Date 2.4.2020
Page 8

normal form	description
VAL	values
DIV/0	stuck at DIV/0
ATM	stuck due to arg type mismatch

e VAL $\Rightarrow$ $e \not\to$

e DIV/0 $\Rightarrow$ $e \not\to$

e ATM $\Rightarrow$ $e \not\to$

SPANNING LEMMA

if $e \not\to$, (1) e VAL, or

(2) e DIV/0, or

(3) e ATM

PROOF

by induction on e

<u>base case</u>

① $e = \bar{n} \Rightarrow e$ VAL

$\times$ e DIV/0

$\times$ e ATM

② $e = \bar{b} \Rightarrow e$ VAL

③ $e = e_1 + e_2$

clearly $e_1 \not\to$

otherwise e is reducible

by IH,

e_1 VAL, or

e_1 DIV/0, or

e_1 ATM

(3.1) (i) $e_1 = \overline{n}_1$

clearly $e_2 \not\hookrightarrow$
otherwise e is reducible

by IH,

e_2 VAL, or

e_2 DIV/0, or

e_2 ATM

(3.1.1) e_2 VAL (2 cases)

$e_2 = \overline{n}_2$

this is not possible, as it ~~is reduci~~
is reducible

$$\overline{n}_1 + \overline{n}_2 \hookrightarrow \overline{n_1 + n_2}$$

$e_2 = \overline{b}_2$

then we build a derivation in ATM.

$$\frac{}{\overline{n}_1 \oplus \overline{b}_2}\ \text{NUM-BOOL}$$

proving $e = \overline{n}_1 \oplus \overline{b}_2$ ATM

(3.1.2) e_2 DIV/0

then we have a deduction in DIV/0

$$\frac{e_1 \text{ VAL} \quad e_2 \text{ DIV/0}}{e_1 \oplus e_2 \text{ DIV/0}} \quad \text{RIGHT}$$

$$\frac{}{\text{DIV/0}} \; e \text{ DIV/0}$$

(3.1.3) e_2 ATM

then we have of e ATM

$$\frac{e_1 \text{ VAL} \quad e_2 \text{ ATM}}{\underbrace{e_1 \oplus e_2}_{e} \text{ ATM}} \quad (\text{RIGHT})$$

(3.1)(ii) $e_1 = \overline{b_1}$

clearly $e_2 \not\mapsto$

otherwise e is reducible.

by IH,

e_2 VAL, or

e_2 DIV/0, or

e_2 ATM

3.1.1 e_2 VAL (2 cases)

$e_2 = \bar{n}_2$

then we have a derivation in ATM.

$$\frac{}{\bar{b}_1 \oplus \bar{n}_2 \text{ ATM}} \text{BOOL-NUM}$$

$\therefore$ e ATM

$e_2 = \bar{b}_2$

then

$$\frac{}{\underbrace{\bar{b}_1 \oplus b_2}_{e} \text{ ATM}} \text{BOOL-BOOL}$$

3.1.2 e_2 DIV/0

then we have a deduction in DIV/0.

$$\frac{e_2 \text{ DIV/0}}{\underbrace{\bar{b}_1 \oplus e_2}_{e} \text{ DIV/0}} \text{RIGHT}$$

(3.1.3) e_2 ATM

then we have a derivation

$$\frac{\bar{5}_1 \text{ VAL} \quad e_2 \text{ ATM}}{\underbrace{\bar{5}_1 \oplus e_2}_{e} \text{ ATM}} \text{ RIGHT}$$

(3.2) e_1 DIV/0

then we have a derivation in DIV/0

$$\frac{e_1 \text{ DIV/0}}{\underbrace{e_1 \oplus e_2}_{e} \text{ DIV/0}} \text{ LEFT}$$

(3.3) e_1 ATM

then we have a derivation in ATM

$$\frac{e_1 \text{ ATM}}{e_1 \oplus e_2 \text{ ATM}} \text{ LEFT}$$

(4) $e = e_1 \oslash e_2$,

this is similar to (3) with.

