Journal of Financial Economics 126 (2017) 689–712

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

Tax uncertainty and retirement savings diversification[☆]

David C. Brown[a,*], Scott Cederburg[a], Michael S. O'Doherty[b]

[a] Eller College of Management, University of Arizona, McClelland Hall, Room 315R, Tucson, AZ 85721, USA
[b] Robert J. Trulaske, Sr. College of Business, University of Missouri, 513 Cornell Hall, Columbia, MO 65211, USA

ARTICLE INFO

Article history:
Received 25 July 2016
Revised 21 September 2016
Accepted 9 November 2016
Available online 6 October 2017

JEL classification:
G11
H31
J32

Keywords:
Tax uncertainty
Asset location
Retirement savings
IRA
Roth

ABSTRACT

We investigate the optimal savings decisions for investors with access to pre-tax (traditional) and post-tax (Roth) versions of tax-advantaged retirement accounts. The model features a progressive tax schedule and uncertainty over future tax rates. Traditional accounts are valuable for hedging retirement account performance and managing current income near tax-bracket cutoffs, whereas Roth accounts allow investors to mitigate uncertainty over future tax schedules. The optimal asset location policy for most households involves diversifying between traditional and Roth vehicles. Contrary to conventional advice, the substantial economic benefits from Roth investments are not limited to investors with low current income.

1. Introduction

Financial decisions have significant effects on household welfare and must often be made within complex economic and regulatory environments (Campbell, 2006; 2016). Among the most important of such decisions are choices related to saving for retirement, including the optimal amount of savings, portfolio allocation across assets, and location of assets within various accounts. In this context, locating retirement savings in tax-advantaged vehicles is of critical importance, and the current United States (US) tax code provides investors with a number of alternatives, such as Individual Retirement Accounts (IRAs) and employer-sponsored 401(k) plans. Moreover, the recent introduction of post-tax Roth versions of 401(k), 403(b), and 457(b) accounts, which have no income limits for eligibility, has greatly expanded the pool of investors who can strategically invest in both pre-tax (traditional) and post-tax (Roth) retirement vehicles.[1] Location decisions across traditional and Roth accounts are likely to have a pronounced impact on the economic outcomes of retirement

☆ We thank Shaun Davies, John Howe, Jennifer Huang, Chris Leach, David Nanigian, Seth Pruitt, David Schreindorfer, Clemens Sialm, Rick Sias, Chester Spatt, Luke Stein, Ashish Tiwari, Mitch Towner, Adam Yore, an anonymous referee, and seminar participants at the Arizona Junior Finance Conference, the University of Missouri, and the California State University at Fullerton for their helpful insights and suggestions.

* Corresponding author.
E-mail addresses: dcbrown@email.arizona.edu (D.C. Brown), cederburg@email.arizona.edu (S. Cederburg), odohertym@missouri.edu (M.S. O'Doherty).

[1] Traditional retirement account investments are made using pre-tax dollars, and any contributions in a given year reduce taxable income in that year. Withdrawals from traditional accounts in retirement are taxed as ordinary income. In contrast, investments in Roth accounts are made using post-tax dollars, but withdrawals in retirement are tax free. Throughout the paper, we use the terms "pre-tax" ("post-tax") and "traditional" ("Roth") interchangeably.

https://doi.org/10.1016/j.jfineco.2017.10.001
0304-405X/

savers given the $14 trillion in tax-advantaged retirement account assets at the end of 2015.[2]

In this paper, we study the optimal retirement savings decisions of households with access to both pre-tax and post-tax accounts. Our modeling approach accounts for investor age, current income, and taxable income from outside sources in retirement. Our analysis emphasizes two aspects of the US tax environment that are often ignored in the retirement savings literature. First, the tax system is progressive (i.e., the marginal tax rate is increasing in taxable income). Marginal tax rates in 2015 ranged from 10% to 39.6%, such that the impact of taxes varies greatly across income levels. This feature is important because it generates both uncertainty over the investor's marginal tax rate in retirement and a positive correlation between this marginal rate and investment performance. Second, the future tax-rate schedule is unknown, and historical tax-rate changes in the US suggest that substantial uncertainty exists about future rates. For example, the marginal rate for married taxpayers with inflation-adjusted income of $100,000 has changed 39 times since the introduction of income taxes in 1913 and has ranged from 1% to 43%.

Prior literature often ignores these two sources of tax-rate uncertainty and considers tax-advantaged accounts in an environment with a known, flat tax rate (see, e.g., Shoven and Sialm, 2003; Dammon, Spatt and Zhang, 2004; Garlappi and Huang, 2006; Amromin, Huang and Sialm, 2007; Huang, 2008; Gomes, Michaelides and Polkovnichenko, 2009; Marekwica, Schaefer and Sebastian, 2013; Fischer and Gallmeyer, 2017). In such a setting, investors are indifferent between traditional and Roth retirement accounts as long as contribution limits are not binding. As such, this literature largely centers on the optimal allocations of assets across and within tax-advantaged and fully taxable accounts. In contrast, we emphasize allocations across traditional and Roth accounts and show that these choices have first-order implications for investor welfare.

In an economy with progressive taxes and tax-schedule uncertainty, both pre-tax and post-tax versions of retirement accounts provide relative advantages to savers, and we find that the optimal policies for most households involve diversifying across these accounts. Roth accounts are specifically valuable for managing uncertainty about the future tax schedule. To understand this point, consider an economy that maintains a flat tax structure but introduces uncertainty about the future tax rate with a mean-preserving spread.[3] Risk-averse investors strictly prefer post-tax Roth accounts to traditional alternatives in this setting, as Roth vehicles allow investors to lock in the known current rate and eliminate the unrewarded exposure to tax risk incurred by traditional savings. Once we introduce a progressive tax structure into the economy, however, many investors begin to allocate savings to traditional accounts.[4] Because investors tend to have relatively higher income during their working lives, contributions to traditional accounts often shift tax payments from points in the life cycle with higher marginal tax rates to points with lower marginal rates. These vehicles are also valuable for reducing consumption risk due to the desirable positive correlation between realized account performance and marginal tax rates paid on traditional savings in retirement. Importantly, the impact of poor realized investment performance on retirement consumption is partially offset by a low marginal tax rate on retirement income in these scenarios.

To formally analyze the retirement savings problem, we solve for the optimal strategies of risk-averse investors who maximize their expected utilities from current and retirement consumption. After being endowed with current and retirement incomes, investors choose how much to save versus consume, how to allocate savings between riskless bonds and stocks, and where to locate these assets. Assets can be located in a pre-tax traditional account or a post-tax Roth account. Given the model's complexity, we numerically solve for investors' optimal strategies. We specify a progressive tax structure designed to reflect important features of the US tax system, and we consider constant and uncertain tax schedules in retirement. To produce distributions of tax rates at a given retirement horizon, we use a bootstrap approach based on historical changes in tax rates that preserves the observed volatility and correlation structure of past rate changes. We model stock returns in a similar fashion, using past return realizations as the basis for the bootstrapped distributions of holding-period returns.

To establish a baseline for our main results, we begin by characterizing investors' optimal strategies when faced with a progressive, but constant, tax schedule. In this case, traditional accounts produce two primary benefits for investors. First, the deductability of current savings in traditional retirement accounts allows investors to manage their current taxable income around tax-bracket cutoffs. Second, the progressive tax rates faced in retirement provide a natural hedge against investment performance. Investors with poor investment results and little wealth in retirement pay a relatively low marginal tax rate, whereas larger tax burdens are borne by investors who become wealthy as a result of good investment performance. With a static tax schedule, Roth accounts are primarily useful for low-income investors who can lock in a low marginal rate by paying taxes in the current period. This finding is consistent with conventional advice from the financial press on the benefits of Roth retirement vehicles. Beyond this group of low-income investors, retirement savers optimally prefer traditional accounts given their benefits within a constant, progressive tax schedule.

Introducing uncertainty about the future tax schedule leads investors to increase their use of Roth accounts. Traditional accounts remain valuable for managing taxable income around tax-bracket cutoffs and hedging investment

[2] See https://www.ici.org/research/stats/retirement/ret_15_q4

[3] Shoven and Sialm (2003), Garlappi and Huang (2006), and Huang (2008) consider flat tax rates that are potentially time-varying but known over the relevant decision-making horizon. These studies investigate the optimal location of highly taxed versus lightly taxed assets between a tax-advantaged account and a fully taxable account.

[4] Zhou (2009, 2012) studies allocations to traditional and fully taxable accounts in an economy with known, progressive tax rates.

performance in the progressive tax system. These benefits, however, must be balanced against the cost of higher consumption risk in retirement resulting from tax-schedule uncertainty. Roth investments allow investors to manage this uncertainty by locking in current tax rates on the associated savings. Future tax rates are more uncertain over longer retirement horizons, and our analysis of historical tax changes also suggests that the rates associated with higher incomes are more variable. As a result, the highest tax-risk exposures occur among younger investors with sufficient traditional account savings to produce taxable income in retirement that exhausts the lower-income brackets. Young, high-income investors who are likely to meet these criteria can manage their exposure to tax-schedule uncertainty by investing a portion of their wealth in Roth accounts. Despite high current marginal tax rates, and contrary to conventional financial advice, these investors achieve substantial benefits from the tax-strategy diversification offered by Roth accounts.

Given that tax-schedule uncertainty is often ignored in the academic literature and financial press, we conduct an equivalent-fee analysis to quantify the benefits of accounting for this uncertainty in retirement planning. We start by considering investors who optimize their portfolios while ignoring tax-schedule uncertainty and then measure their expected utilities after they are exposed to uncertainty about future tax rates. The investors are subsequently allowed to reoptimize under uncertainty, but we place annual fees on their savings and compute the fees that make the investors indifferent between reoptimizing and keeping their original strategies. These fees are, thus, a direct proxy for each investor's value of considering tax-schedule uncertainty in her financial plan. We find that accounting for tax uncertainty is economically important for a wide range of investors, with estimated fees exceeding 2.0% annually in some cases. The fees tend to increase with current and future income and the investor's time to retirement.

The results, to this point, correspond to decision environments in which investors have unrestricted access to traditional and Roth savings. In practice, however, large discrepancies exist in households' abilities to save in tax-advantaged retirement accounts. More than 68 million individuals do not have access to employer-sponsored plans (New York Times, 2015), leaving them with only traditional and Roth IRAs as investment options. Investments in these vehicles are limited to \$5,500 per year and subject to income qualification. This opportunity set stands in stark contrast to that of public university employees, for example, who can contribute to both 403(b) and 457(b) accounts, each having an annual contribution limit of \$18,000 as of 2015. In addition to account access, the availability of post-tax options within employer-sponsored retirement plans differs across investors. On this point, Utkus and Young (2015) find that 44% of employer-sponsored plans through Vanguard do not have designated Roth alternatives.

These constraints have direct implications for investor welfare, and our modeling approach offers a useful framework for assessing their economic importance. As a starting point, we consider constrained households with access to only traditional retirement accounts and estimate the value of granting them access to Roth vehicles using an equivalent-fee analysis. We find that the welfare impact of Roth investments is large for most households, as low-income investors value the ability to lock in low marginal tax rates on current earnings and high-income investors value the ability to mitigate tax-schedule uncertainty. We also apply our model to quantify the importance of tax uncertainty for households facing more realistic investment constraints on account access and contribution limits. Our main conclusions for unconstrained investors also apply to the constrained-investor setting.

Finally, we recognize that our optimal investment policies across traditional and Roth accounts are complex functions of tax status, current income, expected future income, investment horizon, and household preferences. Given the low levels of financial literacy in the population (see, e.g., Campbell, 2016), it seems unlikely that investors would be able to implement the exact strategies proposed in this paper. Motivated by this concern, we introduce a simple asset location strategy that allows investors to approximate the recommendations from our formal analysis.[5] The rule directs investors to allocate all of their savings to Roth accounts if their current taxable income corresponds to a low tax bracket and otherwise invest $(Age + 20)$% of their savings in traditional accounts with the remainder in Roth accounts. We find that the differences in economic outcomes for investors using this simple asset location rule and those following our more complex investment recommendations are generally small, indicating that even households with limited financial capabilities can realize the benefits of tax diversification.

Our study demonstrates the value of tax-strategy diversification in retirement planning with progressive taxes and uncertainty about the future tax schedule. Whereas conventional wisdom largely supports choosing between traditional and Roth accounts by comparing current tax rates with expected future tax rates, the hedging benefits of traditional accounts and the usefulness of Roth accounts in managing tax-schedule uncertainty are important considerations in the optimal savings decision.[6] Poterba (2002, p. 1161) notes that "recognizing 'tax-code uncertainty' and incorporating it in models of household portfolio choice represents a useful avenue for future work." We answer this call by providing investors with initial guidance on considering sources of tax uncertainty in retirement planning. We also demonstrate that our results are of practical importance, as simple tax diversification strategies yield considerable welfare gains for retirement savers.

The paper proceeds as follows. Section 2 provides institutional details and outlines the conventional advice for retirement saving using tax-advantaged accounts. Section 3 develops our model for the consumption and

[5] We thank the referee for this suggestion.

[6] Lachance (2013) also examines allocations to traditional and Roth accounts with progressive taxes and uncertainty about future tax rates. However, the Lechance (2013) model does not include risky assets, such that traditional accounts do not provide hedging benefits in a progressive tax system. Further, our modeling approach offers a more flexible representation of tax-schedule uncertainty that is directly calibrated to match properties of historical tax-rate changes.

savings decisions of an investor with access to retirement accounts. Section 4 presents the optimal investment policies for a range of investors and examines the impact of tax-rate uncertainty on optimal behavior. Section 5 investigates economic environments that incorporate constraints on tax-advantaged savings and limitations on investors' abilities to implement optimal strategies. Section 6 concludes.

2. Retirement savings: background information

Section 2.1 provides an overview of the institutional details on pre-tax and post-tax retirement savings in the US. Section 2.2 outlines the conventional investment advice regarding the location of retirement assets.

2.1. Institutional details

The retirement savings landscape in the US has changed substantially in recent decades as employers have shifted from offering defined benefit pension plans to defined contribution plans (Poterba, 2014). Given this development, most workers use tax-advantaged accounts to invest for retirement. Popular savings options include traditional 401(k) plans and IRAs. These vehicles allow investors to save for retirement on a pre-tax basis and reduce their taxable income by the amount of any contributions. Savings are allowed to grow without incurring taxes on capital gains, dividend distributions, or interest payments. In retirement, withdrawals from the accounts are taxed as ordinary income.

