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Programs to compute magnetization to density ratio and the magnetization inclination from 3-D gravity and magnetic anomalies☆

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Abstract

Vector field formulation based on the Poisson theorem allows an automatic determination of rock physical properties (magnetization to density ratio—MDR—and the magnetization inclination—MI) from combined processing of gravity and magnetic geophysical data. The basic assumptions (i.e., Poisson conditions) are: that gravity and magnetic fields share common sources, and that these sources have a uniform magnetization direction and MDR. In addition, the previously existing formulation was restricted to profile data, and assumed sufficiently elongated (2-D) sources. For sources that violate Poisson conditions or have a 3-D geometry, the apparent values of MDR and MI that are generated in this way have an unclear relationship to the actual properties in the subsurface. We present Fortran programs that estimate MDR and MI values for 3-D sources through processing of gridded gravity and magnetic data. Tests with simple geophysical models indicate that magnetization polarity can be successfully recovered by MDR–MI processing, even in cases where juxtaposed bodies cannot be clearly distinguished on the basis of anomaly data. These results may be useful in crustal studies, especially in mapping magnetization polarity from marine-based gravity and magnetic data.

PACS: PRISM3D; MDRMI

Keywords: Potential fields; Poisson theorem; MDR; MI

1. Introduction

The Poisson theorem can be used to extract information about rock properties directly from processing of gravity and magnetic anomaly data. This theorem provides a simple linear relationship connecting gravity and magnetic potentials and, by extension, field components that are commonly derived from geophysical surveys (Blakely, 1995). The assumptions underlying the Poisson theorem

☆Code available from server at http://www.iamg.org/CGEditor/index.htm

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(or Poisson conditions) are that: (i) causative dense and magnetic sources are common; (ii) magnetization direction is constant; and (iii) magnetization to density ratio (MDR) is constant. For geological structures satisfying such constraints, it is possible to use gravity and magnetic anomalies to estimate the source MDR, which is related to the slope of the linear Poisson relationship, and its magnetization inclination (MI). Poisson-based methods have been applied to interpret many geological problems in marine geophysics (Bott and Ingles, 1972; Cordell and Taylor, 1971) and regional studies of continental crust (Chandler et al., 1981; Hildebrand, 1985; Chandler and Malek, 1991).

Two main approaches have been used to apply Poisson theorem. In the first approach, filtering is applied to produce the vertical derivative of gravity anomaly and the magnetic anomaly reduced to pole (vertically polarized) (Bott and Ingles, 1972; Chandler et al., 1981; Chandler and Malek, 1991). The second approach (Baranov, 1957; Cordell and Taylor, 1971) filters the magnetic anomaly solely, to produce a pseudo-gravity anomaly, which involves magnetic reduction to the pole and its integration along the magnetization direction. In either case, transformed anomalies should be coincident if Poisson conditions are upheld. If the direction of magnetization is known, MDR values can be readily calculated by either approach through a linear regression of the transformed gravity and magnetic data. Both approaches can also be used to estimate an unknown magnetization direction by a trying range of trial and error directions until a maximum correlation is attained. However, such trial and error methods lack the efficiency of a one-step approach and are prone to error when Poisson conditions are not upheld.

A one-step method that estimates both MDR and MI values from profile data has been developed by using a vector field formulation for the Poisson theorem (Mendonça, 2004). In this approach, the gravity anomaly is processed to furnish its gradient, and the magnetic anomaly processed to give its parent anomalous vector field. MDR is then obtained by rationing the amplitude of these fields, and MI by taking their inner product. For 2-D sources, this approach provides accurate MDR and MI estimations with no prior knowledge of magnetization direction and no need of iterative procedures. Sources violating Poisson conditions lead to apparent MDR and MI values that do not truly reflect subsurface sources, but they nonetheless can be used to discern geological contacts and infer spatial variations in rock physical properties (Mendonça, 2004). Unfortunately, little is known about the significance of apparent MDR and MI values that are derived in this way from sources that are actually 3-D.

