Chapter 2

July 25, 2019

```
In [511]: import numpy as np
          import params
          from scipy.optimize import root
          from matplotlib import pyplot as plt
          import time

          beta = .442
          beta1 = .55
          delta = 0.6415
          sigma = 3
          A = 1
          alpha = .35
          nvec = np.array([1,1,.2])
          L = np.sum(nvec)
          SStol = 1.E-10
          xi = .4
          T = 40


          params1 = (.55, sigma, nvec, L, A, alpha, delta, SStol)
          params2 = (beta, sigma, nvec, L, A, alpha, delta, SStol, 1000, xi, 45)
```

0.0.1 Exercise 2.1

```
In [493]: f_params = (nvec, A, alpha, delta)
          def feasible(f_params, bvec_guess):
              '''
              Purpose:
                  Determine the feasibility of a given steady state guess based on a set of weal
              Inputs
                  f params = (nvec, A, alpha, delta)
                  bvec guess = np.array([scalar, scalar])
              Outputs
                  b_cnstr (boolean list, len=2) [which element of bvec guess is
                      likely responsible for any of the consumption nonnegativity
                      constraint violations identified in c cnstr]
                      - If the first element of c cnstr is True, then the first element of b cns
```

```
                - If the second element of c cnstr is True, then both elements of b cnstr
                - if the last element of c cnstr is True, then the last element of b cnstr
            c_cnstr (boolean list, len=3) [True if cs ≤ 0]
            K_cnstr (boolean list, len=1) [True if K ≤ 0]
        '''
        nvec, A, alpha, delta = f_params
        # First we want to check whether the savings are collectively nonnegative
        K = np.sum(bvec_guess)
        K_cnstr = np.array([K <= 0])

        # Calculate wage and rental rate parameter
        r = alpha * A * ( (np.sum(nvec) / abs(K)) ** (1-alpha) )
        w = (1-alpha) * A * ( ( abs(K) / np.sum(nvec)) ** (alpha) )
        if (K <= 0):
            r = r * -1
            w = w * -1



        # Each Generations Consumption
        c = np.array([0.0,0.0,0.0])
        c[0] = w*nvec[0] - bvec_guess[0]
        print
        c[1] = w*nvec[1] - bvec_guess[1] + bvec_guess[0]*(1+r)
        c[2] = w*nvec[2] + bvec_guess[1]*(1+r)
        c_cnstr = np.array(c <= 0)

        # Why are certain generations getting negative consumption
        b_cnstr = np.array((False, False))
        if (c_cnstr[0]):
            b_cnstr[0] = True
        if (c_cnstr[1]):
            b_cnstr[0] = True
            b_cnstr[1] = True
        if (c_cnstr[2]):
            b_cnstr[1] = True
        return [b_cnstr, c_cnstr, K_cnstr]
```

In [494]:
```
guess1 = np.array([1.0, 1.2])
b_cnstr, c_cnstr, K_cnstr = feasible(f_params, guess1)
print("K_cnstr", K_cnstr)
print("c_cnstr", c_cnstr)
print("b_cnstr:", b_cnstr)
```

```
K_cnstr [False]
c_cnstr [ True False False]
b_cnstr: [ True False]
```

In [495]:
```
guess2 = np.array([0.06, -.001])
```

```
b_cnstr, c_cnstr, K_cnstr = feasible(f_params, guess2)
print("K_cnstr", K_cnstr)
print("c_cnstr", c_cnstr)
print("b_cnstr:", b_cnstr)
```

```
K_cnstr [False]
c_cnstr [False False False]
b_cnstr: [False False]
```

```
In [496]: guess2 = np.array([0.1, 0.1])
          b_cnstr, c_cnstr, K_cnstr = feasible(f_params, guess2)
          print("K_cnstr", K_cnstr)
          print("c_cnstr", c_cnstr)
          print("b_cnstr:", b_cnstr)
```

```
K_cnstr [False]
c_cnstr [False False False]
b_cnstr: [False False]
```