The Taxpayer Relief Act of 1997 introduced an alternative retirement savings account, the Roth IRA. The primary difference between traditional and Roth investments centers around the timing of tax payments. Contributions to Roth IRAs are not tax-deductible, but principal and earnings withdrawn in retirement are tax-free.[7] Whereas post-tax retirement savings were originally allowed only in IRAs, the Economic Growth and Tax Relief Reconciliation Act of 2001 expanded the types of accounts that accept post-tax contributions. This act created Roth versions of 401(k) and 403(b) accounts, which became available to investors in 2006. Similarly, the Small Business Jobs Act of 2010 created a Roth version for governmental 457(b) plans.

Although a wide variety of account options exists, regulations limit contribution levels and access. For example, single (married) households can typically contribute up to \$5,500 (\$11,000) to a Roth IRA, but only if income is below regulatory limits (e.g., filers in 2015 can make a full contribution as long as modified adjusted gross income (MAGI) is below \$116,000 for single or \$183,000 for married taxpayers). Similar contribution limits also exist for traditional IRAs, but the tax deductibility of contributions is allowed only at sufficiently low incomes for households with access to a retirement plan at work. In 2015, for example, single (married) filers meeting this description could take a full deduction with MAGI of \$61,000 (\$98,000) or less and a partial deduction with MAGI below \$71,000 (\$118,000). Access to 401(k), 403(b), and 457(b) plans is restricted to employees in certain sectors, and these accounts are also subject to annual contribution limits, which in 2015 were \$18,000 per account.[8]

A given household's ability to contribute to tax-advantaged accounts is, therefore, a function of filing status, employment status of household members, employer characteristics and plan offerings, income level, and other factors. Importantly, the current retirement savings environment reflects substantial disparities in account access across investors. For instance, nearly 68 million workers in the US are without access to an employer-sponsored plan (New York Times, 2015). Tax-advantaged savings for many individual investors in this setting is limited to \$5,500 per year in an IRA. In contrast, certain public university employees could save \$50,000 or more through a combination of 401(a), 403(b), and 457(b) plans. Married-filing-jointly households with access to multiple workplace accounts also have considerably more favorable access to tax-advantaged investment options.

For workplace accounts, access to pre-tax versus post-tax options is largely determined by the employer. That is, a qualified plan sponsor must elect to sponsor a particular vehicle, such as a Roth 401(k), for employees to have access. Many employers offer a limited selection of account options. Only 56% of employer-sponsored plans at Vanguard, for example, make a post-tax option available to employees (Utkus and Young, 2015).[9]

2.2. Conventional wisdom

One can easily find advice from online brokerages, wealth managers, financial periodicals, and other sources regarding the relative merits of traditional versus Roth investments. Roth accounts are primarily deemed attractive options for investors whose current tax rates are lower than their expected tax rates in retirement, and traditional accounts are favored under the reverse scenario. Based on this criterion, Roth accounts are typically recommended to low-income savers and young investors who expect their real incomes to grow. The conventional advice is, therefore, largely based on expected differences in current and retirement income and often emphasizes one type of account to the exclusion of the other. However, this advice fails to account for an investor's exposures to two sources of uncertainty in future tax rates. First, in a progressive tax system, an individual's future marginal tax rate is unknown but is plausibly correlated with retirement account performance. In particular, the taxable retirement income from traditional account withdrawals is likely to produce a positive relation between investment outcomes and realized marginal tax rates. Second, the overall tax schedule,

[7] Traditional and Roth IRAs also differ in terms of eligibility income limits, eligibility age limits, penalties for early withdrawals, and minimum required distributions.

[8] Individuals over 50 are allowed catch-up contributions that exceed normal limits. In 2015, these provisions allowed eligible investors to save an additional \$6000 in each account.

[9] Although our analysis is motivated by US tax laws and retirement options, other countries provide investors with access to pre-tax and post-tax savings options. Canadians, for example, have access to the post-tax Tax-Free Savings Account (TFSA) as well as the pre-tax Registered Retirement Savings Plan (RRSP).

including the number of tax brackets and the corresponding rates, can change significantly over time.

Although our focus is on characterizing the importance of progressive taxes and uncertainty about future rates on optimal investment policy, other factors need to be considered in choosing between traditional and Roth investments. For example, Roth accounts allow households to effectively save more for retirement when contribution limits are binding (see, e.g., Burman, Gale and Weiner, 2001; Beshears, Choi, Laibson and Madrian, 2017). Our analysis of constrained investors in Section 5 incorporates this difference between traditional and Roth accounts. Other differences, such as the conversion option of traditional IRAs and the withdrawal flexibility of Roth investments, are not modeled in this study. We provide a detailed discussion of these factors in Section 3.4.

3. Model

In this section, we model the optimal consumption and savings decisions of an investor with access to both pre-tax (i.e., traditional) and post-tax (i.e., Roth) retirement accounts. We consider a two-period economy in which asset location and allocation decisions are made in the first period and all accumulated wealth is consumed in the second period. The key features of the economy are a progressive tax-rate structure and investor uncertainty regarding tax rates applied to income and traditional retirement account savings in the second period. Section 3.1 introduces the model, and Section 3.2 details the model's parameters. Section 3.3 outlines our approach to modeling the distributions of asset returns and future tax rates. Section 3.4 discusses aspects of the retirement savings environment that are omitted from our model and left to future research.

3.1. Investor's problem

We consider an investor who maximizes the sum of utility over consumption today, c_0, and the discounted expected utility from retirement consumption T years in the future, c_T. The investor's utility function takes the power utility form,

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma}, \tag{1}$$

in which γ is the coefficient of relative risk aversion. Expected utility over future consumption is discounted by a subjective discount factor, β^T. Finally, the investor is endowed with pre-tax income today, e_0, and guaranteed pre-tax income in retirement, e_T.

The investor chooses consumption today as well as the location and asset allocation for her savings. Regarding location, the investor has access to two tax-advantaged retirement accounts, and the amount of savings in the pre-tax (post-tax) retirement account is given by s_{Trad} (s_{Roth}). We initially consider an investor who is unconstrained with respect to investment limits in these accounts. In extensions, we impose retirement account contribution limits and allow the investor to save in a third account with less favorable tax features.

Within each account, the investor also chooses the allocation of her assets between a stock portfolio and a riskless bond. We denote the equity weights in the traditional and Roth accounts as ϕ_{Trad} and ϕ_{Roth}, respectively. To simplify the presentation of our results and highlight the most relevant tensions, we restrict the asset allocations to be equal in the traditional and Roth accounts (i.e., $\phi_{Trad} = \phi_{Roth}$). As discussed in Section 4.2, relaxing this constraint does not materially affect the results. The T year holding period returns on the riskless bond and the stock portfolio are denoted $r_{f,\,T}$ and $\tilde{r}_{m,T}$, respectively. We detail our approach to modeling the distribution of stock returns in Section 3.3.

The investor considers taxes in making her optimal consumption and savings decisions. The tax schedule in the model is designed to reflect key features of the US tax system. In particular, income taxes are progressive, with higher marginal rates for higher levels of taxable income. The tax schedule is divided into three tiers with low, middle, and high rates in a given period:

$$\tau_t \equiv \{\tau_{L,t}, \tau_{M,t}, \tau_{H,t}\}. \tag{2}$$

As we focus on real consumption, the income thresholds between tiers, Γ_1 and Γ_2, are constant. We model tax uncertainty by allowing the tax rates for each bracket to vary over time. Let $\mathscr{T}(I_t, \tau_t)$ be a function mapping taxable income and tax rates into income that is available for consumption and savings in a Roth account:

$$\mathscr{T}(I_t, \tau_t) = \begin{cases} (1-\tau_{L,t})I_t & \text{if } I_t < \Gamma_1 \\ (1-\tau_{M,t})(I_t - \Gamma_1) & \\ \quad + (1-\tau_{L,t})\Gamma_1 & \text{if } \Gamma_1 \le I_t < \Gamma_2 \\ (1-\tau_{H,t})(I_t - \Gamma_2) & \\ \quad + (1-\tau_{M,t})(\Gamma_2 - \Gamma_1) & \\ \quad + (1-\tau_{L,t})\Gamma_1 & \text{if } I_t \ge \Gamma_2. \end{cases} \tag{3}$$

At $t = 0$, current tax rates are known and future tax rates, $\tilde{\tau}_T \equiv \{\tilde{\tau}_{L,T}, \tilde{\tau}_{M,T}, \tilde{\tau}_{H,T}\}$, are random variables. As discussed in Section 3.3, we model the distribution of random future taxes based on the observed historical changes in US tax rates.

Year t taxable income, I_t, depends on the investor's location decision between traditional and Roth accounts. The distinction between these accounts in the model is the timing of tax payments. At $t = 0$, traditional retirement savings produce a one-for-one reduction in taxable income, whereas the investor's tax liability is unaffected by Roth savings. In retirement ($t = T$), accumulated savings in Roth accounts are consumed tax-free and traditional savings are taxed as ordinary income. Taxable income is, therefore, given by

$$I_t = \begin{cases} e_0 - s_{Trad} & \text{if } t = 0 \\ e_T + s_{Trad}\left[1 + r_{f,T} + \phi_{Trad}(\tilde{r}_{m,T} - r_{f,T})\right] & \text{if } t = T. \end{cases} \tag{4}$$

Considering the tax implications of the Roth and traditional accounts, the investor's optimization problem can be represented as

$$\max_{s_{Roth}, s_{Trad}, \phi_{Roth}, \phi_{Trad}} \quad u(c_0) + \beta^T E[u(c_T)], \tag{5}$$

$$\text{s.t.} \quad c_0 = \mathscr{T}(I_0, \tau_0) - s_{Roth}, \tag{6}$$

Table 1
Model parameters.
The table summarizes the parameters used in our numerical analyses.

Parameter	Value
β	0.99
γ	5
T	{10, 30}
e_0	[\$25,000, \$250,000]
e_T	{\$25,000, \$50,000, \$75,000}
$r_{f,T}$	2% (annualized)
$\tilde{r}_{m,T}$	Described in Section 3.3.1
Γ_1	\$50,000
Γ_2	\$100,000
τ_0	{15%, 25%, 33%}
$\tilde{\tau}_T$	Described in Section 3.3.2

$$c_T = \mathscr{T}(I_T, \tilde{\tau}_T) + s_{Roth}\left[1 + r_{f,T} + \phi_{Roth}(\tilde{r}_{m,T} - r_{f,T})\right], \tag{7}$$

$$s_{Roth} \geq 0, \tag{8}$$

$$s_{Trad} \geq 0, \tag{9}$$

$$0 \leq \phi_{Roth} \leq 1, \tag{10}$$

$$0 \leq \phi_{Trad} \leq 1, \tag{11}$$

$$\phi_{Roth} = \phi_{Trad}. \tag{12}$$

The asset location and allocation choices in combination with the progressive, dynamic tax schedule prevent analytic tractability. As a result, we present numerical solutions for the investor's optimal decisions.

3.2. Model parameters

Table 1 provides model parameters. We set the tax-rate thresholds, Γ_1 and Γ_2, at \$50,000 and \$100,000, consistent with our historical tax-rate analysis in Section 3.3. Initial tax rates are set at 15%, 25%, and 33% to reflect the current marginal tax rates for married-filing-jointly households at income levels within each of the three tax brackets. We allow the investor's pre-tax income at $t = 0$ to vary from \$25,000 to \$250,000, and we consider time T incomes of \$25,000, \$50,000, and \$75,000. From a qualitative perspective, the lowest level of retirement income is roughly intended to represent Social Security income, and higher levels correspond to those households with larger guaranteed income through defined benefit pensions, rental income, and non-retirement savings. We set the risk-free rate at 2% per year. The investor has a coefficient of relative risk aversion of five and a subjective discount factor of 0.99.[10] In our analysis, we focus on investment horizons of $T = 10$ and $T = 30$ years.

[10] We also consider a coefficient of relative risk aversion of two. Optimal allocations to traditional and Roth accounts are similar to our base case.

3.3. Simulation methods

Our numerical procedure uses one million random draws of stock market holding period returns and future tax rates to solve for the investor's optimal policies. We search for the combination of savings choices and asset allocations that maximizes average utility over the simulations. We generate draws of returns and tax rates using bootstrap methods. The simulation procedures are designed to capture salient aspects of the distributions of returns and taxes while minimizing distributional assumptions by using the historical record to produce draws of these variables.

3.3.1. Stock market returns

We produce a distribution of horizon T stock market holding period returns by compounding bootstrapped monthly returns. Specifically, we use monthly returns on the Center for Research in Security Prices (CRSP) value-weighted index from July 1926 to June 2015 in excess of the risk-free rate.[11] Our investors are assumed to have access to a riskless bond that pays a real interest rate of 2% per year, so we add the monthly excess stock market returns to the real risk-free rate of $\frac{1}{12} \times 2\%$ to produce 1068 monthly stock market returns. To generate a horizon T holding period return, we draw $T \times 12$ monthly stock market returns with replacement from this empirical distribution. Finally, we calculate a holding period return by compounding these monthly return draws. We repeat this process for each of the one million iterations.[12]

Fig. 1 shows the distributions of cumulative stock market returns for horizons of ten years and 30 years. As a reference point, the cumulative return on the riskless bond is shown with a vertical dotted line. The top row of each panel in Table 2 reports the mean, standard deviation, and percentiles of the cumulative return distribution for the ten-year horizon (Panel A) and the 30-year horizon (Panel B).

3.3.2. Future tax rates

We generate a joint distribution of future tax rates for three tax brackets. Using US data from 1913 to 2015, we find married-filing-jointly marginal tax rates associated with real taxable income levels of \$50,000, \$100,000, and \$250,000 in 2013 dollars.[13] Fig. 2 shows the historical tax

[11] Excess returns on the CRSP value-weighted index are from Kenneth French's website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. We thank Kenneth French for making these data available.

[12] For robustness, we also implement a block bootstrapping approach that draws sequences of consecutive returns to account for persistence in expected returns and volatility. Block lengths are drawn from a geometric distribution with an average block length of five years. Long-horizon returns are somewhat less volatile using this approach, which could reflect mean reversion in returns, but our results and inferences about the allocation of funds to traditional and Roth accounts are not affected by this change.