This paper presents two Fortran programs that use gridded gravity and magnetic data to estimate MDR and MI values for 3-D sources. Program PRISM3D computes responses over simple 3-D models and gives some guidance for interpreting field-acquired data sets. Program MDRMI can be used to process field-acquired gravity and magnetic data sets to estimate MDR and MI values for the causative sources. MATLAB scripts to visualize program outputs are also included.

2. Poisson relationship in vectorial form

Poisson theorem (Blakely, 1995) relates the magnetic potential, V_m, to gravity potential, U, as

$$V_m = -p\frac{\partial U}{\partial m}, \tag{1}$$

under the assumption that Poisson conditions are held. By Eq. (1), magnetization direction is given by unit vector $\mathbf{m}$,

$$p = c\frac{\Delta\mathscr{M}}{\Delta\rho}, \tag{2}$$

where $\Delta\mathscr{M}$ is the intensity of the magnetization contrast $\Delta\mathbf{M} = \Delta\mathscr{M}\mathbf{m}$ (bold types denoting vectors), $\Delta\rho$ is the source density contrast, $c = \mu_0/4\pi G$, where $\mu_0 = 4\pi 10^{-7}\,\mathrm{H/m}$ is vacuum magnetic permeability and $G = 6.67\times10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ is the gravitational constant.

Gravity anomaly, g^z, is obtained from gravity potential U as $g^z = \mathbf{z}\cdot\nabla U$, with ∇ denoting the gradient operator, and $\mathbf{z}$ a unit vector aligned to the vertical (positive downward). Total field magnetic anomaly, T_m^t, is obtained from magnetic potential V_m as $T_m^t = -\mathbf{t}\cdot\nabla V_m$, where $\mathbf{t}$ is the unit vector along the local magnetic field.

For 2-D sources (Mendonça, 2004), the MDR, $r \equiv \Delta\mathscr{M}/\Delta\rho$, is obtained by

$$r = \frac{1}{c}\frac{|\mathbf{T}_m|}{|\nabla g^z|} \tag{3}$$

and MI, α, such that

$$\sin(\alpha) = \frac{\mathbf{T}_m\cdot\nabla g^z}{|\mathbf{T}_m||\nabla g^z|}. \tag{4}$$

For 3-D sources, the vector quantities related in Eqs. (3) and (4) are such that

$$\mathbf{T}_m = T_m^x \mathbf{e}_x + T_m^y \mathbf{e}_y + T_m^z \mathbf{e}_z \tag{5}$$

and

$$\nabla g^z = \frac{\partial g^z}{\partial x}\mathbf{e}_x + \frac{\partial g^z}{\partial y}\mathbf{e}_y + \frac{\partial g^z}{\partial z}\mathbf{e}_z. \tag{6}$$

For real 3-D sources, substitution of fields (5) and (6) in Eqs. (3) and (4) yields apparent values for quantities r and α (MDR and IM, respectively), which eventually may reflect the true values for the underlying sources.

3. MDR–MI from 3-D models

The basic building block of our formulation is a vertical prism with a horizontal top and bottom. By compounding blocks we can readily compute the field components and derivatives that reflect a complex, multi-source data set. Vector fields $\mathbf{T}_m$ and ∇g^z can be evaluated simply by adding magnetic field components and gravity derivatives from adjacent prisms. Once obtained, these fields enter Eqs. (3) and (4) to give MDR and MI.

