The Future and the

Blockchain Interplanetary Superhighway

(The Lagrange Interplanetary Superhighway)

Nicolaus Copernicus, 1473 - 1543.

Around the turn of the 15th century, Copernicus came up with the revolutionary idea that the planets orbit around the Sun. Although this new view of the world was received with hostility in some quarters, notable scientists, including Kepler and Galileo about a century later, came to accept it. But how exactly did the planets move? Did they describe circles, as Copernicus thought, or ellipses, as observation seemed to suggest?

It took 150 years and the brilliant mind of Isaac Newton to come up with a mathematically rigorous answer. He considered a simplified problem in which just two massive bodies orbit around each other, each exerting a gravitational pull on the other. Using his new technique of calculus, Newton took his equation describing the force of gravitational attraction and integrated it. His solution showed that the path of a planet is always an ellipse.

In fact, what Newton considered was the way in which an object falls through a gravitational field, and he found that, along with the ellipse (or the special case of a circle), there are another two possible paths for a falling object. These are the well-known parabola of a cannonball, and the hyperbola of an object with enough velocity to completely escape the gravitational pull - such as the Voyager 2 probe now leaving the solar system at nearly 45,000 mph. These three trajectories are known as conic sections, as they are also the curves produced by cutting a cone along different planes.

Newton had shown that the "two body problem" was integrable, and so could be solved exactly. His magic formula can tell you precisely where the Earth is in its orbit around the Sun for all time, no matter how far in the past or future - as long as you assume that the Earth and the Sun are the only celestial bodies. After this triumph of cutting-edge mathematics (back in the seventeenth century) the next natural step was to ask about the "three body problem" - could another precise "analytical" solution be found for a solar system composed of the Sun, Earth and Jupiter? This question is much more difficult, as the changing gravitational tug of all three bodies produces very complex behaviour. In fact, without quite realising it, mathematicians at the time had stumbled across one of the first examples of chaos. They soon realised that the three body problem is not integrable, which means that no exact solution can ever be found.

Isaac Newton, 1643 - 1727.

If certain simplifications are made, however, the problem can once again be solved analytically. The "restricted three body problem" assumes that one of the three masses is negligible, and so exerts no gravitational influence on the other two. The problem is now like tracking the path of a mote of dust as it falls through the gravity field of the Earth and Moon.

Two great mathematicians, Euler and Lagrange, shared a prize offered by the Paris Academy of Sciences in 1772 for their solutions to this problem. Their analytical solution showed that there exist special regions in the space surrounding two bodies like the Earth-Moon system within which a particle, or spaceship, can orbit naturally while maintaining the same position with respect to the other two.

Euler found three such locations, but Lagrange's analysis was more thorough and he discovered an additional two less obvious points. Not that Euler should feel cheated however, as he had already lost sight in both eyes and completed the entire solution in his head! But neither of them could have had any idea how important their work would become 200 years in the distant future, with our spacecraft now routinely voyaging through the inky blackness of the solar system.

Lagrange points

These unique regions in space became known as Lagrange points, with a total of five in every two body system. To understand the mechanics of these five points, let's take as an example the system arising from the Earth, the Moon and a spaceship. To keep things simple, let's assume that the Moon and the spaceship both orbit the Earth in perfect circles, and that the spaceship exerts no gravitational pull on the other two.

Leonhard Euler, 1707-1783.

Joseph-Louis Lagrange, 1736 - 1813.

Newton's first law of motion says that an object moving at a certain speed into a certain direction will keep moving in that same direction and at the same speed, unless there is some force which diverts it off its path or changes its velocity. If an object, like the Moon or our spaceship, is to describe a circular motion, then we know it is likely experiencing a force which continually pulls it towards the centre of the circle. This is a "centripetal force" defined by the formula below

$$F_c = mv^2$$

where is the mass of the object, v is its velocity and r he circle's distance from the center. The Earth's gravitational pull supplies the centripetal force in our example, which according to Newton's law of gravitation is

$$F_g = GMm$$

where M is the mass of the Earth, G the gravitational constant, and m and r are, as before, the mass of the object and its distance to the centre of the Earth. If an object of mass m orbits the Earth at distance r then the gravitational pull of the Earth must supply exactly the centripetal force needed to keep it in orbit, so

$$GMm/r^2 = m\imath$$

Rearranging, this gives

$$v = \sqrt{GM}$$

When an object is orbiting Earth at distance r, its velocity is expressed by

$$v = \sqrt{GM}$$

Any slower and the Earth's pull will make it spiral in until it reaches its natural orbit, any faster and it will migrate outwards a short way.

This formula also tells us that the further an orbiting object is from the Earth, the longer it will take to complete one full turn around the Earth: v decreases as r increases. A spaceship orbiting Earth further out than the Moon will take more time for a complete turn than the Moon, while one further in will take less.

An Influence by the Moon's gravity on the spaceship was ignored in this example. Suppose, that the spaceship sits on the straight line segment connecting Earth and Moon. Then the gravitational pull of the Moon will counteract that of the Earth. The net force on the spaceship reduces; the centripetal force also weakens. This means that the spaceship orbits slower than it would if the Moon wasn't there. In fact, there is one point on the line between Earth and Moon where the spaceship orbits Earth at exactly the same speed as the Moon, and this in one of the Lagrange points.