[13] Historical inflation-adjusted tax brackets from 1913 to 2013 are available from the Tax Foundation at http://taxfoundation.org/article/us-federal-individual-income-tax-rates-history-1913-2013-nominal-and-inflation-adjusted-brackets. We update the data for the 2014 and 2015 tax years using the Internal Revenue Service tax brackets and US Consumer Price Index.

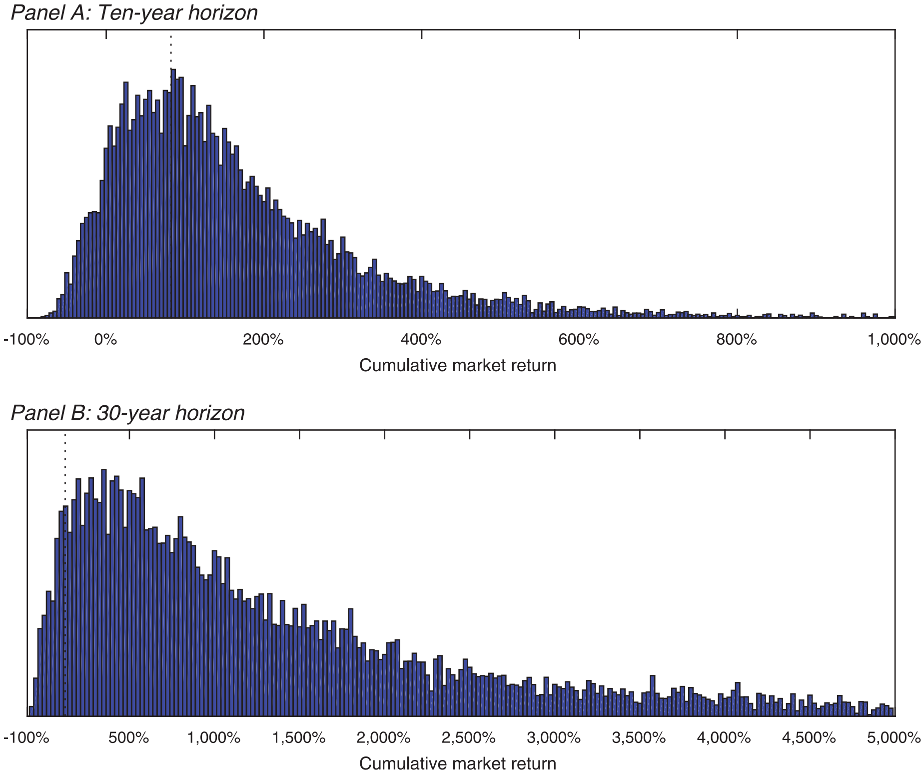

Fig. 1. Cumulative stock market return distributions for ten-year and 30-year horizons. This figure shows bootstrapped distributions of holding-period returns over ten years (Panel A) and 30 years (Panel B). Holding-period returns are formed by compounding randomly drawn (with replacement) monthly returns from July 1926 to June 2015. These monthly returns are the sum of realized stock market excess returns and a 2% annualized risk-free rate. The concurrent cumulative returns on a risk-free asset are given by the dashed vertical lines. The distributions represent the results from one million iterations at each horizon.

Table 2

Distributional statistics: holding-period returns and future tax rates.

The table reports distributional statistics for holding-period returns and future tax rates at a ten-year horizon in Panel A and at a 30-year horizon in Panel B. The first row in each panel summarizes the distribution of holding-period returns, first providing the mean and standard deviation, and then values at the 1st, 5th, 25th, 50th, 75th, 95th, and 99th percentiles. The subsequent three rows provide similar statistics for future tax rates, beginning with the lowest tax bracket and progressing to the highest tax bracket.

			Percentile of distribution						
Variable	Average	Standard deviation	1	5	25	50	75	95	99
Panel A: Ten-year horizon									
$\tilde{r}_{m,T}$	164%	165%	−44%	−15%	52%	124%	230%	481%	747%
$\tilde{\tau}_{L,T}$	14.9%	5.9%	0.0%	5.2%	11.4%	15.0%	18.0%	25.2%	31.0%
$\tilde{\tau}_{M,T}$	24.8%	8.7%	3.2%	10.3%	19.4%	24.9%	30.2%	39.2%	45.9%
$\tilde{\tau}_{H,T}$	33.5%	12.1%	4.5%	13.2%	26.3%	33.5%	40.5%	54.0%	64.5%
Panel B: 30-year horizon									
$\tilde{r}_{m,T}$	1,789%	2,465%	−5%	105%	478%	1,041%	2,125%	5,952%	11,610%
$\tilde{\tau}_{L,T}$	14.6%	9.7%	0.0%	0.0%	7.4%	14.0%	20.9%	31.9%	40.0%
$\tilde{\tau}_{M,T}$	24.8%	14.0%	0.0%	2.3%	14.5%	24.2%	34.2%	49.0%	60.0%
$\tilde{\tau}_{H,T}$	35.4%	19.2%	0.0%	5.1%	21.5%	34.4%	48.1%	69.1%	85.3%

rates for these income levels as well as the tax rate for the top income bracket. The US has maintained a progressive tax-rate schedule since the introduction of the permanent income tax in 1913. Another interesting feature of the historical record is that high-income tax rates experience larger absolute year-over-year changes compared with low-income rates. This pattern results in larger variance for tax rates at higher income levels over the 1913–2015 period, as the marginal tax rate for a $50,000 income level ranged from 1% to 26% and the rate for $250,000 in income ranged from 1% to 62%. The top marginal tax rate shows even more variation, ranging from 7% to 94%. Our boot-

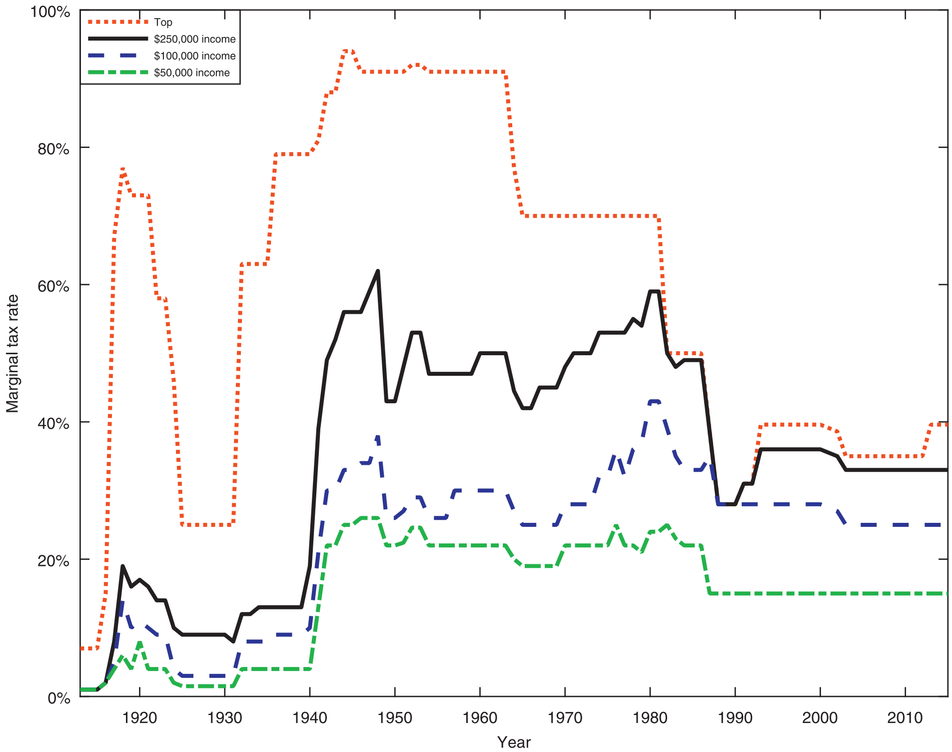

Fig. 2. Historical tax rates. This figure shows historical marginal tax rates at income levels of $50,000, $100,000, and $250,000 (adjusted for inflation), as well as the highest marginal tax rate for each year. Tax rates correspond to the married-filing-jointly filing status.

strap procedure is designed to capture these features of the tax rates for the $50,000, $100,000, and $250,000 income levels. Historical tax rates for single filers at these income levels are, on average, higher than the corresponding rates for married taxpayers, but the marginal rates otherwise have similar time series features across the two filing statuses. In particular, the historical standard deviations of rate changes and degrees of marginal rate progressivity are nearly equivalent across the single and married tax schedules, such that the main conclusions of our analysis using married taxpayer rates translate to single filers.

To draw year T tax rates, year 0 rates are first set to the 2015 levels of 15%, 25%, and 33%. We then draw tax rates for year t ($t = 1, \ldots, T$) by adding a random draw of annual rate changes to the rates in year $t - 1$. Annual tax-rate changes at each income level are calculated from the historical record. Given the overall increase in rates over the 1913–2015 period, we subtract the average rate change from the actual rate changes to limit expected drift in future tax rates. This procedure results in 102 sets of annual tax-rate changes for the three brackets. The draw of rate changes for each year in the holding period is from the joint distribution of changes in the rates for the three tax brackets to preserve cross-correlations in tax rates. In some draws, tax rates for lower taxable income levels in a period can be larger than rates at higher income levels. In these instances, we preserve a (weakly) progressive tax system by replacing both rates with their average. Further, tax-rate bounds of 0% and 100% are enforced. Using the bootstrap procedure, we simulate paths for the three tax rates between years 0 and T in this manner. In the end, we produce one million draws from the joint distribution of year T tax rates for the three income levels.[14]

Fig. 3 shows the distributions of the future tax rates at ten- and 30-year horizons, and Table 2 reports summary statistics for these rates. The means and medians of future tax rates are close to their starting levels such that the expected drift in tax rates is small.[15] The distributions reflect substantial uncertainty about future tax rates. The standard deviations of tax rates for the $50,000, $100,000, and $250,000 income levels are 6%, 9%, and 12%, respectively, at a ten-year horizon and 10%, 14%, and 19% at a 30-year horizon. Notably, the highest marginal rate is more uncertain than the lower tax rates at both horizons. At a 30-year horizon, for example, the 90% interval for the future tax rate for the lowest income level is 0–32%, whereas the highest tax rate has a 90% interval of 5–69%. This feature

[14] We also consider a block bootstrapping approach that draws random block lengths with a geometric distribution and a five-year average block length. Future tax rates are more uncertain in this case, reflecting serial correlation in tax rate changes in the historical data. Investors using the implied distribution of future tax rates are more favorably inclined toward Roth accounts relative to our base case, and the economic significance of considering tax-rate uncertainty increases using this approach.

[15] The low and middle tax rates tend to be slightly lower on average than their starting rates, and the average high tax rate is slightly above the 33% starting rate. The small drifts result from interactions between the asymmetric jumps in past tax rates (i.e., increases in the high tax rate have tended to be somewhat larger in magnitude than rate decreases) with the imposition of a progressive tax-rate structure and tax-rate bounds of 0% and 100%.

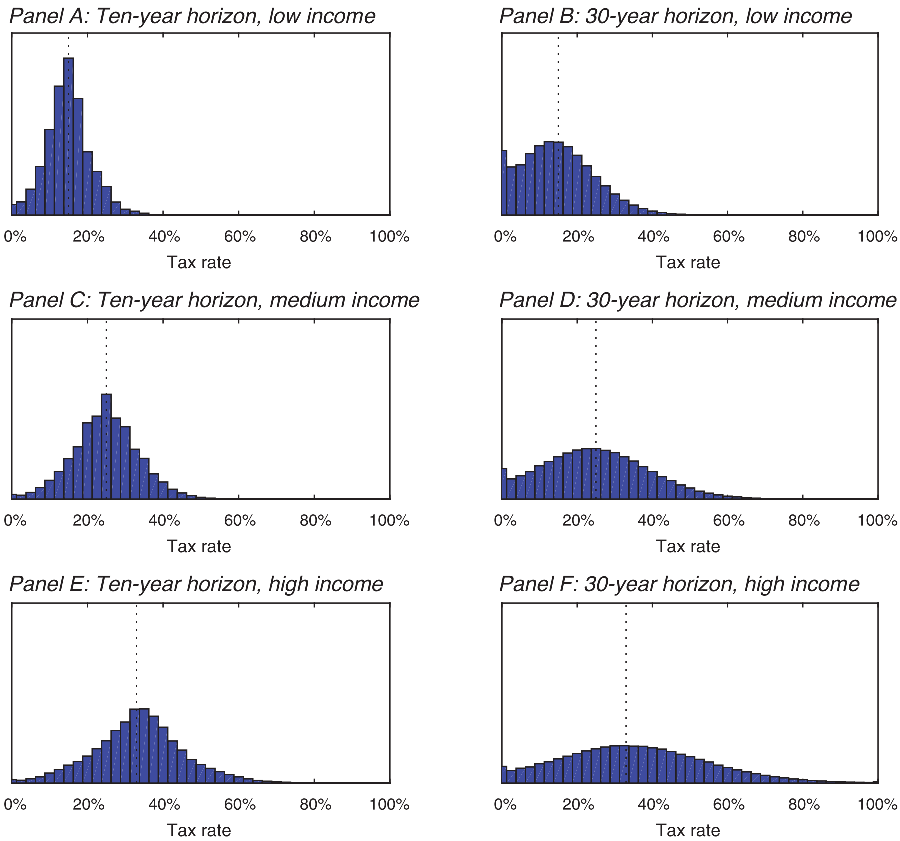

Fig. 3. Tax-rate distributions of ten-year and 30-year horizons. This figure shows bootstrapped distributions of the three brackets of future tax rates at ten-year (Panels A, C, and E) and 30-year (Panels B, D, and F) horizons. Future tax rates are formed by randomly drawing demeaned tax-rate changes from the historical tax rates shown in Fig. 2. We enforce tax-rate bounds of 0% and 100% and preserve a (weakly) progressive tax system. Current tax rates are given by the dashed vertical lines. The distributions represent the results from one million iterations at each horizon.

is important for our analysis, as higher-income households face substantially greater uncertainty about their future tax rate compared with lower-income investors.

