Probabilistic approaches to investment strategies on stock markets

Abstract

Making profit from stock investment involves two main decisions, decide when to buy a stock and when to sell it. For the sake of definiteness, this research focused on the moment of selling the stocks. Different probabilistic strategies for determining this moment in the way that maximizes long-run profits were proposed and estimated: **"golden rule" of decision making, estimation of expected maximum and their modified versions.** All of the algorithms were tested and estimated on real historical stocks prices data. The best results were obtained for the modified binomial algorithm: the average profit of **7.8%** in quarter comparing to an average annual market return of **10%** [1].

Contents

I. Introduction 4

II. Problem statement 5

III. "Golden Rule" for decision making 6

A. Classic "Golden Rule' 6

B. Method modification: minimizing the rank 7

IV. Expected maximum approach 9

A. Expected maximum calculation 9

1. Binomial pricing model 9

2. Deriving the formula via reflection principle 10

3. Parameters scaling for applying binomial theory 12

4. Parameter estimation 13

B. Method modification: constant parameter update 13

V. Experiments 15

VI. Results 16

VII. Conclusions 18

References 19

Appendix A: Parameter estimation 20

Appendix B: Stocks used 21

Appendix C: Detailed experiment results 26

I. INTRODUCTION

Investment process consists of two important sub-processes, formating and managing of the investment portfolio. Key decisions investors make are **"when to buy stocks"** and **"when to sell stocks"**. The successful guessing of those moments is a pledge of profits.

The problem of determining the optimal moment of selling stock has a lot of analogies in real life: buying the ticket on Ryanair website, hiring an employee, getting married. Actually, those are different formulations of the classical problem of the optimal stopping theory. Should I buy the ticket now, or wait a few more days? From the one side, if the price seems not low enough, the decision to wait seems reasonable, but by waiting too long, you would probably miss the best deals.

The classical formulation of the problem is guessing the maximum or minimum in the sequence of random numbers based on previous observations of numbers in the sequence. According to Ferguson, this task was first published in 1960 in the column of Martin Gardner in the journal "Scientific American" [2]. The type of problem became very popular in the field of applied probability, game theory and statistical-based decision making.

The problem of guessing the moment to sell stocks reminds us of another well-known problem in this field: **secretary problem**. Imagine an employer running the sequence of interviews with the main goal to hire the best secretary out of n rankable candidates. The hypothetical employees appear one at the time in random order. The employer has to make the decision right after the interview and is not aware of the quality of unseen candidates.

By making analogies, I assumed that we might consider the problem of guessing the optimal moment to buy or to sell stocks as an extended version of the classical problem. We treat the actions of an investor during the investment period as a continuous decision-making process. He aims to choose the most profitable price with no precise information about the future prices only having the data about the previous days. Investing period is limited, so it is important to sell the stock in time. If the period is over and the decision has not been made yet, the only remaining option is to sell the stock at the very last moment.

II. PROBLEM STATEMENT

The problem of guessing the optimal moment to sell stocks reminds a classical problem that involves optimal stopping theory. Every moment investor should make a selling decision and his expected profit depends on the strategy he uses.

The main aim of the research is **to suggest, analyze and estimate possible probabilistic approaches to form an investment strategy**. Mainly, I focused on determining the moment to sell the stocks for gaining the maximum possible profit. It is always possible to consider the reversed problem as well, guessing the moment of minimum price in a sequence to buy the stock and even to combine it with that of guessing the moment of maximum price to sell it.

III. “GOLDEN RULE“ FOR DECISION MAKING

A. Classic “Golden Rule‘

The optimal stopping problem is stated as follows: a known number of elements is presented to the observer one by one in random order. At any moment, the observer can arrange the part of the items that he has already viewed in order of desirability. As each item is presented, he must either accept it (in this case the process stops) or reject it (next item is presented). If the observer rejects all the items and the last item is reached, it should be accepted. The observer's aim is to choose the best value among all possible elements.

In our case, the observes the trader; he is interested in finding the best moment to sell stocks during the investment period. Let's discuss some strategies to gain maximum profit. We denote by W the event of his win. So:

$$W = \{\text{PREDICTING EXACTLY THE MAXIMUM PRICE OF STOCK}\}$$

What is the probability of W? Actually, it depends on the decision making strategy. Let's denote by S the primitive strategy, that is always choosing the first day of the period. In such case the probability of winning:

$$P(W|S) = \frac{1}{N}$$

Not very impressive. That is why we decided to use “A Golden Rule for Decision Making”, proposed by Sardelis and Valahas [2]. This rule is about getting some information from observations before making the final decision. Our task was to choose a perfect selling moment for every investment period, where the price was at its peak.

Out of all N days of the investment period, the trader lets n days pass so that $1 < n < N - 2$, ranks prices of those days from highest to lowest, and watches for the prices next days. As the first price with the higher rank, i.e., the first price higher than before appears, he selects it. In case such price does not appear, he selects the last-day price. The essential problem here is determining how many days should the trader let to pass. Let's denote by S_n the strategy of observing stock prices exactly n days. This strategy wins if:

1. The best price hasn't appeared in one of the first n days. Otherwise, we just passed it.

2. Price ranks on the days after the n^{th} preceding the best one do not exceed those of the first n prices.

Let's denote by B_k the event that the best price appears in some day k. Here we somewhat ignore the market rules and assume that all orderings of prices are equally likely. Then, considering the condition 1 above:

$$P(B_k) = \frac{1}{N}, \text{k} > \text{n}$$

Let R_k denote the event described by condition 2 above. R_k occurs if the maximum of the first $k-1$ days appears in the first n days, so:

$$P(R_k) = \frac{n}{k-1}$$

B_k and R_k are independent and probability of success of strategy S_n, denoted by $P_N(S_N)$ can be calculated as follows:

$$P_N(S_N) = \sum_{k=n+1}^{N} P(B_k \cap R_k) = \sum_{k=n+1}^{N} P(B_k)P(R_k) = \frac{n}{N} \sum_{k=n+1}^{N} \frac{1}{k-1}$$

As it was proved [2]:

$$P_N(W|S_N) \to max, \text{ as } \frac{n}{N} \to e^{-1}$$

$$\lim_{N\to\infty} P_N(S_N|n = e^{-1}) = e^{-1}$$

In other words, probability of W is the highest (e^{-1}) when $n \approx Ne^{-1}$. So, before moving to decision making, we collect 37% of the period prices data for future comparison.

B. Method modification: minimizing the rank

After performing experiments with "golden rule" on real stock prices data, we detected some disadvantages of this method. For example, if the best price appears in the first n days allocated for observation, the opportunity to get a good result is immediately limited to a situation where the last-day price is quite high, which is not always true. The "Golden Rule" solves the problem of searching exactly the best result (maximum price), considering all the results different from the maximum equally bad. Setting a task as "look for the maximum price for a period" can be justified and sounds ambitious, however, given that no significant changes occur in the price of the stock daily, it is worth considering a slightly different algorithm for finding the moment of profitable sale.

The trader will also win if he sells his share at the second or third best price. That is why it is expedient for this problem to make a decision based not only on information about the best possible price but the position that the stock price takes in the price ranking for the period. In this case, we need to find such n (number of days allocated for observation), which would minimize the rank of the sought price in period prices ranking rather than maximizing the probability of obtaining exactly the maximum price, as in the previous case.

To determine the particle $\frac{n}{N}$, which minimizes the rank of the price, we ran experimental simulations with permutations of numbers (from 1 to N). As a result, we determined that it is reasonable to observe 12% of prices in the beginning, i.e., $n = 0.12N$. Since 'golden rule' requires randomness and independence of candidates, and stock pricing data do not satisfy these conditions, we decided to calculate $\frac{n}{N}$ separately for every kind of stock.

IV. EXPECTED MAXIMUM APPROACH

A. Expected maximum calculation

The ideal case for an investor whose aim is to sell a stock is knowing an exact maximum stock price on period. The stock prices can be affected by many unpredictable factors for which it is almost impossible to construct a deterministic mathematical model. The next suggested approach for building an investment strategy consists in calculating the expected price maximum within the binomial pricing model proposed by Cox, Ross, and Rubinstein that well describes the discrete process of price changes [3]. **The algorithm of decision making is fully intuitive: if today stock price is greater or equals to expected maximum, sell it!**

1. Binomial pricing model

In finance, the binomial pricing model has been widely used for pricing options. The basis of the model is the principle of a random walk, which describes a path consisting of a sequence of random steps. The model can be represented as follows: at each minute with some probability p the traveler will take a step on the west and with probability $1-p$ on the east. What is the expected maximum distance he will take on west in the time interval of N minutes?

The same approach can be applied to describe price changes. The model assumes that at each moment of the period there is probability p that price goes up and $1-p$ that it does down. The changes at two different time moments are independent. For now we assume that price can either go up by one or down by one, so the random walk is symmetric with step $s \pm 1$. Let us denote by D the random variable equals to the maximal price over the period of n days(can be treated as maximum distance the price takes up during that period); we find that:

$$E[D] = \sum_{d=0}^{n} dP(D = d) = \sum_{d=0}^{n} P(D \geq d)$$

By denoting by N^+ the number of days on which the price goes up and conditioning on $N^+ = i$, we get:

$$E[D] = \sum_{d=0}^{n} \sum_{i=d}^{n} P(D \geq d | N^+ = i) P(N^+ = i) \tag{1}$$

2. Deriving the formula via reflection principle

In this section we will calculate the expected value of (1). N^+ follows a binomial distribution:

$$N^+ \sim \beta(n, p)$$

$$P(N^+ = i) = \binom{n}{i} p^i (1-p)^{n-i} \tag{2}$$

We start by calculating $P(D \geq d | N^+ = i)$. We know that the number of days the price goes up $N^+ = i$, so that the number of days it goes down is $n - i$. Denoting by x_j the price in day j and by $v = x_n$ the price in the last day of the period, one gets:

$$v = i - (n - i) = 2i - n$$

There are two possible scenarios:

$$v \geq d \implies i \geq \frac{d+n}{2} \tag{3}$$

$$v < d \implies i < \frac{d+n}{2} \tag{4}$$

For (3) the inequality $D \geq d$ holds automatically, as the price at the end of period becomes greater than d, whence:

$$P(D \geq d | N^+ = i) = 1 \tag{5}$$

The second case is more interesting, and here we apply **the reflection principle (Fig1)**:

The number of paths from x_1 to v that touch or intersect the line $y = d$ equals the number of all paths from x_1 to v', where v' is the symmetric reflection of v with respect to $y = d$.