To evaluate magnetic fields from an isolated prism, we use formula adapted from Plouff (1976),

$$\begin{aligned} T_m^t = \frac{\mu_0}{4\pi}\Delta\mathscr{M}\Bigg[&(mN+nM)\frac{1}{2}\log\left(\frac{R-x}{R+x}\right) \\ &+(lN+nL)\frac{1}{2}\log\left(\frac{R-y}{R+y}\right) \\ &+(lM+mL)\frac{1}{2}\log\left(\frac{R-z}{R+z}\right) \\ &+lL\tan\left(\frac{yz}{xR}\right)+mM\tan\left(\frac{xz}{yR}\right) \\ &+nN\tan\left(\frac{xy}{zR}\right)\Bigg], \end{aligned} \tag{7}$$

in which (L, M, N) and (l, m, n) are cosine directors, respectively, for unit vectors $\mathbf{t}$ (geomagnetic field direction) and $\mathbf{m}$ (magnetization direction) and $R = \sqrt{(x^2 + y^2 + z^2)}$. For a local field with inclination I and declination D, $L = \cos(I)\cos(D)$, $M = \cos(I)\sin(D)$, and $N = \sin(I)$. For a magnetization direction with inclination i and declination d, $l = \cos(i)\cos(d)$, $m = \cos(i)\sin(d)$, and $n = \sin(i)$. Using Eq. (7), the field components T_m^x, T_m^y and T_m^z can be evaluated by assigning cosine directions (L, M, N), respectively, equal to $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

For gravity field evaluation, we start from Banerjee and Das Gupta (1977) formula,

$$\begin{aligned} g^z = \frac{1}{2}G\Delta\rho\Bigg[&x\log\left(\frac{R-y}{R+y}\right)+y\log\left(\frac{R-x}{R+x}\right) \\ &+2z\tan\left(\frac{xy}{zR}\right)\Bigg] \end{aligned} \tag{8}$$

obtaining gravity derivatives as

$$\begin{aligned} \frac{\partial g^z}{\partial x} &= \frac{1}{2}G\Delta\rho\log\left(\frac{R-y}{R+y}\right), \\ \frac{\partial g^z}{\partial y} &= \frac{1}{2}G\Delta\rho\log\left(\frac{R-x}{R+x}\right), \\ \frac{\partial g^z}{\partial z} &= G\Delta\rho\tan\left(\frac{xy}{zR}\right). \end{aligned} \tag{9}$$

To our knowledge, the expressions in (9) were not published previously and they were consequently tested by comparing their results with numerically evaluated derivatives that were derived through a finite difference scheme with Eq. (8).

As shown in Eqs. (7)–(9), field components and derivatives are evaluated at the origin of the coordinate system. To be evaluated at a generic position (x_0, y_0, z_0), terms (x, y, z) are substituted by $(x_0 - x_i, y_0 - z_i, z_0 - z_i)$, $i = 1, 2$, in which terms (x_i, y_i, z_i) denote the coordinates for a prism vertices.

4. MDR–MI from data processing

Current potential field data provide only a single component for underlying vector anomalous fields. In magnetic surveys, the measured quantity commonly is regarded as being the total field anomaly, T_m^t, which means the field component along the direction $\mathbf{t}$, of the local geomagnetic field (Blakely, 1995). Gravity surveys carried out with common gravimeters provide the vertical component, g^z, of the anomalous gravity field. Therefore, a processing scheme is required to evaluate magnetic field components and gravity derivatives in Eqs. (5) and (6), and by extension in Eqs. (3) and (4). It can be done by exploiting well-known properties of potential fields, which allow to compute field components and derivatives by applying a suitable set of linear transforms (Gunn, 1975; Blakely, 1995).

In the wavenumber domain, a potential field $C(x, y)$ measured at a constant height can be transformed into a field $D(x,y)$ by making

$$\mathscr{F}\{D(x,y)\} = F(k_x,k_y)\mathscr{F}\{C(x,y)\}, \tag{10}$$

in which $\mathscr{F}$ denotes the Fourier transform, $F(k_x,k_y)$ is a mathematical (or filter) expression for the desired transformation, written in terms of wavenumbers k_x and k_y corresponding to directions x and y of a Cartesian system. For operations involving a component change from a measured total field anomaly, filter $F(k_x,k_y)$ assumes a general form of

$$F(k_x,k_y) = \frac{A(k_x,k_y)}{\mathrm{i}Lk_x + \mathrm{i}Mk_y + N|k|}, \tag{11}$$

with $A(k_x,k_y)$ being equal to $\mathrm{i}k_x$, $\mathrm{i}k_y$, and $|k| \equiv (k_x^2+k_y^2)^{1/2}$ to, respectively, compute x, y, and z components of $\mathbf{T}_m$. To compute derivatives of gravity anomaly, term $F(k_x,k_y)$ is made equal to $\mathrm{i}k_x$, $\mathrm{i}k_y$, and $|k|$, respectively, to provide derivatives along x, y, and z directions. A flow chart to compute MDR and MI is presented in Fig. 1.