Our tax modeling approach reflects our focus on the roles of progressive taxes and tax uncertainty in retirement saving. Our framework can incorporate additional features of future tax rates, such as drift in expected future tax rates and correlation between future tax rates and stock return realizations. In line with conventional wisdom, investors who believe that rates in the tax schedule are likely to drift upward (downward) prior to retirement should increase their allocations to Roth (traditional) accounts relative to our analysis. In addition, taxes and asset prices could be interrelated (see, e.g., Sialm, 2006; 2009), whereas we assume independent distributions for stock returns and tax rates. A positive (negative) correlation between tax-rate changes and realized stock market returns produces an additional relative advantage for traditional (Roth) accounts.

3.4. Unmodeled features of the retirement savings decision

Our analysis focuses on the roles of progressive taxes and tax uncertainty on optimal retirement savings strategies. As such, we examine a simplified, two-period investment problem that allows us to highlight the economic importance of these effects. An alternative approach would be to study consumption and savings decisions in a dynamic, multi-period setting that includes, for example, life-cycle effects in expected pre-tax income, periodic investment decisions during a work phase, and optimal account withdrawals during a retirement phase. We opt for our parsimonious specification, as including these additional features would sacrifice tractability and distract from the focus of the paper. Our model also omits some aspects of Roth and traditional accounts that could be of importance to investors.

First, differences exist in the rules governing withdrawals from Roth and traditional IRAs. Although withdrawals can be made from both accounts without penalty at age $59\frac{1}{2}$, Roth IRAs have no mandatory withdrawals during retirement. Assets in Roth accounts, therefore, provide flexibility in managing retirement income and associated taxes, whereas mandatory withdrawals from traditional accounts create taxable income in retirement.[16] Early withdrawals from the two accounts are also treated differently. Both accounts allow withdrawals for the purchase of a first

[16] See, for example, Horan (2006) and DiLellio and Ostrov (2017) for discussions of optimal retirement-period withdrawal strategies.

home ($10,000 lifetime maximum), qualified educational expenses, disability, death, large unreimbursed medical expenses, qualified military reservist deployment, payment of Internal Revenue Service levies, a series of substantially equal periodic payments (SEPP program), and health insurance premiums if the account owner is unemployed. Other early withdrawals from traditional IRAs, however, are taxed as ordinary income and incur a 10% penalty. In contrast, Roth IRAs allow penalty-free withdrawals up to the total contributions that have been made, such that Roth IRAs have more attractive liquidity features relative to traditional IRAs.

Second, traditional and Roth accounts differ with respect to inheritance and estate planning. An investor who shields income from taxes with traditional accounts is likely to achieve a higher estate value relative to a similar investor using Roth accounts, which could result in substantial estate taxes. Roth account withdrawals are tax-free for descendants, whereas traditional withdrawals are taxed as income. Optimal strategies for estate planning depend on the likely estate value and relative tax rates of the household and its beneficiaries.

Finally, whereas Roth accounts are relatively more attractive for gaining flexibility over both mandatory withdrawals in retirement and early withdrawals, traditional accounts include a potentially valuable conversion option. In 2010, as part of the Tax Increase Prevention and Reconciliation Act of 2005, investors gained the ability to convert traditional accounts to Roth accounts regardless of income. Conversions allow investors to pay ordinary income taxes on the amount of the conversion and eliminate subsequent taxes on the account. The conversion option can be used strategically, for example, when investors have temporarily low income due to underemployment or early retirement.[17]

4. Optimal investment policies

In this section, we investigate the optimal savings behavior of households with access to traditional and Roth retirement accounts. Our primary focus is on the effects of progressive taxes and future tax-rate uncertainty on optimal investment policies. As previously noted, investors in our model would be indifferent between pre-tax and post-tax retirement savings options in an economy with constant, flat tax rates. Section 4.1 isolates the effect of a progressive tax schedule by analyzing an economy with progressive, but known, future tax rates. Section 4.2 introduces uncertainty in the tax schedule and examines the impact on optimal behavior. Section 4.3 analyzes the economic value of considering this uncertainty in developing investment policies, and Section 4.4 investigates the value of access to a Roth account.

4.1. Optimal policies with progressive taxes

We begin our analysis by considering an economy characterized by a progressive tax schedule but no uncertainty in future tax rates. The tax rate for each bracket is known and equal to the current rate for the bracket. For a given investment policy, the distribution of year T consumption is, therefore, governed solely by the assumed distribution of asset returns. Within this economy, we consider investors with horizons of ten and 30 years, current incomes ranging from $25,000 to $250,000, and taxable outside incomes in retirement of $25,000, $50,000, and $75,000. The tax-bracket cutoffs are $50,000 and $100,000, such that investors with higher levels of outside retirement income tend to pay relatively high marginal tax rates on any withdrawals from pre-tax accounts.

Fig. 4 presents optimal consumption and savings decisions as a function of horizon, current income, and future taxable income. For each investor, the figure shows current consumption, the savings amounts in traditional and Roth accounts, and the dollar amount of taxes paid on current income. In general, the asset location decisions displayed in the graphs are a reflection of several underlying economic drivers. In line with conventional wisdom, investors with low current income prefer to locate retirement savings in Roth accounts, as this option allows them to lock in the lowest possible tax rate on these investments. Investors with current-period income in the highest tax bracket, in contrast, favor traditional retirement accounts.

Investors with intermediate levels of pre-tax income face a more nuanced policy decision. Roth accounts allow these investors to eliminate uncertainty about the tax rate paid on savings. Traditional accounts are desirable under a progressive tax structure because realized marginal tax rates tend to provide a hedge for investment performance. That is, marginal tax rates on traditional savings are low when realized returns are poor. Traditional retirement vehicles are also attractive for investors with current-period, pre-tax income just above tax-bracket cutoffs, as these accounts allow investors to manage current-period taxes by reducing taxable income.

Panel A of Fig. 4 shows the optimal policies of investors with ten-year horizons and $25,000 in future taxable income. Both consumption and total savings for retirement are increasing in current income. The optimal investment levels in traditional and Roth accounts, however, vary substantially across investors. Any investor with current income up to $50,000 pays the lowest marginal tax rate in

[17] Dammon (2009), Dammon, Spatt and Zhang (2012), and Brown and Leach (2017) consider optimal strategies for Roth IRA conversions and recharacterizations. Dammon, Spatt and Zhang (2012) emphasize that the recharacterization option's value is enhanced by tax uncertainty. We do not anticipate that incorporating a conversion option into our model, along with life-cycle effects and income variability, would substantially alter our main results. As our subsequent analyses show, most investors optimally use a mix of Roth and traditional accounts, and investors retain the conversion option for their traditional account balance. Having a portion of their assets in traditional accounts allows investors to capture the most valuable portions of the conversion option. For example, within a given year, larger conversions are subject to higher marginal tax rates, so the first dollars converted are the most valuable. Across years, low-income years allow more dollars to be converted at low tax rates and are therefore the most valuable conversion opportunities. Because larger conversions become more expensive due to progressive tax rates and strategic opportunities to convert at low incomes are likely limited, many investors could be unable to convert all of their assets from traditional to Roth accounts. As a result, the marginal value of the conversion option is decreasing in investors' traditional assets, such that having a mix of traditional and Roth accounts provides investors with the most valuable portion of the conversion option.

Fig. 4. Optimal consumption and savings with no uncertainty about future tax rates. This figure shows investors' optimal policy choices when future tax rates are known. Each panel plots current consumption, savings into Roth and traditional retirement accounts, and the resulting current tax liability as functions of current income. Panels A, C, and E (Panels B, D, and F) show policies corresponding to a ten-year (30-year) retirement horizon. The graphs present results for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income.

the current period regardless of the retirement account type for savings. If these low-income investors choose to exclusively invest through a Roth account, their future taxable income will be $25,000, such that they will also pay the lowest marginal tax rate in retirement. On the other hand, these investors could choose to invest using a traditional account. In this case, the investors would still pay the lowest marginal tax rate in the current period. If investments in the traditional account do well over the ten-year period before retirement, however, the combined taxable retirement income from the traditional account and outside sources can exceed $50,000, leading to a higher marginal tax rate in retirement. The low-income investors thus optimally choose Roth accounts for their retirement savings.

Investors with incomes just above the cutoff for the lowest tax bracket begin to favor traditional accounts. Income invested through a traditional account is not taxable in the current period, such that an investor can manage current-period taxes by allocating capital to these accounts. Over a range of income from $50,001 to $66,500, investors increase their allocations to traditional accounts dollar-for-dollar with increases in income to keep current taxable income at $50,000. This increase in traditional account savings reflects a substitution effect of traditional for Roth accounts. Over this range of income, an additional dollar invested in a Roth account would push the investor's taxable income in the current period into the middle tax bracket. Investing the same dollar in a traditional account produces some uncertainty about the investor's realized marginal tax rate in retirement, but the progressive tax structure generates a natural hedge against investment performance. Panel A of Fig. 5 illustrates the dependence of the retirement tax rate on the realized cumulative return on the stock market for the investor with $66,500 in current income. Given her optimal decisions, this investor will pay the lowest marginal tax rate in retirement in the 25% of outcomes with the worst stock market performance compared with an 11% chance of paying the highest marginal rate with a good market realization. This investor strongly prefers to invest an incremental dollar in the traditional account, because this account offers both a lower expected tax rate and a hedge against investment performance.

Panel A of Fig. 4 shows that investors exclusively rely on pre-tax savings for incomes between $66,500 and about $106,000. Investors with incomes from $106,000 to about $132,500 begin to invest marginal savings dollars in the Roth account, and they lock in the middle tax bracket on these investments. To better understand this optimal decision, Panel B of Fig. 5 shows marginal tax rates in retirement as a function of stock returns for an investor

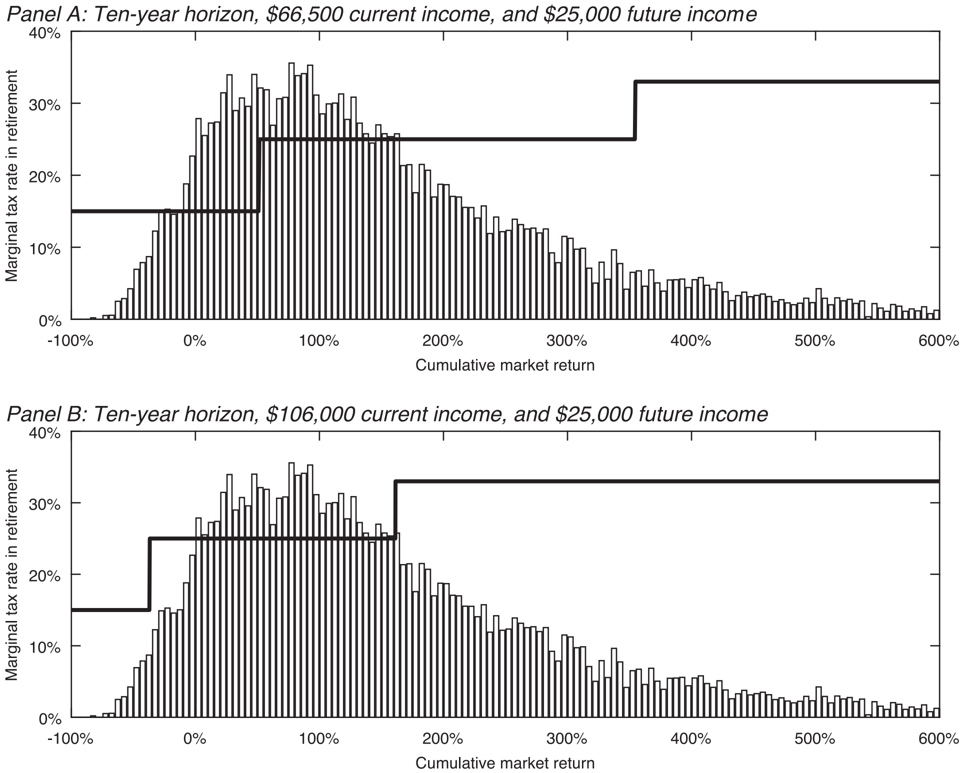

Fig. 5. Investment performance and marginal tax rates in retirement. This figure shows the marginal tax rate in retirement as a function of the realized cumulative stock market return for investors with a ten-year horizon, $66,500 (Panel A) or $106,000 (Panel B) in current income, and $25,000 in guaranteed future income in retirement. The bootstrapped distribution of ten-year holding period stock returns is also shown in each panel.

with $106,000 of current income. This investor has relatively large traditional retirement savings, which increases the probability of realizing a high marginal tax rate in retirement. The traditional account continues to provide a hedge against poor investment performance, but less than a 2% chance exists that the investor's taxable income will fall into the lowest tax bracket. The probability of a high future tax rate accompanying high consumption is sufficiently larger than the probability of a low tax rate in a low-consumption state such that the investor chooses to invest some wealth in a post-tax account to lock in the middle tax rate.

As income levels increase beyond $132,500, the investors increase allocations to the traditional account to maintain a taxable income of $100,000 and avoid paying a higher marginal tax rate. At an income level of $153,500, the investor consumes $80,000, pays taxes of $20,000, and invests $53,500 in a traditional account with no investment in a Roth account. Above this taxable income level, any allocation to a Roth account would be taxed at the highest marginal tax rate. Investments in a traditional account would be taxed at a lower future rate in states with poor performance in the retirement account. The traditional account is thus preferred for all investors with high income levels when future tax brackets are known with certainty.

Investors with a ten-year horizon and future income of $50,000 or $75,000 face similar tensions. These investors optimally choose higher consumption and lower savings levels compared with the investors with $25,000 of future income. Higher future income also reduces or eliminates the possibility of falling into a low tax bracket in retirement regardless of investment account performance, such that the post-tax option is more attractive for investors with relatively low current income. Investors with sufficiently high current income must pay the top tax rate in the current period, however, such that the traditional account is preferable for higher-income investors who could end up in a lower tax bracket if the stock market performs poorly.

We also consider longer-term investors with a 30-year investment horizon. For given current and future income levels, the 30-year investors consume more and save less than the corresponding investors with ten-year horizons because the expected growth of retirement savings is greater. The longer-term investors make optimal allocations that are similar in spirit to those of the shorter-term investors. The reduced need for retirement savings and increased consumption, however, tend to increase an investor's current taxable income such that the Roth retirement option is relatively less desirable.