Let us denote by l the number of trajectories that exist from x_1 (starting price) to v':

$$2l - n = 2d - i + n$$

$$l = d - i + n$$

So, in the case (4):

$$P(D \geq d | N^+ = i) = \binom{l}{n} / \binom{i}{n} \tag{6}$$

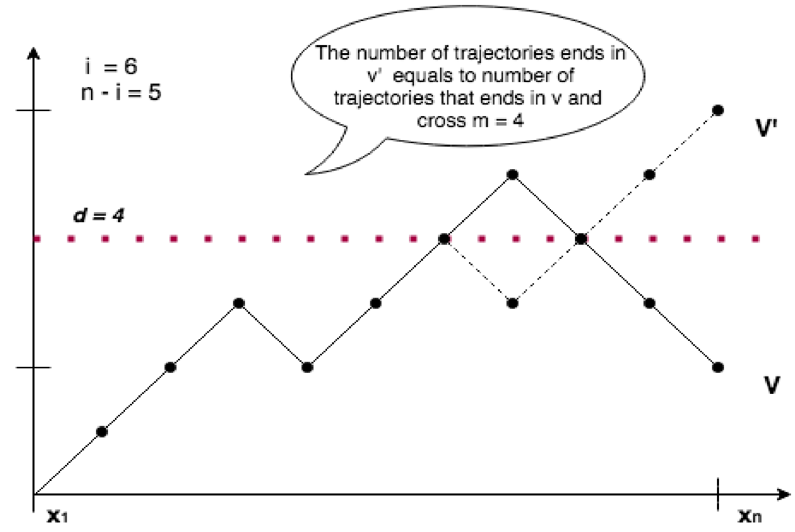


Figure 1: Illustration of the reflection principle

The relation in (6) treated as the relation between all paths to v with touching or crossing $y = d$ and the total number of paths to v. By combining (2), (5) and (6):

$$E[D] = \sum_{i=0}^{i<\frac{d+n}{2}} \binom{l}{n} p^i(1-p)^{n-i} + \sum_{i\geq\frac{d+n}{2}}^{n} \binom{i}{n} p^i(1-p)^{n-i} \tag{7}$$

Important that **the formula in (7) treated as the expected maximum distance the price ever over the period approaches up with step equals to one**. On the Figure 1 $d = 5$ corresponds to such maximum distance.

Now, when we know the maximum value distance the price approaches up on an interval, one can calculate the maximum expected price.

Let's denote by S_n the price of the stock at the moment n. The binomial model assumes that during the next period, the price of shares may either increase in comparison with the previous one or decrease with certain probabilities. In other words,

$$S_{n+1} = \epsilon_n S_n \tag{8}$$

where ϵ is a following random variable :

$$\epsilon_n = \begin{cases} u, & \text{with probability } p \\ d, & \text{with probability } 1-p \end{cases}$$

Let's denote by M the maxim price on period of length n:

$$M = \max(S_1, S_2, ...S_n)$$

Let's switch to additive model for simplifying the calculations:

$$\log(S_n) = \log(\epsilon) + \log(S_n) \tag{9}$$

$$\log(M) = \max(log(S_1), \log(S_1) + \log(\epsilon), ... \log(S_1) + \sum_{j=1}^{n} \log(\epsilon)) \tag{10}$$

The formula (7) treated as expected maximum the price reaches on period with step equals to 1, similarly to expected maximum distance the traveller covered eastwards. Considering (7) and (9), (10), the maximum expected price over the period can be calculated as:

$$\log(M) = \log(S_1) + \log(u)E[D] \tag{11}$$

3. Parameters scaling for applying binomial theory

One of our basic assumption was the symmetry of random walk, but it is not true for stocks. In this section, we adapt the binomial pricing model for asymmetric changes in prices. Recall that $E[M]$ was calculated assuming the symmetry of random walk. In the case of stocks $\log(u) + \log(d) \neq 0$. We are not able to apply the binomial model as one step up can no longer be offset by one step down. One of the possible ways to fix it is the introduction of a new random variable with parameters that will compensate each other:

$$\epsilon = \widetilde{\epsilon} + \frac{\log(u) + \log(d)}{2}$$

$$\widetilde{\epsilon} = \begin{cases} \frac{\log(u)-\log(d)}{2}, \text{ with probability } p \\ -\frac{\log(u)-\log(d)}{2}, \text{ with probability } 1-p \end{cases} \tag{12}$$

Finally, we can apply the binomial model for above symmetric values for a step up and step down. One thing before we did: one should take into account that in each step our expectations are shifted either greater or lower depending on values of u and d. Let's denote by Δ error in the actual value of the change in price. We assume that the maxim price can appear in one of n days with equal probabilities, then:

$$E[\Delta] = \frac{n}{2} * \frac{log(u) + log(d)}{2} \tag{13}$$

Now, we can express **the expectation of the maximum for the entire time interval**, let's denote it by M through a random variable M with compensated parameters and add the mathematical expectation of a trend of shifts in predictions because of asymmetry:

$$log(E[M]) = log(E(M)) + E[\Delta] \tag{14}$$

Finally, following the approach, one should sell the stock at the moment when the current price is greater or equals to the expected maximum, calculated by (14).

4. Parameter estimation

The binomial model requires estimation of following parameters:

$$p = \{probability\ for\ going\ up\}$$

$$u - 1 = \{the\ percentage\ at\ which\ the\ price\ increases\}$$

$$d - 1 = \{the\ percentage\ at\ which\ the\ price\ decreases\}$$

The above values can be calculated empirically from historical data. For example, you need to know when to sell stocks in April, having data for March. To begin, we calculate the probabilities. Let's denote by N the number of days in the previous period, by N^+ the number of days the price went up. Via maximum likelihood estimation method:

$$\hat{p} = \frac{N^+}{N} \tag{15}$$

One can calculate the estimates $\hat{u}$ and $\hat{d}$ as geometric means of corresponding percentages in price change from previous data. Detailed estimation of $\hat{u}$ and $\hat{d}$ and proof of unbiasedness can be found in *Appendix A*.