5. Program description

We describe here two main programs developed under Fortran 90 environment. The first one, *P*RISM3D, implements Eqs. (7)–(9) to calculate magnetic field components, gravity derivatives, and MDR–MI parameters from a set of juxtaposed prisms. It works as a forward model for MDR and MI quantities in the sense that from a known prism model, one can evaluate MDR–MI values

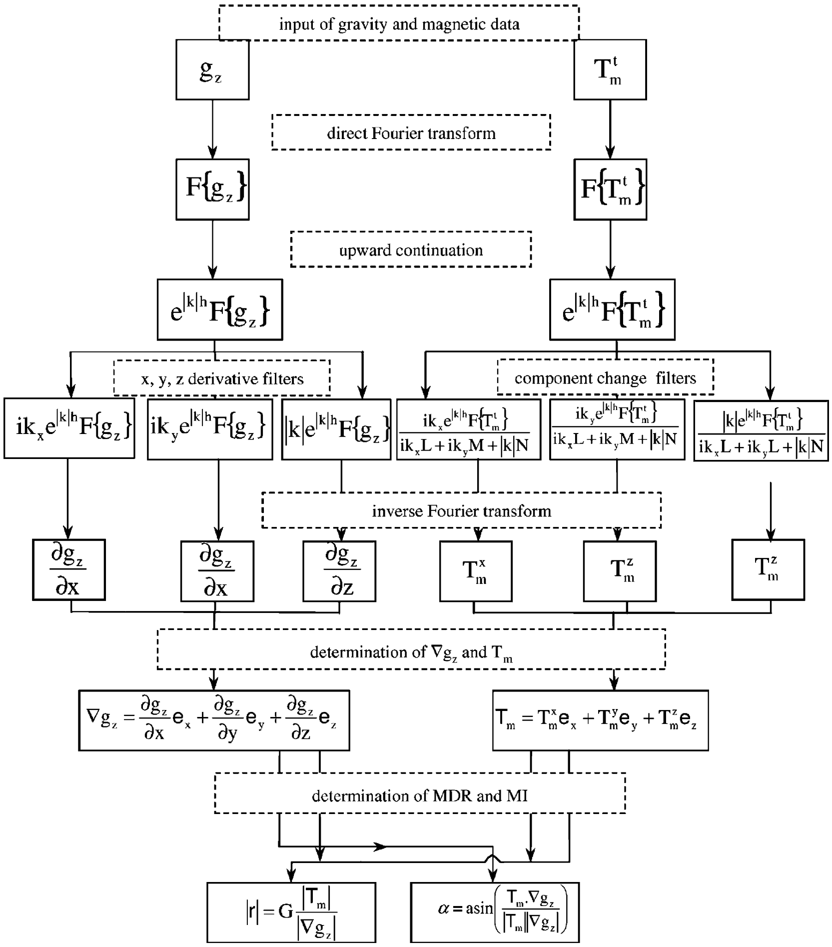

Fig. 1. Flow diagram for MDR–MI processing of gravity anomaly g^z and total field magnetic anomaly T_m^t. T_m^τ, for $\tau = x,y,z,t$, is τ component of the anomalous vector field $\mathbf{T}_m$; $\mathscr{F}\{C\}$ denotes the Fourier transform of C, h is the continuation height (positive downward) for gravity and magnetic fields, k_x and k_y are wavenumber for the x and y directions, $\mathrm{i} = \sqrt{-1}$, and $|k| = (k_x^2+k_y^2)^{1/2}$. Geomagnetic field direction, $\mathbf{t}$, is defined by director cosines (L,M,N), and magnetization direction, $\mathbf{m}$, by director cosines (l,m,n). Unit vectors $\mathbf{e}_\tau$, for $\tau = x,y,z$, define the axis of the coordinate system.

