classmate

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P def fib(n)
fib-iter(n, 0, 1)

$$\text{fib-iter}(i, a, b) = \begin{cases} a & \text{if } i = 0 \\ \text{fib-iter}(i-1, b, a+b) & \end{cases}$$

fibonacci : $\mathbb{N} \to \mathbb{N}$

RTP $\underbrace{\text{fib}(n) \text{ returns}}_{①}$ with $\underbrace{\text{fibonacci}(n)}_{②}$

$F: \mathbb{N} \times \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N} \times \mathbb{N} \times \mathbb{N}$

$F: \langle X_F, X_F^0, U_F, \xrightarrow[F]{}, Y_F, h_F \rangle$

$X_F = (i, a, b)$ where $i, a, b \in \mathbb{N}$

$X_F^0 = (n, 0, 1)$ n = n^{th} fibonacci no. to find

$U_F = \{next\}$

$$(i, a, b) \xrightarrow[F]{next} (i', a', b')$$

iff ① $i > 0$ (term)
$i' = i - 1$

② $a' = b$ (if $i > 0$)
$b' = a + b$

ex $(3,0,1) \rightarrow (2,1,1) \rightarrow (1,1,2) \rightarrow (0,2,3) \not\rightarrow$

$(0,3,5) \not\rightarrow$

thm: F is terminating.

if $(i, a, b) \rightarrow (i', a', b')$ then

① $i > 0$
② $i' = i - 1$

} well ordered
has minimum.

thm: invariant (correctness).

$(n, 0, 1) \rightarrow \dots \quad (0, \text{fibonacci}(n), \text{fibonacci}(n+1))$

P is an invariant if

① $P(x_0)$ is true where $x_0 \in X^0$, and

② if $x \longrightarrow x'$ and $P(x)$, then $P(x')$

ex

5, 0, 1	6, 0, 1	1, 5, 8
4, 1, 1	5, 1, 1	0, 8, 13
3, 1, 2	4, 1, 2	
2,	3, 2, 3	
	2, 3, 5	

6 ←	0	1						
5		1	1					
4			1	2				
3				2	3			
2					3	5		
1						5	8	
0							8	13

$fibonacci^{-1}$

$P(i, a, b) = df = fibonacci(i + fibonacci^{-1}(a))$

$$\begin{aligned} P(x_0) &= P(n, 0, 1) \\ &= fibonacci(n + fibonacci^{-1}(0)) \\ &= fibonacci(n + 0) \leftarrow \\ &= fibonacci(n) \checkmark . \end{aligned}$$

assume $(i, a, b) \longrightarrow (i', a', b')$

assume $P(i, a, b)$

show $P(i', a', b')$

$$\begin{aligned} & fibonacci(i' + fibonacci^{-1}(a')) \\ &= fibonacci(i' + fibonacci^{-1}(b)) \\ &= fibonacci(i' + fibonacci^{-1}(a) + 1) \\ &= fibonacci(i - 1 + 1 + fibonacci^{-1}(a)) \\ &= fibonacci(i + fibonacci^{-1}(a)) \\ &= P(i, a, b) \checkmark ?? \end{aligned}$$

for $i = 0$

$$P(i, a, b) = fibonacci(0 + fibonacci^{-1}(a))$$
$$= a$$

$\therefore a = fibonacci(n).$

23.1.2020

ex check if string is palindrome.

p if $i = n/2$
palind
pal' (a, i+1, p & match'

a c a d a c a

match (i, j) = true if $a_i = a_j$
false otherwise

match' (i) = match(

match (a, i, p) $\longrightarrow$ match (a', i', p') off

- p is true,
$i < \lfloor n/2 \rfloor$ $\quad i' = i+1$
$a' = a.$

~~match~~ palindrome( )

invariant ? p & $a[i \ldots n-i]$

$P(x_0) = P(a, 0, \cancel{false}$ true$)$ = palindrome $(a[0 \ldots n-1])$ ✓
= palindrome (a)

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$(a, i^*, p^*) \longrightarrow (a', i', p')$

assume $P(a, i, p)$

$P(a', i', p')$

$= \cancel{p} \ p.$

$P(a, i, p)$

$= p \ \& \ \text{palindrom}(a[i \ldots n-i-1])$

$P(a', i', p')$

$= p' \ \& \ \text{palindrome}(a[i+1 \ldots n-i-2])$

$= p \ \& \ (\text{match}(a(i), a(n-i-1))) \ \& \ p$

$= p \ \& \ (\text{pal} \ldots)$

$= P(a, i, p).$