Overall, investors who optimize under a static and progressive tax schedule face a trade-off of paying a known current tax rate on Roth retirement savings or a future tax rate that varies with the investment performance of their traditional account. The fact that retirement account

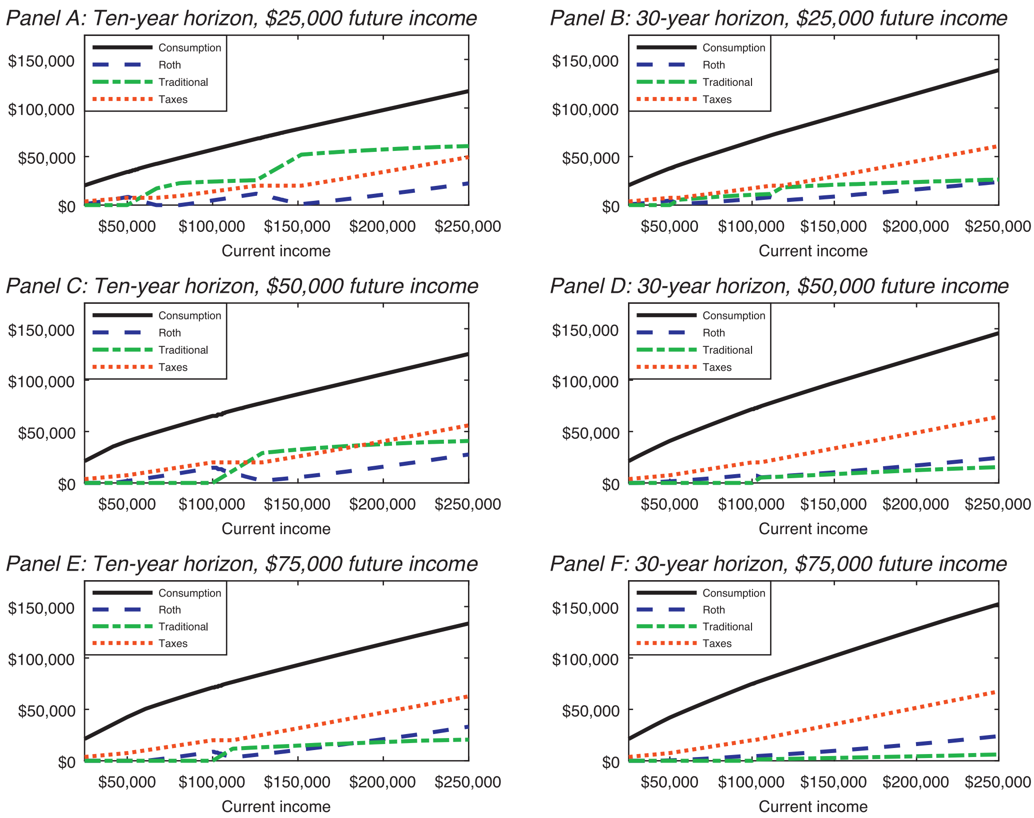

Fig. 6. Optimal consumption and savings with uncertainty about future tax rates. This figure shows investors' optimal policy choices when future tax rates are unknown. Each panel plots current consumption, savings into Roth and traditional retirement accounts, and the resulting current tax liability as functions of current income. Panels A, C, and E (Panels B, D, and F) show policies corresponding to a ten-year (30-year) retirement horizon. The graphs present results for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income.

investment performance determines future consumption as well as the investor's retirement tax bracket introduces an advantage for pre-tax retirement savings. That is, if the investor experiences poor investment performance and low retirement consumption, then the investor tends to pay a lower income tax rate on pre-tax retirement savings. The Roth retirement vehicle, on the other hand, is primarily useful as a mechanism to lock in a low current tax rate when income is expected to increase.

4.2. Optimal policies with progressive taxes and uncertain future rates

We continue our analysis by introducing an economy characterized by uncertainty in the schedule of future tax rates. We again consider investors with horizons of ten and 30 years, current incomes ranging from $25,000 to $250,000, and taxable outside incomes in retirement of $25,000, $50,000, and $75,000. Tax-bracket cutoffs remain at $50,000 and $100,000. Year *T* tax rates within brackets, however, are allowed to vary following the process outlined in Section 3.3, resulting in the tax-rate distributions displayed in Fig. 3.

Fig. 6 shows the optimal consumption and savings decisions for investors with each combination of horizon, current income, and future taxable income. The main tensions that investors face without uncertainty in future rates are still in effect in the presence of tax uncertainty. Across retirement horizons and income levels, investors contribute to traditional accounts to manage their taxable incomes around the tax-bracket cutoffs. Investors also use traditional accounts to take advantage of potentially being in lower tax brackets in the future, which continues to be particularly valuable due to the relation between retirement account performance and the realized future tax bracket. Introducing tax-schedule uncertainty, however, dampens the relative importance of these factors, and investors increase their contributions to Roth accounts.

To highlight the role of uncertainty about future rates, Fig. 7 presents the differences in optimal consumption, savings, and taxes between the investors in Fig. 6 and the corresponding investors in Fig. 4. The differences are expressed as percentages of current pre-tax income. From Fig. 4, in the absence of tax-schedule uncertainty, Roth investments are primarily attractive for investors with low current income relative to expected future income as a means to lock in low tax rates. Introducing additional tax uncertainty increases the variability of investors' future consumption, and Roth accounts eliminate a portion of this risk for investors regardless of current income.

The most pronounced effects across the six panels in Fig. 7 are seen at higher current incomes. High-income

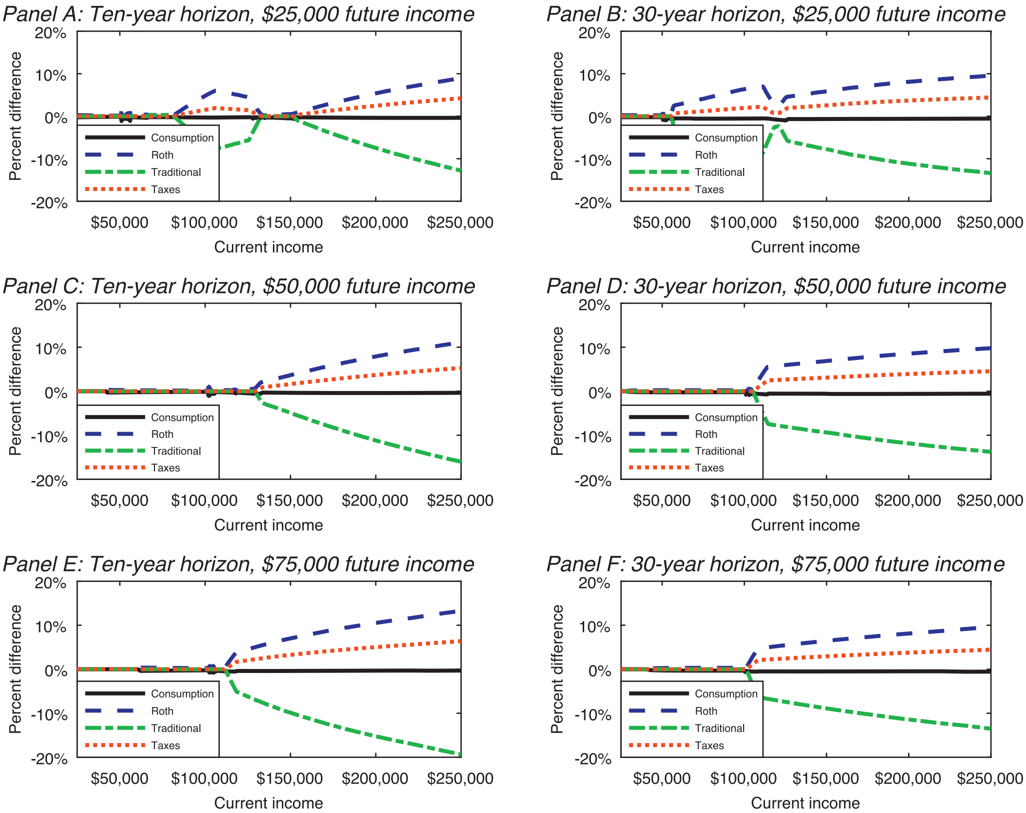

Fig. 7. Differences in policies for investors with and without tax uncertainty. This figure shows the differences in investors' optimal policy choices between the cases in which future tax rates are unknown and known. The reported quantities are the differences between the policies in Fig. 6 and Fig. 4 as percentages of current income. Each panel plots the percentage differences for current consumption, savings into Roth and traditional retirement accounts, and the resulting current tax liability as functions of current income. Panels A, C, and E (Panels B, D, and F) show policy differences corresponding to a ten-year (30-year) retirement horizon. The graphs present results for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income.

investors increase their allocations to Roth accounts when faced with uncertainty about future tax rates. At these income levels, reducing consumption risk in retirement by locking in tax rates today is more valuable than realizing a potentially lower tax bracket in the future. Fig. 7 also shows that the difference in percentage allocation to the Roth account is generally increasing in guaranteed future income. This result is straightforward, as investors with high levels of future earnings are highly exposed to tax uncertainty. When guaranteed future income exceeds $100,000 (not shown in Figs. 4–7), investors pay the highest marginal tax rate in the future with certainty and, as a result, use Roth accounts for 100% of their savings. Finally, the investors with $25,000 in future income (i.e., Panels A and B) use Roth accounts more intensively at moderate current income levels further away from tax-bracket cutoffs, where incentives to manage taxable income are weaker.

To provide additional insight into the economic outcomes of the investors, Table 3 considers the resulting retirement consumption of an investor who forms her optimal policies while either ignoring or considering tax-schedule uncertainty. For this example, we study an investor with 30 years to retirement, $130,000 of current income, and $50,000 of taxable income in retirement. The second column of Table 3 shows that, unconditionally, accounting for uncertainty in the future tax schedule results in higher retirement consumption. In the median cases, the investor ignoring tax uncertainty (Panel B) consumes $174,100 in retirement, and the investor considering tax uncertainty (Panel C) consumes about $190,300. The remaining columns show retirement consumption conditional on realized tax rates. Although the investor ignoring tax-schedule uncertainty outperforms if future tax rates are very low, the investor who considers uncertainty outperforms in almost all other scenarios. If tax rates rise to extreme levels and overall consumption is low (i.e., marginal utility is high), planning for tax uncertainty results in median consumption of $149,000 compared with $69,400 for the investor who ignores uncertainty. Considering tax uncertainty allows the investor to effectively hedge against tax rate increases, providing higher consumption in relatively bad states of the world.

For completeness, Fig. 8 presents the optimal percentage allocations to equity for the economies with and without uncertainty over future tax rates. Throughout our analysis, we have restricted the investors' asset allocation decisions to be the same in their traditional and Roth accounts (i.e., $\phi_{Roth} = \phi_{Trad}$). For the combinations of current income, future income, and investment horizon considered in Fig. 8, investors tend to favor equity investments over riskless bonds, although we do see some positive

Table 3
Distributional statistics: future tax rates and retirement consumption.
The table reports average future tax rates and retirement consumption statistics in thousands of dollars for an investor with current income of $130,000 and retirement income of $50,000 in 30 years. Column 1 reports unconditional averages. Columns 2–6 report averages conditional on realizations of the highest tax rate (i.e., $\tilde{\tau}_{H,T}$). Panel A reports the frequency of realizations and average tax rates for each of the tax brackets. Panel B presents retirement consumption values resulting from ignoring tax uncertainty in forming consumption and investment decisions. The three rows of values show the 10th, 50th, and 90th percentiles of retirement consumption realizations. Panel C reports results from considering tax uncertainty in those decisions, also showing the 10th, 50th, and 90th percentiles of retirement consumption realizations.

		Realization of $\tilde{\tau}_{H,T}$				
	Unconditional (1)	0–20% (2)	20–40% (3)	40–60% (4)	60–80% (5)	80–100% (6)
Panel A: Future marginal tax rates						
Frequency	100.0%	22.6%	38.4%	28.3%	9.0%	1.7%
Average $\tilde{\tau}_{L,T}$	14.6%	4.8%	12.7%	19.7%	27.3%	36.5%
Average $\tilde{\tau}_{M,T}$	24.8%	7.9%	21.9%	33.7%	45.1%	57.7%
Average $\tilde{\tau}_{H,T}$	35.4%	10.8%	30.3%	48.6%	67.5%	88.1%
Panel B: Retirement consumption of investor who ignores tax uncertainty						
10th percentile	77.6	97.2	85.2	74.5	63.2	44.8
50th percentile	174.1	230.5	190.0	151.9	113.4	69.4
90th percentile	538.0	714.4	568.2	431.5	292.2	145.4
Panel C: Retirement consumption of investor who considers tax uncertainty						
10th percentile	82.3	91.5	84.4	78.3	71.9	64.5
50th percentile	190.3	211.8	195.3	179.8	164.8	149.0
90th percentile	571.5	643.3	584.7	530.1	475.9	417.3