expected from fields observable at the ground surface. The input of a particular geophysical model is done by file *mod*par.txt, by entering basic information on grid size, number of reading points, observation height, local geomagnetic field, and number and size of prisms. For each prism, 11 parameters are required; two to define prism position, six to define size and depth, four to assign physical properties and one to establish the rotation angle with respect to northward pointing *x*-axis. *P*RISM3D was validated in comparison with the gravity and magnetic anomalies evaluated with programs *g*box and *m*box developed by Blakely (1995, p. 377 and 379). Besides a joint evaluation of gravity and magnetic fields, *P*RISM3D evaluates fields from rotated prisms, which is not implemented in published versions of *g*box and *m*box.

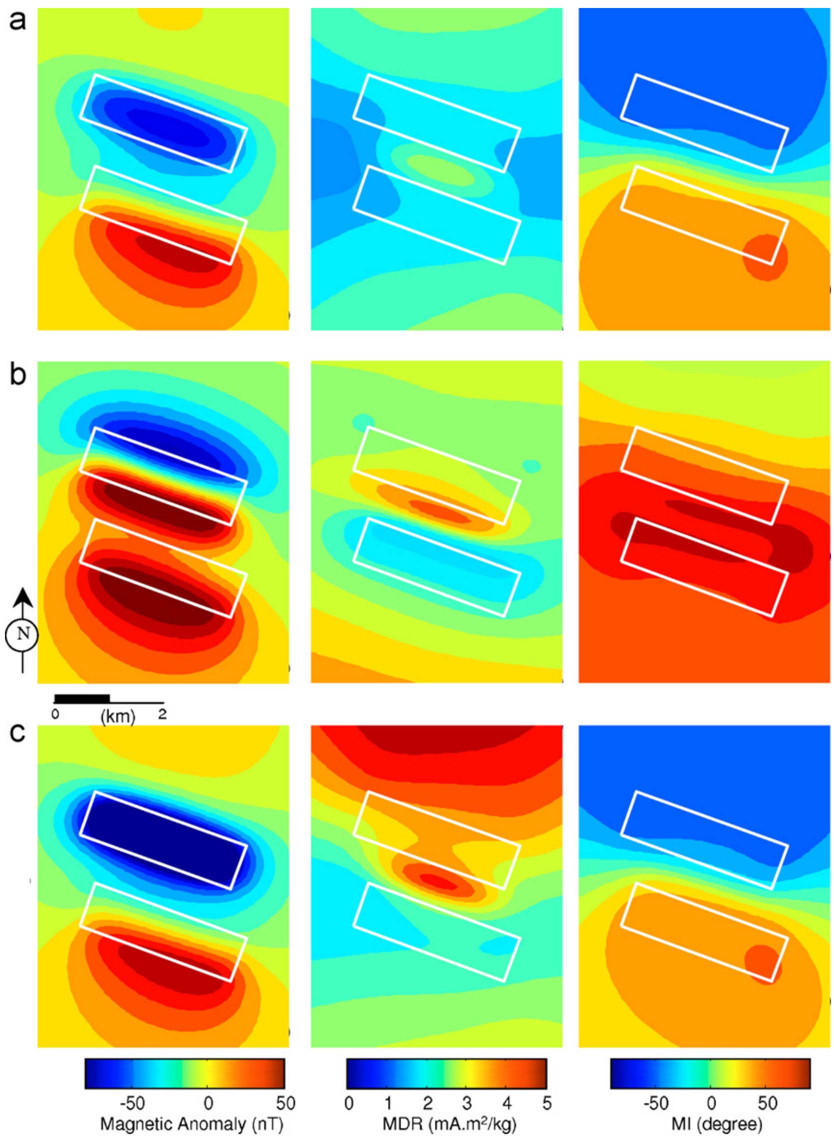

Fig. 2. Magnetic anomaly, MDR and MI evaluated from *P*RISM3D program for models with two adjacent prisms, 2.5 km tick and depth to the top at 1 km. In all cases, the magnetization of the southern prism is 0.25 A/m, with inclination of 40° and declination of 10°, coincident with the local geomagnetic field. The northern prism, otherwise, assumes variable magnetization intensity of 0.25, 0.50, and 0.50 A/m, respectively, in Cases (a), (b), and (c), as well as variable direction (reversed, normal, reversed) in Cases (a), (b), and (c). Both prisms have a density contrast of 0.1 g/cm^3. MDR values for Cases (a), (b), and (c) are then 2.5, 5.0, and 5.0 mA.m^2/kg^3.