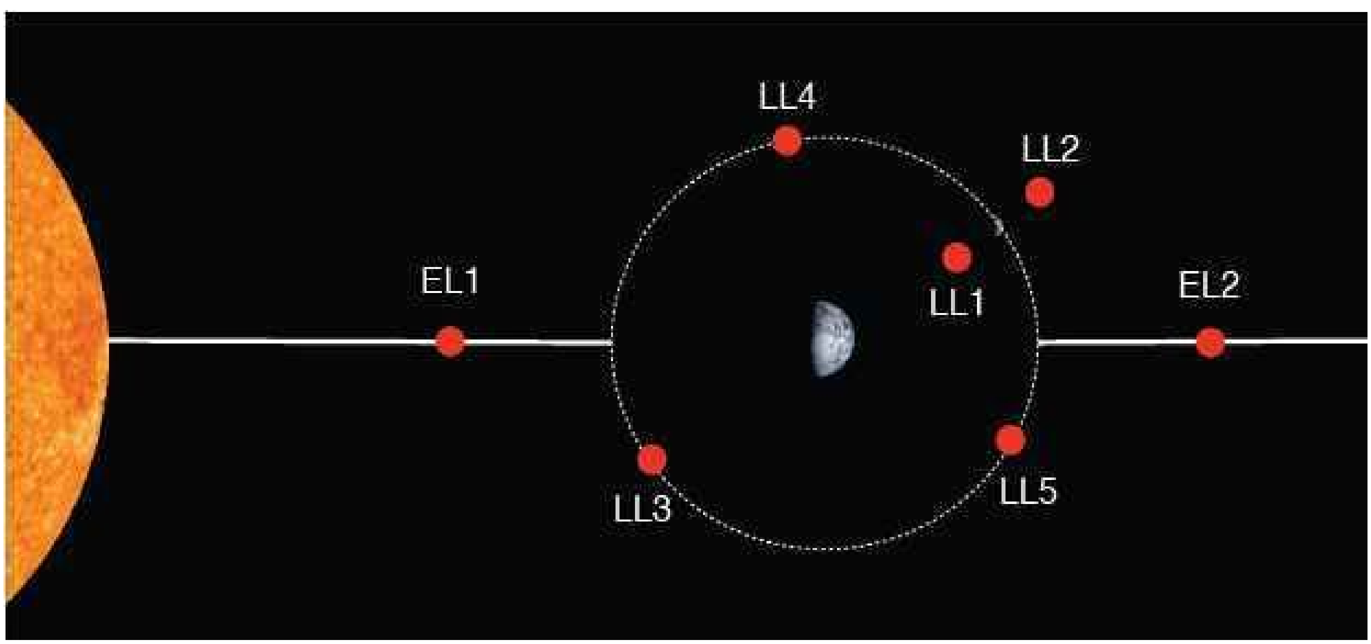


Figure 1 - not drawn to scale.

There are four other points at which the joint gravitational pull of Earth and Moon exactly balances the centripetal force. As the solar system turns, a spaceship sitting at these points always maintains the same place with respect to Earth and Moon.

The diagram in figure 1 shows the Lagrange points in Earth's neighbourhood - made up of all five due to the Moon, and two of those from the Sun. The first three Lunar Lagrange points (LL1, LL2 and LL3) all lie on the line joining the Earth and the Moon and were the ones found by Euler. LL4 and LL5 each form the third point of an equilateral triangle and so always keep 60° in front and behind of the Moon as it circles the Earth.

These Lagrange points are not merely mathematical curiosities, though, as they are already being used by spacecraft exploring the solar system. Of course our model above is very simplified: more than just two massive bodies in the solar system, the planets' orbits are elliptical and not circular, and other forces at work, not just gravity. In a real-life space travel scenario, calculations along similar lines as the ones above come close to enough to calculate real scenarios.

At any of the five Lagrange points, a spaceship can maintain a fixed place with respect to the two larger bodies with relatively little effort, and so they are perfect for long-duration space missions. The EL1 point is now orbited by the Solar and Heliospheric Observatory Satellite (SOHO) as it offers an unrestricted view of the Sun, and WMAP is observing the left-over radiation of the Big Bang from its orbit around EL2.

Different kinds of equilibrium

The exact dynamics around some of the Lagrange points, however, is far more complicated than Euler and Lagrange could have predicted. Understanding it requires the full power of modern dynamical systems theory. The two points L4 and L5 are "stable equilibria". In other words, if a spacecraft placed at this spot becomes nudged, or perturbed, it will naturally return to its original point. It is like a marble placed in a wash basin. The slightest displacement away from the bottom moves the marble up the curved side. It might roll round in circles for a bit, but it will always eventually return to the plug hole. Similarly, a spacecraft can stably orbit around the empty space at L4 or L5.

The first three Lagrange points L1, L2 and L3, however, are "unstable equilibria". They are examples of "saddle points". Instead of a wash basin, think of a horse-riding saddle. It curves upward in one direction but downward along another axis. This means that even the slightest force exerted on the marble even minutely away from the equilibrium point, it starts falling down the gradient and will never return. Similarly, a spaceship at L1, L2 and L3 will drift off at the slightest disturbance. SOHO and WMAP both need to fire their thrusters about once a fortnight to keep themselves orbiting their unstable Lagrange point.