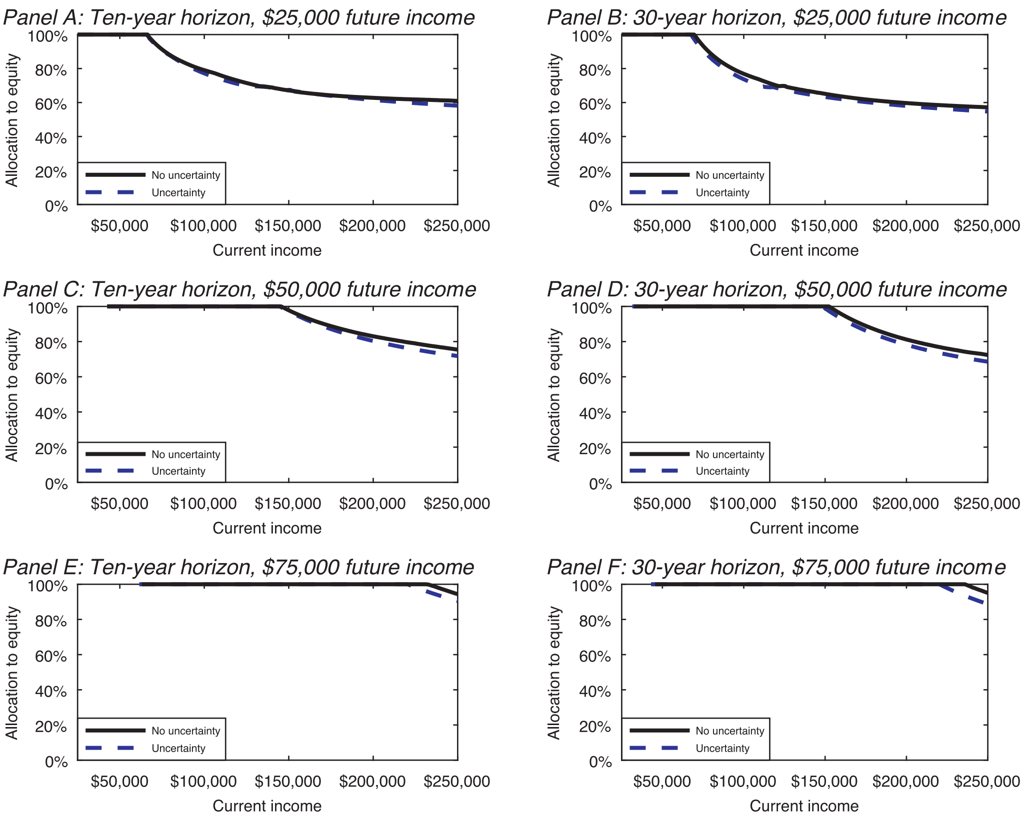

Fig. 8. Allocation to equity without and with tax uncertainty. This figure shows investors' optimal equity allocations when future tax rates are either known or unknown. Each panel plots the percentage allocated to equity under each scenario as a function of current income. Panels A, C, and E (Panels B, D, and F) show policies corresponding to a ten-year (30-year) retirement horizon. The graphs present results for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income.

allocations to bonds at higher ratios of current income to future income. In addition, tax-schedule uncertainty has a limited impact on optimal allocation decisions. The introduction of tax uncertainty causes investors to shift their allocations slightly more toward riskless bonds, but the observed differences from the no-uncertainty case are not economically large.[18]

The optimal equity allocations shown in Fig. 8 are relatively high compared with common financial advice. To investigate the sensitivity of our findings to alternative asset allocation decisions, we also study investors who follow a simple age-based allocation rule. We assume that our ten-year and 30-year investors are currently 55 and 35 years old, respectively, and we set their respective equity allocations to 40% and 50% to reflect common financial advice that investors decrease their allocations to stocks as they near retirement. Results in the Online Appendix show that optimal decisions with respect to consumption and the location of assets in traditional and Roth accounts are insensitive to this change in portfolio makeup. Investors optimally save slightly more for retirement in this scenario, but the relative usages of traditional and Roth accounts are very similar to our base specification.

Overall, optimal retirement planning in the presence of progressive taxes and tax-schedule uncertainty involves balancing allocations between traditional and Roth accounts. Post-tax accounts allow investors to eliminate future tax-rate risk on a portion of their savings, giving their portfolios tax-rate diversification. We find that high-income investors make considerable use of Roth accounts to reduce tax-rate risk. These results stand in direct contrast to popular investment advice that instructs wealthy investors to avoid Roth accounts due to their high current marginal tax rates. In the following section, we consider the welfare implications of failing to account for uncertainty in the future tax schedule while forming investment policies.

4.3. Economic value of considering tax uncertainty

We measure the importance of considering tax-schedule uncertainty by finding the annual percentage fees that investors would be willing to pay on their savings in exchange for being allowed to consider this source of uncertainty. To calculate these fees, we begin with the investors in Section 4.1 who form their optimal policies under known, progressive tax rates. We then expose these investors to uncertainty about the future tax schedule and measure their expected utility. Finally, the investors are allowed to reoptimize considering all sources of uncertainty, but we place an annual fee on their savings. We find the fee that makes the investors indifferent between reoptimizing and keeping their original strategy. Investors would thus prefer to consider tax-schedule uncertainty and form new consumption and savings policies at any annual fee less than the fee we report.

Fig. 9 shows the annual fees associated with considering uncertainty in future tax rates. Each panel displays fees for investors with current incomes of \$25,000 to \$250,000 and future outside incomes of \$25,000, \$50,000, and \$75,000. The fee is plotted for investors who save at least 2% of their current income in retirement accounts.

Analysis of Fig. 9 produces several interesting findings. First, tax-schedule uncertainty is of great economic importance. For the wealthiest investors who we consider, the annual fee to account for tax uncertainty reaches 0.68% for investors with ten-year horizons and 2.10% for investors with 30-year horizons. Second, fees for investors with relatively low current incomes are small because these investors tend to use Roth accounts whether or not they face tax-schedule uncertainty. Investors with higher current incomes, however, focus primarily on pre-tax investments when ignoring this type of uncertainty. The economic benefit of reoptimizing after being introduced to tax-rate uncertainty is large for these investors as they attempt to reduce or eliminate their exposure to tax-rate risk.

Third, the benefit of considering tax uncertainty is generally largest for investors with high future income levels. Income from a traditional account is taxed at uncertain future rates. Given that outside income is also taxed at these uncertain rates, investors whose wealth primarily consists of outside income and pre-tax retirement income are highly exposed to realized future tax rates. When tax rates are uncertain, investors with higher levels of outside income thus strongly prefer to add post-tax savings to their retirement portfolios.

Finally, the fees that investors are willing to pay to consider tax-schedule risk are increasing in the degree of uncertainty about future taxes. As illustrated by Fig. 3, future tax rates are more uncertain over longer retirement horizons and for higher tax brackets. These features of tax-schedule uncertainty contribute to the relatively higher fees that we calculate for younger investors as well as retirement savers with high current and future incomes. In fact, the largest fees are paid by young, high-income investors who wish to mitigate tax risk using Roth accounts, even though they lock in the highest current tax rate on this portion of savings.

4.4. Economic value of Roth account access

Conventional wisdom suggests that retirement savers with relatively low current incomes benefit most from access to Roth accounts. We find, however, that higher-income investors have greater exposures to tax-schedule uncertainty, which can be managed using post-tax Roth options. For perspective on which investors benefit most from Roth account access, we calculate the annual percentage fees that investors who are restricted to save

[18] We also examine the effect of relaxing the constraint of identical allocations across the traditional and Roth accounts. Allowing investors to separate their asset allocation decisions in the two retirement accounts has little impact on optimal consumption, total traditional account investment, or total Roth investment. To quantify the economic importance of this constraint, we follow the approach introduced in Section 4.3 and compute the maximum annual fees that investors would be willing to pay on their retirement savings to have differing allocations to equity in their traditional and Roth accounts. For the economy with tax uncertainty, the largest fee that any investor in Fig. 6 would be willing to pay is 4.3 basis points per year. Many investors would not be willing to pay any fee, and willingness to pay fees above 2.0 basis points is rare. Given the nature of these results, we focus on solutions to optimization problems with allocations to equity constrained to be equal across retirement accounts.

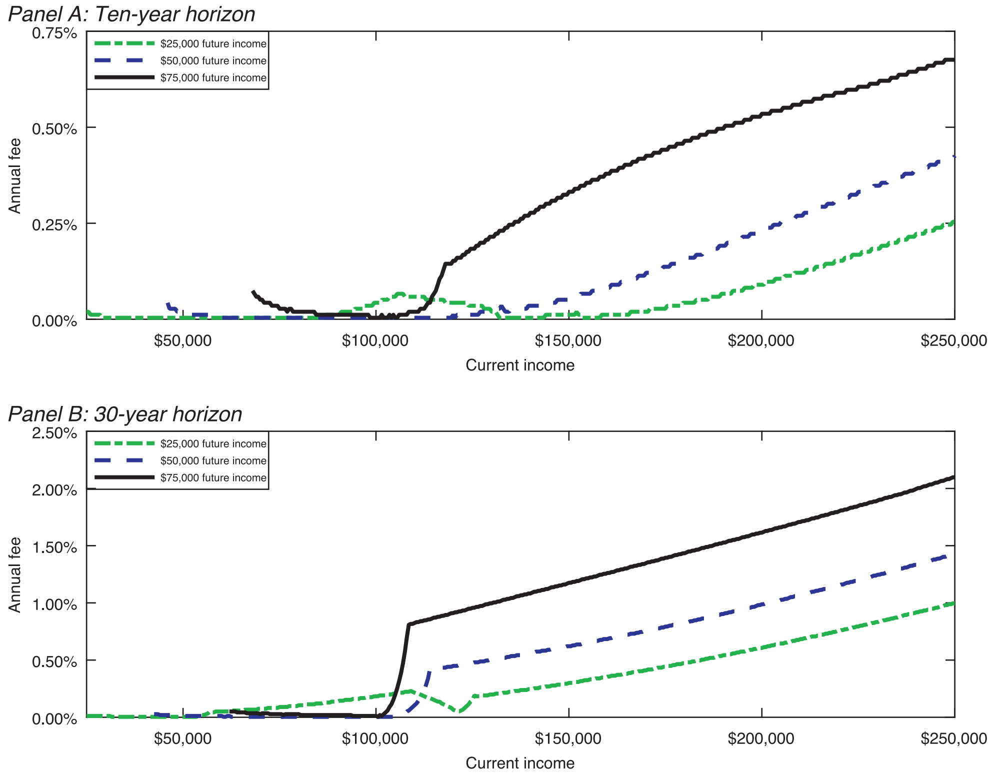

Fig. 9. Annual fees that investors are willing to pay to form policies that consider tax uncertainty. This figure shows the annual percentage fees that investors would be willing to pay on their savings in exchange for being allowed to consider tax-schedule uncertainty while making allocations. To calculate these fees, we begin with the investors in Section 4.1 who form their optimal policies under known, progressive tax rates. We then expose these investors to uncertainty about the future tax schedule and measure their expected utility. Finally, the investors are allowed to reoptimize considering all sources of uncertainty, but we place an annual fee on their savings. We find the fees that make the investors indifferent between reoptimizing and keeping their original strategies. Each panel plots fees as functions of current income, and the three lines in each panel show fees for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income. Panel A (Panel B) presents results corresponding to a ten-year (30-year) retirement horizon. We report fees for investors who save at least 2% of their current income.

using only traditional accounts would pay to gain Roth options. We first measure the expected utility of investors who optimize using traditional retirement vehicles. We then find the fee that makes investors indifferent between optimizing using both types of accounts and maintaining the traditional-only policy.

Fig. 10 shows the fees that investors are willing to pay for Roth account access. We plot fees for investors who save at least 2% of their current income. The figure demonstrates that the ability to save for retirement using post-tax accounts is economically important for most of the investors that we consider. Beginning with ten-year investors in Panel A, the ability to lock in low current marginal tax rates on Roth investments is important for retirement savers with relatively low current income.[19] Higher-income investors also value Roth accounts for reducing tax-schedule uncertainty, and the annual fees are generally similar in magnitude for investors with low and high levels of current income. For example, investors with $25,000 in outside retirement income and current incomes of $25,000 and $250,000 are willing to pay the same fee of 0.25% per year to access a Roth account.

Panel B of Fig. 10 displays fees for 30-year investors. Many young investors with relatively low current incomes are willing to pay fees to lock in current tax rates with Roth options, consistent with conventional wisdom. The largest fees in Fig. 10, however, are paid by 30-year investors with high levels of current income. This finding reflects the economic importance of managing exposure to tax-schedule uncertainty using Roth accounts. The recent introduction of Roth versions of 401(k), 403(b), and 457(b) plans without regulatory limits on income provides high-income investors with the means to mitigate tax risk. In sum, although our analysis ignores potentially important features of the real world such as contribution limits and different withdrawal rules for traditional and Roth accounts, the results highlight large potential welfare gains from Roth investments across a wide spectrum of current income levels and retirement horizons.

5. Retirement account constraints and simple investment rules

To this point, our analysis ignores two important features of more realistic decision environments. First, the

[19] To maintain consistent scaling across figures, the fees for ten-year investors with $50,000 in future income and current income between $46,500 to $51,500 are not visible in Fig. 10. In this range, fees are between 0.88% and 1.56%.

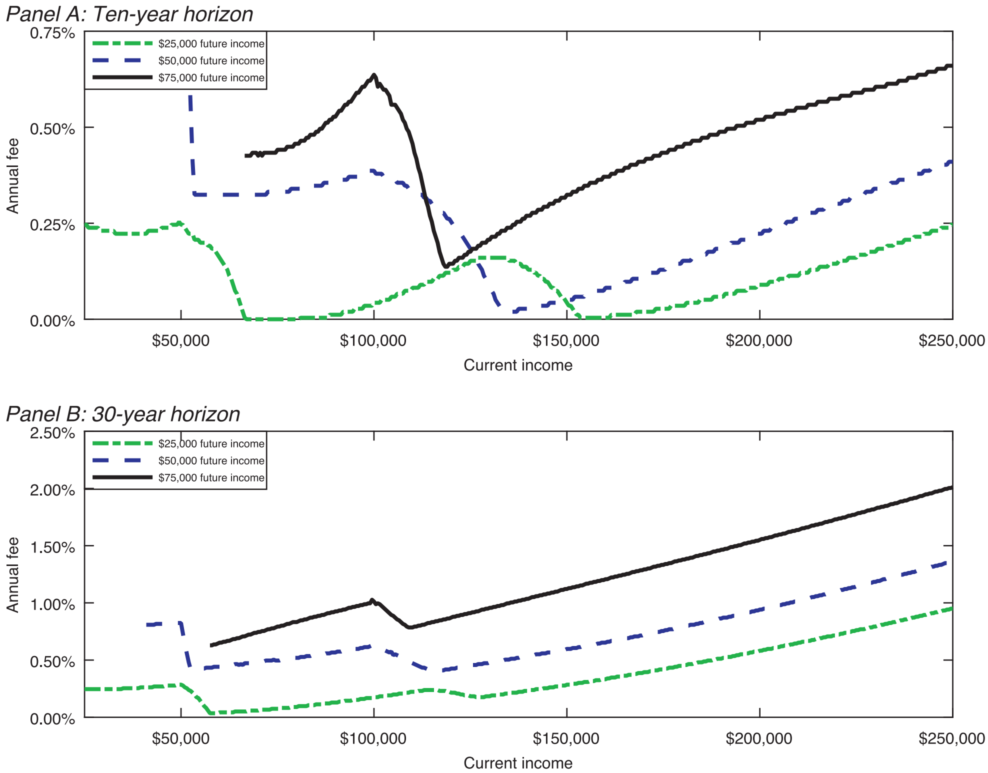

Fig. 10. Annual fees that investors are willing to pay to gain access to a Roth account. This figure shows the annual percentage fees that investors would be willing to pay on their savings in exchange for being allowed to invest in a Roth account. To calculate these fees, we begin with investors who are constrained to invest only through traditional accounts. The investors are then granted access to a Roth account and are allowed to reoptimize using both traditional and Roth accounts, but we place an annual fee on their savings. We find the fee that makes the investors indifferent between reoptimizing and keeping the traditional-only strategy. Each panel plots fees as functions of current income, and the three lines in each panel show fees for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income. Panel A (Panel B) presents results corresponding to a ten-year (30-year) retirement horizon. We report fees for investors who save at least 2% of their current income.

models estimated in Section 4 do not restrict the total investment in tax-advantaged retirement vehicles. As noted in Section 2.1, however, both statutory and sponsor-imposed limitations exist on retirement savings in these accounts, and these restrictions vary widely across households. In Section 5.1, we impose additional constraints on the investor's optimization problem and assess the value of tax-schedule uncertainty in these settings. Second, investors with limited financial literacy are unlikely to be able to implement the decision rules prescribed in Section 4, which are complex functions of tax status, current income, expected future income, and age. As such, we introduce simpler decision rules in Section 5.2 and assess whether or not households can achieve the economic benefits of tax diversification by following these rules.