0.0.2 Exercise 2.2

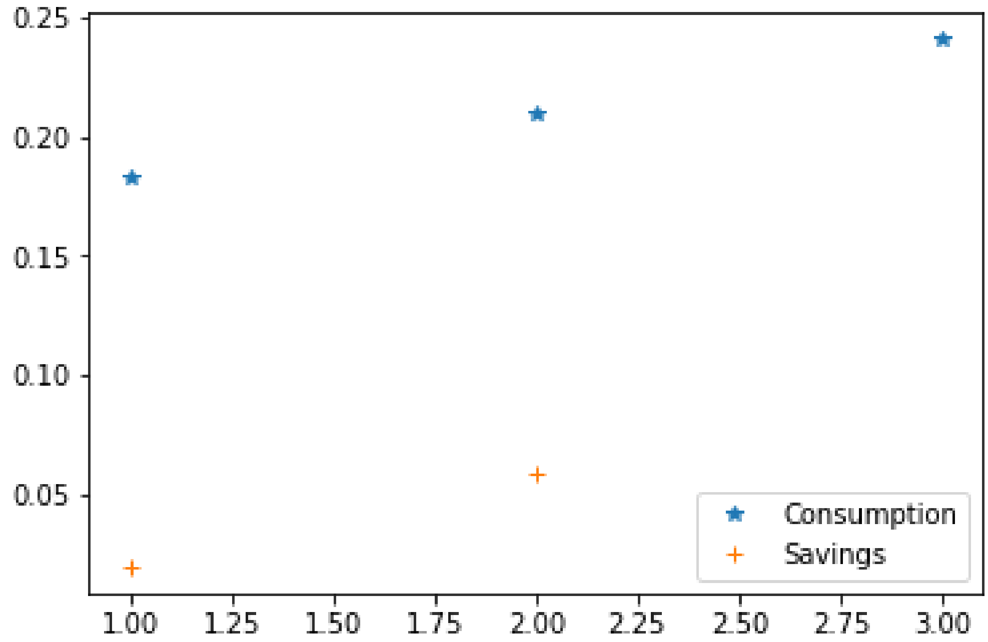
```
In [497]: params = (beta, sigma, nvec, L, A, alpha, delta, SStol)
          def funcfact(params):
              def utility(c):
                  '''
                  utility function
                  Inputs:
                      consumption
                  Outputs
                  '''
                  if (c <= 0):
                      return -9999999
                  else:
                      u = ((c ** (1-sigma)) - 1)/(1-sigma)
                      return u
              def du(c):
                  '''
                  utility function
                  Inputs:
                      consumption
                  Outputs
                  '''
                  if (c <= 0): return -99999999
                  else:
                      return (c ** (-sigma))

              return utility, du
```

```
    u, du = funcfact(params)
```

In [498]:
```
def modeldef(v, beta, sigma, nvec, L, A, alpha, delta):
    '''
        Defines the model
        Inputs:
            v - vector of solutions
            beta - discount rate
            sigma - rate of relative risk aversion
            nvec - how much each population works
            L - aggregate labor
            A - technology
            alpha - capital share
            delta - depreciation
        Return:
            list of functions
    '''
    b2, b3 = v
    k = b2 + b3
    r = ( ( alpha * A * ( (np.sum(nvec) / k))**(1-alpha) )) - delta
    w = (  (1-alpha) * A * ( ( k / np.sum(nvec))**(alpha) ) )
    c1 = w*nvec[0] - b2
    c2 = w*nvec[1] + (1+r)*b2 - b3
    c3 = w*nvec[2] + (1+r)*b3
    rv = []
    rv.append( du(c1) - ( (beta*(1+r))*du(c2) ) )
    rv.append( du(c2) - ( (beta*(1+r))*du(c3) ) )
    return rv



def get_SS(params, bvec_guess, SS_graphs):
    '''
    Purpose:
        Get the steady state of the function from a guess
    Inputs:
        params = beta, sigma, nvec, L, A, alpha, delta, SS_tol
        bvec_guess = np.array([scalar, scalar])
        SS_graphs (boolean to display graphs if true)
    Outputs
        ss_output: dictionary containing the model parameters in steady state.
    '''
    start_time = time.perf_counter()
    ret = root(modeldef, bvec_guess, method = 'hybr', args=(params[:7]) )
    b2, b3 = ret.x
```