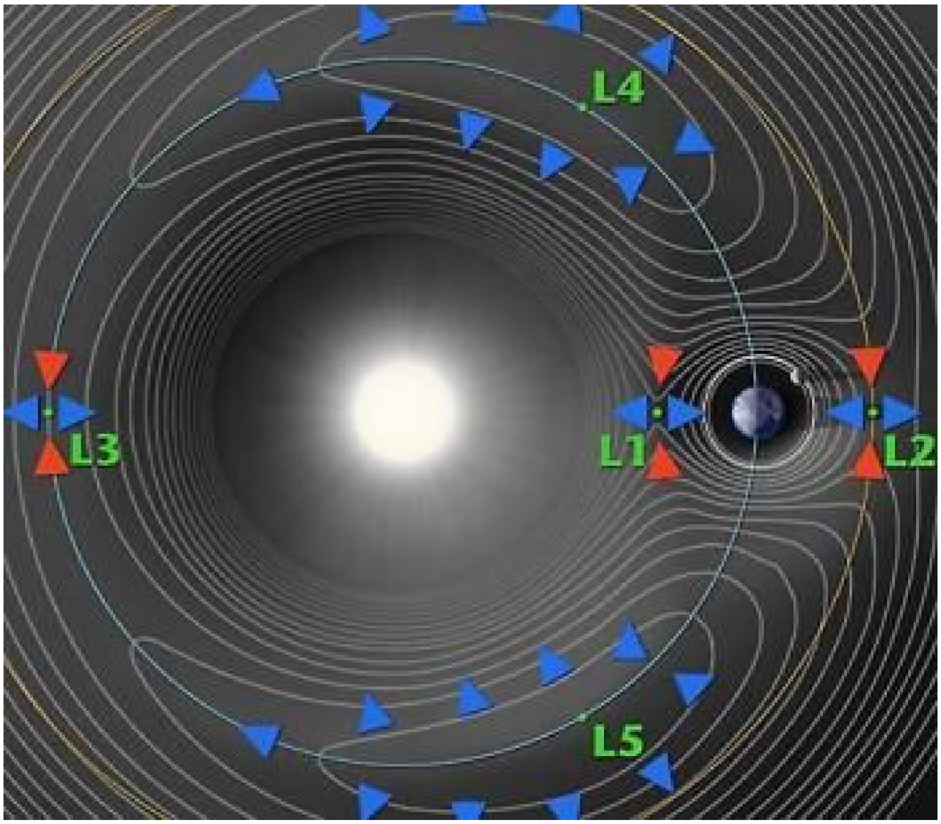


Figure 2 - not drawn to scale. Image courtesy of NASA.

Figure 2 shows a map of the gravity field of the Sun-Earth restricted three body problem. The contours show that the steepest gradients surround the Earth and Sun, with the five Earth Lagrange Points in equilibrium regions with relatively gentle gradient. L1-L3 are unstable saddle points, and spacecraft positioned here will always drift away from the equilibrium. L4 and L5 are stable equilibria, and objects can orbit here indefinitely. The blue arrows show that L4 and L5 are actually atop a potential hill - it is the further effect of the "Coriolis force" that makes them stable.

So some of the most advanced machines ever built are using special equilibria points that were found two centuries ago, along with some more recent mathematics in the form of stability analysis and dynamical theory. The unstable Lagrange points and their importance does not stop here. As we've seen, analysis with calculus can find exact solutions to the equations describing only two massive bodies. How can we ever predict anything about the motions of all nine planets within our solar system?

Until recently mathematicians simply ignored the interactions between the planets' gravitational forces, an assumption that works fine to a certain level of accuracy. The Supercomputer advancement, mathematicians can use a much more brute-force approach and calculate numerical solutions. Instead of attempting to solve the equations exactly, they model a complex system step-by-step, crunching through an enormous amount of separate integration calculations to see how the system behaves over time. The Lagrange points help us discover about the dynamics of our own solar system using this technique. But first, we need to learn a bit more about space travel.

Ultra-low energy trajectories

Moving between different positions within the gravity field of the solar system requires energy. The space shuttle must obviously consume an enormous amount of fuel to launch itself against the pull of gravity into an orbit 400 km over the Earth's surface. The Apollo spacecraft had to fire rockets to lift their astronauts even further up to get to the Moon, and then again to bring them home afterwards. Even though there is no air friction, moving between two positions in the solar system usually requires a rocket burst to provide a change in velocity, or "delta V" as it is known. The delta V needed to reach a low Earth orbit (LEO) from the ground is 9.7 km/s (about 22,000 mph). The delta V to lift a spacecraft from LEO to the Lunar Lagrange point LL1 is a further 3.15 km/s (much less because the strength of gravity decreases with distance).

Now, it just so happens that the delta V needed for the one and a half million kilometre journey from LL1 to EL2 is only 0.014 km/s - no faster than a cyclist! The fact that the energy levels of this Lunar Lagrange and Earth Lagrange point are so similar is purely a coincidence, but it opens up a fabulous opportunity for space exploration. There is an ultra-low energy pathway between these two crucial points in space - meaning no "uphill struggle" in either direction and spacecraft can travel between the two with almost no fuel.