(4.1.1) $e_1 = \overline{n_1}$
$e_2 = \overline{n_2}$ } (2 cases)

(i) $n_2 = 0$

in this case we have

$$\overline{\underbrace{\overline{n_1} \oslash \overline{n_2}}_{e} \quad \text{DIV}/0}$$

(ii) $n_2 \neq 0$

in this case we have

$$\underbrace{\overline{n_1} \oslash \overline{n_2}}_{e} \longrightarrow \overline{n_1 \div n_2}$$

contradicting $e \not\to$

(5) $e = \text{if}\ e_1\ e_2\ e_3$

Clearly $e_1 \not\to$

otherwise e is reducible.

(5.1) e_1 VAL

(i) $e_1 = \overline{n_1}$

then we have a deduction in ATM

$$\frac{}{\underbrace{\text{if}\ \overline{n_1}\ e_2\ e_3}_{e}\ \text{ATM}}\ \text{IF}$$

(ii) $e_1 = \overline{b_1}$

$e_1 = \text{true}$

then we have a derivation in RED

$$\frac{}{\text{if}\ \text{true}\ e_2\ e_3 \longrightarrow e_2}\ \text{TRUE}$$

contradicting the assumption that $e \not\to$

$e_1 = \text{false}$

similar to true case

(5.2) e_1 DIV/0

then we have a derivation in DIV/0

$$\frac{e_1\ \mathrm{DIV/0}}{\underbrace{\mathrm{if}\ e_1\ e_2\ e_3}_{e}\quad \mathrm{DIV/0}}\ \mathrm{IF}$$

(5.3) e_1 ATM

then we have a derivation in ATM.

$$\frac{e_1\ \mathrm{ATM}}{\mathrm{if}\ e_1\ e_2\ e_3\ \mathrm{ATM}}\ \mathrm{IF}$$

QED

this completes the proof

$e \not\mapsto \;\Rightarrow\;$ e VAL, or

e DIV/0, or

e ATM

DISJOINTNESS LEMMA

VAL,
DIV/0,
ATM

are disjoint. exercise

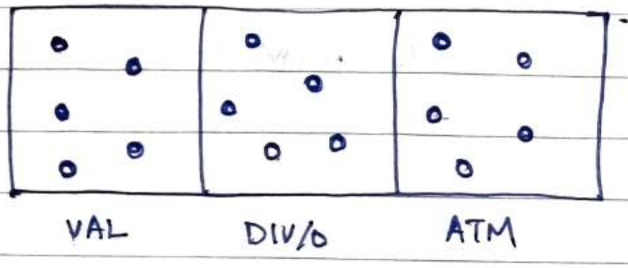
PARTITION THEORM FOR NORMAL FORMS

the sets VAL, DIV/0, ATM partition the set of normal forms of $\rightarrow$

PROOF

direct consequence of the spanning lemma & disjointness lemma for $\rightarrow$ normal forms.

normal forms under $\rightarrow$



SIMPLIFICATION

$\boxed{e \xrightarrow{*} e'}$

$$\vdash_{SIM} e \xrightarrow{*} e'$$

$$\frac{}{e \xrightarrow{*} e}\ \text{REFL}$$

$$\frac{e \rightarrow e'}{e \xrightarrow{*} e'}\ \text{RED}$$

$$\frac{e \rightarrow e' \qquad e' \xrightarrow{*} e''}{e \xrightarrow{*} e''}\ \text{TRANS}$$

$\boxed{e \ \ \text{VAL}^*}$

$$\frac{e \xrightarrow{*} e' \qquad e' \ \text{VAL}}{e \ \ \text{VAL}^*}$$

$\boxed{e \ \ \text{DIV/0}^*}$

$$\vdash_{\text{DIV/0}^*} e \ \ \text{DIV/0}^*$$

$$\frac{e' \ \text{DIV/0} \qquad e \xrightarrow{*} e'}{e \ \ \text{DIV/0}^*}$$

e simplifies to an expression stuck due to DIV/0 error ex $(2+2) + 3 / (1 + -1)$ DIV/0*

$$\Big\downarrow *$$

$$4 + 3/(1+-1)$$

$$\Big\downarrow *$$

$$4 + 3/0$$

$\boxed{e \text{ ATM*}}$

$$\frac{\quad}{\text{ATM*}} \quad e \text{ ATM*}$$

$$\frac{e' \text{ ATM} \quad e \xrightarrow{*} e'}{e \text{ ATM*}}$$

Simplification of e results in an expression that is stuck due to an ARG TYPE MISMATCH