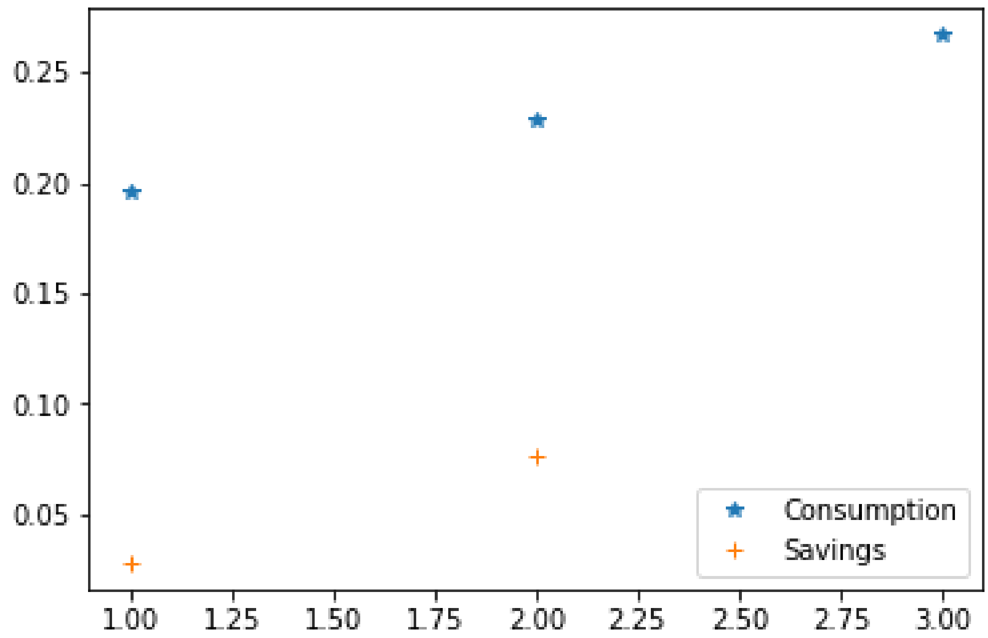
```python
        e1, e2 = ret.fun
        beta, sigma, nvec, L, A, alpha, delta, SStol = params

        k = b2 + b3
        r = ( (np.sign(k) * alpha * A * ( (np.sum(nvec) / abs(k))**(1-alpha) )) - delta)
        w = ( np.sign(k) * (1-alpha) * A * ( ( abs(k) / np.sum(nvec))**(alpha) ) )
        c1 = w*nvec[0] - b2
        c2 = w*nvec[1] + (1+r)*b2 - b3
        c3 = w*nvec[2] + (1+r)*b3
        C = c1 + c2 + c3
        Y = A * (L**(1-alpha)) * (k**alpha)


        rv = {}
        rv['b_ss'] = [b2, b3]
        rv['c_ss'] = [c1, c2, c3]
        rv['w_ss'] = w
        rv['r_ss'] = r
        rv['K_ss'] = k
        rv['Y_ss'] = Y
        rv['C_ss'] = C
        rv['EulErr_ss'] = [e1, e2]
        rv['RCerr_ss'] = Y - C - delta*k


        if (SS_graphs == True):
            fig = plt.figure()
            age = np.array((1,2,3))
            plt.plot(age, rv['c_ss'], '*', label="Consumption")
            plt.plot(age[:2], rv['b_ss'], '+', label="Savings")
            plt.legend(loc="lower right")
        return rv

    d = get_SS(params, [.1, .1], True)
    for key, item in d.items():
        print("{:10s}".format(key), ":", item)
```

```
b_ss       : [0.019312529832027088, 0.05841109592113676]
c_ss       : [0.18241212755849867, 0.20961468030898922, 0.24087386507209851]
w_ss       : 0.20172465739052575
r_ss       : 2.433062339127069
K_ss       : 0.07772362575316386
Y_ss       : 0.6827603788602411
C_ss       : 0.6329006729395864
EulErr_ss  : [-1.3926637620897964e-12, 2.6290081223123707e-12]
RCerr_ss   : 8.326672684688674e-17
```




```
In [499]: params = (beta1, sigma, nvec, L, A, alpha, delta, SStol)
          d = get_SS(params1, [.1, .1], True)
          for key, item in d.items():
              print("{:10s}".format(key), ":", item)
```

```
b_ss       : [0.028176918915182085, 0.07686545131079861]
c_ss       : [0.19597527701928497, 0.2286159413839988, 0.26669307195186887]
w_ss       : 0.22415219593446706
r_ss       : 1.8863765057189819
K_ss       : 0.1050423702259807
Y_ss       : 0.7586689708551193
C_ss       : 0.6912842903551526
EulErr_ss  : [6.87805368215777e-12, 2.5295321393059567e-12]
RCerr_ss   : 9.71445146547012e-17
```




```
In [516]: def hhsol3(V, beta, n, w, wp, wpp, r, rp, rpp, b20):
              '''
              Corner Solution
              '''
              b21, b31, b32 = V
              rv = []
              rv.append( du(w*n[0] - b21) - ( beta*(1+rp)*du(wp*n[1] + (1+rp)*b21 - b32) ) )
              rv.append( du(wp*n[1] + (1+rp)*b21 - b32) - ( beta*(1+rpp)*du(wpp*n[2] + (1+rpp)*b

              rv.append( du(w*n[1] + (1+r)*b20 - b31) - ( beta*(1+rp)*du(wp*n[2] + (1+rp)*b31 )
              return rv


          def hhsol2(V, beta, n, w, wp, wpp, rp, rpp):
              '''
              Standard Solution
              '''
              b21, b32 = V
              rv = []
              rv.append( du(w*n[0] - b21) - ( beta*(1+rp)*du(wp*n[1] + (1+rp)*b21 - b32) ) )
              rv.append( du(wp*n[1] + (1+rp)*b21 - b32) - ( beta*(1+rpp)*du(wpp*n[2] + (1+rpp)*b
              return rv


          def TPI(current , parameters):
              '''
```