There are plans for satellites destined for the Earth Lagrange points. If any of them were to break down so far from home they would be almost impossible to retrieve or repair. Sending astronauts up to fix the Hubble Space Telescope in Earth orbit was difficult enough. Getting astronaut mechanics out to EL2 would need not only a rocket as big as that which took men to the Moon but a voyage time of three months one-way, all the while exposed to the harsh space radiation outside Earth's magnetic field.

No faster than a cyclist

However, due to this low-energy pathway, faulty spacecraft could be commanded to coast back to LL1, practically nearly free. NASA has been seriously considering LL1 as an ideal place for a permanent space-station. Not only would a space habitat be an astounding human achievement, but it could serve as a repair station for faulty satellites returning along the EL2-LL1 pathway.

But how do these low energy pathways arise? Can we find more of them and so enable our spacecraft to tour the solar system almost for free? The foundations for the discovery of such a network was laid in the late 19th century by the mathematician Jules-Henri Poincaré. Poincaré worked on the three body problem. His crucial observation was that although it is impossible to precisely predict the trajectories of particles near the unstable Lagrange points, you can separate out families of trajectories that behave similarly. These similar trajectories together form the surface of a tube. A particle that starts out on such a tube will move along its surface, spiralling away from the Lagrange point. For each such outbound tube, called an "unstable manifold", there is an in-bound tube, called a "stable manifold", along which particles move towards a region around the Lagrange point. Near each unstable Lagrange point, there is a multitude of such pairs of tubes, winding around each other in a very complex way, but ultimately going off into wildly different directions.

Theoretically, a spaceship could hitch a free ride to the region close to the Lagrange point on an in-bound tube. More exciting, what if an outbound tube coming from the region around a lunar Lagrange point intersects an in-bound tube to a region around a Lagrange point of, say, the Sun-Jupiter system? Then a spacecraft could travel from one to the other practically nearly free, as long as it switches manifolds at the right moment.

It wasn't until the 1980s that the idea of exploiting manifolds for space travel was given serious consideration. But over the last few years, NASA mathematician [Martin Lo] and his team have built an elaborate theory around this idea. With increased computing power, these scientists used numerical experiments and did indeed find some such low energy paths. Each of these is like a valley carving through the potential energy landscape of the solar system. Spacecraft could flow along these channels practically effortlessly - barely needing thrusters to struggle against gravity.

There is a stellar interconnection Earth's Lagrange points and those of Mars, or Saturn, by these minimal energy trajectories. Jupiter has Lagrange points associated with each of its many bodies like our moon on earth, all joined up into an interlinked web, which itself connects to the Earth system via the Jupiter-Sun Lagrange points. Spacecraft travelling along routes within this tube would be able to efficiently reach their destination. Any spacecraft that fly beyond the manifold, however, would enter the "unstable region" and their trajectory would inexorably stray further and further away from the low-energy tube. This collection of low-energy trajectory families spreads across the entire solar system, regularly interconnecting at Lagrange points into a vast system of tunnels. And the system is not stationary: the tubes forming the stable and unstable manifolds move with the planets, an ultra complex heap of writhing interplanetary spaghetti.

Jules Henri Poincaré, 1854 - 1912.

This network is as ancient as the solar system, but is completely invisible and would have lain undiscovered were it not for the power of modern mathematics and numerical integration on fast computers. The mathematicians that discovered this system of low-energy trajectories have named it the *Interplanetary Superhighway*, or *IPS*.

The fact that the behaviour of trajectories near unstable Lagrange points is unpredictable, or chaotic, is only a minor nuisance for spacecraft stationed here. A total delta V of only a few m/s per year would be needed to keep nudging the space-station back towards the equilibrium point. But, according to Martin Lo, this inherent instability provides a remarkable opportunity for human exploration of the solar system

The Lunar Gateway

In Chaos Theory, systems like the LL1 Lagrange point are known as "highly nonlinear dynamical regions". If an object close to LL1 gets nudged , it will drift away, like the marble falling off the downward slopes of the saddle. Even a slight alteration to a trajectory passing close to LL1 will take it off into a different direction and lead to a large change in the eventual path of the spacecraft. This is more popularly known as the "Butterfly Effect" - in chaotic regions of space a small perturbation results in a huge difference in outcome.

The upshot of all of this is that a spacecraft swinging past LL1 can easily push itself from one low-energy trajectory onto another that leads to a completely different destination. Thus a probe launched from Earth could, theoretically, be sent to LL1, fire its thrusters at a precisely calculated time, and efficiently switch from the LL1-EL2 tunnel into the one leading to Mars. The neighbourhood around LL1 is like a vast highway interchange, allowing spacecraft to choose between different IPS pathways. This means that a human habitat at LL1 could not only be used as a service-station for spacecraft needing repairs, but as a departures terminal for missions throughout the solar system. The space-station would very literally serve as the Gateway into the tunnel network of the Interplanetary Superhighway.