B. Method modification: constant parameter update

To improve the results of determining the optimal stopping point was implemented a minor modification of the above model. The disadvantage of the previous method is that we calculate the expected maximum at the starting day of a period from previous data and completely ignore the data of the current investment period when evaluating the parameters of the model. The modified approach suggest updating these settings daily using price information from the previous k days. For every next day, one should recalculate the mathematical expectation of maximum prices using updated estimates for $\hat{u}$, $\hat{d}$, and $\hat{p}$. The formula (14) remains the same.

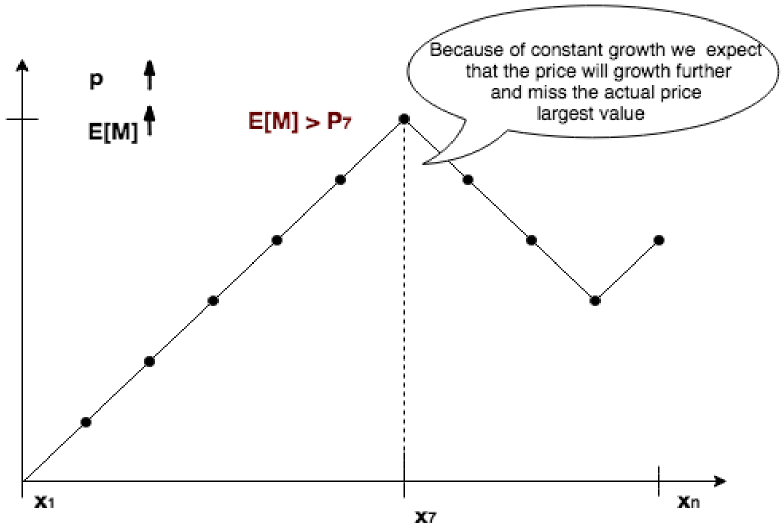


Figure 2: Common situation in the presence of a clear peak value of price

One of the possible negative situation illustrated in Figure 2. Imagine, that you observe the constant growth of stock price and recalculate you expected maximum price every day. In the model, the estimate of probability $\hat{p}$ was also constantly increasing, that caused the constant increase of our expectation, and we will miss the maximum price. It is better to ignore the last 4-5 days for estimating the expected maximum.

V. EXPERIMENTS

All the methods of determining the best moment for selling, mentioned in the previous sections, were implemented in **R** and tested on real-world data. We used 10 different stocks from various industries, of companies of different sizes, which included:

- Apple Inc. [AAPL] (technology)
- Microsoft [MSFT] (technology)
- PVH Corp. [PHV] (apparel)
- Biogen Inc. [BIIB] (biotech)
- Advanced Micro Devices Inc. [AMD] (semiconductor)
- Green Plains Inc. [GPRE] (ethanol fuel producer)
- McDonald's [MCD] (fast food)
- American Airlines, Inc. [AAL] (airline)
- Tesla Inc. [TSLA] (automotive and energy)
- Foot Locker, Inc. [FL] (athletic retailer)

We collected daily data about these stocks prices during the last 10 years (Jan 2008 - Jan 2018) from Yahoo! Finance [5] [6] and used the closing price to run the experiments. *Appendix B* contains aggregated data and plots for the mentioned stocks over chosen years.

Each year was divided into 3 periods of 4-months duration, which we took as our investment periods. As a result, we got 30 investment periods for each stock type and tested each of them using 4 strategies described in the previous sections.

For the strategies that require parameter estimation, the average of previous periods' parameters was used as estimates for the current one. For example, to determine the best length of observation interval for the n^{th} period when using modified 'golden rule' (method III.B), we calculated lengths of observation intervals for all the preceding periods, i.e. from 1^{st} to $(n-1)^{th}$ and took the average value of them as the estimated length for the n^{th} period. Parameter estimation for method IV was described in detail in the method explanation.

To compare the effectiveness of each method, we also implemented the simulation of the most primitive strategy, which is a random selection of the selling day.

VI. RESULTS

To compare the results of different methods, we used the following parameters: the rank of the predicted price in the overall prices rating for the period, and relative profits for each method (calculated as the relative difference between the predicted maximum price (selling price) and the price of the first day of the month (purchase price).

If it turns out that the foreseen price takes the first position in the rating - we are in a win-win situation since the model has returned the best possible result, the real maximum price of the stock. Similarly, the greater the relative profit we get, the better.

Below is an aggregated comparative table based on the data collected from all the experiments described in the study, where mean values of the results for 10 stocks are represented.

Method	**average profit**	**profit std. deviation**	**confidence intervals**	
			95 lower	**95 upper**
golden rule	3.88%	19.73%	-2.70%	11.06%
modified golden rule	5.42%	21.02%	-2.35%	13.19%
expected maximum	4.72%	21.26%	-3.11%	12.82%
modified exp. max.	7.80%	24.24%	-1.78%	16.07%
primitive strategy	1.47%	17.31%	-3.55%	7.77%

Table I. Comparison of profits generated by applying different strategies

For the set of 10 stocks mentioned in section **V**, using price data over 2008-2018.