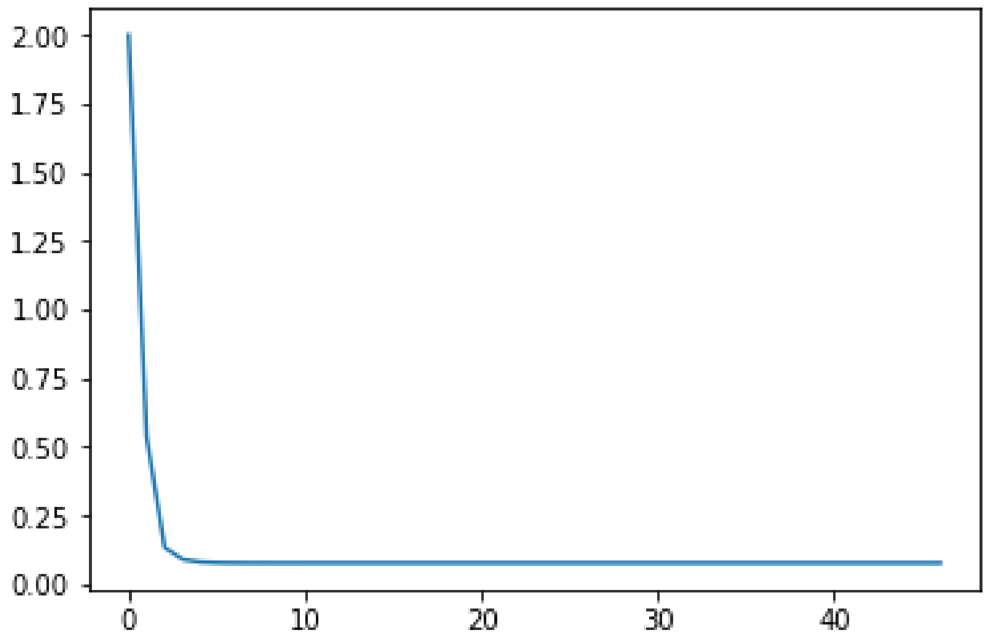
```
A function which solves for the optimal transition path through value function ite
Inputs:
    parameters = beta, sigma, nvec, L, A,
    alpha, delta, SS_tol, max_iterations, xi
Outputs:
    Ideal capital path
'''
beta, sigma, nvec, L, A, alpha, delta, SS_tol, max_iters, xi, T = parameters
# Solve for SS
SS = get_SS(parameters[:8], (.1, .1), False)
# Guess T Economy will be in SS
currk = sum(current)
K_i = np.linspace(currk, SS['K_ss'], T+1)
K_i = np.append(K_i, K_i[-1])

for i in range(1, max_iters):
    # Guess K_i = {K_i1, K_i2, ... K_iT} which implies
    # r_i = {r_i1, r_i2, ... r_iT} and w_i = {w_i1, w_i2, ... w_iT}
    r = alpha * A * ((L / K_i)**(1-alpha)) - delta
    r = np.append(r, r[-1])
    w = (1-alpha)*A*(K_i / L) **(alpha)
    w = np.append(w, w[-1])
    b2 = np.zeros_like(K_i)
    b3 = np.zeros_like(K_i)
    Kp = np.copy(K_i)
    EuErrs = np.zeros((2, len(K_i)))
    b2[0] = current[0]
    b3[0] = current[1]
    Kp[0] = currk
    # HH solution to optimal b2, and b3 + Calculate K'
    g = beta, nvec, w[1], w[2], w[3], r[1], r[2], r[3], current[0]
    ret = root(hhsol3, [.1, .1, .1], method = 'hybr', args=(g) )
    b2[1], b3[1], b3[2] = ret.x
    EuErrs[0,1], EuErrs[1,2], EuErrs[1,1] = ret.fun
    Kp[1] = b2[1] + b3[1]

    for t in range(2, T+1):
        g = beta, nvec, w[t], w[t+1], w[t+2], r[t+1], r[t+2]
        ret = root(hhsol2, [.1, .1], method = 'hybr', args=(g) )
        b2[t], b3[t+1] = ret.x
        EuErrs[0,t], EuErrs[0,t] = ret.fun
        Kp[t] = b2[t] + b3[t]

    totaldiff = np.sum(((K_i - Kp)/K_i)**2)
    if (totaldiff < SS_tol):
        print("Total difference:", totaldiff)
        print("Function Converged after {} iterations".format(i),
              "with epsilon {} in total squared deviations".format(SStol))
```

```
                print("Yeet XD")
                break
            else:
                K_i = xi*Kp + (1-xi)*K_i

        r = alpha * A * ((L / Kp)**(1-alpha)) - delta
        w = (1-alpha)*A*(Kp / L) **(alpha)
        for i in range(4):
            r = np.append(r, r[-1])
            w = np.append(w, w[-1])
            Kp = np.append(Kp, Kp[-1])

        return (Kp, EuErrs, r, w)

    params = (beta, sigma, nvec, L, A, alpha, delta, SStol, 1000, xi, T)

    kp, errs, r, w = TPI([.5, .5] , params)
```

```
Total difference: 8.137488997864269e-11
Function Converged after 56 iterations with epsilon 1e-10 in total squared deviations
Yeet XD
```