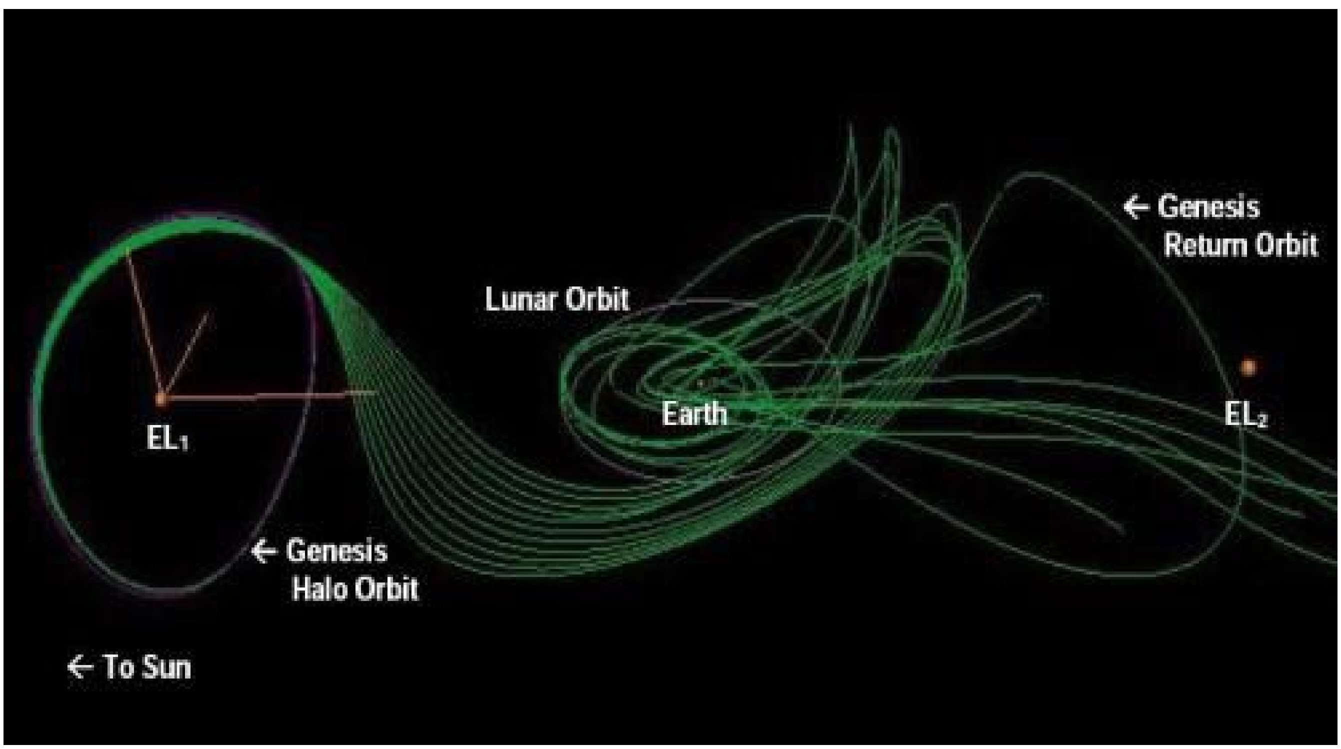


Figure 3: a perspective view of different paths through Earth's local IPS. The Genesis trajectory is only one of a large set of similar paths - it orbits EL1 five times, and returns to Earth via EL2. The other trajectories shown leaving EL1 each orbit the Earth in a different way, and some even interchange at the LL1 Gateway and exit the Earth-Moon system entirely - shown winding off past EL2 to the right. Image courtesy of NASA.

Most of these interplanetary journeys are still in the realms of theory, and the scientists admit that most of these meandering routes would take much more time than a trip using more conventional methods. But if you think that all of this is high-octane science fiction of a distant future, then you're wrong. The Interplanetary Superhighway has already been travelled by one of our spacecraft. "Genesis" was a probe launched in 2001 under the direction of Martin Lo. Its mission was to collect samples of the solar wind - the stream of charged particles flowing off the Sun. After launch the spacecraft was injected into a precisely pre-planned course along the local IPS in Earth's neighbourhood. Genesis spent two and a half years orbiting around EL1 before looping once around EL2 and then re-entering Earth's atmosphere in the afternoon skies over Utah, USA. And all of this without needing any rockets to change course (aside from very minor thrusts to keep it on the planned path) - the entire trajectory was ballistic like a ball thrown in the air. The whole mission, right down to the precise time of arrival back on Earth, had been calculated years in advance using chaos mathematics and computers. It has even been mathematically proven that, in principle, virtually any tour around Earth's neighbourhood could be designed, no matter how convoluted, along low-energy trajectories of the IPS. The spacecraft can be sent around the Sun, then the EL1, then the Earth, then EL2, with any desired number of orbits at each stage.

Moon Colonies: The first 𝔁Blockchain deployed

We can see the first elements of the Blockchain Superhighway expresses in figure 4. When we place a HEPTANODE Cluster in a straight line at position **L𝔁** connecting the earth and the moon, the gravitational pull of the earth will counteract that of the moon. The net force exerted on the HEPTANODE Cluster reduces; the centrifugal force also weakens, resulting on the HEPTANODE Cluster to orbit slower. In fact the HEPTANODE is at equilibrium at a Lagrange point as both bodies orbit at exactly the same speed around the earth.