5.1. Economic value of tax uncertainty for constrained investors

We examine investors with the same combinations of current incomes, future incomes, and investment horizons as in Section 4. For each investor, we solve for the optimal consumption and portfolio rules under additional investment constraints intended to provide a close approximation of the options available to large subsets of retirement savers. In reality, retirement account access and contribution limits depend on several factors including tax filing status, employment status, and employer account offerings. We focus on the following four sets of constraints associated with the married-filing-jointly tax status to match our use of historical tax-rate information for this filing class.

1. IRA only. Households are allowed to invest up to $11,000 across pre-tax and post-tax retirement accounts. The post-tax account phases out linearly over the current income range of $183,000 to $193,000, such that investors with greater than $193,000 in current income are restricted to invest only in a traditional account. This set of constraints closely matches the IRA options for married-filing-jointly households without access to a workplace account.
2. Pre-tax workplace account plus IRA. This investor type represents households that can invest up to $18,000 in a pre-tax workplace account (e.g., a traditional 401(k), 403(b), or 457(b) plan). In addition, the household can invest up to $11,000 in IRAs. To mimic statutory restrictions described in Section 2.1, the traditional (Roth) version of the IRA is phased out linearly over the income range of $98,000 to $118,000 ($183,000 to $193,000).

3. Pre-tax or post-tax workplace account plus IRA. Investors of this type are similar to the preceding group, except that the workplace retirement account can be invested in traditional or Roth options with a combined limit of $18,000. These investors are designed to represent households in which one worker has access to both traditional and Roth options in her workplace retirement account.
4. Multiple workplace accounts. These households have access to multiple workplace retirement accounts with a combined limit of $50,000 and are allowed to invest in pre-tax and post-tax versions of these accounts. The aggregate limit of $50,000 roughly corresponds to maximum investments in multiple retirement account types, such as households in which both spouses have access to traditional and Roth workplace accounts or investors with the broad access granted to many state university employees.

Given that the constraints limit access to traditional and Roth accounts, we introduce a third account option in the model to permit additional savings. The contribution rules and tax features of this account are designed to mimic those of an after-tax 401(k). Investments in the account are made with post-tax dollars, and any gains in the account are taxed as income in retirement. The balance of the after-tax 401(k) account is thus taxed somewhat differently in retirement compared with the pre-tax retirement account. Whereas the full amount of the traditional account is taxed, only the difference between the account balance and the initial investment is taxed in the after-tax 401(k) account. We also allow for a full deduction of losses in the after-tax 401(k) account, and restricting capital loss deductions has little impact on the analysis.

We assess the economic importance of tax-schedule uncertainty for constrained households following the approach introduced in Section 4.3. That is, we first compute the optimal consumption and savings decision for an investor who ignores this source of uncertainty but faces one of the four sets of constraints on tax-advantaged savings. We then calculate the expected utility associated with this policy after introducing tax uncertainty. Finally, we allow the investor to pay an annual fee on her savings to reoptimize and report the fee that makes the investor indifferent between this new strategy and her original choice.

Fig. 11 presents the annual fees that investors with a ten-year horizon are willing to pay to reconsider their optimal policies and condition on uncertainty over future tax rates. Each panel in the figure corresponds to one of the four sets of investment constraints. The value of considering tax uncertainty is generally small for investors in Panels A and B with relatively limited access to retirement account options (i.e., investors with access to only IRAs or access to IRAs and traditional 401(k) accounts). These results are straightforward, as investors primarily value the reoptimization option when their portfolios would shift toward Roth investments in the presence of tax-schedule uncertainty. Low-income investors in Panels A and B of Fig. 11 prefer Roth over traditional accounts with or without tax uncertainty. High-income investors who would like to shift to Roth when faced with tax uncertainty are constrained from doing so by their investment options and the phase-out restrictions on post-tax IRAs. The exceptions are investors in Panel A with $75,000 in future income and current income ranging from about $105,000 to $190,000. These investors exhibit a marginal preference for Roth over traditional accounts in the presence of tax uncertainty and are willing to pay fees of up to 0.21% per year to adjust their portfolios.

Uncertainty about the tax schedule becomes more economically relevant for some higher-income investors as the constraints on Roth accounts are relaxed. In Panel C of Fig. 11, investors with $25,000 and $50,000 in guaranteed future income exhibit optimal consumption and savings policies that are similar with or without tax uncertainty. As such, the maximum fees to reoptimize are negligible. In contrast, constraints on Roth IRA investments are binding for higher-income investors with $75,000 in future income. These investors face substantial tax uncertainty because their taxable base in retirement is larger and they are more likely to end up in the highest tax bracket with the most variability in marginal rates. At current income levels above about $160,000, the optimal investment policy includes more than $11,000 in Roth contributions, such that the Roth 401(k) allows investors to overcome the Roth IRA investment limits. Access to a Roth 401(k) is particularly important for households with current income that exceeds the $193,000 limit for Roth IRA eligibility.

Finally, the fees for investors with total combined access to traditional and Roth accounts of $50,000 (i.e., Panel D of Fig. 11) are typically higher in comparison with the other three cases. These relatively less constrained investors with future incomes of $25,000, $50,000, and $75,000 are willing to pay up to 0.07%, 0.22%, and 0.58%, respectively, for the option to reoptimize.

Fig. 12 shows the fee results for the 30-year investment horizon. In comparison with the ten-year investors in Fig. 11, investors in Panels A and B (i.e., IRA only and IRA plus traditional 401(k)) with moderate and high levels of future income are willing to pay substantial fees to shift allocations to post-tax accounts in the range of current income just before the Roth option phases out. For example, investors with $75,000 in future income and access to an IRA only (IRA and traditional 401(k)) sacrifice as much as 1.16% (1.25%) per year in returns across their traditional, Roth, and after-tax 401(k) investments to consider tax uncertainty.

Panels C and D of Fig. 12 show maximum fees for 30-year investors facing relatively less restricted access to tax-advantaged accounts. At higher levels of current income, all of these investors exhibit a desire to shift their savings into Roth accounts when faced with tax uncertainty. The associated fees for these investors are positive and larger than the corresponding totals for the ten-year investors in Fig. 11. As in Fig. 9, the fees are strongly increasing in future income.

In summary, the main takeaways from our analysis of unconstrained retirement savers in Fig. 9 continue to hold under common investment constraints. Tax uncertainty is most economically relevant for households with high current income, as these agents save relatively more in Roth vehicles when the future tax schedule is uncertain. Low-

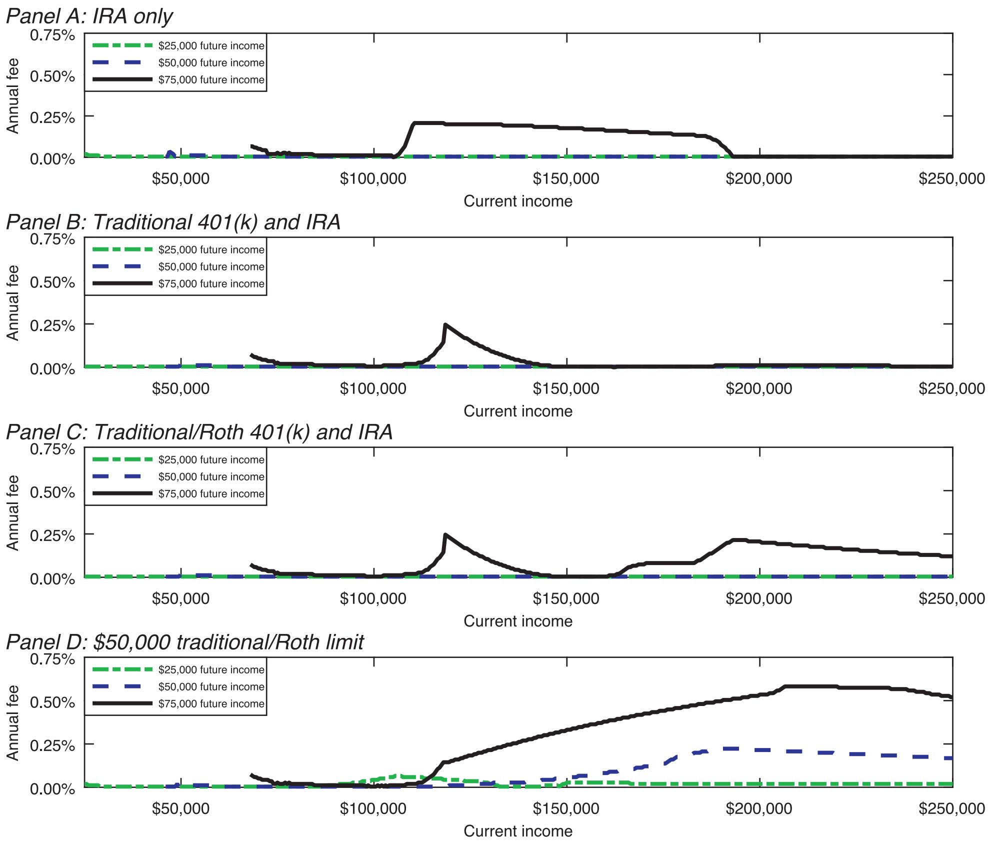

Fig. 11. Annual fees that ten-year constrained investors are willing to pay to form policies that consider tax uncertainty. This figure shows the annual percentage fees that constrained investors with a ten-year horizon would be willing to pay on their savings in exchange for being allowed to consider tax-schedule uncertainty while making allocations. The four panels consider investors facing different investment constraints: Individual Retirement Account (IRA) only (Panel A), traditional 401(k) and IRA (Panel B), traditional or Roth 401(k) and IRA (Panel C), and $50,000 of either traditional or Roth accounts (Panel D). Each panel plots fees as functions of current income, and the three lines in each panel show fees for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income. We report fees for investors who save at least 2% of their current income.

current-income investors, in contrast, prefer Roth accounts with or without tax uncertainty. Further, the economic value of considering tax risk increases in investment horizon and the level of expected future taxable income. Investors with limited access to Roth accounts, however, are largely unwilling to pay fees to account for uncertainty given their constraints.

5.2. Simple investment policies for tax diversification

Implementing the investment policies discussed in Section 4.2 would require households to solve complex optimization problems that account for current and future income levels, tax status, investment horizon, and preference parameters. In actual applications, the optimal mix of traditional and Roth investments for a given investor would also be a function of account access, contribution limits, and a number of other features. Investors with limited financial literacy, therefore, would likely be unable to exactly implement the optimization methods prescribed above. Given this limitation, we propose an alternative policy for locating assets across traditional and Roth accounts. Our objective is to introduce a heuristic for investors that allows them to benefit from tax diversification without explicitly considering all of the complexities of their decision environment. We then assess how close the economic outcomes of these investors are to the outcomes from following the optimal policies from Section 4.2.