Our second program, *M*DRMI, implements the algorithm presented in Fig. 1, by calling a set of subroutines. Subroutine *t*mag evaluates the magnetic fields components T_m^x, T_m^y and T_m^z from processing the total field anomaly T_m^t. It has a same structure of program *s*ignal, which evaluates x, y, and z derivatives for gravity anomaly g^z. Auxiliary programs *f*mdr and *f*inc compute Eqs. (3) and (4) to estimate MDR and MI values. Subroutine *f*ourn from Press et al. (1990) was used to compute the discrete Fourier transform for a grid data set in both *t*mag and *s*ignal routines. Subroutine *s*ignal to compute $|\nabla g^z|$ was kindly provided by Richard Blakely (personal communication). Functions *d*ircos (to compute direction cosines from inclination and declination of an unitary vector) and *k*value (to evaluate wavenumber coordinates for subroutine *f*ourn) are found in Blakely (1995).

Grid gravity and magnetic data entering in *M*DRMI is read by program *r*ead_matlab, in a format able to be plotted by using *l*oad and *p*color functions from MATLAB. Output from programs *P*RISM3D and *M*DRMI can be displayed by MATLAB script files *p*rism3d.m and *m*drmi.m.

6. Numerical experiments

We apply program *P*RISM3D to obtain MDR and MI estimates over a model composed by two juxtaposed prismatic bodies. The southern body of the model has induced magnetization only whereas the northern one has a variable magnetization as illustrated by Cases (a), (b), and (c) in Fig. 2. In Case (a), magnetization intensity is constant in all bodies but the northern one is reversely magnetized. In Case (b), magnetization direction is the same for