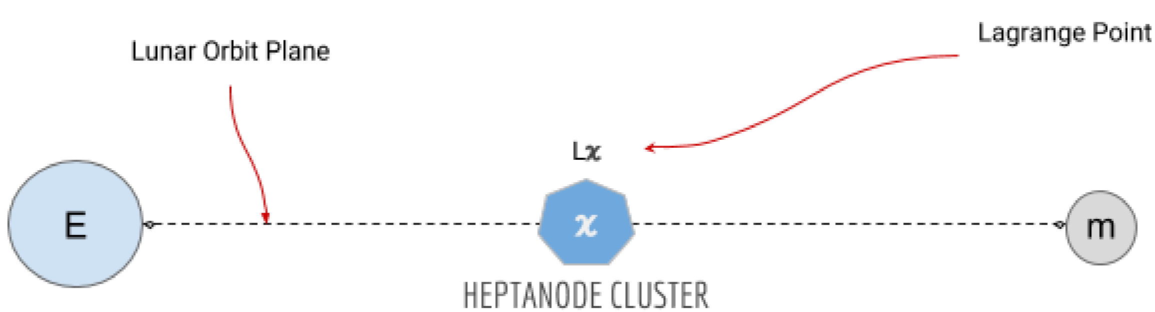


Figure 4.

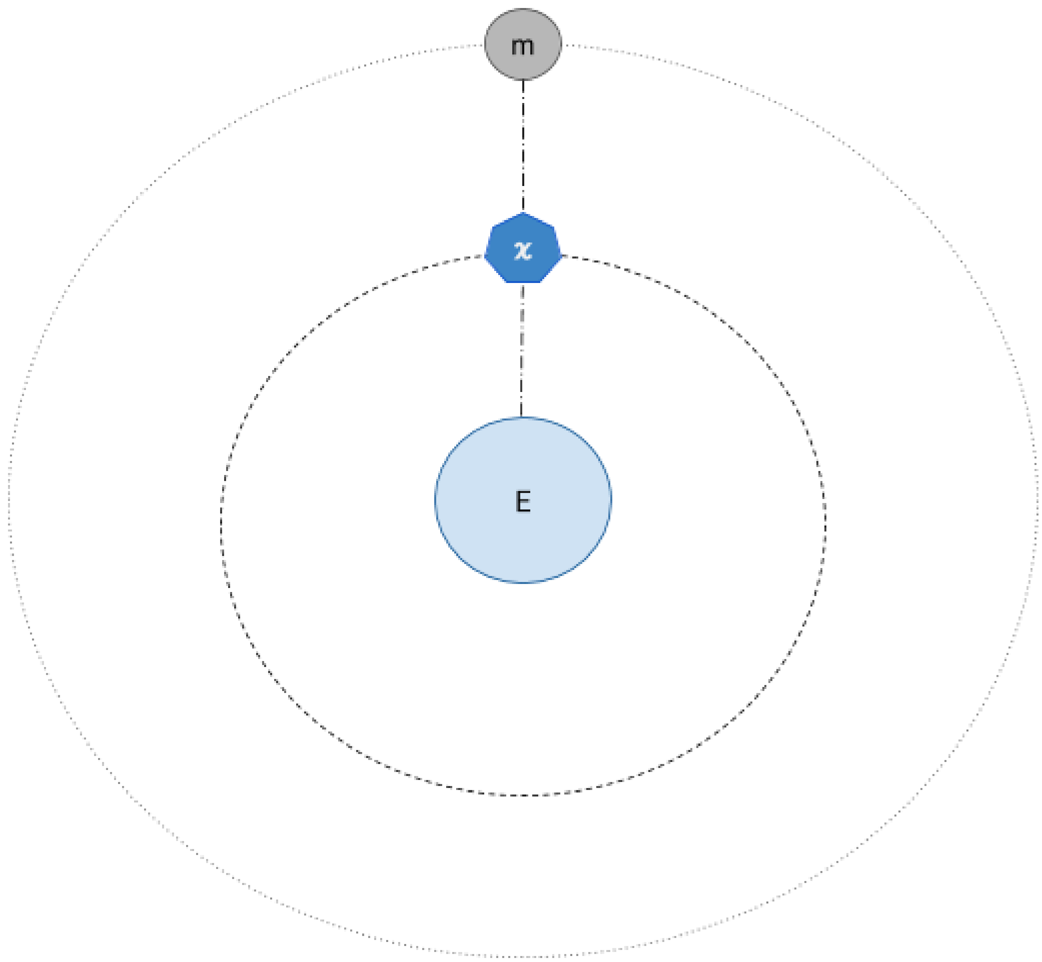


Figure 5.