The results of each of 4 strategies are better than those for the primitive one. In both cases, method modifications performed better and produced greater profits. More detailed results, which include profits for every stock and strategy, are shown in *Appendix C.*

On the next page you can see the visualization of profits generated over all periods:

- for the best and the worst method (Figure III)
- for two methods that performed the best (Figure IV)

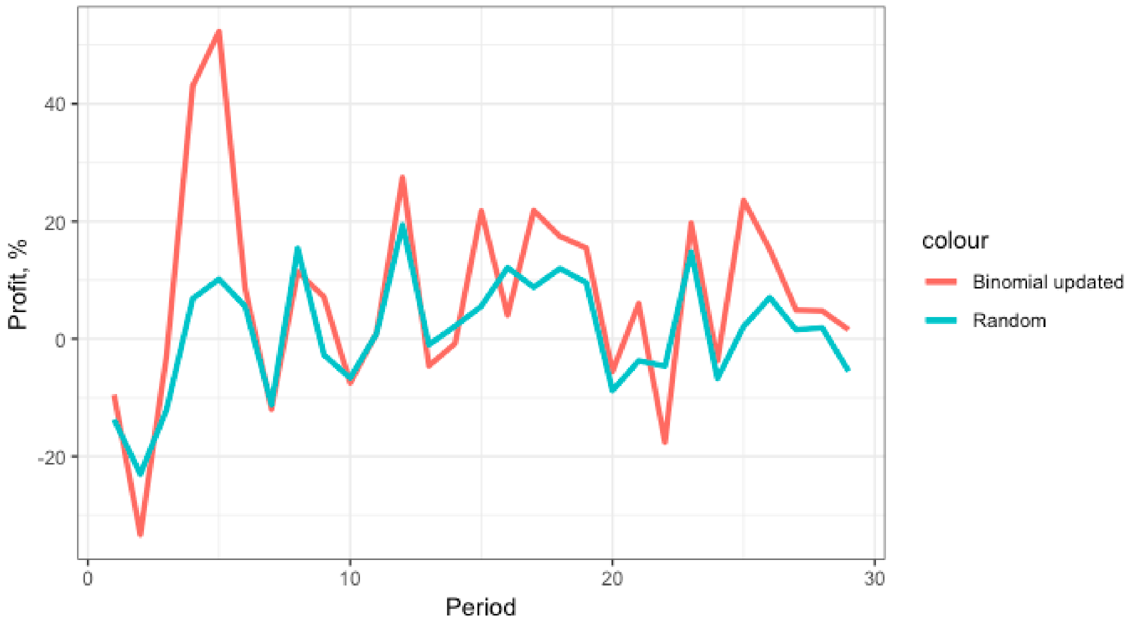


Figure III. Profits generated over time with primitive strategy and modified binomial model (calculating the expected maximum with constant parameter update).

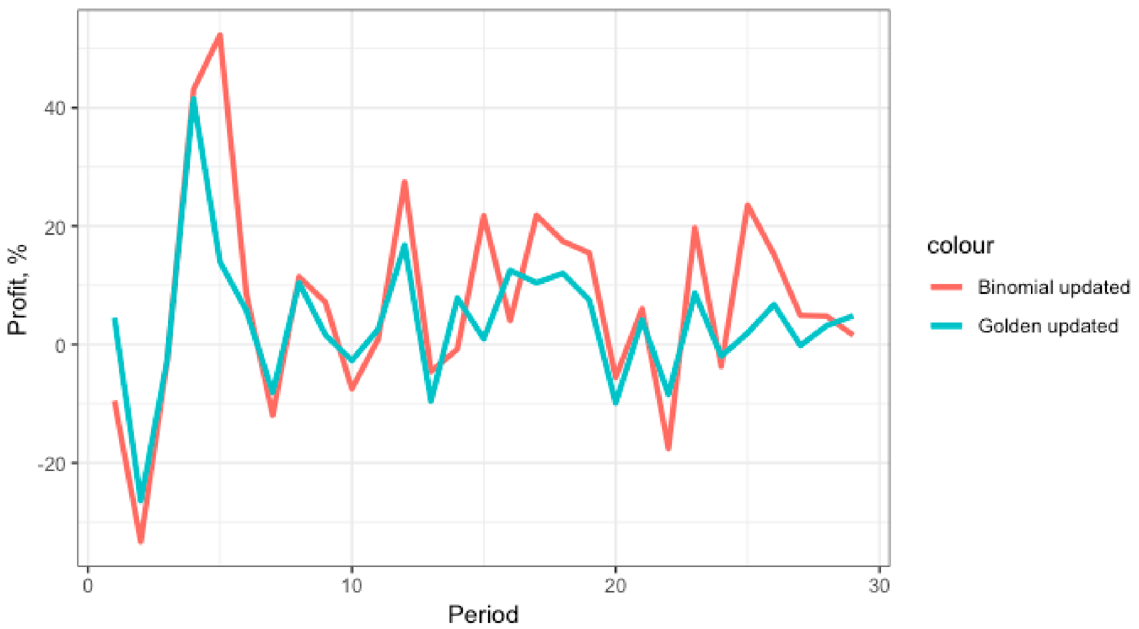


Figure IV. Profits generated over time with modified 'golden rule' and modified the binomial model.

VII. CONCLUSIONS

The study has shown that the use of purely probabilistic approaches for predicting an optimal moment of sale for stocks is efficient, **applying the algorithm based on the formula (14) derived in the research showed up to 6.33% better performance than non-strategic approach**.