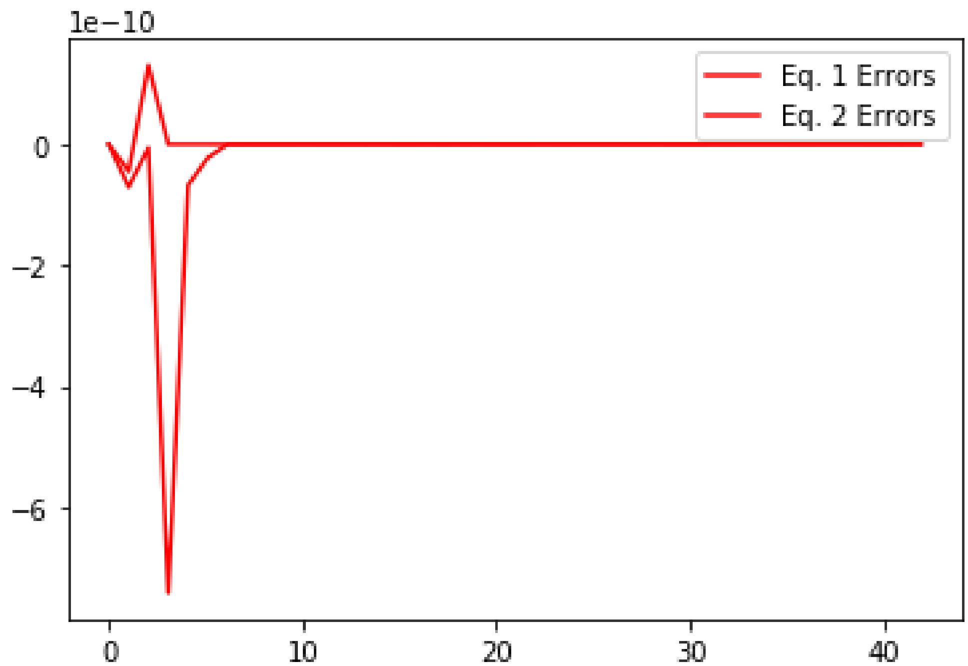
In [513]:
```
fig = plt.figure
x = np.linspace(0, T+6, T+6)
plt.plot(x, kp)
print(kp)
```

```
[2.         0.53863168 0.13479665 0.09130508 0.08151059 0.07882706
 0.07804931 0.07782016 0.07775231 0.0777322  0.07772624 0.07772448
 0.07772396 0.0777238  0.07772376 0.07772374 0.07772374 0.07772374
 0.07772373 0.07772373 0.07772373 0.07772373 0.07772372 0.07772372
 0.07772372 0.07772372 0.07772371 0.07772371 0.07772371 0.07772371
 0.0777237  0.0777237  0.0777237  0.07772369 0.07772369 0.07772369
 0.07772368 0.07772368 0.07772367 0.07772367 0.07772366 0.07772363
 0.07772363 0.07772363 0.07772363 0.07772363]
```