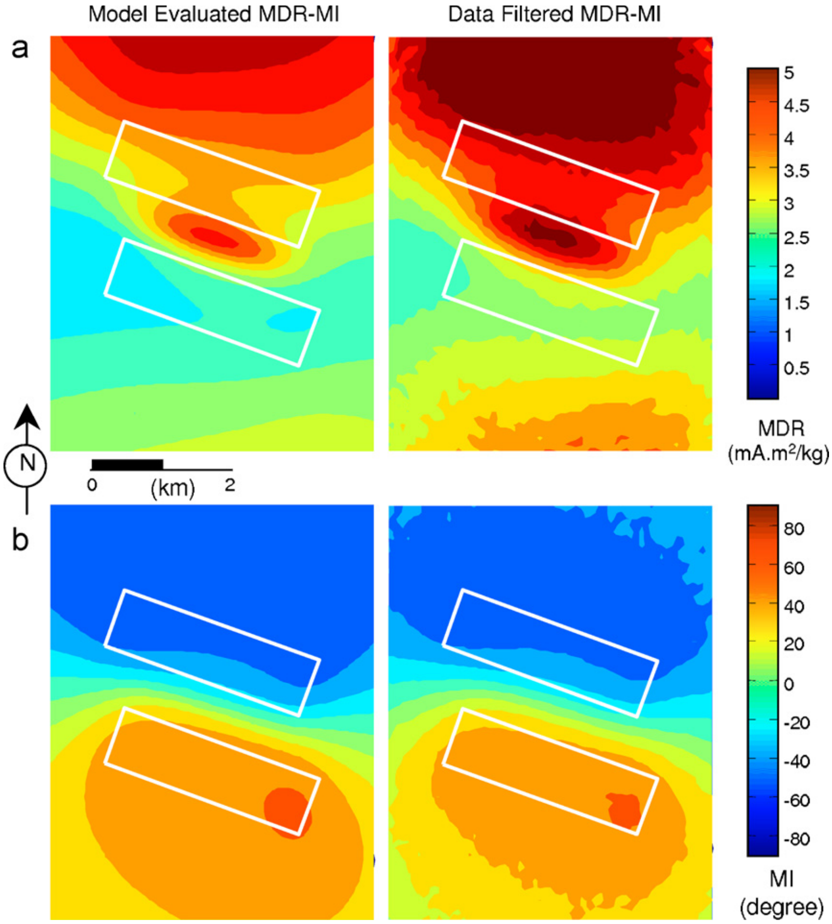

Fig. 3. MDR and MI evaluated from the model in Case (c) of Fig. 2 (left column) and obtained by applying program *M*DRMI on corrupted noise data for both gravity and magnetic anomalies. Additive uniformly distributed noise with amplitude equal to 1% of the peak-to-peak anomalies was applied.

the two bodies but magnetization intensity is twice as much in the northern body. Finally, in Case (c), the northern body has a doubled magnetization intensity and a reversed magnetization direction relative to the southern body. MDR for the southern body is $2.5\,\mathrm{mA\,m^2/kg^3}$, whereas for the northern body it is equal to 2.5, 5.0, and $2.5\,\mathrm{mA\,m^2/kg^3}$, respectively. Magnetization polarity for northern body is reversed, normal, and reversed, respectively. An induced magnetization with inclination of 40° was assumed, normal (positive) polarity and reversed (negative) polarity are relative to this direction.

Fig. 2 demonstrates that even when the two sources cannot be clearly discriminated from the magnetic anomaly data, they are readily discernable in the MDR and MI data. In all cases magnetization polarity is well recovered from MI mapping and good MDR estimates are obtained over the central portions of the prisms. These results suggests a potential application of MDR–MI mapping in crustal studies, especially with regard to mapping the magnetization polarity of the ocean floor in marine geophysics.

Fig. 3 illustrates the capacity of the method in recovering suitable MDR and MI values from gravity and magnetic anomalies in the presence of noise. The left column of the figure shows MDR and MI values evaluated from the model by using forward program *P*RISM3D program; its right column shows corresponding values obtained with processing program *M*DRMI applied to noise-corrupted data. To simulate noise in data, gravity and magnetic anomalies were disturbed with additive, uniformly distributed noise, with amplitude equal to 1% of the peak-to-peak anomalies.

7. Concluding remarks

Results from MDR–MI analysis can be degraded in two major ways: (1) the computation of gravity anomaly gradient involves the application of a noise enhancing derivative filter, and (2) the evaluation of magnetic z component for fields measured at low magnetic latitudes. The latter operation exhibits the same sort of instability previously recognized in reducing to the pole, a total field magnetic anomaly measured close to the magnetic equator. In this transformation, the denominator of filter expression in Eq. (11) tends to zero at a particular direction in the k_x–k_y plane, thus unboundedly enhancing data spectral content along this direction. For field in low magnetic latitudes, specialized procedures (Silva, 1986; Mendonça and Silva, 1993) to stabilize the operation of component change must be applied.

Noisy data should be low-pass filtered prior to the application of MDRMI or upward continued to a suitable height. Upward continuation attenuates the data's short wavelength content, thus indirectly diminishing noise effects. Having managed noise amplification in gravity and magnetic data processing, we believe programs presented here can be used to process real data sets in continental and marine studies.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version, at doi:10.1016/j.cageo.2007.09.013

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