The classic "golden rule" can be improved by determining the optimal number of days monitored for stock prices, which increases the average return by 1.54%.

The method of determining the expected maximum was the best, and the modification with dynamic parameters update helped to increase the average profit from 4.72% to 7.8%.

The results of the research can be used in practical investment decision making for both short and long periods. Each of the studied methods is simple, understandable and can be easily implemented using the modern software.

[1] James Royal Arielle O'Shea, *What Is the Average Stock Market Return?*, nerdwallet, NerdWallet Compare Inc., March 25, 2019, https://www.nerdwallet.com/blog/investing/average-stock-market-return/.

[2] Dimitris A. Sardelis and Theodoros M. Valahas, *Decision Making: A Golden Rule.* The American Mathematical Monthly. Vol. 106, No. 3 (1999). pp. 215-226.

[3] Cox J.C., Ross S.A., Rubinstein M., *Options Pricing: A Simplified Approach.* Journal of Financial Economics. Vol. 7 (1979). pp. 229–263.

[4] Feller William , *An Introduction to Probability Theory and Its Applications.* John Wiley and Sons, Inc. Vol. 1, 2nd Edition (1960).

[5] Yahoo! Finance, *finance. yahoo. com*. Provider of financial news, data and commentary including stock quotes, press releases, financial reports, etc.

[6] Quantmod , *quantmod. com*. Quantitative Financial Modelling & Trading Framework for R.

[7] Bruce C. N. Greenwald, Paul D. Sonkin *Value Investing: From Graham to Buffett and Beyond.* Wiley; 1 edition (2004).

Appendix A: Parameter estimation

Suppose I_1 is an indicator that in that day the price went up and I_2 the price went down.

$$I_1(x_i) = \begin{cases} 1 & \text{i} \in N^+ \\ 0 & \text{otherwise} \end{cases}$$

$$I_2(x_i) = 1 - I_1(x_i)$$

$$\hat{u} = \sqrt[\sum_i I_1(x_i)]{\prod_i (\frac{x_i}{x_{i-1}})^{I_1(x_i)}},\ i > 1$$

$$\hat{d} = \sqrt[\sum_i I_2(x_i)]{\prod_i (\frac{x_i}{x_{i-1}})^{I_2(x_i)}}\ i > 1$$

It is important that we calculate u and d as relative values, so we go in different direction not by some value, but some **percentages**. Let's suppose $|N^+| = k$, so $|N^-| = n-k$ and we always go up by u, go down by value d. One can prove that $\hat{u}$ and $\hat{d}$ are unbiased estimators, i > 1:

$$E[\hat{u}] = E(\sqrt[\sum_i I_1(x_i)]{\prod_i (\frac{x_i}{x_{i-1}})^{I_1(x_i)}}) = E(\sqrt[k]{u^k}) = u$$

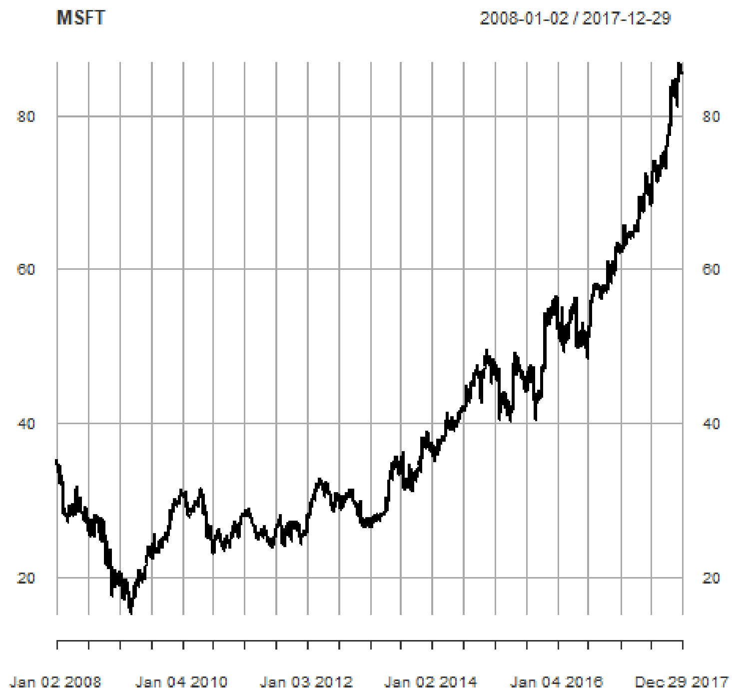
$$E[\hat{d}] = E(\sqrt[\sum_i I_2(x_i)]{\prod_i (\frac{x_i}{x_{i-1}})^{I_2(x_i)}}) = E(\sqrt[n-k]{d^{n-k}}) = d$$

The estimators û and $\hat{\mathbf{d}}$ are unbiased! Now one can easily calculate the estimates using, for example, R and previous data.