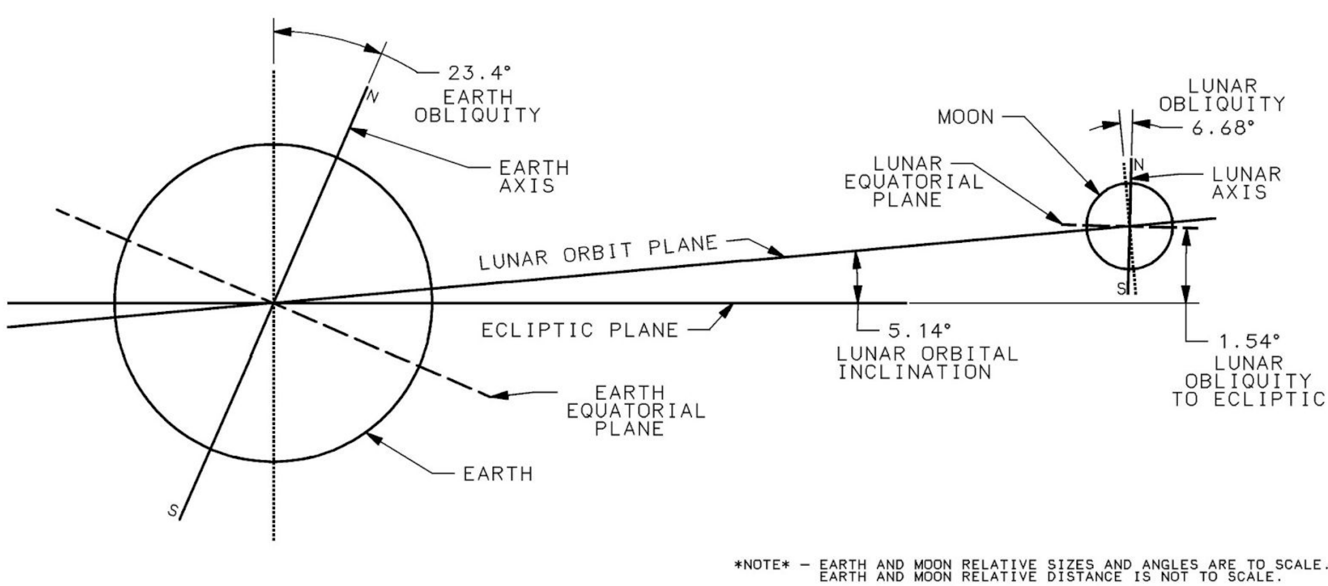


Figure 6.

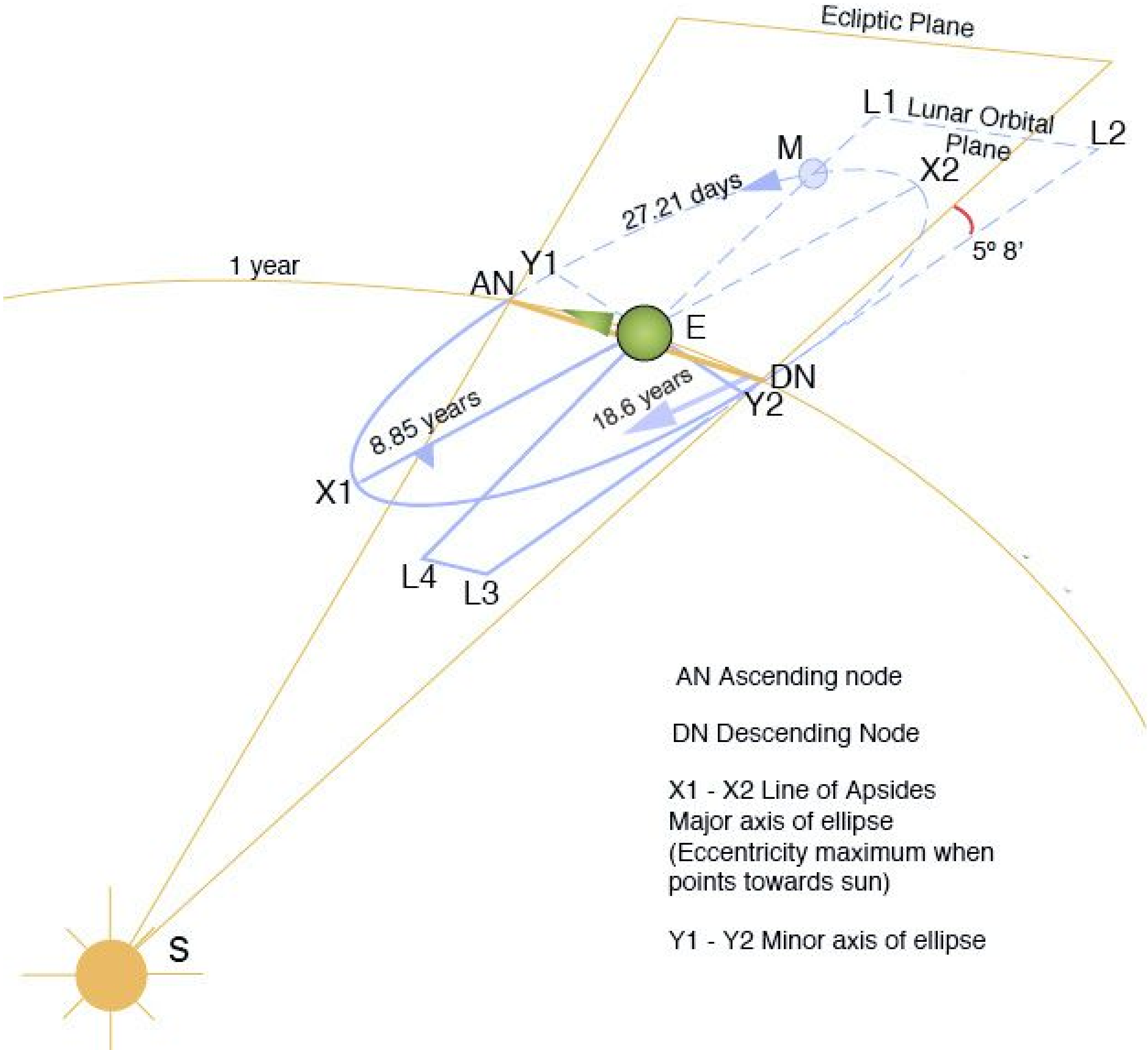

Ecliptic Plane
L1
Lunar Orbital Plane
L2
M
X2
27.21 days
5° 8'
1 year
AN
Y1
E
DN
Y2
18.6 years
8.85 years
X1
L4
L3
S
AN Ascending node
DN Descending Node
X1 - X2 Line of Apsides
Major axis of ellipse
(Eccentricity maximum when
points towards sun)
Y1 - Y2 Minor axis of ellipse

Figure 7.

