in RED

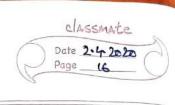
(if) true &2 &3 -> B2

contradicting the assumption that

e, = false similar to true case



then we have a derivation in Divide C1 DIVIO C1 DIVIO C2 DIVIO C3 Eq ATM Then we have a derivation in ATM. C1 ATM C1 ATM C1 ATM C1 C2 C3 ATM C1 ATM C2 C3 DIVIO C3 DIVIO C4 ATM C5 C4 ATM C6 C4 C5 C5 C6 ATM C7 C5 C5 C6		
then we have a derivation is DIV/o C1 DIV/o C2 DIV/o C3 DIV/o C4 ATM C4 ATM C1 ATM C1 ATM C1 C2 C2 ATM C1 C3 DIV/o C1 ATM C2 ATM C4 ATM C5 C4 ATM C5 C4 ATM C6 C6 C7	6.2	e, DIV/O
En		then we have a derivation in Divid
En		0 5
e 63) eq ATM Then we have a derivation in ATM. E1 ATM [F] (if) e1 e2 e3 ATM Phis completes the proof B \$\frac{1}{2}\$ \(\text{P} \) \		G DIVIO
Hen we have a derivation in ATM. Completes the proof B >> COMPLETED Completes the proof		(16) e1 e2 e3 DIV/0
then we have a derivation in ATM. En ATM IF (if) en en en en ATM ALM ALM ALM ALM ALM ALM ALM		ě
then we have a derivation in ATM. E1 ATM [if) E1 E2 E2 ATM This completes the proof B \$\frac{1}{2}\$ \Rightarrow VAL on E DIV/O on		
then we have a derivation in ATM. En ATM IF (if) en en en en ATM AED This completes the proof B A >> B VAL on B DIV/O on	6.3	& ATM
this completes the proof B +> > C VAL on C DIV/O on		
this completes the proof B +> > C VAL on C DIV/O on		4 T T T T T T T T T T T T T T T T T T T
this completes the proof B		
this completes the proof B >> & VAL on B DIV/O on		(if) e1 e2 e3 ATM
this completes the proof B >>> & VAL or B DIV/O on		
this completes the proof B >> & VAL on B DIV/O on		0 - D
B A >> & VAL on B DIV/O on	-	Q E-D
B A >> & VAL on B DIV/O on		
B A >> & VAL on B DIV/O on		this completes the proof
& DIN/O 200		2718 CON 1 1 0
& DIN/O 200		e / >> e VAL on
		· ·
e ATM		& DIV/D SON
e atm		
		e ATM



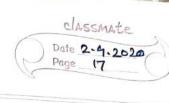
DISTOINTNESS	LEMMA

DIV/O,	1	nei o	lisjoint.	exerce en exercite
	¥.			
	THEORM	FOR	MORMAL F	DRMS
the sets	VAL		partition	the set of
	DIV/O		normal	forms of
1	ATM		→	•

direct consequence of the spanning lemma & disjointness, lemma for .

normal forms under ->

PROOF



SIMPLIFICATION

e *>e'

REFL

REFL

TRANS

DIV/0*

RED

e *> e

e +> e1

e -> e' e' *> e"

e VAL*

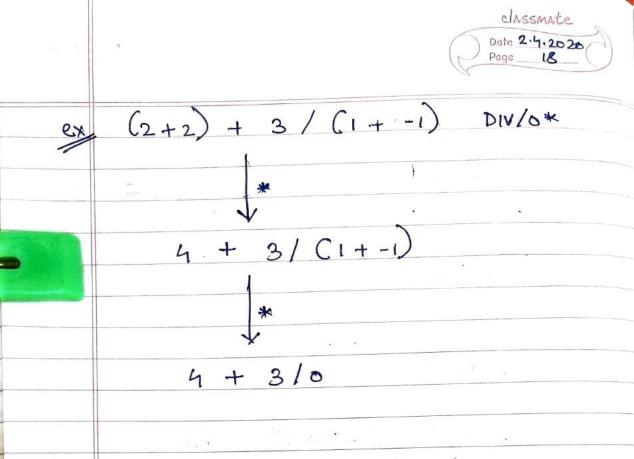
e DIV/0*

e DIV/o*

e *> e' e' VAL

e' DIV/0 e *> e'

e simplifies to an expression stuck



e ATM* e AT

e' ATM e → e' e ATM*

expression that is stuck due to an ARG TYPE MISMATCH