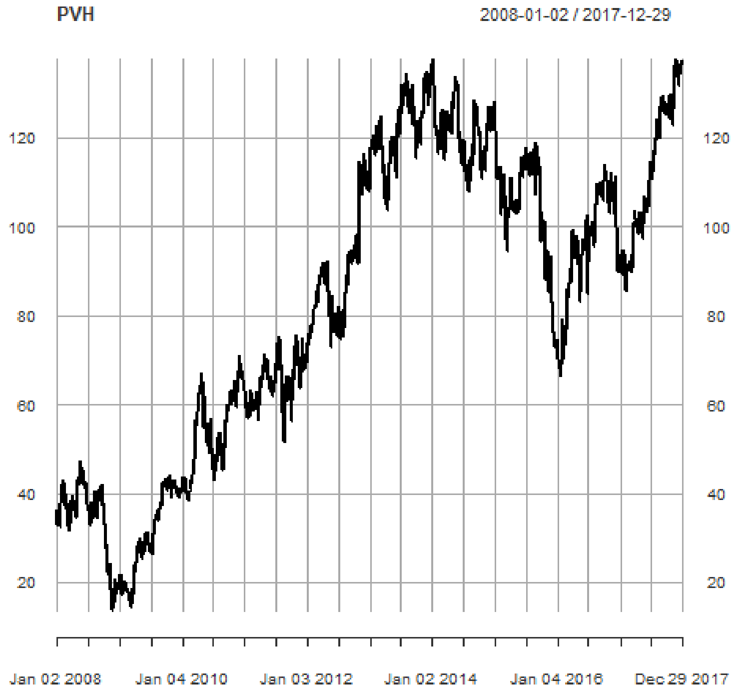
Appendix B: Stocks used

Apple Inc., volatility: 7.06%

Microsoft, volatility: 5.24%



PVH Corp., volatility: 8.62%

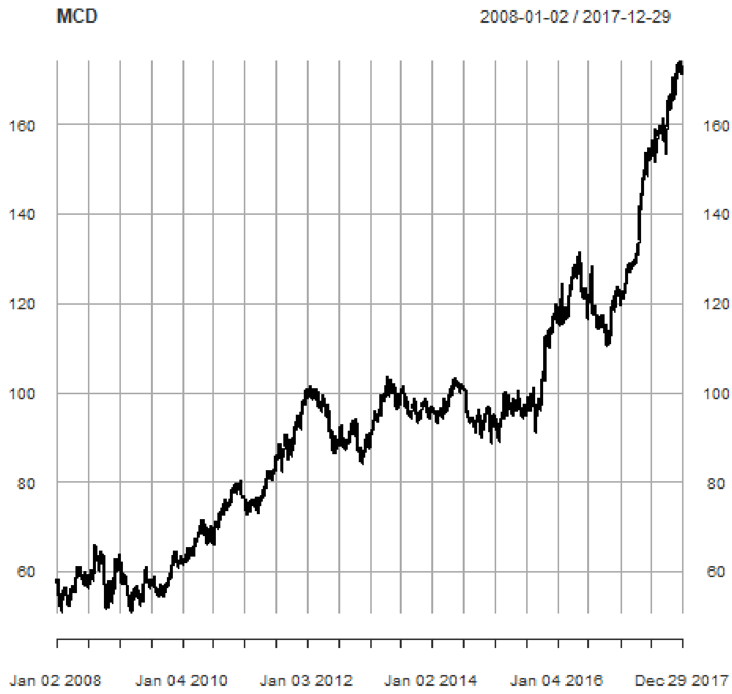


Biogen Inc., volatility: 6.29%

Advanced Micro Devices Inc., volatility: 13.99%

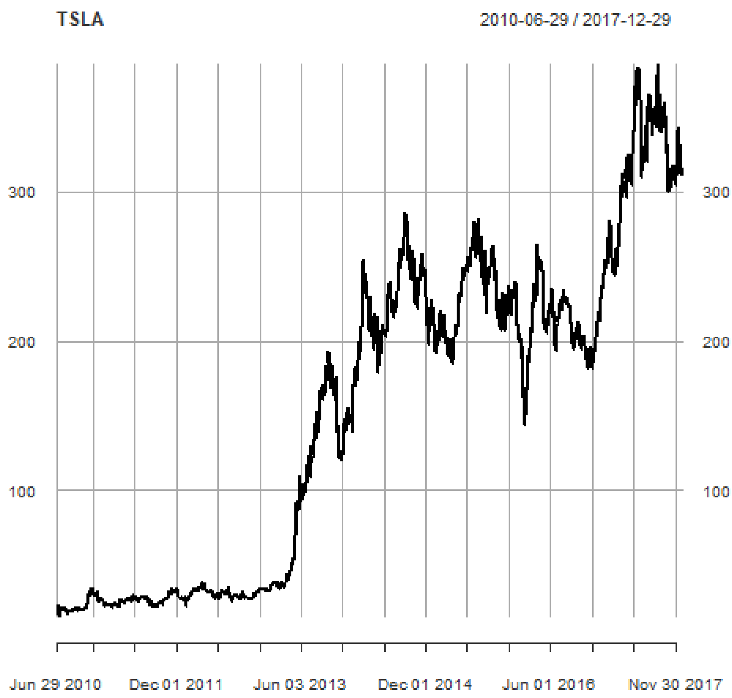
Green Plains Inc., volatility: 13.99%

McDonald's, volatility: 3.40%

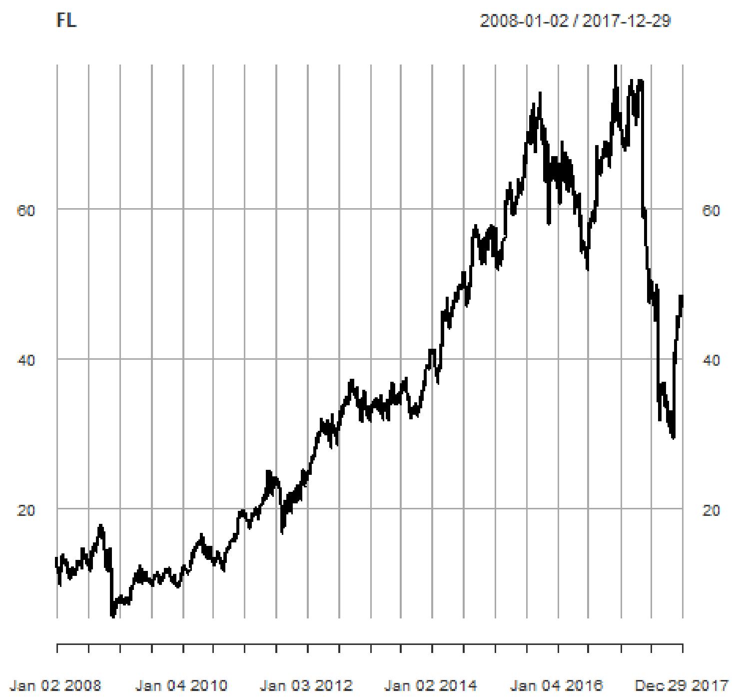


American Airlines, Inc., volatility: 13.32%

Tesla Inc., volatility: 10.84%



Foot Locker, Inc., volatility: 8.42%



Appendix C: Detailed experiment results

Profits generated using classic 'golden rule' (section III.A)

Stock	**average profit**	**profit std. deviation**	**confidence intervals**	
			95 lower	**95 upper**
AAPL	3.41%	14.78%	-1.87%	8.70%
MSFT	0.57%	10.65%	-3.24%	4.38%
BIIB	5.78%	6.43%	3.48%	8.08%
AMD	1.09%	27.15%	-8,62%	10.82%
GPRE	4.20%	48.50%	-13.16%	21.57%
MCD	1.44%	5.96%	-0.68%	3.57%
AAL	9.98%	29.66%	-0.63%	20.60%
TSLA	11.73%	19.27%	3.68%	19.78%
FL	0.16%	17.98%	-6.28%	6.59%
PVH	0.44%	16.91%	-5.61%	6.50%
average	3.88%	19.73%	-2.70%	11.06%

Profits generated using modified 'golden rule', which minimizes the rank. (section III.B)

Stock	**average profit**	**profit std. deviation**	**confidence intervals**	
			95 lower	**95 upper**
AAPL	4.32%	15.71%	-1.40%	10.03%
MSFT	3.58%	10.04%	-0.07%	7.24%
BIIB	6.09%	12.23%	1.64%	10.54%
AMD	3.26%	30.07%	-7.69%	14.20%
GPRE	7.17%	49.40%	-10.81%	25.15%
MCD	2.95%	7.77%	0.12%	5.77%
AAL	11.22%	28.10%	1.00%	21.45%
TSLA	11.18%	19.02%	3.04%	19.31%
FL	2.50%	19.49%	-4.59%	9.60%
PVH	1.91%	18.36%	-4.77%	8.59%