Figure 8.

Figure 9.

Ancient Travellers

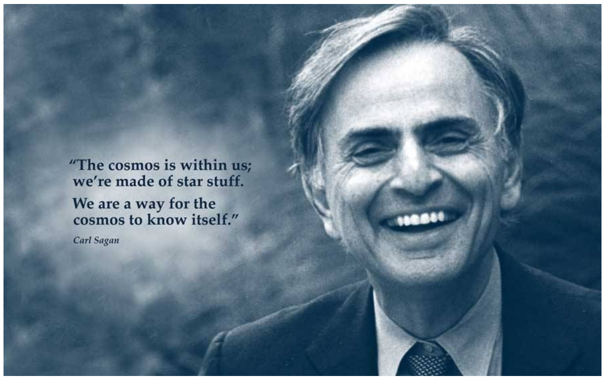
"Genesis" demonstrates well the power of mathematical analysis using computers and the advantages of chaotic regions. But it was not the first explorer, not by a long way, to have used the Interplanetary Superhighway. Some comets have travelled along the ethereal tunnels for aeons, and the asteroid that wiped out the dinosaurs 65 million years ago. Genesis has shown us it may have reached Earth through the IPS. For some scientists, these near-Earth asteroids present as much of an opportunity as they do a threat. Many scenarios of dealing with such a potential killer involve trying to destroy it with nuclear weapons or landing rockets to push it away into a wider orbit. But using the nonlinear dynamics of the IPS such passers-by could be captured and parked in an orbit around one of the Lagrange points. Such a huge lump of iron would offer humanity with an enormous supply of raw materials for space industrial complexes.

This newly found Interplanetary Superhighway is a perfect example of the overlap between classic analysis and modern numerical techniques. The genius minds of Euler and Lagrange used the new technique of calculus to solve the restricted three body problem. Now we know intriguing equilibrium points in exist in space. Now, 212 years later, we are employing our own ground-breaking methods using dynamical systems theory and supercomputers, and taking our first steps along the invisible tunnels stretching through the solar system and ultimately build the blocks that carry the Human Blockchain in outer edge of the universe.

Further reading

- This page by John Baez contains more information on Lagrange points.
- This article by Neil Cornish gives a technical derivation of the positions of Lagrange points.
- The article "The Interplanetary Superhighway and the Origins Program" by Martin Lo describes the IPS and some of the maths behind it.
- Wikipedia's entry on the IPS has a long list of references.

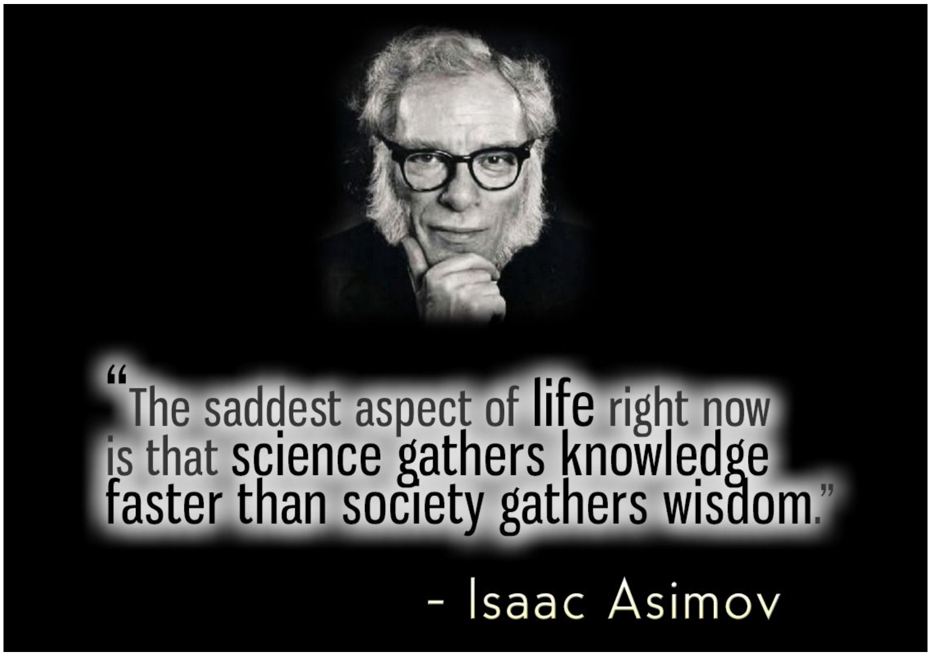
DEDICATED TO MY HEROES



CARL SAGAN

The man who helped me see beyond the earthly bounds,
and far beyond the Cosmos...

&



ISAAC ASIMOV

The man who helped me to dream when I read his book *I Robot*...
As a young boy and the endless possibilities of the human mind!