Our simplified asset location rule is designed to reflect key features of the policies presented in Fig. 6. We focus on two patterns across the six panels in this figure. First, investors with current income in the lowest tax bracket (i.e., less than $50,000) locate assets exclusively in Roth accounts to lock in low tax rates on their retirement savings, whereas most other investors implement strategies that involve diversifying between account types. Second, older investors with a shorter investment horizon tend to locate a greater proportion of their assets in traditional accounts. These investors have relatively less exposure to tax uncertainty and often favor the hedging benefits of traditional accounts within the progressive tax system. Based on these results, we propose a rule that calls for investors to allocate all of their savings to Roth accounts if current taxable income corresponds to the lowest tax bracket and otherwise invest $(Age + 20)$% of their savings in traditional accounts with the remainder in Roth accounts. For the two investment horizons considered in this paper, we think of the ten-year (30-year) investors as being 55 (35)

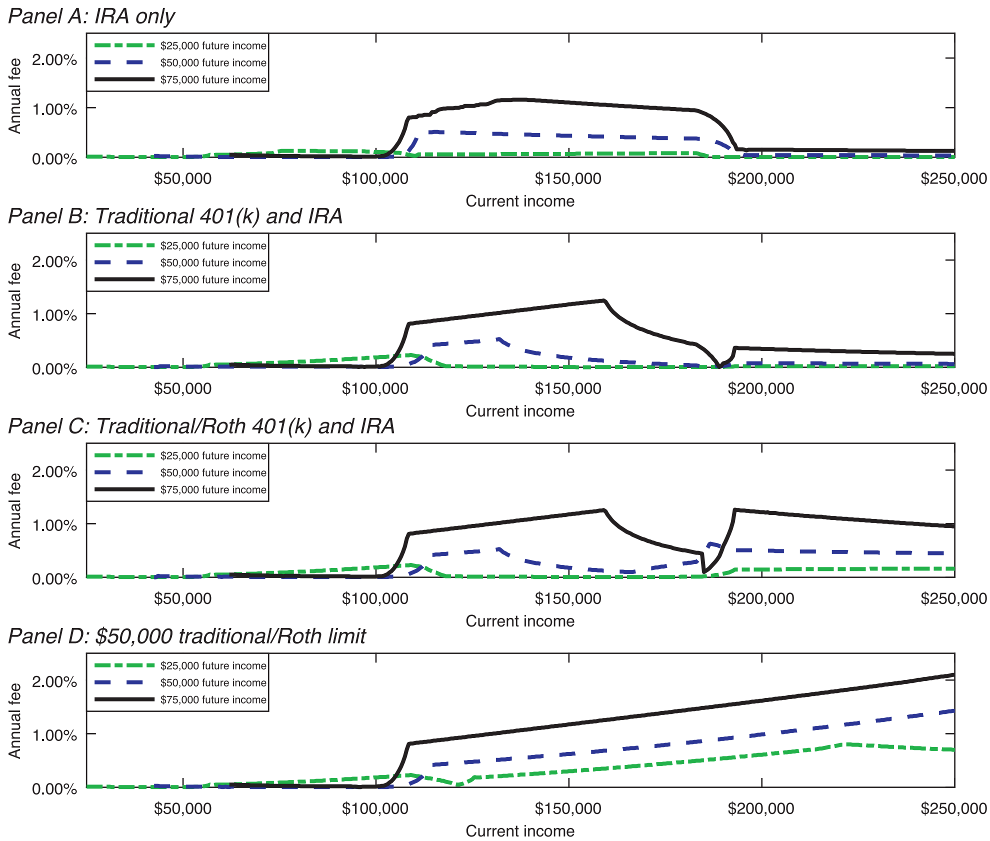

Fig. 12. Annual fees that 30-year constrained investors are willing to pay to form policies that consider tax uncertainty. This figure shows the annual percentage fees that constrained investors with a 30-year horizon would be willing to pay on their savings in exchange for being allowed to consider tax-schedule uncertainty while making allocations. The four panels consider investors facing different investment constraints: Individual Retirement Account (IRA) only (Panel A), traditional 401(k) and IRA (Panel B), traditional or Roth 401(k) and IRA (Panel C), and $50,000 of either traditional or Roth accounts (Panel D). Each panel plots fees as functions of current income, and the three lines in each panel show fees for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income. We report fees for investors who save at least 2% of their current income.

years old. As such, the investment rule dictates that these investors locate 75% and 55% of their assets in traditional accounts, respectively, as long as current taxable income exceeds $50,000.[20]

We quantify the economic impact of relying on the simple investment policy instead of the optimal policy using two equivalent fee analyses. First, we consider the fees that the traditional-only investors from Section 4.4 would pay to gain access to Roth accounts and subsequently implement the $(Age + 20)\%$ rule. We then compare these fees with those in Fig. 10 that measure the benefit of fully optimizing using Roth and traditional savings. Across the investor types we consider, the average (median) difference in fees between using the simple rule and fully optimizing is six (three) basis points per year. Even among the investor types with above-median fee differences, the rule-based fees average 65% of the magnitude of the corresponding fees from the optimized case. Overall, households are able to realize much of the potential gain from tax-strategy diversification by following the $(Age + 20)\%$ rule.

Our second equivalent fee analysis focuses on the remaining differences between the $(Age + 20)\%$ rule and a fully optimal strategy. In particular, we allow investors to optimize using the $(Age + 20)\%$ rule and record their expected utilities. We then calculate the annual fee that would make these investors indifferent between fully optimizing with the fee assessed and relying on the $(Age + 20)\%$ rule. The results are presented in Fig. 13. For most investors, the estimated fees are small, suggesting that the economic outcomes of investors following the $(Age + 20)\%$ rule are not materially different from those adopting the optimal policies. The fees for the ten-year investors in Panel A, for example, are typically between zero and 20 basis points per year. The most notable exceptions are for investors with about $50,000 to just over $100,000 in current income and either $50,000 or $75,000 in guaranteed future income. These investors have a relatively high probability of facing high marginal tax rates in retirement and, as such, prefer to locate a substantial portion of their savings in Roth accounts rather than follow the simple investment policy. The maximum fees over this range of current income are still relatively small, however, at 0.21% and

[20] The Online Appendix presents results for the $(Age + 20)\%$ asset location rule for investors with 20-year and 40-year horizons. This Appendix also considers the impact of alternative age-based and constant-proportion asset location strategies.

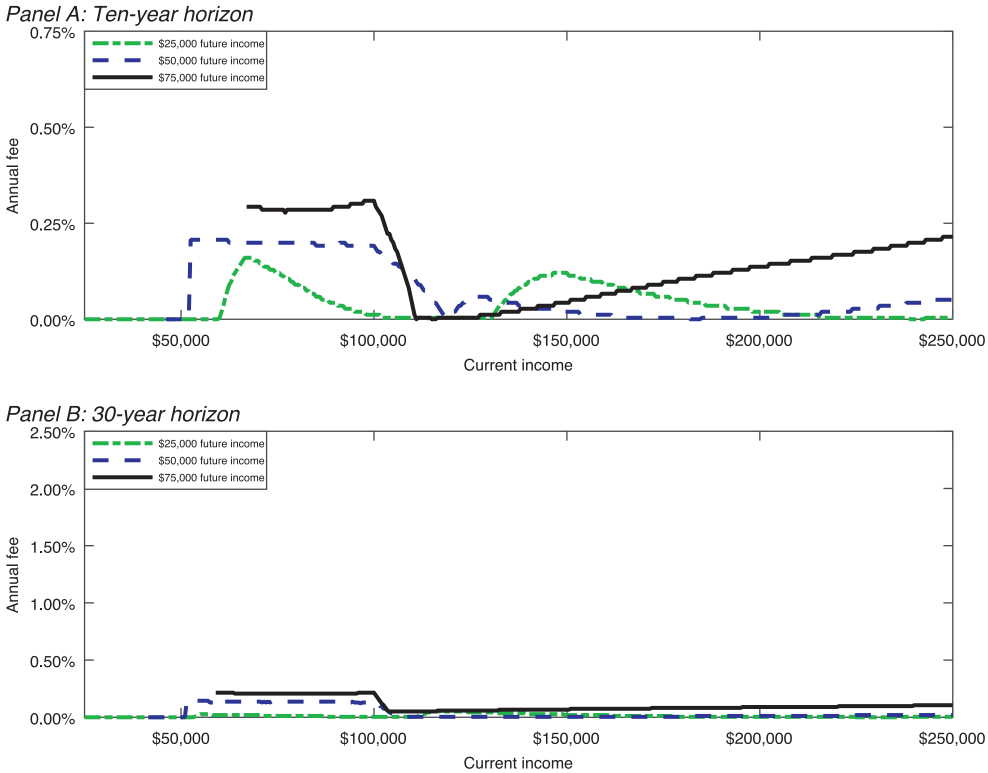

Fig. 13. Annual fees that investors are willing to pay to optimize instead of follow the $(Age + 20)\%$ rule. This figure shows the annual percentage fees that investors would be willing to pay on their savings in exchange for being allowed to fully optimize instead of using the rule proposed in Section 5.2. The rule dictates that investors allocate all of their savings to Roth accounts if their current taxable income corresponds to the lowest tax bracket and otherwise invest $(Age + 20)\%$ of their savings in traditional accounts with the remainder in Roth accounts. To calculate these fees, we begin with investors who follow this retirement savings rule and measure their expected utility. We then allow the investors to reoptimize using any desired allocation across traditional and Roth accounts, but we place an annual fee on their savings. We find the fee that makes the investors indifferent between reoptimizing and keeping their original strategy. Each panel plots fees as functions of current income, and the three lines in each panel show fees for investors with $25,000, $50,000, or $75,000 of guaranteed retirement income. Panel A (Panel B) presents results corresponding to a ten-year (30-year) retirement horizon. We report fees for investors who save at least 2% of their current income.

0.31% per year for the investors with $50,000 and $75,000 in future income, respectively. The only other case in Panel A of Fig. 13 in which investors experience meaningful economic losses from following the simple investment rule is for households with $75,000 in future income and very high levels of current income. Intuitively, these investors are the most exposed to tax uncertainty and would optimally invest considerably less in traditional accounts than the 75% weight prescribed by the $(Age + 20)\%$ rule (see, e.g., Fig. 6). Similar to the results for ten-year investors, the estimated fees for most 30-year investors are economically small. The largest observed fees correspond to households with current income ranging from $50,000 to just over $100,000 and moderate-to-high future income. The maximum fee for investors with $50,000 ($75,000) in future income reaches 0.14% (0.21%) per year. Investors with low future income, in contrast, are able to achieve most of the benefits of tax diversification by following the $(Age + 20)\%$ location rule, with maximum fees for these investors at just 0.06% per year.

We also assess how our simple asset location policy performs in the presence of the four sets of investment constraints introduced in Section 5.1. The constrained investors follow the rule outlined above to the extent possible given their account access, but the investors can deviate from the rule once constraints are binding. For example, a household with a traditional 401(k) and sufficient income to be disqualified from investing in a Roth IRA must invest 100% of its tax-advantaged retirement savings in the traditional account. Table 4 reports the maximum fee for each combination of investment horizon and future income level over current incomes from $25,000 to $250,000. Panel A shows fees for the unconstrained case, such that these results correspond to those in Fig. 13, and Panel B displays fees for constrained investors. We find that the magnitudes of the fees are generally very similar for unconstrained and constrained investors. The only cases in which the constrained fees exceed those in Panel A correspond to households without access to workplace retirement accounts. The maximum fees for ten-year investors with $50,000 in future income (0.36%) and 30-year investors with $75,000 in outside retirement income (0.23%) occur when the investors would prefer to fully invest in Roth IRAs, whereas the location rule restricts them to an $(Age + 20)\%$ allocation. Roth accounts allow for a higher effective savings limit because households can consume the full savings amount in retirement instead of the after-tax value. This feature of Roth accounts is particularly

Table 4
Maximum annual fees for the $(Age + 20)\%$ rule among unconstrained and constrained investors. The table reports the maximum annual percentage fees that unconstrained and constrained investors would be willing to pay on their savings in exchange for being allowed to fully optimize instead of using the rule proposed in Section 5.2. The rule dictates that investors allocate all of their savings to Roth accounts if their current taxable income corresponds to the lowest tax bracket and otherwise invest $(Age + 20)\%$ of their savings in traditional accounts with the remainder in Roth accounts. For each investor type, we report the maximum fee across investors with \$25,000 to \$250,000 in current income who save at least 2% of their current income. To calculate these fees, we begin with investors who follow this retirement savings rule and measure their expected utility. We then allow the investors to reoptimize using any desired allocation across traditional and Roth accounts, but we place an annual fee on their savings. We find the fee that makes the investors indifferent between reoptimizing and keeping their original strategy. Panel A shows fees for unconstrained investors, and Panel B reports results for investors facing different investment constraints: Individual Retirement Account (IRA) only, traditional 401(k) and IRA, traditional or Roth 401(k) and IRA, or \$50,000 of either traditional or Roth accounts.

	Ten-year horizon			30-year horizon		
	Future income			Future income		
Investor type	\$25,000	\$50,000	\$75,000	\$25,000	\$50,000	\$75,000
Panel A: Maximum fees for unconstrained investors						
Unconstrained	0.16%	0.21%	0.31%	0.06%	0.14%	0.21%
Panel B: Maximum fees for constrained investors						
IRA only	0.13%	0.36%	0.31%	0.03%	0.14%	0.23%
IRA and traditional 401(k)	0.16%	0.21%	0.31%	0.06%	0.14%	0.21%
IRA and traditional/Roth 401(k)	0.16%	0.21%	0.31%	0.06%	0.14%	0.21%
\$50,000 traditional/Roth	0.16%	0.21%	0.31%	0.06%	0.14%	0.21%

valuable for the most constrained investors we consider. Overall, however, the results in Table 4 suggest that our proposed rule allows investors to gain much of the diversification benefits of traditional and Roth accounts regardless of constraints.

In sum, the analysis in this section supports the use of a simple heuristic for diversifying across retirement savings accounts. We propose a practical rule with two steps to determine the allocation of savings across traditional and Roth accounts.

1. Households that currently fall into a low tax bracket (e.g., the 10% or 15% brackets in the 2015 tax schedule) should invest 100% of their savings in Roth accounts.
2. Other households should allocate $(Age + 20)\%$ of their retirement savings in traditional accounts with the remainder in Roth vehicles, subject to constraints on account access and investment limits.

A careful analysis of household-specific information regarding current and future income levels, account access, and other considerations could allow investors to improve their outcomes by tailoring the rule-based policy to their specific circumstances.[21]

6. Conclusion

We study the effects of progressive tax rates and uncertainty about the future tax schedule on optimal retirement savings. In an economy with these features, both traditional and Roth accounts offer advantages for investors. Roth investments allow investors to eliminate tax risk on a portion of retirement savings. In contrast to conventional wisdom, we find that this benefit is important for high-income households. Traditional savings are valuable for managing current-period taxable income and creating a favorable correlation structure between investment performance and marginal tax rates in retirement. We find that, given these benefits, the optimal asset location policy for most investors involves diversifying between traditional and Roth accounts. We demonstrate a large economic impact of optimally investing across traditional and Roth retirement accounts, and these effects remain significant in the presence of realistic contribution limits and constraints on account access. Moreover, we offer a simple age-based heuristic that allows investors to achieve most of the potential gains from tax strategy diversification.

In addition to the benefits for managing tax uncertainty, a mix of account types allows investors to achieve favorable balances across unmodeled account features and tax code risks. Traditional and Roth versions of IRAs and employer-sponsored plans have alternative rules for pre- and post-retirement withdrawals, conversions, and estate taxes. Maintaining assets across a variety of accounts can provide households with additional flexibility in managing their consumption throughout the life cycle. In addition, we do not explicitly model the risk of a structural change in the tax code or the regulations for a particular retirement vehicle. For instance, a shift to a consumption tax system would favor traditional accounts relative to Roth, whereas a flat income tax structure would reduce the tax benefit of traditional investments and make Roth accounts more desirable. Diversifying with traditional and Roth savings can help households balance these risks.

[21] The Online Appendix provides further information and guidance regarding tailoring our simplified asset location rule to more specific circumstances.

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