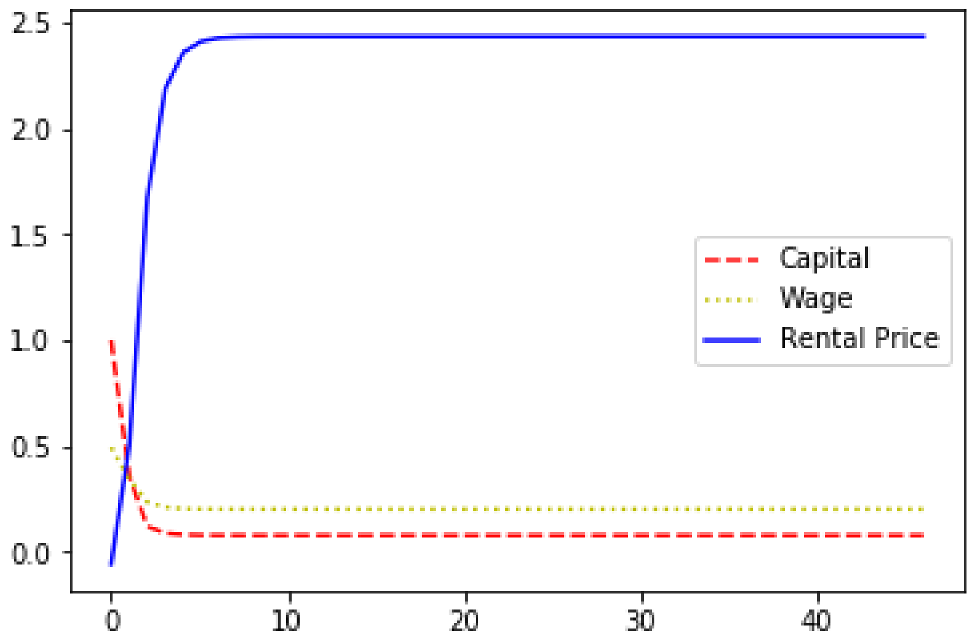

```
In [517]: fig = plt.figure()
          plt.plot(x[:T+2], errs[0], 'r-', label="Eq. 1 Errors")
          plt.plot(x[:T+2], errs[1], 'r-', label="Eq. 2 Errors")
          plt.legend()
```

```
Out[517]: <matplotlib.legend.Legend at 0x10215e3a58>
```



In [518]:
```
fig = plt.figure()
plt.plot(x, kp, 'r--', label="Capital")
plt.plot(x, w, 'y:', label="Wage")
plt.plot(x, r, 'b-', label="Rental Price")
plt.legend()
```

Out[518]: <matplotlib.legend.Legend at 0x1022321c18>



After 12 periods the capital value was never farther than .00001 away from the steady state value. This is becuase without shocks the system was effectively in steady state and since the it was effectrively already in the steady state, the adjustments made by agents asymptotically slowed as they tried to get closer and closer to the perfect steady state. However, it was always above the steady state until machine precision made it appear equivalent and throughout that whole time it was monotonically decreasing

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