average	5.42%	21.02%	-2.35%	13.19%

Profits generated using binomial model and expected maximum calculation (section IV.A)

Stock	**average profit**	**profit std. deviation**	**confidence intervals**	
			95 lower	**95 upper**
AAPL	5.70%	15.00%	0.14%	11.20%
MSFT	1.99%	11.47%	-2.18%	6.18%
BIIB	3.00%	9.60%	-0.40%	6.50%
AMD	4.56%	33.00%	-7.45%	16.58%
GPRE	7.50%	47.00%	-9.50%	25.00%
MCD	3.20%	8.00%	0.35%	6.00%
AAL	5.75%	23.51%	-2.80%	14.31%
TSLA	12.64%	27.00%	0.72%	24.50%
FL	1.50%	19.00%	-5.50%	8.60%
PVH	2.40%	19.00%	-4.50%	9.30%
average	4.72%	21.26%	-3.11%	12.82%

Profits generated using binomial model and expected maximum calculation (section IV.B)

Stock	**average profit**	**profit std. deviation**	**confidence intervals**	
			95 lower	**95 upper**
AAPL	8.70%	18.00%	1.90%	15.50%
MSFT	4.20%	14.50%	-0.10%	9.30%
BIIB	6.40%	15.70%	0.70%	12.17%
AMD	6.30%	41.00%	-8.00%	21.20%
GPRE	11.10%	52.00%	-8.10%	30.40%
MCD	3.70%	9.10%	0.50%	7.10%
AAL	9.90%	31.00%	-1.40%	21.00%
TSLA	20.00%	17.00%	5.00%	20.00%
FL	4.50%	23.00%	-3.90%	13.00%
PVH	3.20%	21.10%	-4.40%	11.00%
average	7.80%	24.24%	-1.78%	16.07%

Profits generated using primitive random strategy.

Stock	**average profit**	**profit std. deviation**	**confidence intervals**	
			95 lower	**95 upper**
AAPL	0.25%	0.20%	-2.10%	7.80%
MSFT	-1.00%	0.09%	-4.30%	2.60%
BIIB	2.00%	0.08%	-0.90%	5.00%
AMD	0.17%	0.27%	-0.80%	2.50%
GPRE	4.70%	0.39%	-9.60%	19.00%
MCD	1.50%	0.06%	-0.80%	3.50%
AAL	4.50 %	0.26%	-4.90%	14.00%
TSLA	1.80%	4.00%	-0.60%	4.40%
FL	0.70%	4.70%	-2.10%	3.60%
PVH	1.70%	19.00%	-5.40%	8.80%
average	1.47%	17.31%	-3.55%	7.77%